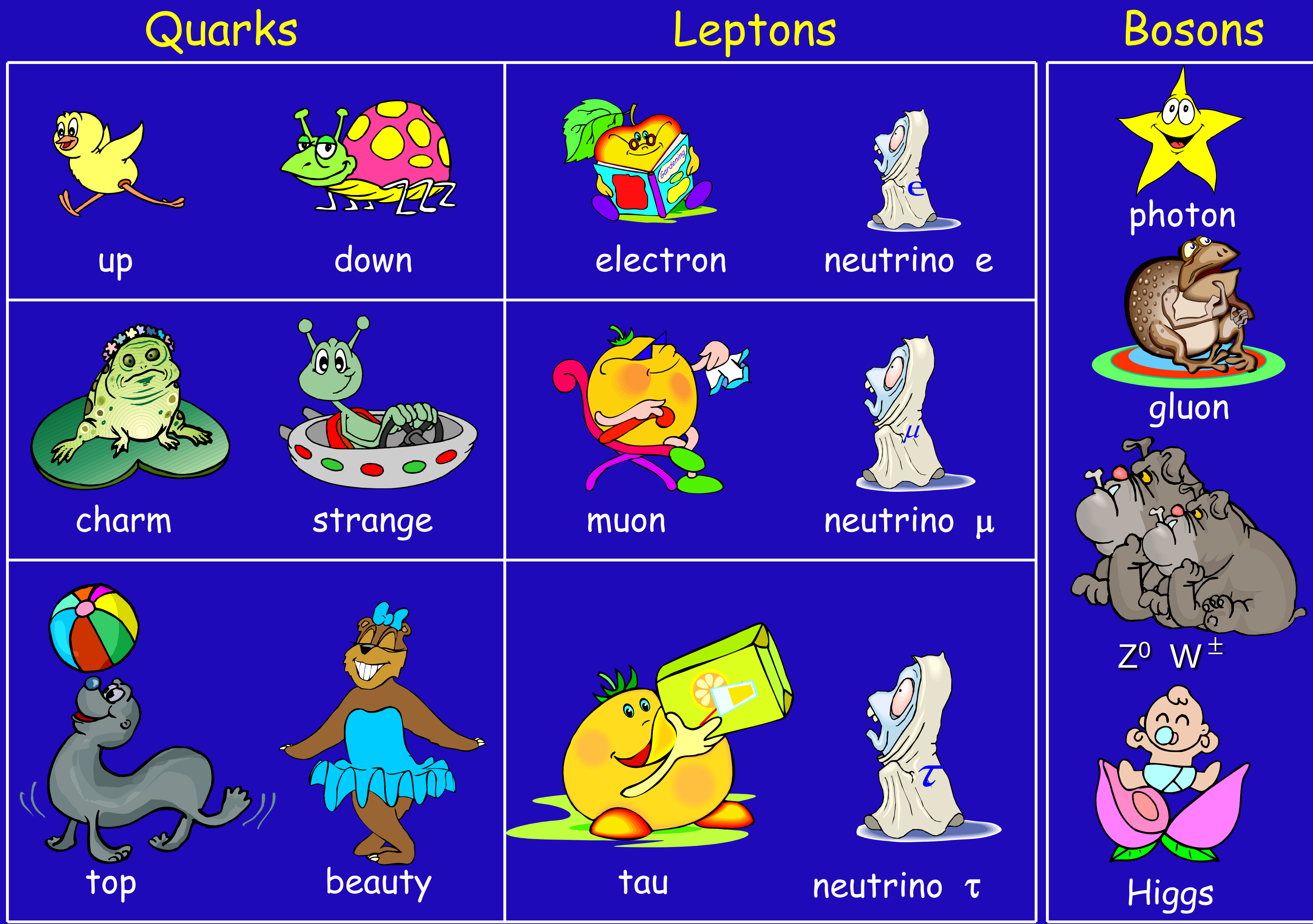
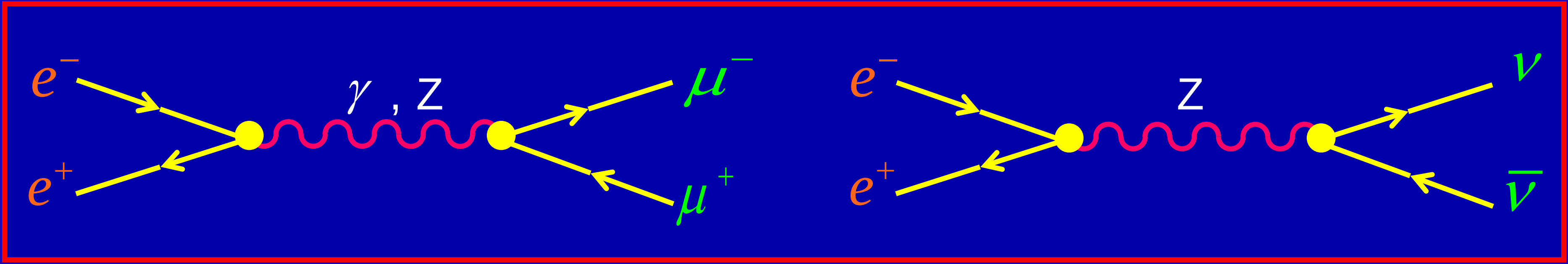
2. Electroweak Unification

- **Experimental Facts**
- $SU(2)_L \otimes U(1)_Y$ **Gauge Theory**
- **Charged Current Interaction**
- **Neutral Current Interaction**
- **Gauge Self-Interactions**



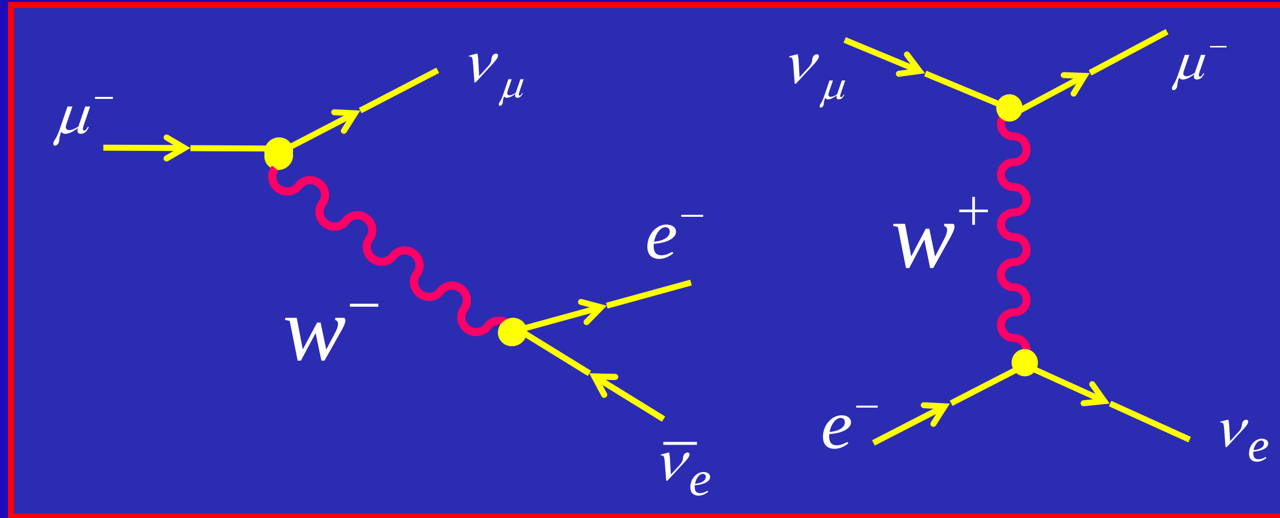
Quarks
Leptons
Bosons
up
down
electron
neutrino e
photon
gluon
charm
strange
muon
neutrino μ
Z0 W±
top
beauty
tau
neutrino τ
Higgs

NEUTRAL CURRENTS

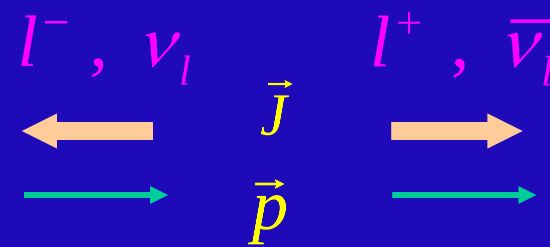


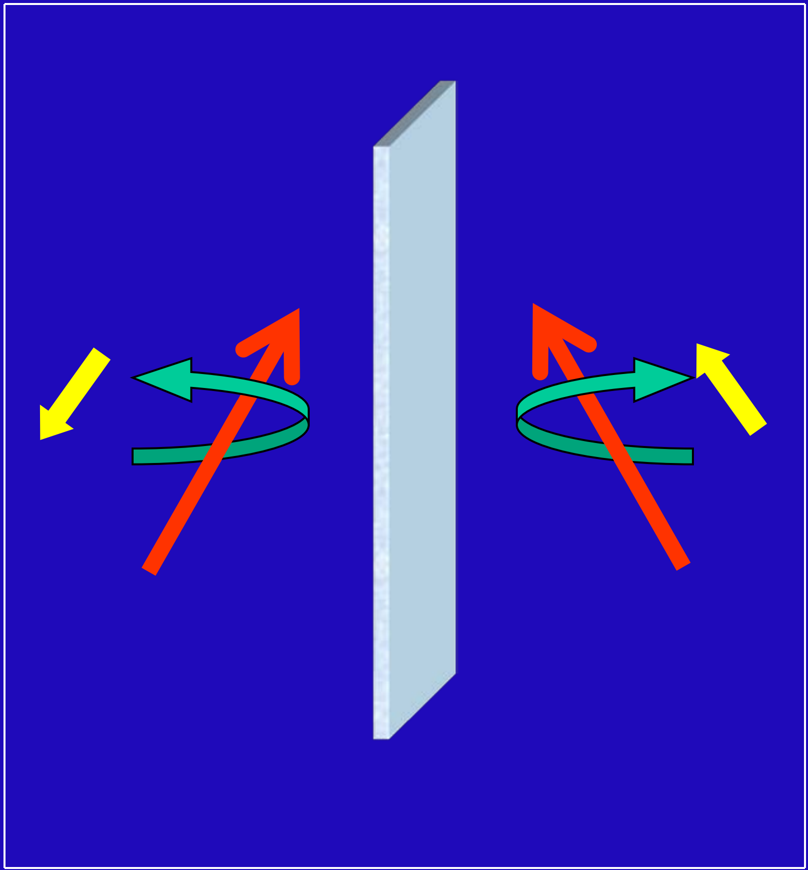
- Flavour Conserving $\mu \not\to e\,\gamma$; $Z \not\to e^{\mp}\mu^{\pm}$
- $g_\gamma \sim Q_l$ $(Q_e = Q_\mu = Q_\tau \;;\; Q_\nu = 0)$
- Same γ interaction for both lepton helicities
- NC Universality: $g_{Zee} = g_{Z\mu\mu} = g_{Z\tau\tau} \neq g_{Z\nu\nu}$
- Different Z coupling to l_R and l_L
- Left-handed neutrinos only
- 3 Families with light (nearly massless) neutrinos

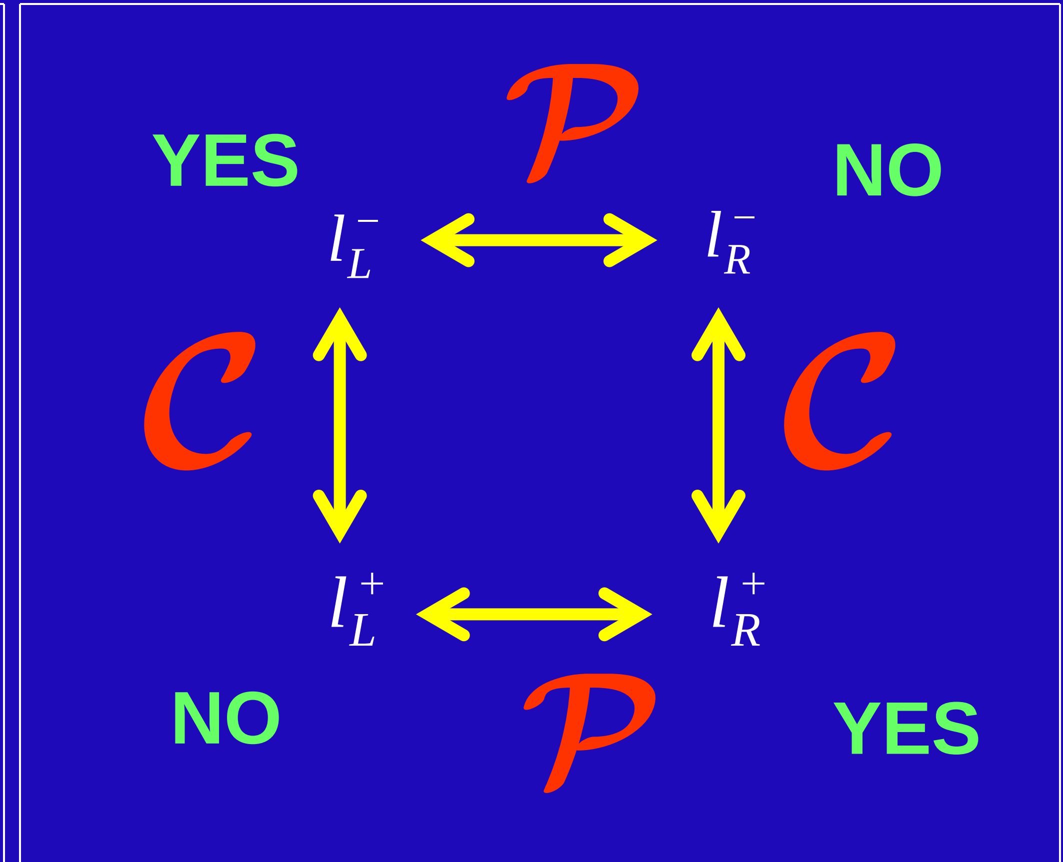
CHARGED CURRENTS



- Left-handed leptons (Right-handed antileptons)



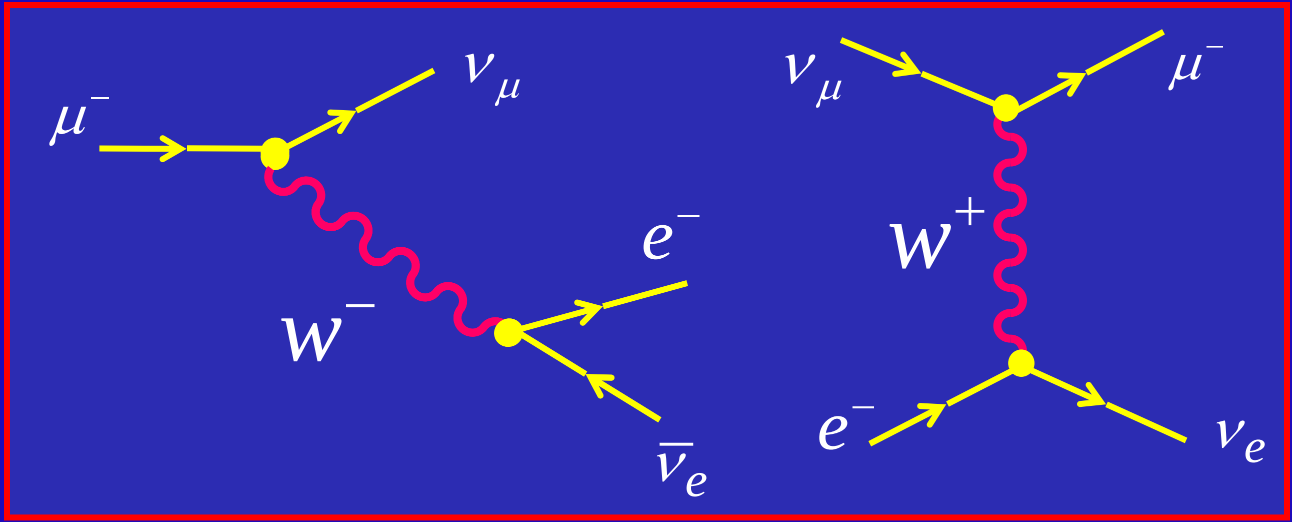




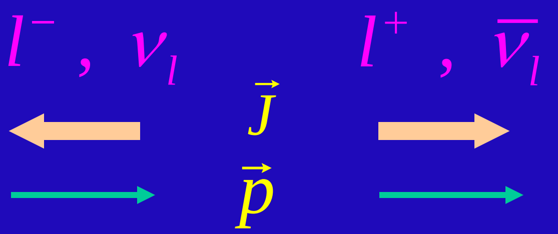
$\not{\mathcal{P}}$ and $\not{\mathcal{C}}$ in Weak Interactions

$\mathcal{CP}$ still a good symmetry

CHARGED CURRENTS



- Left-handed leptons (Right-handed antileptons)



- Doublet partners: $l^- \Leftrightarrow \nu_l$

$$\nu_\mu X \to \mu^- X' \quad ; \quad \nu_\mu X \not\to e^- X'$$

- Universal Strength

$$T(l \to \nu_l\, l' \bar{\nu}_{l'}) \sim \frac{g_W^2}{M_W^2 - q^2} \xrightarrow{q^2 << M_W^2} \frac{g_W^2}{M_W^2} \sim G_F \implies \Gamma(l \to \nu_l\, l' \bar{\nu}_{l'}) \sim G_F^2\, m_l^5$$

$$\Gamma(\tau \to \nu_\tau\, e\, \bar{\nu}_e) / \Gamma(\mu \to \nu_\mu\, e\, \bar{\nu}_e) \approx (m_\tau / m_\mu)^5$$

CHIRALITY

Chirality Projectors: $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}$; $(\gamma_5)^2 = I_4$

$$P_R \equiv \frac{1+\gamma_5}{2} \quad ; \quad P_L \equiv \frac{1-\gamma_5}{2}$$

$$\mathrm{P}_R^2 = \mathrm{P}_R \quad ; \quad \mathrm{P}_L^2 = \mathrm{P}_L \quad ; \quad \mathrm{P}_R\,\mathrm{P}_L = \mathrm{P}_L\,\mathrm{P}_R = 0$$

$$\psi(x) = \left(P_L + P_R\right)\psi(x) \equiv \psi_L(x) + \psi_R(x)$$

$$\bar{\psi} \equiv \psi^\dagger \gamma^0$$

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu\partial_\mu - m)\,\psi$$

$$= \bar{\psi}_L\, i\gamma^\mu\partial_\mu\,\psi_L + \bar{\psi}_R\, i\gamma^\mu\partial_\mu\,\psi_R - m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$$

EXPERIMENTAL FACTS

Three Families

$$\begin{bmatrix} \nu_e & u \\ e^- & d' \end{bmatrix} \quad , \quad \begin{bmatrix} \nu_\mu & c \\ \mu^- & s' \end{bmatrix} \quad , \quad \begin{bmatrix} \nu_\tau & t \\ \tau^- & b' \end{bmatrix}$$

Family Structure

$$\begin{bmatrix} \nu_l & q_u \\ l^- & q_d \end{bmatrix} \equiv \left\{ \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L , \; (\nu_l)_R \; , \; l_R^- \right\} \; ; \; \left\{ \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L , \; (q_u)_R \; , \; (q_d)_R \right\}$$

Charged Currents $\quad W^\pm$

- Left–handed Fermions only
- Flavour Changing: $\nu_l \Leftrightarrow l \quad , \quad q_u \Leftrightarrow q_d$

Neutral currents $\quad \gamma \, , \, Z \quad$ Flavour Conserving

Universality $\quad$ (Family – Independent Couplings)

$$(\nu_l)_R \qquad ?$$


standard
model
E.M
Weak
W,Z
Quarks
Leptons

$SU(2)_L \otimes U(1)_Y$

GAUGE THEORY

Fields	$\psi_1(x)$	$\psi_2(x)$	$\psi_3(x)$
Quarks	$\begin{pmatrix} q_u \\ q_d \end{pmatrix}_L$	$(q_u)_R$	$(q_d)_R$
Leptons	$\begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L$	$(\nu_l)_R$	$(l^-)_R$

Free Lagrangian for Massless Fermions:

$$\mathcal{L}_0 = \sum_j i \, \bar{\psi}_j \, \gamma^\mu \partial_\mu \psi_j$$

$SU(2)_L \otimes U(1)_Y$ **Flavour Symmetry:**

$$\mathbf{U}_L \equiv \exp\left\{ i \, \frac{\vec{\sigma}}{2} \, \vec{\alpha} \right\}$$

$$\psi_1 \to e^{i y_1 \beta} \, \mathbf{U}_L \, \psi_1 \quad ; \quad \psi_2 \to e^{i y_2 \beta} \, \psi_2 \quad ; \quad \psi_3 \to e^{i y_3 \beta} \, \psi_3$$

$$\bar{\psi}_1 \to \bar{\psi}_1 \, \mathbf{U}_L^\dagger \, e^{-i y_1 \beta} \quad ; \quad \bar{\psi}_2 \to \bar{\psi}_2 \, e^{-i y_2 \beta} \quad ; \quad \bar{\psi}_3 \to \bar{\psi}_3 \, e^{-i y_3 \beta}$$

Gauge Principle: $\vec{\alpha} = \vec{\alpha}(x) \quad , \quad \beta = \beta(x)$

$$\mathbf{D}_\mu \psi_1 \equiv \left[\partial_\mu + i\, g\, \mathbf{W}_\mu(x) + i\, g'\, y_1\, B_\mu(x)\right] \psi_1 \;\to\; e^{i\, y_1\, \beta(x)}\, \mathbf{U}_L(x)\, \mathbf{D}_\mu \psi_1$$

$$\mathbf{D}_\mu \psi_k \equiv \left[\partial_\mu + i\, g'\, y_k\, B_\mu(x)\right] \psi_k \;\to\; e^{i\, y_k\, \beta(x)}\, \mathbf{D}_\mu \psi_k \qquad (k = 2, 3)$$

$$B_\mu(x) \;\to\; B_\mu(x) - \frac{1}{g'}\, \partial_\mu \beta(x)$$

$$\mathbf{W}_\mu(x) \;\to\; \mathbf{U}_L(x)\, \mathbf{W}_\mu(x)\, \mathbf{U}_L^\dagger(x) + \frac{i}{g}\, \partial_\mu \mathbf{U}_L(x)\, \mathbf{U}_L^\dagger(x)$$

$$\mathbf{U}(x) \equiv \exp\left\{ i\, \frac{\vec{\sigma}}{2}\, \vec{\alpha}(x) \right\} \quad ; \quad \mathbf{W}_\mu(x) \equiv \frac{\vec{\sigma}}{2}\, \vec{W}_\mu(x) \quad ; \quad \delta W_\mu^i = -\frac{1}{g}\, \partial_\mu(\delta\alpha^i) - \varepsilon^{ijk}\, \delta\alpha^j\, W_\mu^k$$

4 Massless Gauge Bosons $\quad W_\mu^\pm \;,\; W_\mu^3 \;,\; B_\mu^0$

CHARGED CURRENTS

$$\sum_j i\,\bar{\psi}_j\,\gamma^\mu\,D_\mu\psi_j \quad \longrightarrow \quad -g\;\bar{\psi}_1\,\gamma^\mu\,\mathbf{W}_\mu\,\psi_1 \;-\; g'\;B_\mu\sum_j y_j\;\bar{\psi}_j\,\gamma^\mu\,\psi_j$$

$$\mathbf{W}_\mu \equiv \frac{\vec{\sigma}}{2}\cdot\vec{W}_\mu = \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}\,W_\mu^\dagger \\ \sqrt{2}\,W_\mu & -W_\mu^3 \end{pmatrix} \quad ; \quad W_\mu \equiv \left(W_\mu^1 + i\,W_\mu^2\right)/\sqrt{2}$$

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}}\;W_\mu^\dagger\;\left[\;\bar{q}_u\,\gamma^\mu\,(1-\gamma_5)\,q_d \;+\; \bar{\nu}_l\,\gamma^\mu\,(1-\gamma_5)\,l\;\right] \;+\; \text{h.c.}$$

Quark / Lepton Universality

Left – Handed Interaction

NEUTRAL CURRENTS

$$\mathcal{L}_{\rm NC} = -g\, W_\mu^3\ \bar\psi_1 \gamma^\mu \frac{\sigma_3}{2} \psi_1 - g'\, B_\mu \sum_j y_j\, \bar\psi_j \gamma^\mu \psi_j$$

Massless Fields ➡ Arbitrary Combination

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

A_μ has the QED Interaction IF $\quad g\,\sin\theta_W = g'\cos\theta_W = e$

$$y_1 = Q_u - \frac{1}{2} = Q_d + \frac{1}{2} \quad ; \quad y_2 = Q_u \quad ; \quad y_3 = Q_d$$

Electroweak Unification

$$\mathcal{L}_{\rm NC} = -e\, A_\mu \sum_j \bar\psi_j \gamma^\mu Q_j \psi_j + \mathcal{L}_{\rm NC}^Z$$

$$Q_1 = \begin{pmatrix} Q_u & 0 \\ 0 & Q_d \end{pmatrix} \quad ; \quad Q_2 = Q_u \quad ; \quad Q_3 = Q_d$$

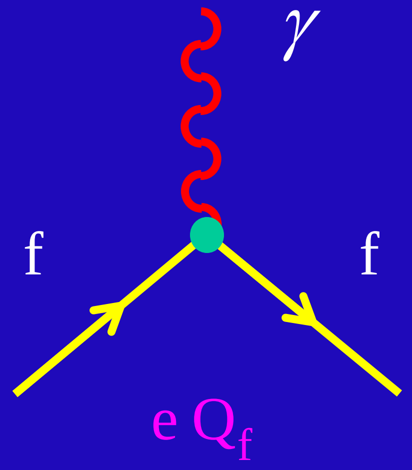
$$\mathcal{L}_{NC}^{Z} = -\frac{e}{\sin\theta_W \cos\theta_W} Z_\mu \left\{ \bar{\psi}_1 \gamma^\mu \frac{\sigma_3}{2} \psi_1 - \sin^2\theta_W \sum_j \bar{\psi}_j \gamma^\mu Q_j \psi_j \right\}$$

$$= -\frac{e}{2\sin\theta_W \cos\theta_W} Z_\mu \sum_f \bar{f} \gamma^\mu \left[v_f - a_f \gamma_5 \right] f$$

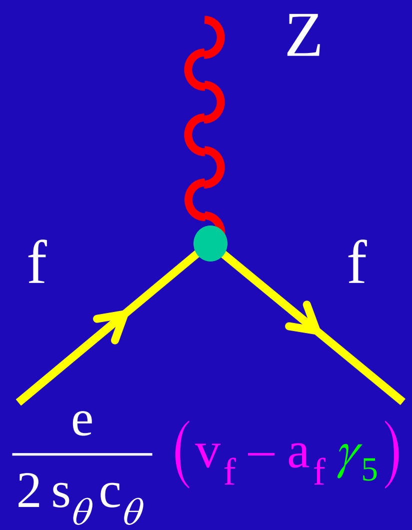
	q_u	q_d	ν_l	l^-
$2\, v_f$	$1 - \frac{8}{3}\sin^2\theta_W$	$-1 + \frac{4}{3}\sin^2\theta_W$	1	$-1 + 4\sin^2\theta_W$
$2\, a_f$	1	-1	1	-1

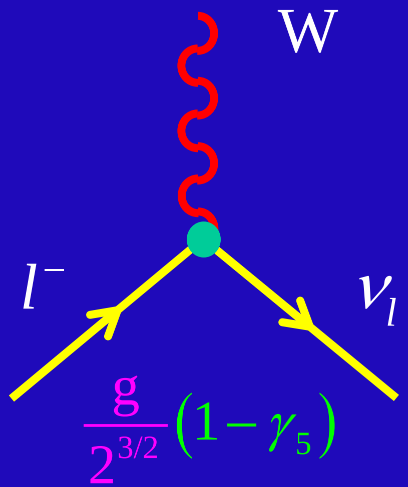
IF ν_R do exist: $y(\nu_R) = Q_\nu = 0$ ➡ No ν_R Interactions

Sterile Neutrinos

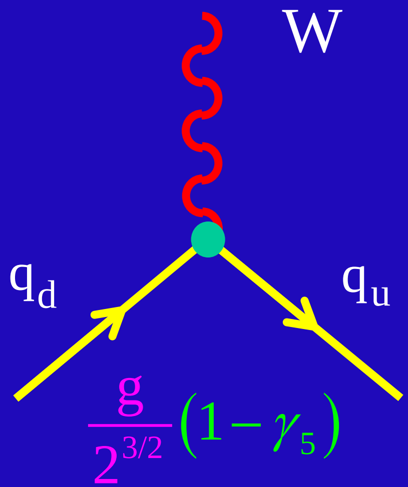


NEUTRAL CURRENTS





CHARGED CURRENTS



$$\mathbf{W}_{\mu\nu} \equiv -\frac{i}{g}\left[\mathbf{D}_\mu , \mathbf{D}_\nu\right] \equiv \frac{\vec{\sigma}}{2}\cdot\vec{W}_{\mu\nu} \;\rightarrow\; \mathbf{U}_{\mathrm{L}}\,\mathbf{W}_{\mu\nu}\,\mathbf{U}_{\mathrm{L}}^\dagger \qquad ; \qquad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \;\rightarrow\; B_{\mu\nu}$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\,\varepsilon^{ijk}\,W^j_\mu W^k_\nu$$

$$\mathcal{L}_{\mathrm{K}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}\mathrm{Tr}\left(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right) = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}\vec{W}_{\mu\nu}\vec{W}_{\mu\nu} = \mathcal{L}_{\text{kin}} + \mathcal{L}_3 + \mathcal{L}_4$$

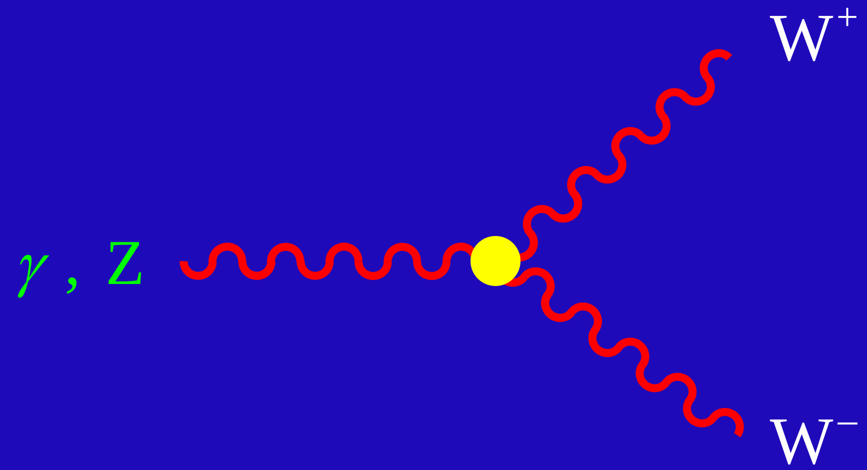
$$\mathcal{L}_3 = i\,e\,\cot\theta_W \left\{(\partial^\mu W^\nu - \partial^\nu W^\mu)\,W^\dagger_\mu Z_\nu - (\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger})\,W_\mu Z_\nu + W_\mu W^\dagger_\nu\,(\partial^\mu Z^\nu - \partial^\nu Z^\mu)\right\}$$

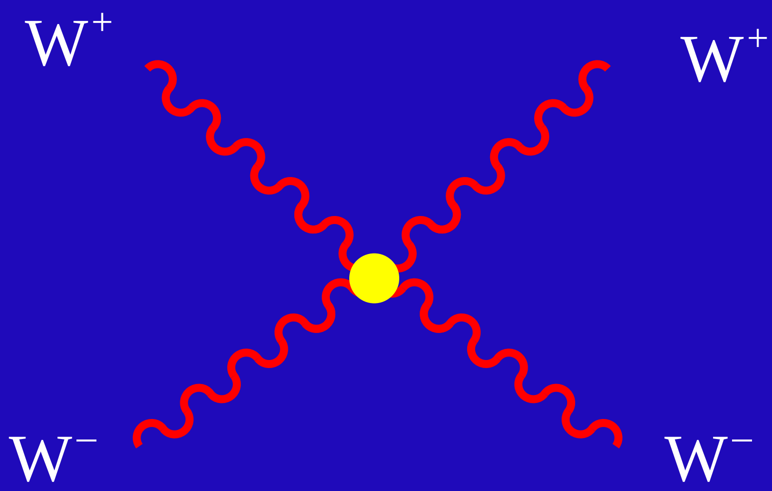
$$+\,i\,e\left\{\left(\partial^\mu W^\nu - \partial^\nu W^\mu\right)W^\dagger_\mu A_\nu - \left(\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}\right)W_\mu A_\nu + W_\mu W^\dagger_\nu\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right)\right\}$$

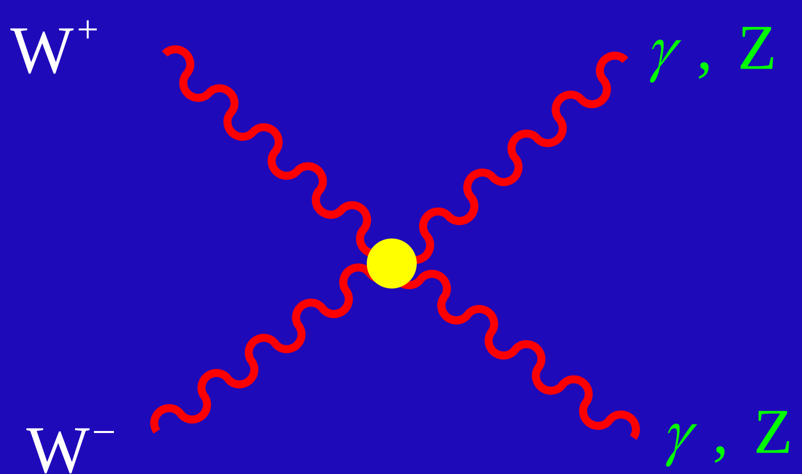
$$\mathcal{L}_4 = -\frac{e^2}{2\sin^2\theta_W}\left\{\left(W^\dagger_\mu W^\mu\right)^2 - W^\dagger_\mu W^{\mu\dagger} W_\nu W^\nu\right\} - e^2\cot^2\theta_W\left\{W^\dagger_\mu W^\mu Z_\nu Z^\nu - W^\dagger_\mu Z^\mu W_\nu Z^\nu\right\}$$

$$-e^2\cot\theta_W\left\{2W^\dagger_\mu W^\mu Z_\nu A^\nu - W^\dagger_\mu Z^\mu W_\nu A^\nu - W^\dagger_\mu A^\mu W_\nu Z^\nu\right\} - e^2\left\{W^\dagger_\mu W^\mu A_\nu A^\nu - W^\dagger_\mu A^\mu W_\nu A^\nu\right\}$$

GAUGE SELF-INTERACTIONS







PROBLEM WITH MASS SCALES

Gauge Symmetry → $m_\gamma = 0$ **Good**

$M_W = M_Z = 0$ **Bad!**

$M_W = 80.40 \text{ GeV}$

$M_Z = 91.19 \text{ GeV}$

Moreover $\mathcal{L}_{m_f} \equiv -m_f \, \bar{f} f = -m_f \, (\bar{f}_L f_R + \bar{f}_R f_L)$

Also Forbidden by Gauge Symmetry → $m_f = 0 \qquad \forall f$

All Particles Massless

Quarks Leptons Bosons