http://arxiv.org/pdf/0705.4264

Field Theory &

The Standard Model

Antonio Pich

IFIC, CSIC – Univ. Valencia

Antonio.Pich@ific.uv.es

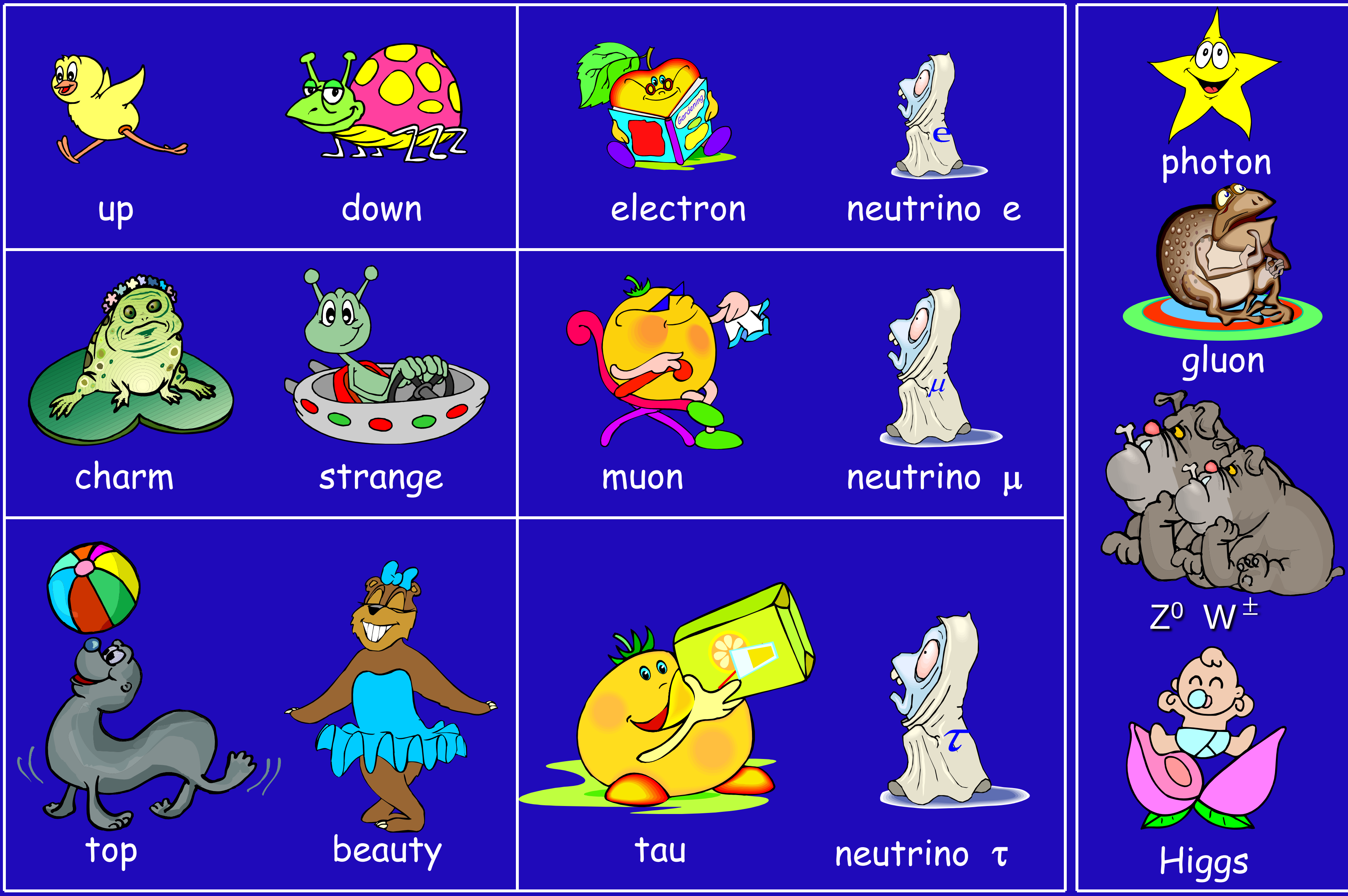
The 2010 European School of High-Energy Physics

Raseborg, Finland, 20 June - 3 July 2010

The Standard Model

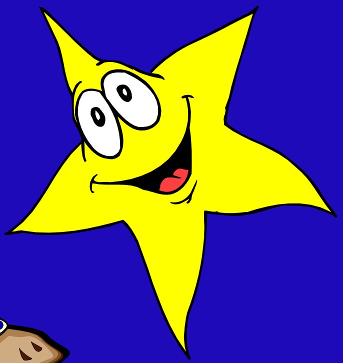
- Gauge Invariance: QED, QCD
- Electroweak Unification: $SU(2)_L \otimes U(1)_Y$
- Symmetry Breaking: Higgs Mechanism
- Electroweak Phenomenology
- Flavour Dynamics

Quarks   Leptons   Bosons




standard
model
E.M
Strong
Weak
W,Z
Quarks
Leptons

1. Gauge Invariance



- Field Theory
- Classical Electrodynamics
- Quantum Electrodynamics (QED)
- SU(N) Gauge Theory
- Quantum Chromodynamics (QCD)

QUANTUM MECHANICS WAVE EQUATIONS

Non Relativistic:

Schrödinger equation

$$\vec{p} = -i\vec{\nabla} \quad ; \quad E = i\frac{\partial}{\partial t} \qquad E = \frac{\vec{p}^{\,2}}{2m} \quad \longrightarrow \quad \boxed{i\frac{\partial}{\partial t}\Psi = -\frac{\vec{\nabla}^2}{2m}\Psi}$$

Relativistic:

Klein-Gordon equation

$$p^\mu = i\partial^\mu = i\, g^{\mu\nu}\frac{\partial}{\partial x^\nu} \qquad E^2 = \vec{p}^{\,2} + m^2 \quad \longrightarrow \quad \boxed{(\Box + m^2)\,\phi = 0}$$

$$\Box \equiv \partial^\mu \partial_\mu = \frac{\partial^2}{\partial t^2} - \vec{\nabla}^2$$

Spin $\frac{1}{2}$:

$$\boxed{(i\gamma^\mu \partial_\mu - m)\,\psi = 0}$$

Dirac equation

$$-(i\gamma^\nu \partial_\nu + m)\,[(i\gamma^\mu \partial_\mu - m)\,\psi] = 0 \;\equiv\; (\Box + m^2)\,\psi \quad \longrightarrow \quad \left\{\gamma^\mu, \gamma^\nu\right\} = 2g^{\mu\nu}$$

spin matrices $(D=4)$

DIRAC ALGEBRA

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu}$$

$$\gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix} \quad ; \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; ; \; \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \; ; \; \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\left\{\sigma^i, \sigma^j\right\} = 2\delta^{ij} \quad ; \quad \left[\sigma^i, \sigma^j\right] = 2i\varepsilon^{ijk}\sigma^k$$

$$\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$$

Particle
Antiparticle } Spinors

Chirality Projectors:

$$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix} \quad ; \quad (\gamma_5)^2 = I_4$$

$$P_R \equiv \frac{1+\gamma_5}{2} \quad ; \quad P_L \equiv \frac{1-\gamma_5}{2}$$

$$\mathrm{P}_R^2 = \mathrm{P}_R \quad ; \quad \mathrm{P}_L^2 = \mathrm{P}_L \quad ; \quad \mathrm{P}_R\,\mathrm{P}_L = \mathrm{P}_L\,\mathrm{P}_R = 0$$

LAGRANGIAN FORMALISM

$$S = \int d^4x \;\; \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\delta S = 0 \quad \longrightarrow \quad \frac{\partial \mathcal{L}}{\partial \phi} - \partial^\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi)} \right) = 0$$

Eq. Motion

Klein-Gordon: (spin 0)

$$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - m^2 \phi^* \phi \quad \longrightarrow \quad (\Box + m^2)\,\phi = 0$$

Dirac: (spin $\frac{1}{2}$)

$$\bar{\psi} \equiv \psi^\dagger \gamma^0$$

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi \quad \longrightarrow \quad (i\gamma^\mu \partial_\mu - m)\,\psi = 0$$

CLASSICAL ELECTRODYNAMICS

Maxwell´s Equations:

$$\vec\nabla\cdot\vec E = \rho \qquad ; \qquad \vec\nabla\cdot\vec B = 0$$

$$\vec\nabla \times \vec B - \frac{\partial \vec E}{\partial t} = \vec J \qquad ; \qquad \vec\nabla \times \vec E + \frac{\partial \vec B}{\partial t} = \vec 0$$

Potentials:

$$\vec E = -\vec\nabla V - \frac{\partial \vec A}{\partial t} \qquad ; \qquad \vec B = \vec\nabla\times\vec A$$

Covariant Notation:

$$A^\mu \equiv (V, \vec A) \qquad ; \qquad J^\mu \equiv (\rho, \vec J)$$

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix} \longrightarrow \partial_\mu F^{\mu\nu} = J^\nu$$

Gauge Invariance:

$$A^\mu \longrightarrow A'^\mu \equiv A^\mu + \partial^\mu \Lambda$$

Same Physics described by many different A^μ

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\partial_\mu F^{\mu\nu} = \Box A^\nu - \partial^\nu (\partial_\mu A^\mu) = J^\nu$$

CONSERVED ELECTROMAGNETIC CURRENT:

$$\partial_\mu \partial_\nu F^{\mu\nu} = 0 \quad \longrightarrow \quad \partial_\mu J^\mu = 0$$

Lorentz Gauge $(\partial_\mu A^\mu = 0)$ and $J^\mu = 0$ $\longrightarrow$ $\Box A^\mu = 0$

Klein-Gordon equation with $m = 0$ MASSLESS PHOTON

Residual Invariance: $A^\mu \longrightarrow A^\mu + \partial^\mu \Lambda \quad ; \quad \Box \Lambda = 0$

$\mu = 0, 1, 2, 3,$; 2 arbitrary constraints

$\longrightarrow$ TWO PHOTON POLARIZATIONS

FREE Dirac fermion:

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi$$

Phase Invariance: $\psi \;\to\; \psi' = \mathrm{e}^{iQ\theta}\,\psi \quad ; \quad \bar{\psi} \;\to\; \bar{\psi}' = \mathrm{e}^{-iQ\theta}\,\bar{\psi}$

Absolute phases are not observable in Quantum Mechanics

GAUGE PRINCIPLE: $\theta = \theta(x)$

Phase Invariance should hold LOCALLY

BUT $\partial_\mu \psi \;\to\; \mathrm{e}^{iQ\theta}\,(\partial_\mu + i\,Q\,\partial_\mu\theta)\,\psi$

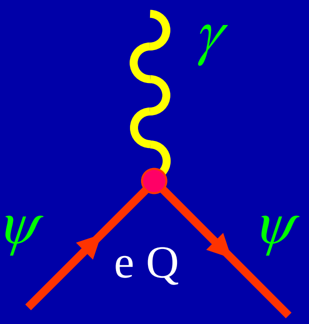
SOLUTION: Covariant Derivative

$$D_\mu\psi \equiv (\partial_\mu + i\,e\,Q\,A_\mu)\,\psi \;\;\to\;\; \mathrm{e}^{iQ\theta}\,D_\mu\psi$$

One needs a spin-1 field A_μ satisfying $A_\mu \;\to\; A_\mu - \frac{1}{e}\,\partial_\mu\theta$

QUANTUM ELECTRODYNAMICS

$$\mathcal{L} = \bar{\psi}\,(i\,\gamma^{\mu} D_{\mu} - m)\,\psi$$
$$= \bar{\psi}\,(i\,\gamma^{\mu}\partial_{\mu} - m)\,\psi \; - \; e\,Q\,A_{\mu}\,(\bar{\psi}\,\gamma^{\mu}\psi)$$



Kinetic term:

$$\mathcal{L}_{K} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad \Longrightarrow \quad \partial_{\mu} F^{\mu\nu} = e Q\,(\bar{\psi}\,\gamma^{\nu}\,\psi)$$

Maxwell

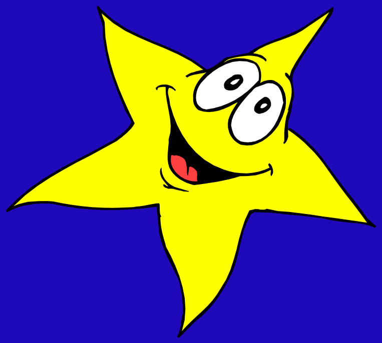
Mass term: [exp: $m_{\gamma} < 1\cdot 10^{-18}$ eV]

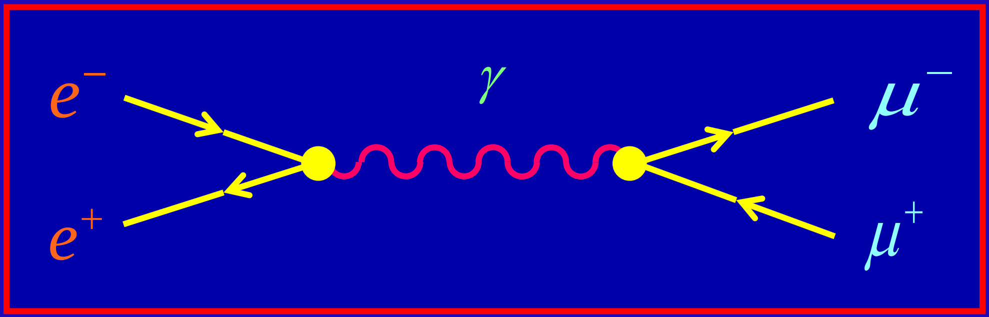
$$\mathcal{L}_{M} = \frac{1}{2} m_{\gamma}^{2}\, A^{\mu} A_{\mu}$$

Not Gauge Invariant ⟹ $m_{\gamma} = 0$

Gauge Symmetry ⟹ QED Dynamics

Successful Theory

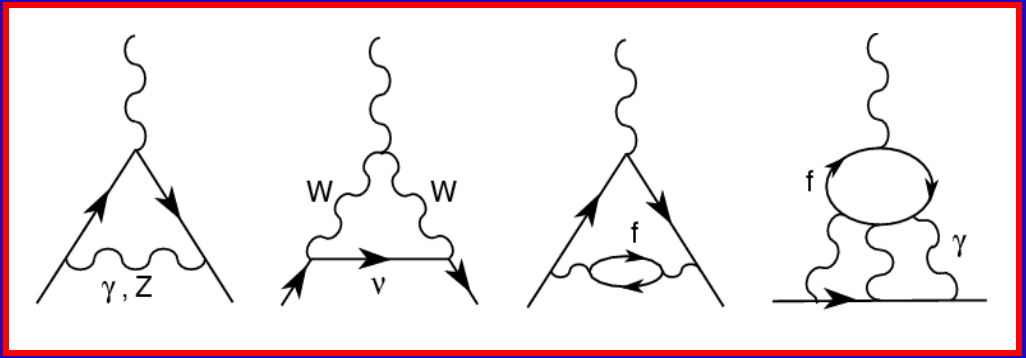




Anomalous Magnetic Moment

$$\mu_l \equiv g_l \, \frac{e}{2\, m_l}$$

$$a_l \equiv \frac{1}{2}\,(g_l - 2)$$



$$a_e = (1\,159\,652\,180.85 \pm 0.76) \times 10^{-12} \;\Rightarrow\; \alpha^{-1} = 137.035\,999\,710 \pm 0.000\,000\,096$$

$$\Rightarrow\quad a_\mu^{\mathrm{th}} = (11\,659\,188 \pm 10) \times 10^{-10} \qquad [\,\mathrm{Exp:}\quad (11\,659\,208.0 \pm 6.3) \times 10^{-10}\,]$$

QUARKS HAVE COLOUR

$\Delta^{++} = u^{\uparrow} u^{\uparrow} u^{\uparrow} \qquad (J = \frac{3}{2},\; J_3 = \frac{3}{2})$

Fermi Statistics ➡ $\Delta^{++} \approx \varepsilon^{\alpha\beta\gamma}\, u^{\uparrow}_{\alpha}\, u^{\uparrow}_{\beta}\, u^{\uparrow}_{\gamma}$

$$B \approx \varepsilon^{\alpha\beta\gamma}\, q^i_\alpha\, q^j_\beta\, q^k_\gamma \quad ; \quad M \approx \delta^{\alpha\beta}\, q^i_\alpha\, \bar{q}^j_\beta \qquad (i,j,k = u,d,s,\ldots\; ;\; \alpha,\beta,\gamma = 1,\ldots,Nc)$$

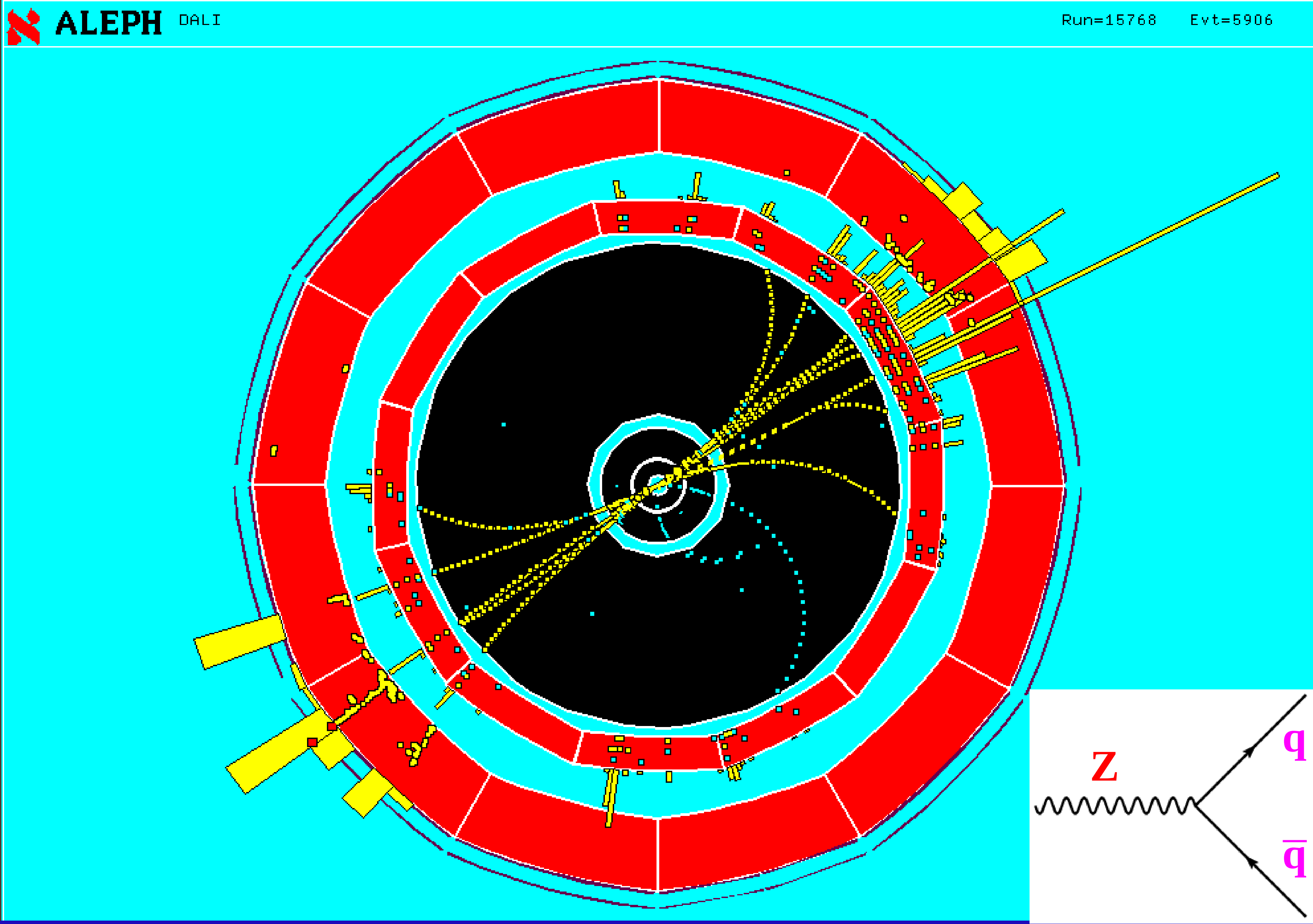
$$N_C = 3$$ ➡ $q^i\, q^i\, q^i$

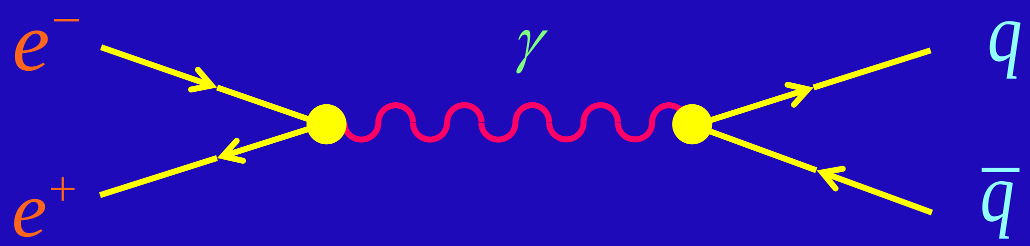
We don´t see Colour Multiplets ➡ Hadrons are Colour Singlets

$(q\,q\,q,\; \bar{q}\,\bar{q}\,\bar{q}$ and $q\,\bar{q}$; BUT NOT $q\,q$ and $q\,q\,q\,q)$

We don´t see Quarks ➡

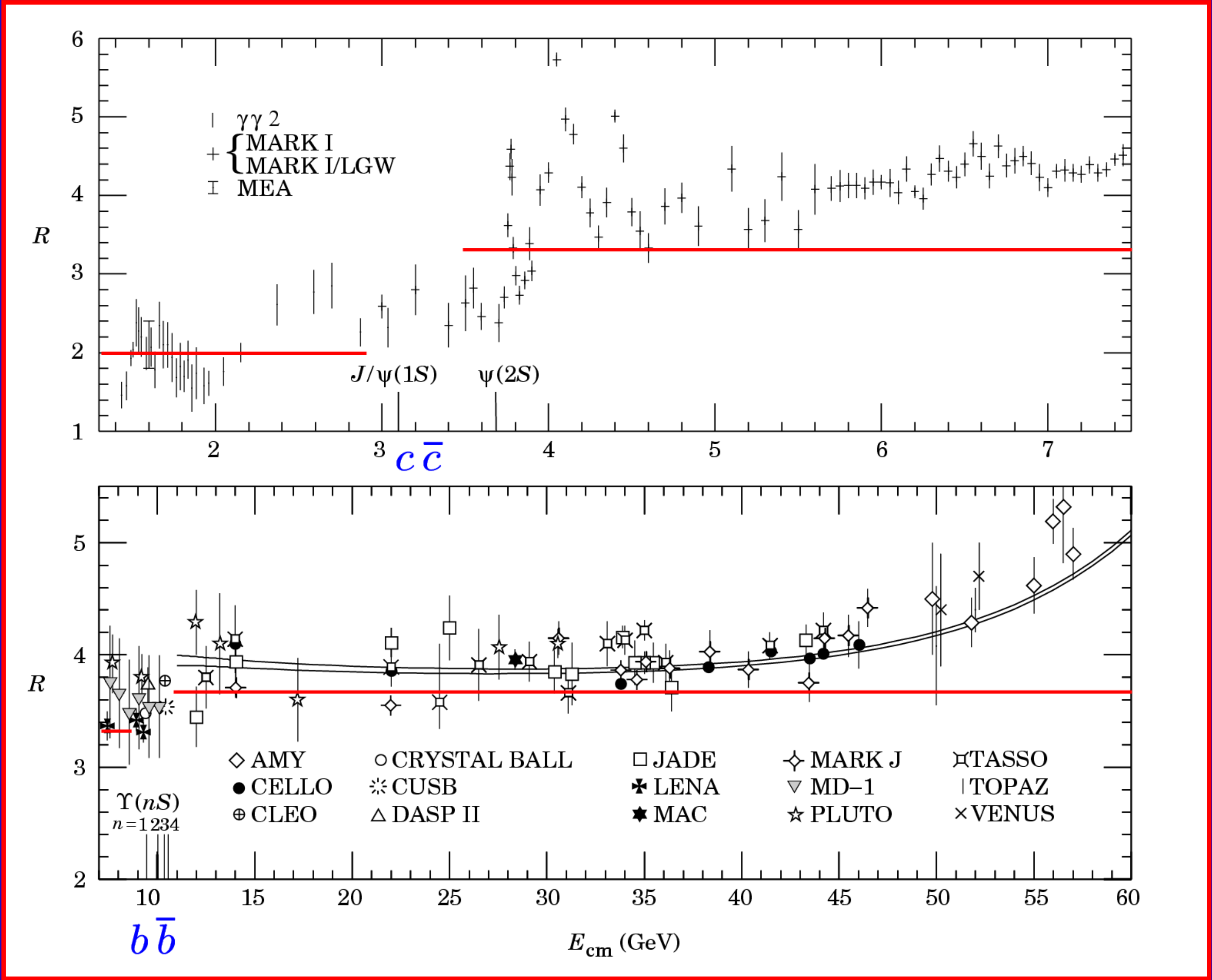
- Don´t exist ?
- **CONFINEMENT**


ALEPH
DALI
Run=15768
Evt=5906
Z
q
$\bar{q}$



$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \simeq \frac{\sum_q \sigma(e^+e^- \to q\,\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} \simeq N_C \sum_q Q_q^2$$

$$= \begin{cases} \frac{2}{3}\, N_C\,, & (u, d, s) \\ \frac{10}{9}\, N_C\,, & (u, d, s, c) \\ \frac{11}{9}\, N_C\,, & (u, d, s, c, b) \end{cases}$$



γγ 2
MARK I
MARK I/LGW
MEA
R
J/ψ(1S)
ψ(2S)
c c̄
AMY
CELLO
CLEO
CRYSTAL BALL
CUSB
DASP II
JADE
LENA
MAC
MARK J
MD–1
PLUTO
TASSO
TOPAZ
VENUS
ϒ(nS)
n = 1234
b b̄
E_cm (GeV)

QUANTUM CHROMODYNAMICS

FREE QUARKS: $N_C = 3$

$$\mathbf{q} \equiv \begin{pmatrix} q \\ q \\ q \end{pmatrix}$$

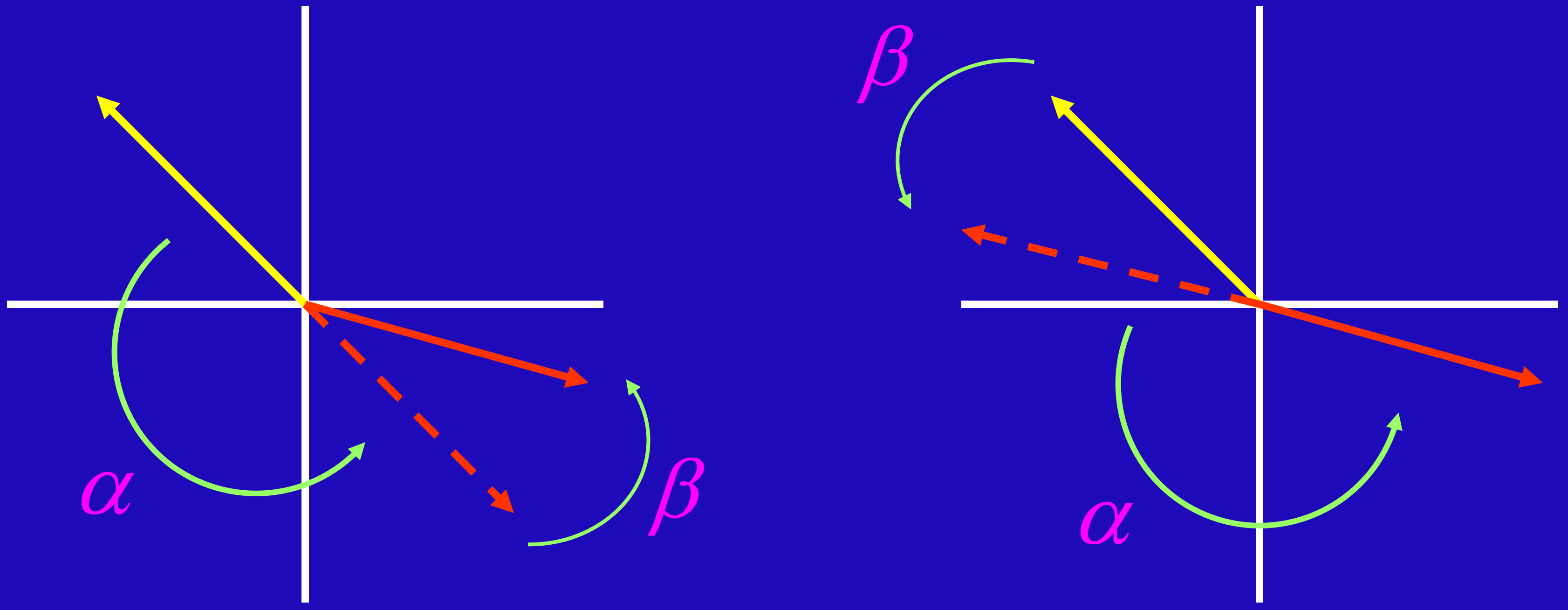
$$\mathcal{L} = \bar{\mathbf{q}} \, [\, i \gamma^\mu \partial_\mu - m \,] \, \mathbf{q}$$

SU(3) Colour Symmetry:

$$\mathbf{q} \;\to\; \mathbf{U}\,\mathbf{q} \qquad ; \qquad \bar{\mathbf{q}} \;\to\; \bar{\mathbf{q}}\,\mathbf{U}^\dagger$$

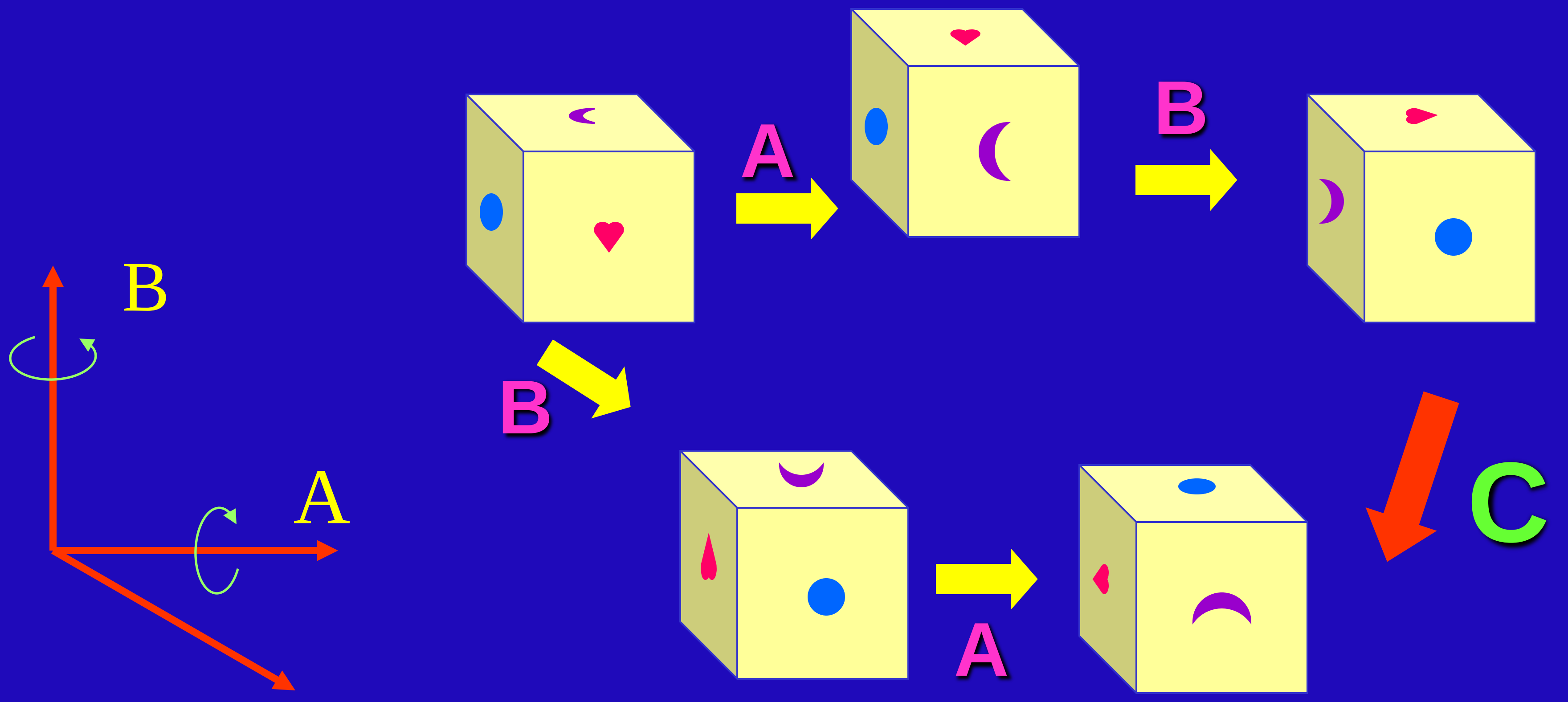
$$\mathbf{U}\,\mathbf{U}^\dagger = \mathbf{U}^\dagger\,\mathbf{U} = \mathbf{1} \qquad ; \qquad \det \mathbf{U} = 1$$

ABELIAN ROTATIONS



NON ABELIAN ROTATIONS

$$AB - BA = C$$



SU(N) ALGEBRA

$N \times N$ matrices: $\mathbf{U}\,\mathbf{U}^\dagger = \mathbf{U}^\dagger\,\mathbf{U} = \mathbf{1}$; $\det \mathbf{U} = 1$

$\mathbf{U} = \exp\left\{ i\, \mathbf{T}^a \theta_a \right\}$; $\mathbf{T}^a = \mathbf{T}^{a\,\dagger}$; $\mathrm{Tr}\,(\mathbf{T}^a) = 0$; $\mathrm{a} = 1, \cdots, N^2 - 1$

Commutation Relation: $\left[\mathbf{T}^a, \mathbf{T}^b \right] = i\, f^{abc}\, \mathbf{T}^c$

Structure Constants f^{abc} real , antisymmetric

Fundamental Representation: $\mathbf{T}_F^a = \frac{1}{2}\lambda^a$ $N \times N$

Adjoint Representation: $(\mathbf{T}_A^a)_{bc} = -i\, f^{abc}$ $(N^2 - 1) \times (N^2 - 1)$

$\mathrm{Tr}\,(\mathbf{T}_F^a\, \mathbf{T}_F^b) = T_F\, \delta_{ab}$; $\mathrm{Tr}\left(\mathbf{T}_A^a\, \mathbf{T}_A^b\right) = C_A\, \delta_{ab}$; $\left(\mathbf{T}_F^a\, \mathbf{T}_F^a\right)_{\alpha\beta} = C_F\, \delta_{\alpha\beta}$

$T_F = \frac{1}{2}$; $C_A = N$; $C_F = \frac{N^2 - 1}{2N}$

SU(2)

2×2 matrices: $\mathbf{U}\,\mathbf{U}^\dagger = \mathbf{U}^\dagger\,\mathbf{U} = \mathbf{1}$; $\det \mathbf{U} = 1$

$\mathbf{U} = \exp\left\{ i\, \mathbf{T}^a \theta_a \right\}$; $\mathbf{T}^a = \mathbf{T}^{a\dagger}$; $\mathrm{Tr}\,(\mathbf{T}^a) = 0$; $\mathrm{a} = 1, \cdots, 3$

Commutation Relation: $\left[\mathbf{T}^a, \mathbf{T}^b\right] = i\,\varepsilon^{abc}\,\mathbf{T}^c$

Fundamental Representation: $\mathbf{T}_F^a = \frac{1}{2}\sigma^a$ **Pauli**

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad ; \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad ; \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

SU(3)

$$[\mathbf{T}^a, \mathbf{T}^b] = i\, f^{abc}\, \mathbf{T}^c$$

Fundamental Representation: $\mathbf{T}^a_F = \frac{1}{2}\lambda^a$ Gell-Mann

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \;;\; \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \;;\; \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \;;\; \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \;;\; \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \;;\; \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \;;\; \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\frac{1}{2} f^{123} = f^{147} = -f^{156} = f^{246} = f^{257} = f^{345} = -f^{367} = \frac{1}{\sqrt{3}} f^{458} = \frac{1}{\sqrt{3}} f^{678} = \frac{1}{2}$$

QUANTUM CHROMODYNAMICS

FREE QUARKS: $N_C = 3$

$$\mathbf{q} \equiv \begin{pmatrix} q \\ q \\ q \end{pmatrix}$$

$$\mathcal{L} = \bar{\mathbf{q}}\,[\, i\gamma^\mu \partial_\mu - m\,]\,\mathbf{q}$$

SU(3) Colour Symmetry:

$$\mathbf{q} \to \mathbf{U}\,\mathbf{q} = \exp\left\{ i\,\frac{\lambda^a}{2}\,\theta_a \right\}\,\mathbf{q}$$

Gauge Principle: Local Symmetry $\theta_a = \theta_a(x)$

$$\mathbf{D}^\mu \mathbf{q} \equiv (\mathbf{I}_3\,\partial^\mu + i\,g_s\,\mathbf{G}^\mu)\,\mathbf{q} \quad \to \quad \mathbf{U}\;\mathbf{D}^\mu \mathbf{q}$$

$$\mathbf{D}^\mu \to \mathbf{U}\;\mathbf{D}^\mu\;\mathbf{U}^\dagger \qquad ; \qquad \mathbf{G}^\mu \to \mathbf{U}\;\mathbf{G}^\mu\;\mathbf{U}^\dagger + \frac{i}{g_s}\,(\partial^\mu \mathbf{U})\;\mathbf{U}^\dagger$$

$$[\mathbf{G}^\mu]_{\alpha\beta} \equiv \frac{1}{2}\,(\lambda^a)_{\alpha\beta}\;G^\mu_a(x)$$

8 Gluon Fields

Infinitesimal SU(3) Transformation:

$$\delta \mathrm{q}^{\alpha} = i\; \delta\theta_a \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} \mathrm{q}^{\beta} \qquad ; \qquad \delta G_a^{\mu} = -\frac{1}{g_s}\, \partial^{\mu}(\delta\theta_a) - f^{abc}\, \delta\theta_b\, G_c^{\mu}$$

Non Abelian Group:

$$f^{abc} \neq 0$$

- δG_a^{μ} depends on G_a^{μ}
- Universal g_s
- No Colour Charges

Kinetic Term:

$$\mathbf{G}^{\mu\nu} \equiv -\frac{i}{g_s}[\mathbf{D}^\mu, \mathbf{D}^\nu] = \partial^\mu \mathbf{G}^\nu - \partial^\nu \mathbf{G}^\mu + i\, g_s [\mathbf{G}^\mu, \mathbf{G}^\nu] \quad \rightarrow \quad \mathbf{U}\ \mathbf{G}^{\mu\nu}\ \mathbf{U}^\dagger$$

$$\mathbf{G}^{\mu\nu} \equiv \frac{\lambda^a}{2} G_a^{\mu\nu} \quad ; \quad G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s\, f^{abc}\, G_b^\mu\, G_c^\nu$$

$$\mathcal{L}_K = -\frac{1}{2}\mathrm{Tr}\,(\mathbf{G}^{\mu\nu}\mathbf{G}_{\mu\nu}) = -\frac{1}{4} G_a^{\mu\nu}\, G^a_{\mu\nu}$$

Mass Term:

$$\mathcal{L}_M = \frac{1}{2}\, m_G^2\ G_a^\mu\, G^a_\mu$$

Not Gauge Invariant ➡ $m_G = 0$

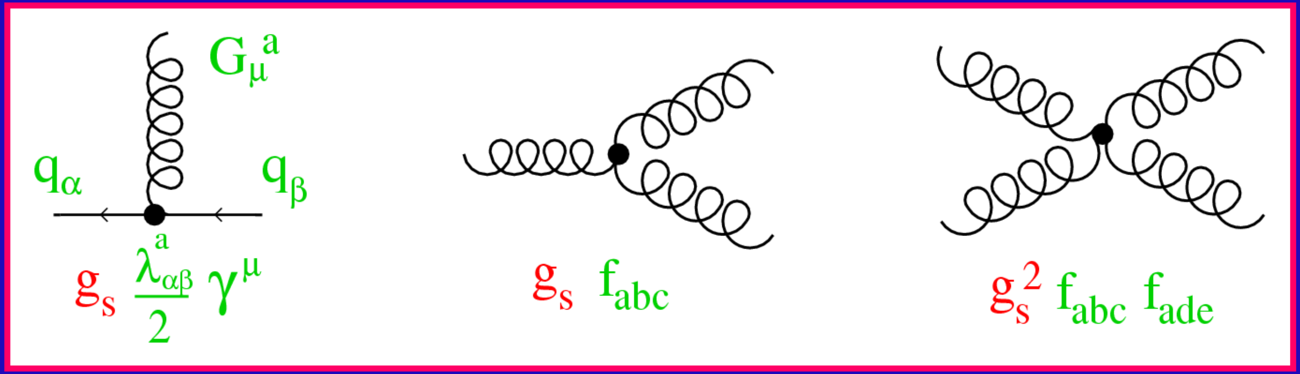
Massless Gluons

$$\mathcal{L}_{QCD} = -\frac{1}{2}\,\mathrm{Tr}\left(\mathbf{G}^{\mu\nu}\,\mathbf{G}_{\mu\nu}\right) + \bar{\mathbf{q}}\,[\,i\,\gamma^{\mu}\,\mathbf{D}_{\mu} - m_q\,]\,\mathbf{q}$$

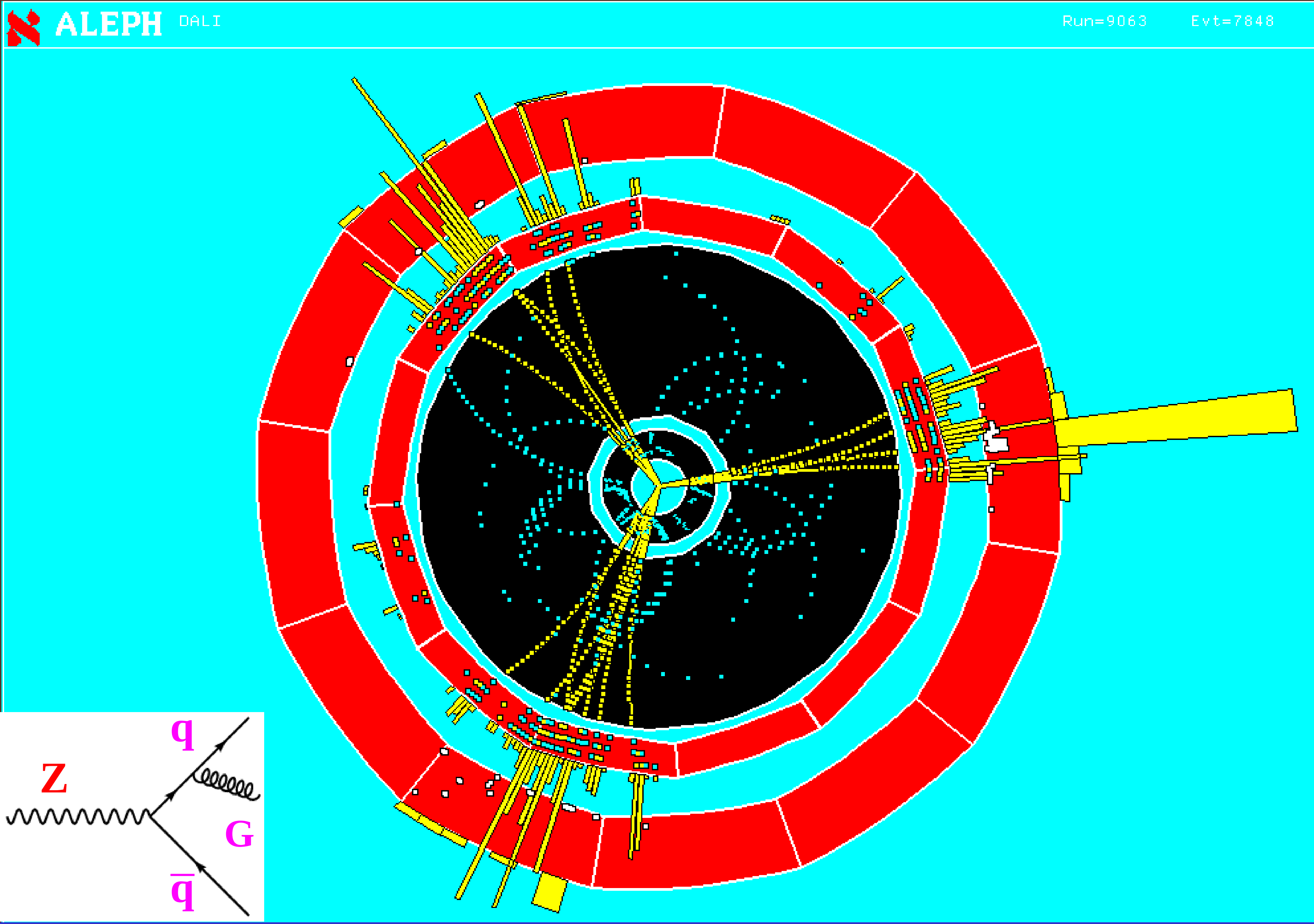
$$= -\frac{1}{4}\left(\partial^{\mu}G^{\nu}_{a} - \partial^{\nu}G^{\mu}_{a}\right)\left(\partial_{\mu}G^{a}_{\nu} - \partial_{\nu}G^{a}_{\mu}\right) + \sum_{q}\bar{q}_{\alpha}\,[\,i\,\gamma^{\mu}\partial_{\mu} - m_q\,]\,q_{\alpha}$$

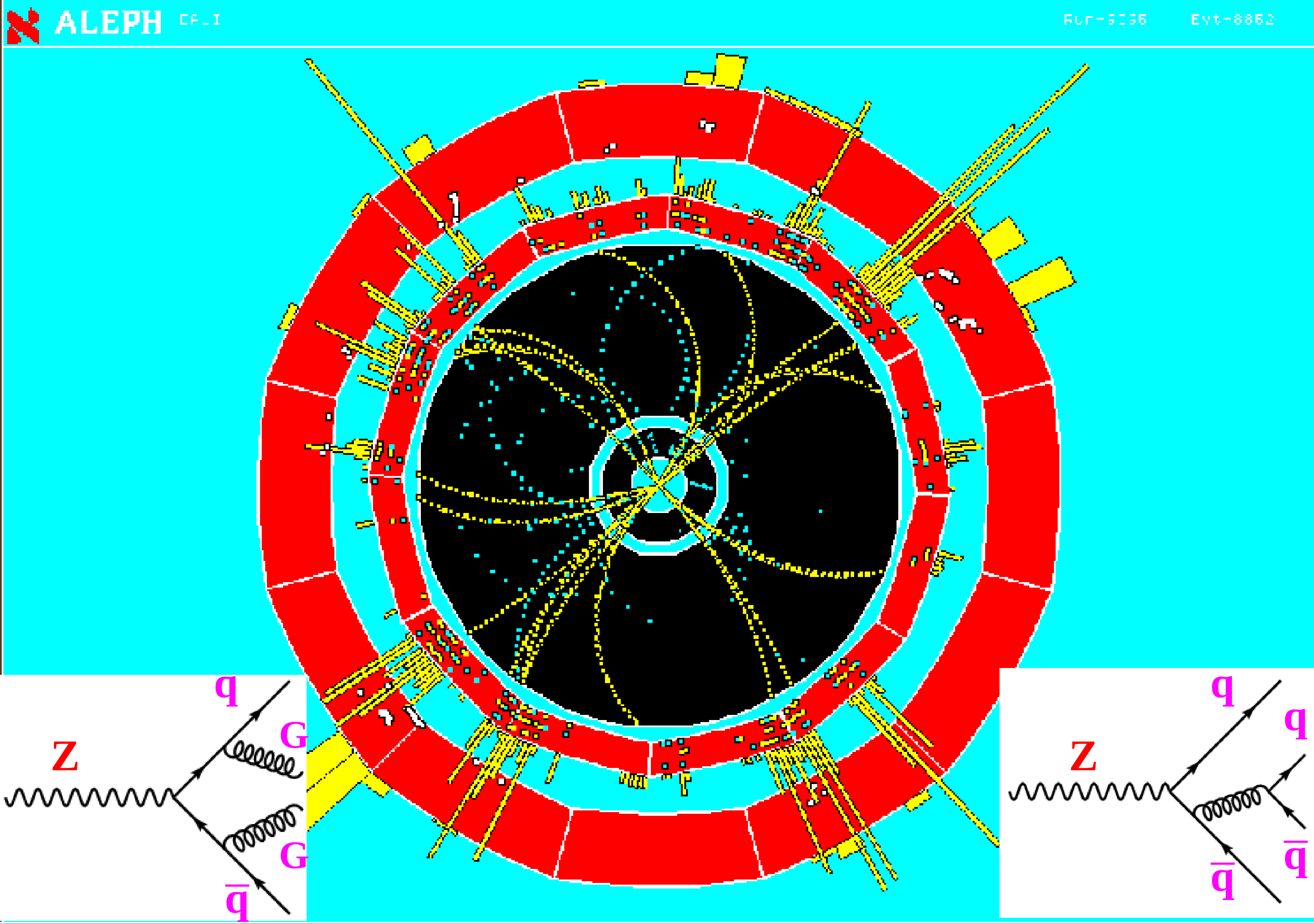
$$- \frac{1}{2}\sum_{q} g_s\,[\bar{q}_{\alpha}\,(\lambda^{a})_{\alpha\beta}\,\gamma^{\mu} q_{\beta}]\,G^{a}_{\mu}$$

$$+ \frac{1}{2} g_s\, f_{abc}\,(\partial_{\mu}G^{a}_{\nu} - \partial_{\nu}G^{a}_{\mu})\,G^{\mu}_{b}\,G^{\nu}_{c} - \frac{1}{4} g_s^2\, f_{abc}\, f_{ade}\, G^{\mu}_{b}\,G^{\nu}_{c}\,G^{d}_{\mu}\,G^{e}_{\nu}$$



- **Gluon Self – interactions** $\mathbf{G^3\ ,\ G^4}$
- **Universal Coupling** $\mathbf{g_s}$ **(No Colour Charges)**


ALEPH
DALI
Run=9063
Evt=7848
q
Z
G
q̄


ALEPH
Z
q
G
G
$\bar{q}$
Z
q
q
$\bar{q}$
$\bar{q}$