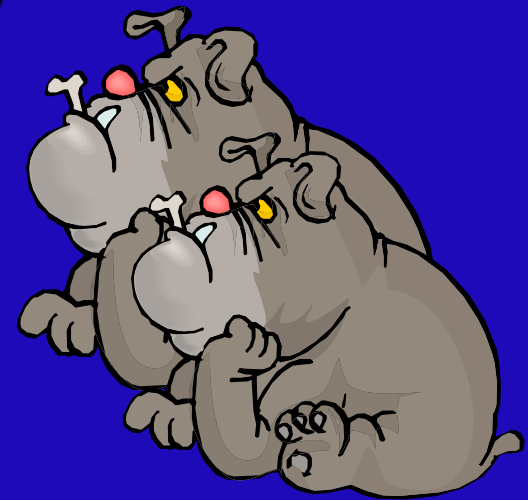


# 3. Symmetry Breaking



- Spontaneous Symmetry Breaking
- Scalar Potential
- Ground State Symmetry
- Higgs Mechanism
- The Higgs Boson
- Fermion Masses



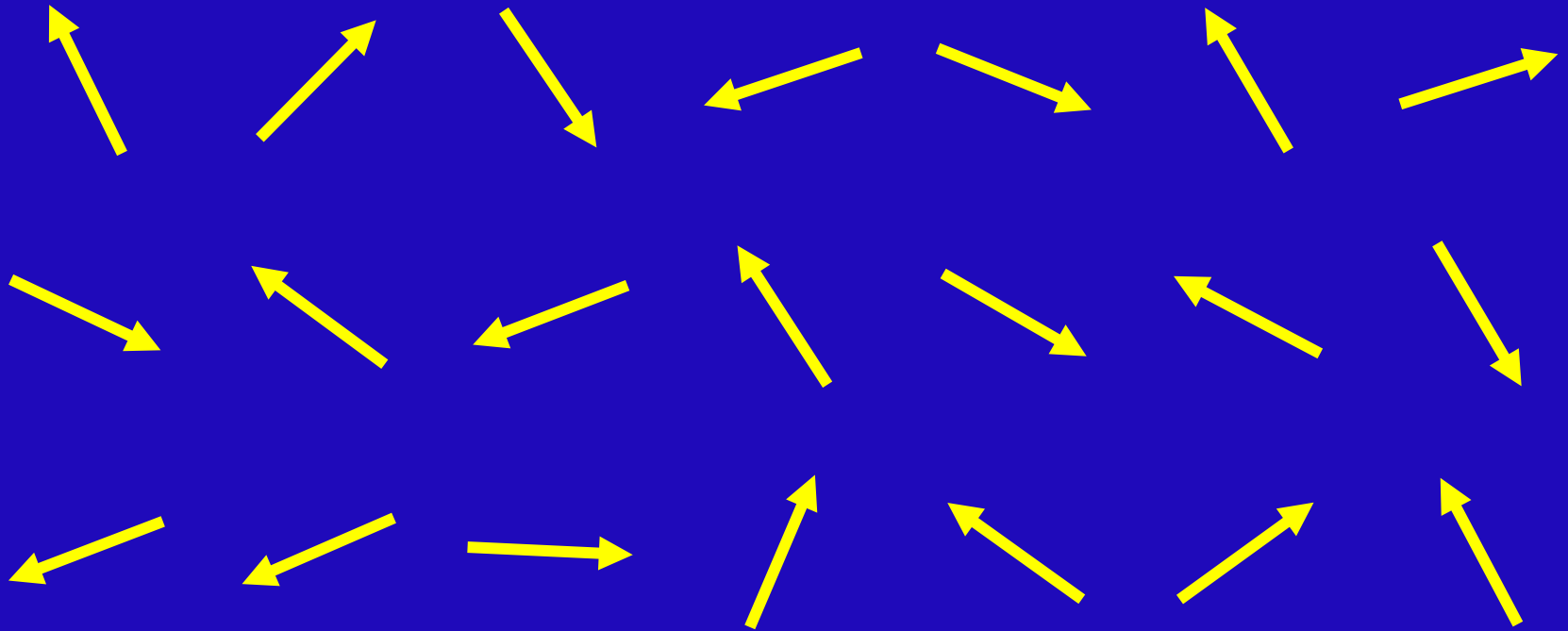






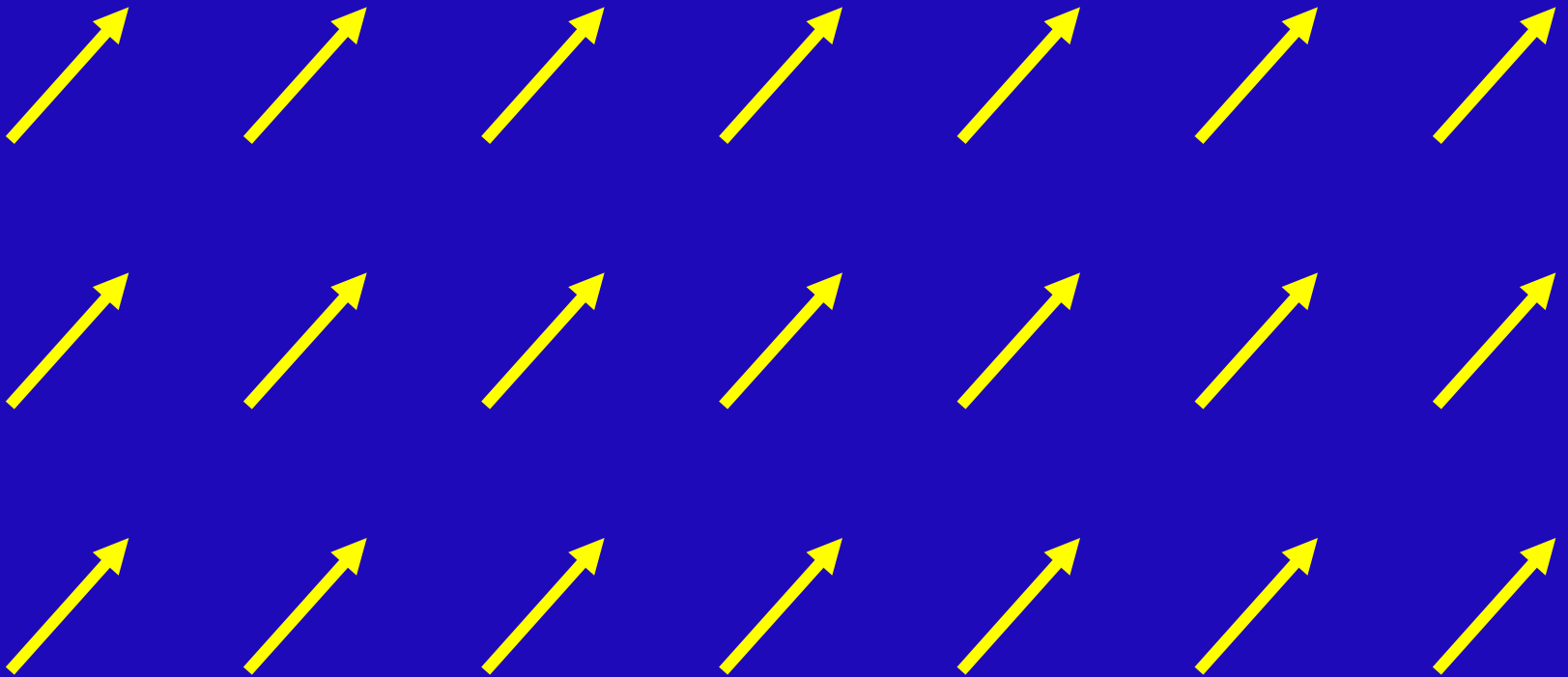
# FERROMAGNET

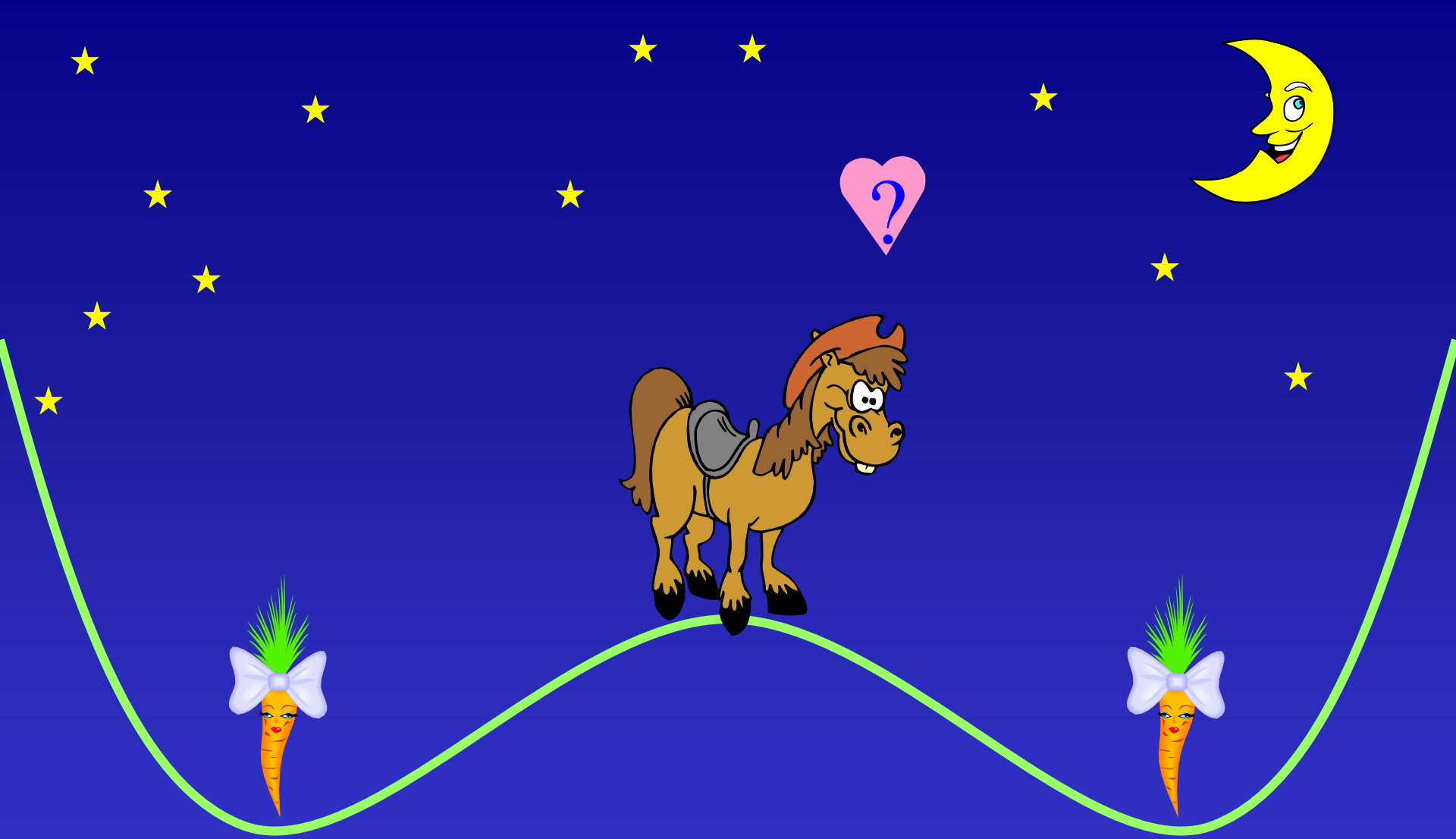
$$T > T_c$$

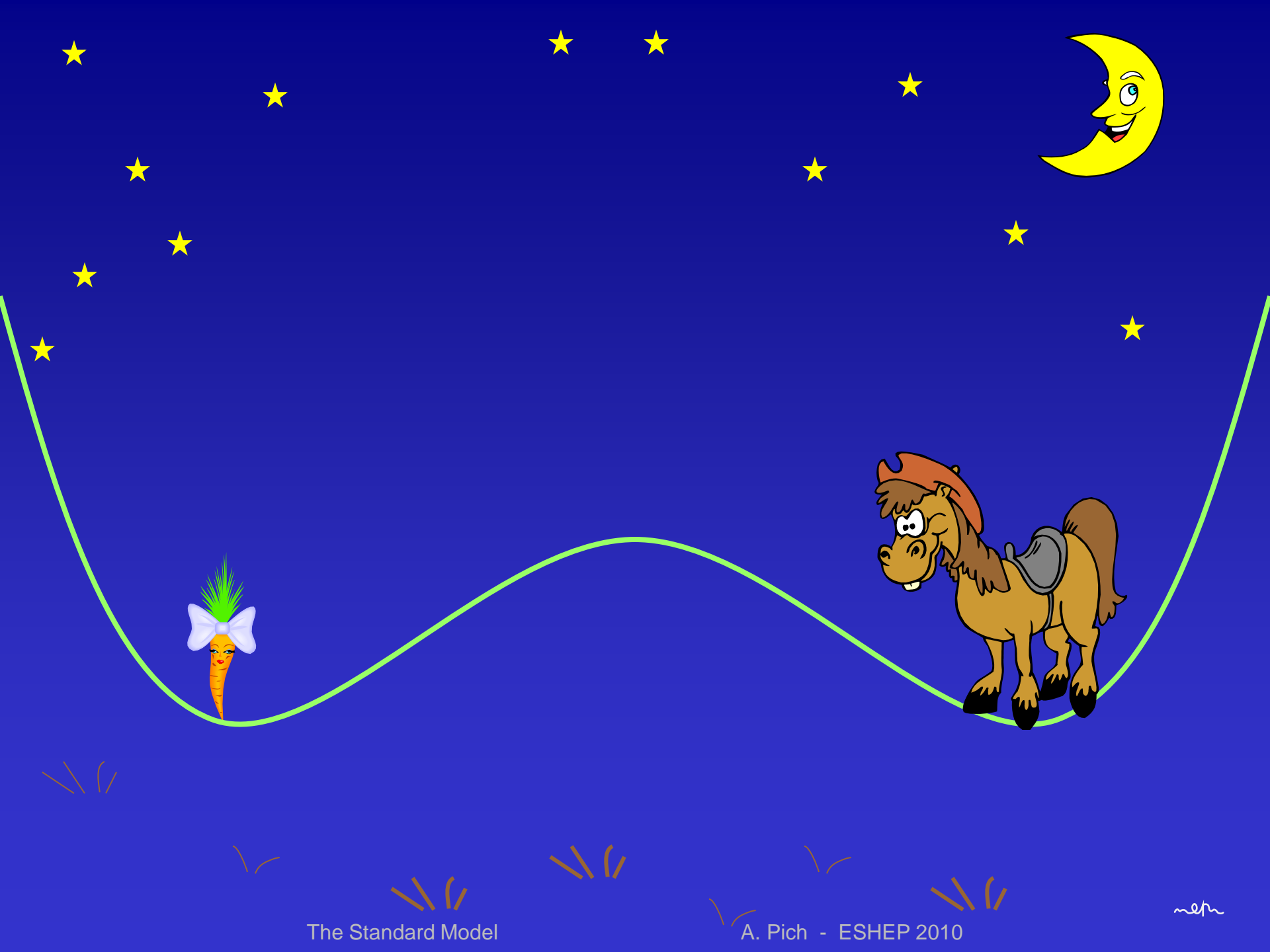


# FERROMAGNET

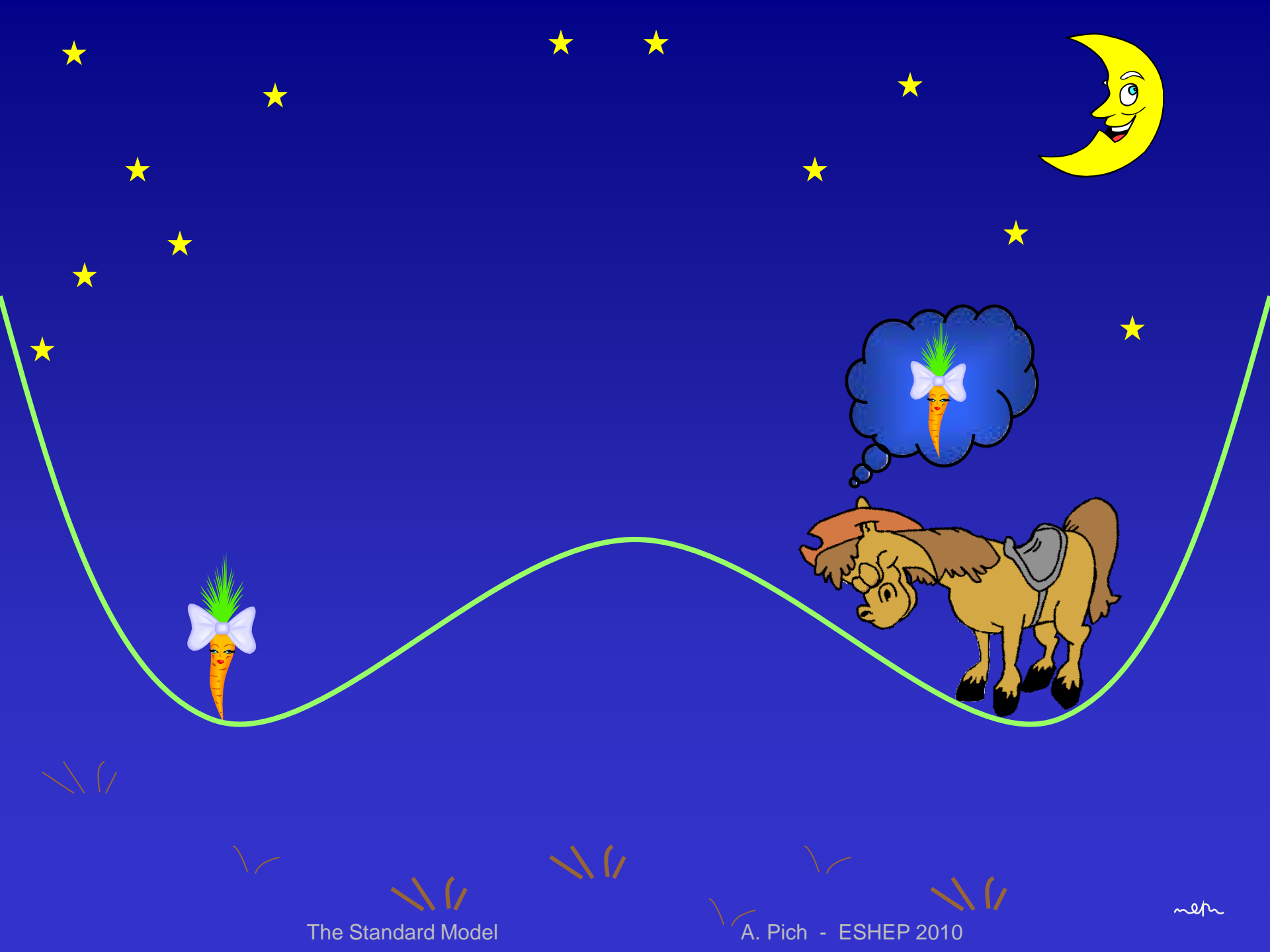
$$T < T_c$$



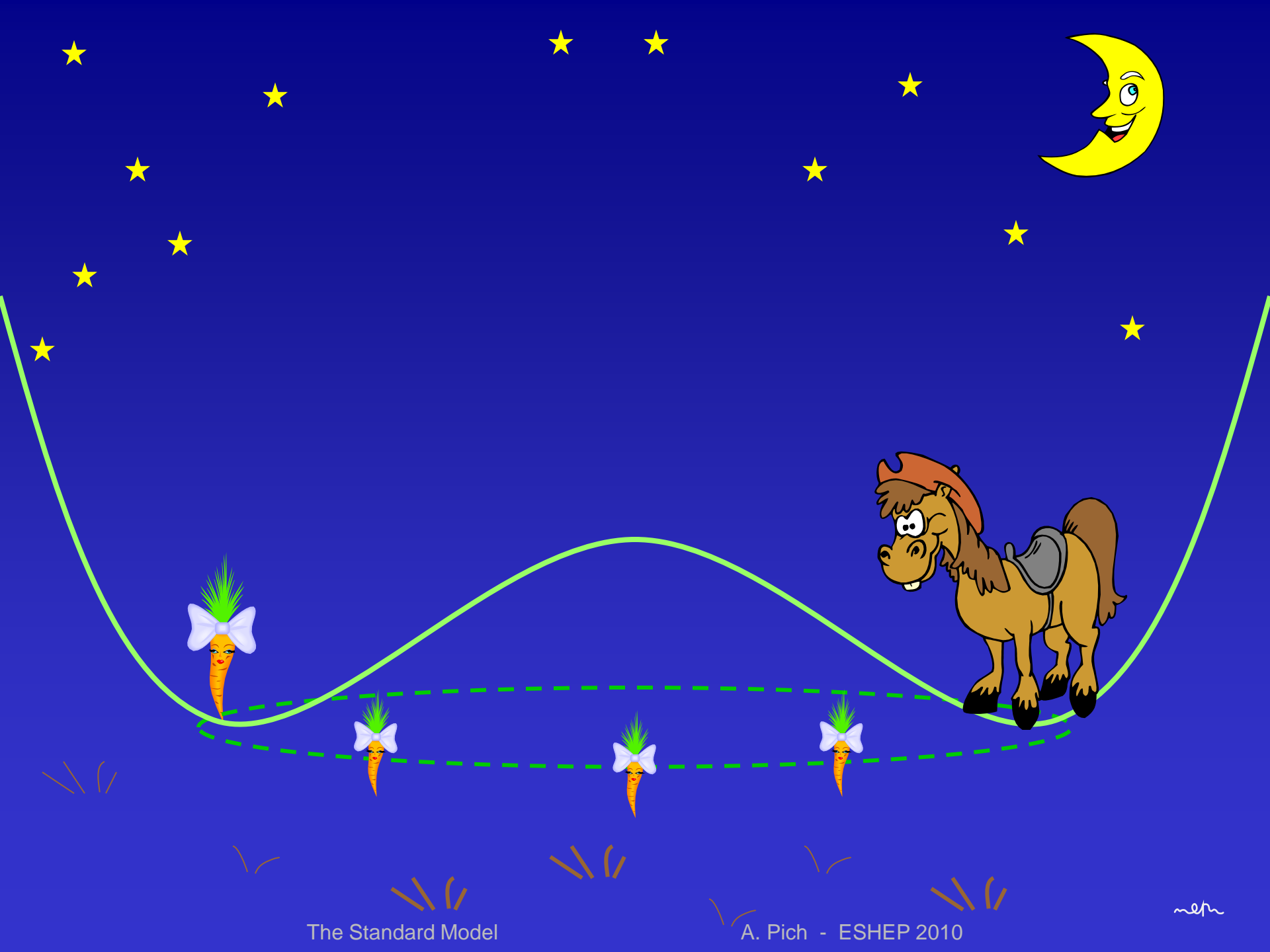


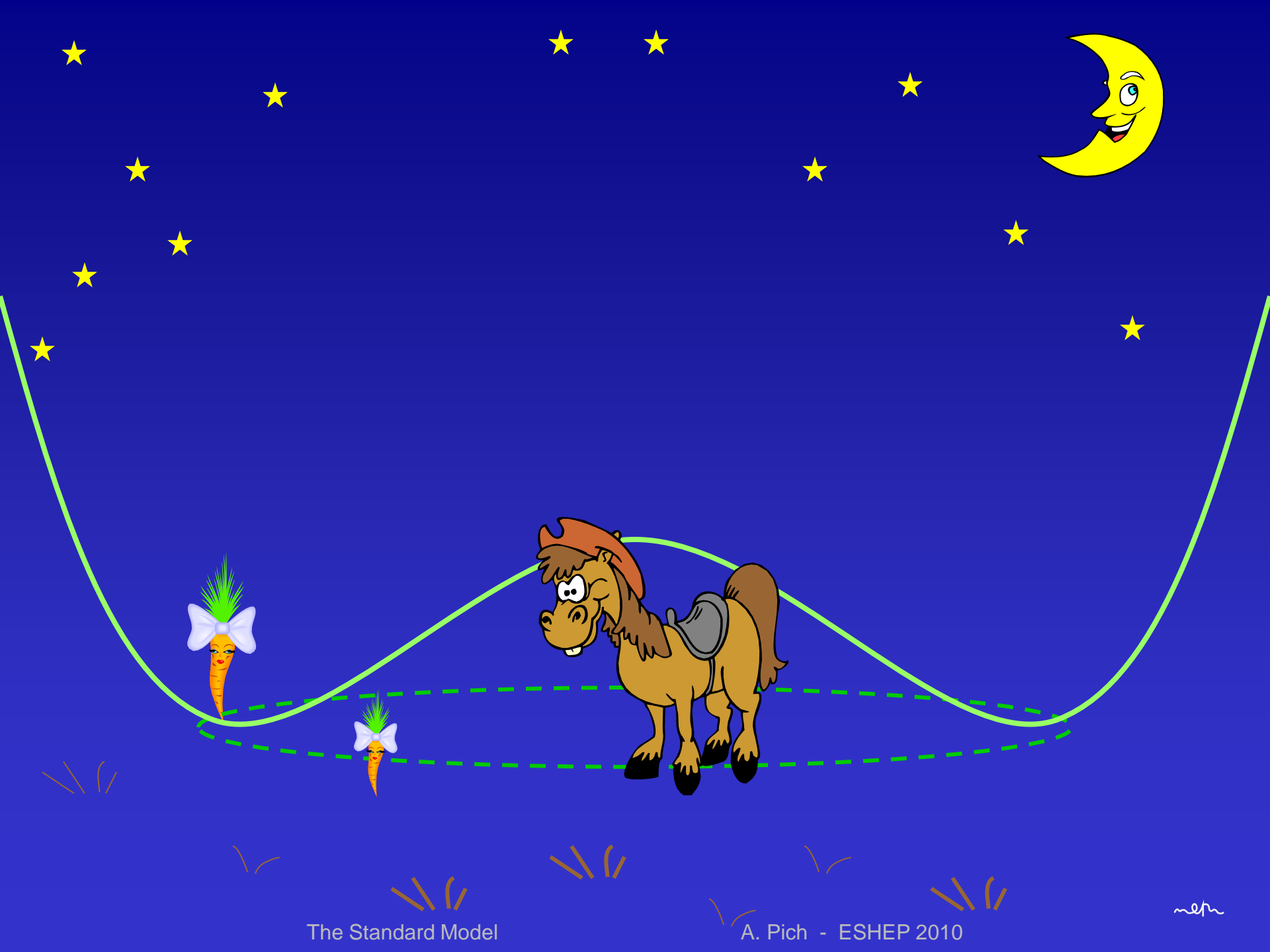












# SCALAR POTENCIAL

$$\mathcal{L}(\phi) = \partial_{\mu}\phi^{\dagger}\partial^{\mu}\phi - V(\phi)$$

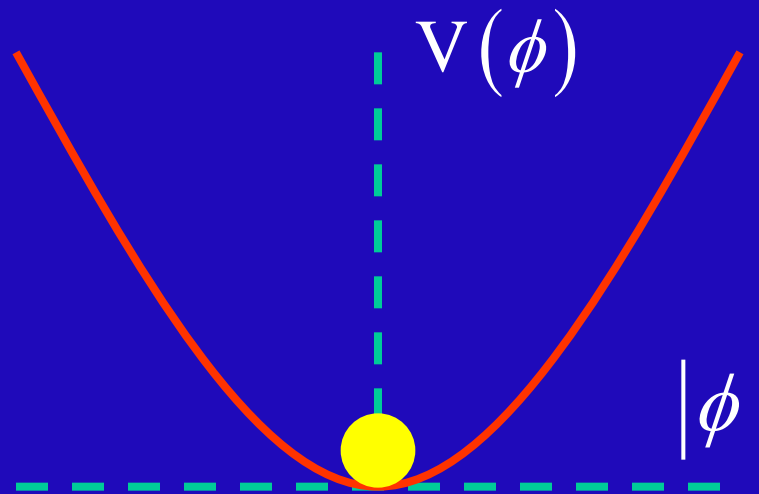
$$V(\phi) = \mu^2\phi^{\dagger}\phi + h(\phi^{\dagger}\phi)^2$$

Phase Symmetry:

$$\phi(x) \rightarrow e^{i\theta}\phi(x)$$

$$h > 0 \quad ; \quad \mu^2 > 0$$

$$M_{\phi} = \mu$$



Trivial Minimum (Ground State / Vacuum):

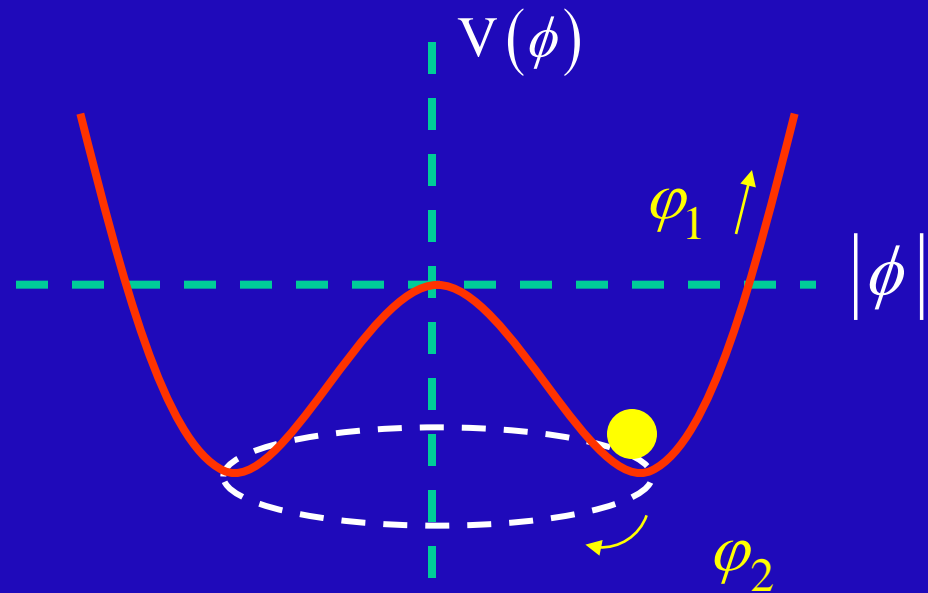
$$\phi = \phi_0 = 0$$

$$V(\phi) = \mu^2 \phi^\dagger \phi + h (\phi^\dagger \phi)^2$$

**Phase Symmetry:**

$$\phi(x) \rightarrow e^{i\theta} \phi(x)$$

$$\mu^2 < 0$$



**Degenerate Minima**  
(Ground State / Vacuum)

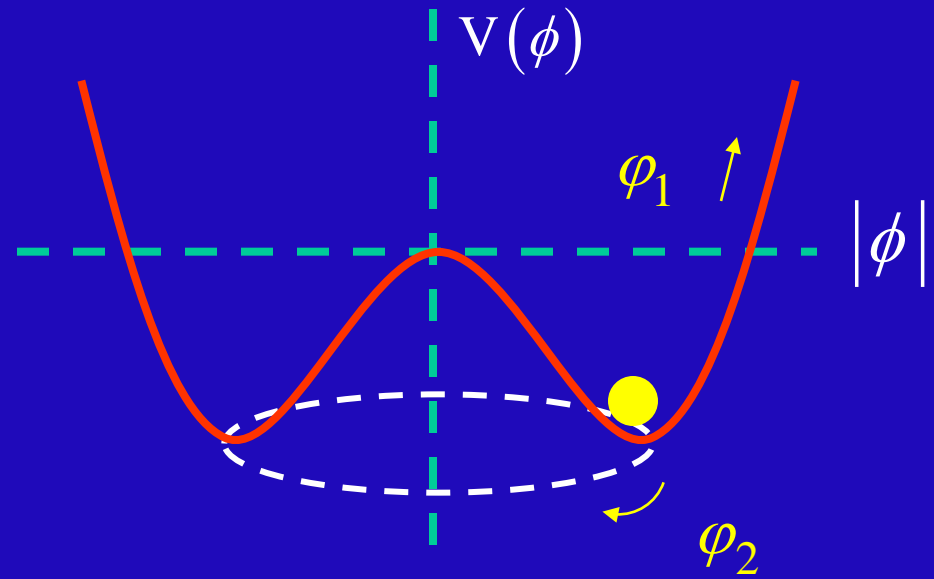
$$|\phi_0| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}} > 0 \quad ; \quad V(\phi_0) = -\frac{1}{4} h v^4$$

**Spontaneous Symmetry Breaking:**  $\phi \equiv \frac{1}{\sqrt{2}} [v + \phi_1(x)] e^{i\phi_2(x)/v}$

**Vacuum Choice**

# Spontaneous Symmetry Breaking

$$\mu^2 < 0$$



$$\phi \equiv \frac{1}{\sqrt{2}} [v + \varphi_1(x)] e^{i\varphi_2(x)/v}$$



$$\mathcal{L}(\phi) = \frac{1}{2} \partial_\mu \varphi_1 \partial^\mu \varphi_1 + \frac{1}{2} \left(1 + \frac{\varphi_1}{v}\right)^2 \partial_\mu \varphi_2 \partial^\mu \varphi_2 - V(\phi)$$

$$V(\phi) = V(\phi_0) + \frac{1}{2} M_{\varphi_1}^2 \varphi_1^2 + h v \varphi_1^3 + \frac{1}{4} h \varphi_1^4$$

$$M_{\varphi_1}^2 = -2\mu^2 > 0 \quad ; \quad M_{\varphi_2}^2 = 0$$

**1 Massless  
Goldstone Boson**

# ELECTROWEAK SSB

**New Scalar Doublet**

$$\phi(x) \equiv \begin{pmatrix} \phi^{(+)}(x) \\ \phi^{(0)}(x) \end{pmatrix} ; \quad y_\phi = Q_\phi - T_3 = \frac{1}{2}$$

$$\mathcal{L}(\phi) = (\mathbf{D}_\mu \phi)^\dagger \mathbf{D}^\mu \phi - \mu^2 \phi^\dagger \phi - h (\phi^\dagger \phi)^2$$

$$\mathbf{D}^\mu \phi = \left[ \partial^\mu - i g \mathbf{W}^\mu - i g' y_\phi B^\mu \right] \phi ; \quad \mathbf{W}^\mu = \frac{\vec{\tau}}{2} \cdot \vec{W}^\mu$$

$SU(2)_L \otimes U(1)_Y$

Symmetry

**Degenerate Vacuum States:**

$$(\mu^2 < 0, h > 0)$$

$$\left| \langle 0 | \phi^{(0)} | 0 \rangle \right| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}}$$

**Spontaneous Symmetry Breaking:**

$$\phi(x) = \exp \left\{ i \frac{\vec{\tau}}{2} \cdot \vec{\theta}(x) \right\} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$



# HIGGS MECHANISM

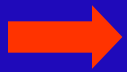
$$\phi(x) = \exp\left\{i\frac{\vec{\tau}}{2} \cdot \vec{\theta}(x)\right\} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \mathbf{v} + H(x) \end{pmatrix}$$

$SU(2)_L$  Invariance  $\rightarrow$   $\vec{\theta}(x)$  Unphysical

**Unitary Gauge:**

$$\vec{\theta}(x) = 0$$

$$(\mathbf{D}_\mu \phi)^\dagger \mathbf{D}^\mu \phi \rightarrow \frac{1}{2} \partial_\mu H \partial^\mu H + \frac{g^2}{4} (\mathbf{v} + H)^2 \left\{ W_\mu^\dagger W^\mu + \frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu \right\}$$



$$M_Z \cos \theta_W = M_W = \frac{1}{2} \mathbf{v} g$$

**Massive  
Gauge Bosons**

# Bosonic Degrees of Freedom

Massless  $W^\pm, Z$

3 x 2 polarizations = 6

+

3 Goldstones  $\vec{\theta}$

SSB



Massive  $W^\pm, Z$

3 x 3 polarizations = 9

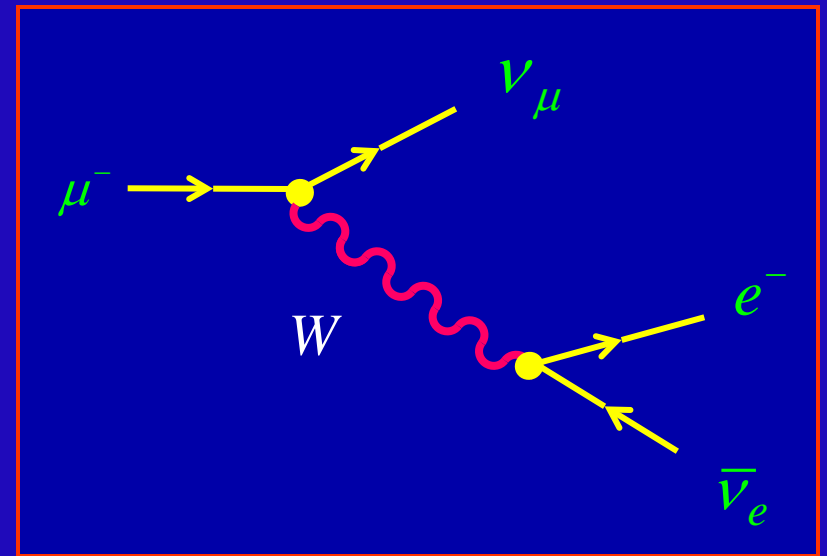
**SAME  
PHYSICS**

$$M_Z \cos \theta_W = M_W = \frac{1}{2} v g$$

$$M_Z = 91.1875 \text{ GeV} > M_W = 80.399 \text{ GeV} \quad \longrightarrow \quad \sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.223$$

$$\frac{g^2}{M_W^2 - q^2} \approx \frac{g^2}{M_W^2} \equiv 4 \sqrt{2} G_F$$

$$\frac{1}{\tau_\mu} \equiv \Gamma = \frac{G_F^2 m_\mu^5}{192 \pi^3}$$



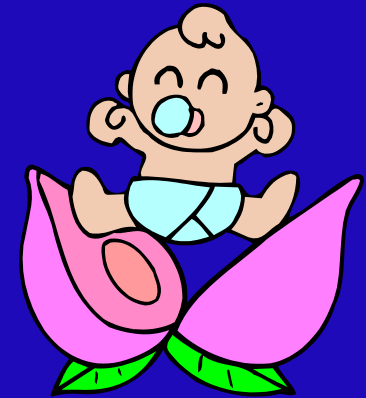
$$G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$$

$$g = \frac{e}{\sin \theta_W}, \quad M_W$$

$$\sin^2 \theta_W = 0.215$$

$$v = \left( \sqrt{2} G_F \right)^{-1/2} = 246 \text{ GeV}$$

# THE HIGGS BOSON



$$\mathcal{L}_S = \frac{h v^4}{4} + \mathcal{L}_H + \mathcal{L}_{HG^2}$$

$$\mathcal{L}_H = \frac{1}{2} \partial_\mu H \partial^\mu H - \frac{1}{2} M_H^2 H^2 - \frac{M_H^2}{2v} H^3 - \frac{M_H^2}{8v^2} H^4$$

$$\mathcal{L}_{HG^2} = \left[ M_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \right] \left\{ 1 + \frac{2}{v} H + \frac{H^2}{v^2} \right\}$$

## 1 Scalar Particle $H^0$ to be Discovered

$$M_H = \sqrt{-2 \mu^2} = \sqrt{2 h} v$$

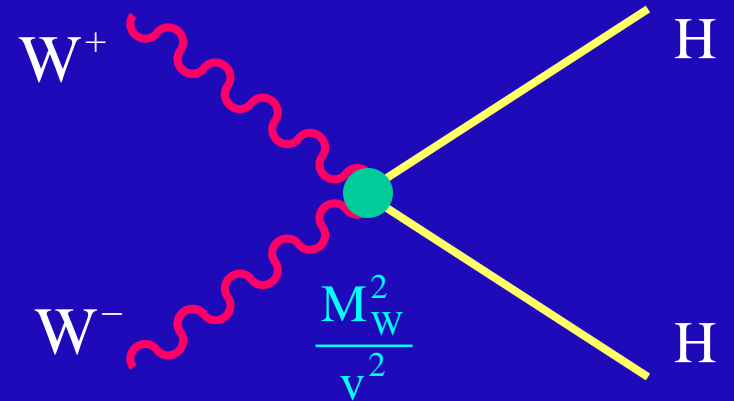
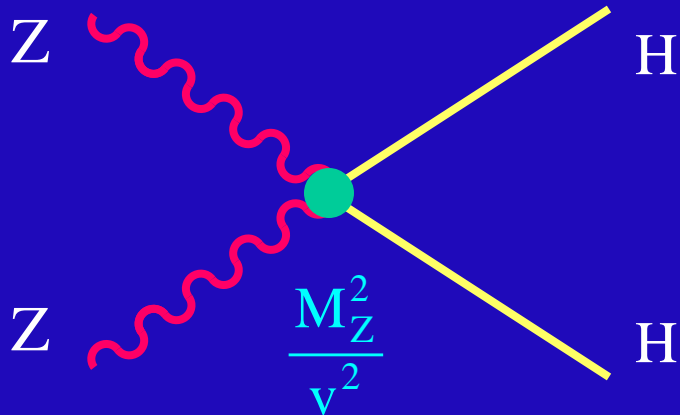
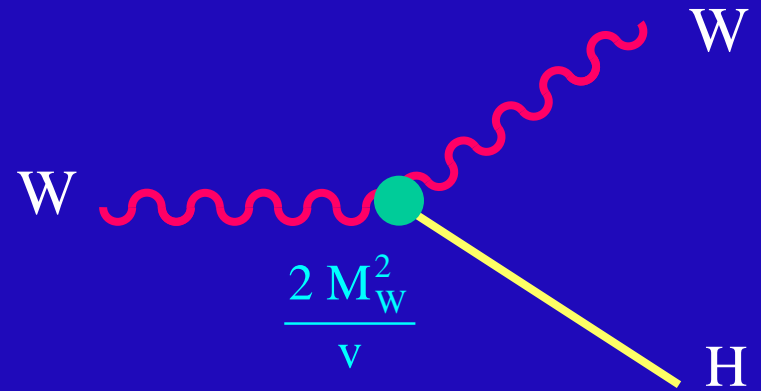
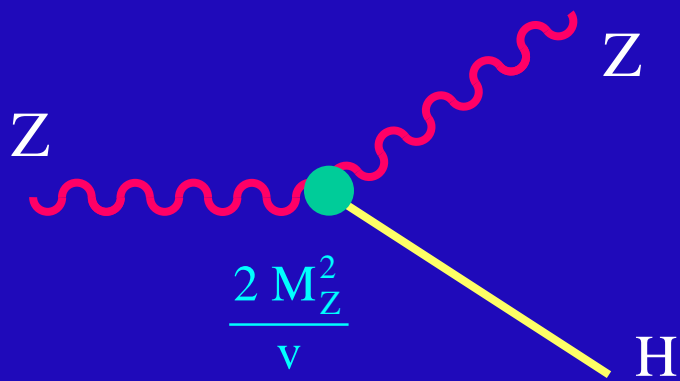
Free Parameter

**LEP:**  $114.4 \text{ GeV} < M_H < 186 \text{ GeV}$  (95% CL)

(Direct)

(Indirect)

# Higgs Couplings $\propto$ Masses



$$v = \left( \sqrt{2} G_F \right)^{-1/2} = 246 \text{ GeV}$$

# FERMION MASSES

Scalar – Fermion Couplings allowed by Gauge Symmetry

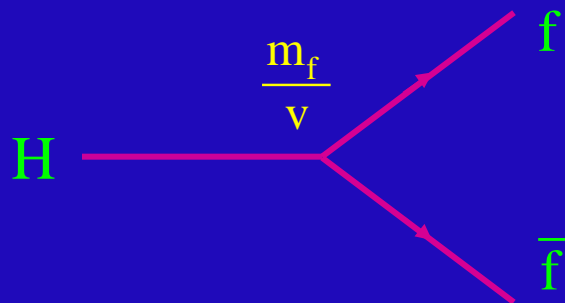
$$\mathcal{L}_Y = (\bar{q}_u, \bar{q}_d)_L \left[ c^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} (q_d)_R + c^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} (q_u)_R \right] + (\bar{\nu}_l, \bar{l})_L c^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l_R + \text{h.c.}$$

SSB

$$\mathcal{L}_Y = - \left( 1 + \frac{H}{v} \right) \left\{ m_{q_d} \bar{q}_d q_d + m_{q_u} \bar{q}_u q_u + m_l \bar{l} l \right\}$$

Fermion Masses are  
New Free Parameters

$$\left[ m_{q_d}, m_{q_u}, m_l \right] = - \left[ c^{(d)}, c^{(u)}, c^{(l)} \right] \frac{v}{\sqrt{2}}$$



Couplings Fixed:

$$g_{Hf\bar{f}} = \frac{m_f}{v}$$

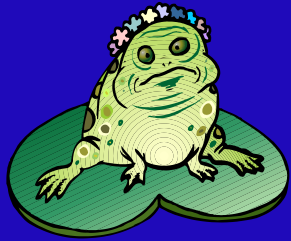
# Quarks



up



down



charm



strange



top



beauty

# Leptons



electron



neutrino  $e$



muon



neutrino  $\mu$



tau



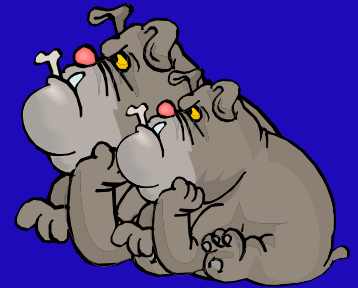
neutrino  $\tau$



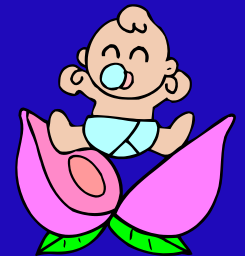
photon



gluon



$Z^0$   $W^\pm$



Higgs



~efm