W/Z-Gamma Production at NLO in Powheg Method

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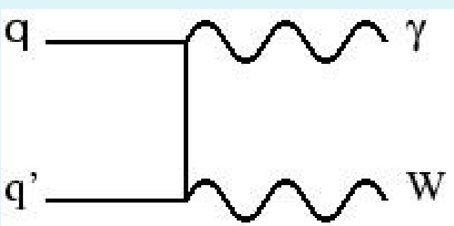
Outline:

- Motivations
- Status of W/Z-Gamma Calculations
- Powheg Method
- Matrix element Calculations
- Results
- Plans

Why Vector Boson-Photon Production?

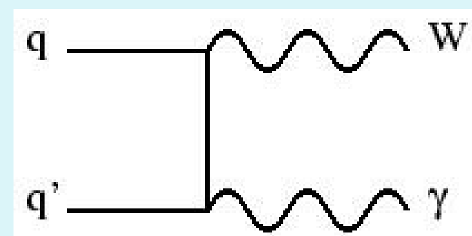
Testing Standard Model:

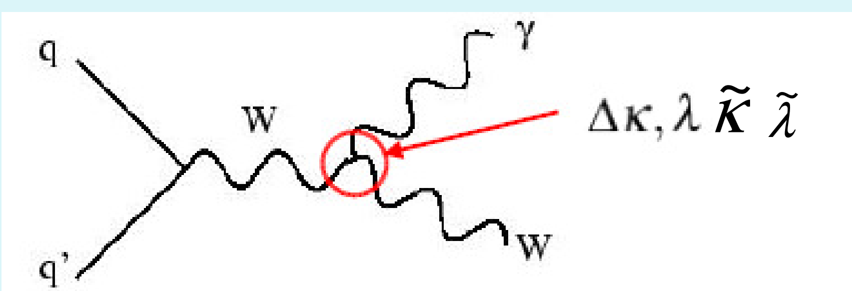
- Non-abelian self-couplings WWγ coupling: CP– conserving κ and λ and CP–violating $\tilde{\kappa}$ and $\tilde{\lambda}$?
- Are there ZZγ or Zγγ couplings? Gauge symmetry breaking!
- Agree well with the standard model and set limits on anomalous couplings in Tevatron

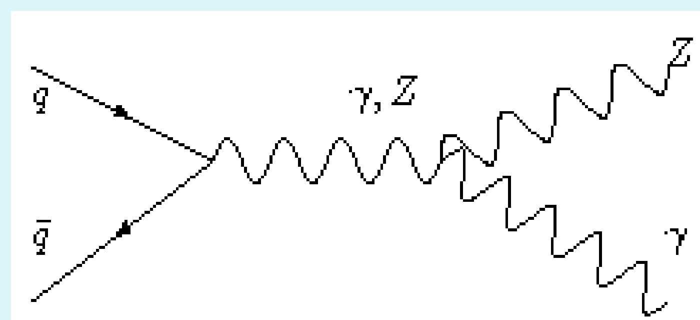


$$\text{CDF} \quad -2.3 < \Delta\kappa < 2.3 \ (\lambda = 0),$$

$$-0.7 < \lambda < 0.7 \ (\Delta\kappa = 0)$$







Why Vector Boson-Photon Production?

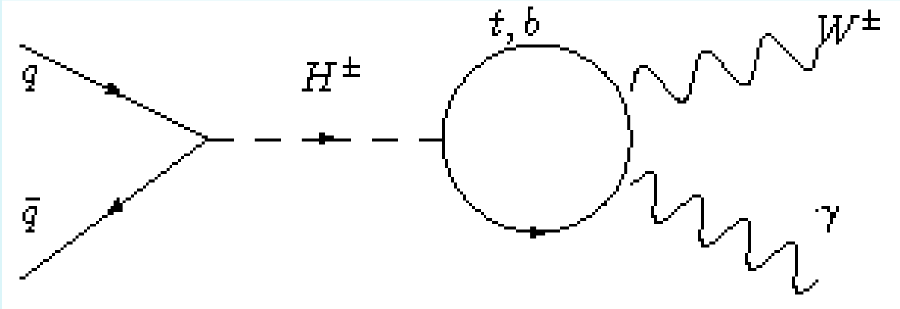
Beyond the Standard Model in LHC?

- Effectively anomalous couplings are remarkable at hight scale $\hat{s}$:

$$\Delta g(\hat{s}) = \Delta g_0 / (1 + \hat{s} / \Lambda_W^2)$$

- Charged Higgs decay to W-Photon in SUSY?

  Not tree level, but loop!



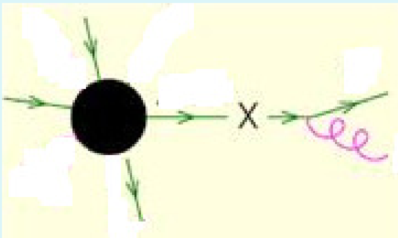
Important backgrounds to New Physics in LHC

Status of Phenomenology Calculations

- Leading Order:
  - Standard Wγ and Zγ, anomalous coupling for Wγ:
  - BHO generator, standalone and implemented in Pythia

Ohnemus, Baur and Han (1993)

- Next to Leading Order:
  - NLO matrix element: real radiations, virtual loop and hard collinear part:
  - Standalone BHO generator
  - Combine with shower MC in Pythia



But Wait!

- LHC: high energy scale and high luminosity: we need precise NLO calculations
- NLO: additional parton radiation + naïve parton Shower ⇒ double counting and invalid in IR phase space region
- Result as inconsistent NLO, i.e. just LO accuracy!
- Soft gluon radiation cut δ_s and photon isolation cut for quark radiation δ_c to cancel the IR divergence

Methods to match NLO matrix element

with NLO parton shower:

MC@NLO and Powheg in Herwig++

Powheg Method

- The hardest radiation is generated first
- NLO accuracy
- p_T order, most case have NLL accuracy, but not so straightwards in angular order
- Generate other parton radiation by Shower Monte Carlo
- Smooth IR region to high p_T region, no phase-space slicing

Powheg Method

- hardest radiation by Sudakov form factor

$$\Delta^{f_b}(\Phi_n, p_T) = \exp\left\{ - \sum_{\alpha_r \in \{\alpha_r | f_b\}} \int \frac{\left[d\Phi_{rad} \, R(\Phi_{n+1}) \, \theta(k_T(\Phi_{n+1}) - p_T) \right]_{\alpha_r}^{\bar{\Phi}_n^{\alpha_r} = \Phi_n}}{B^{f_b}(\Phi_n)} \right\}$$

- The cross-section of Powheg:

$$d\sigma = \sum_{f_b} \bar{B}^{f_b}(\Phi_n) \, d\Phi_n \left\{ \Delta^{f_b}(\Phi_n, p_T^{\min}) + \sum_{\alpha_r \in \{\alpha_r | f_b\}} \frac{\left[d\Phi_{rad} \, \theta(k_T - p_T^{\min}) \, \Delta^{f_b}(\Phi_n, k_T) \, R(\Phi_{n+1}) \right]_{\alpha_r}^{\bar{\Phi}_n^{\alpha_r} = \Phi_n}}{B^{f_b}(\Phi_n)} \right\}$$

where the NLO matrix element is

$$\overline{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \left[\int d\Phi_{rad} \left[R(\Phi_{n+1}) - C(\Phi_{n+1}) \right] + \int \frac{dx}{x} \left[G_a(\Phi_{n,a}, x) + G_b(\Phi_{n,b}, x) \right] \right]^{\overline{\Phi}_n = \Phi_n}$$

Shower Monte Carlo and NLO

- Afterwards shower by Sudakov form factor

$$\Delta_i(t_I, t) = \exp\left[-\sum_{(jk)} \int_t^{t_I} \frac{dt'}{t'} \int dz \frac{\alpha_s(t')}{2\pi} P_{i,jk}(z) \right]$$

- Cross-section

$$d\sigma = d\Phi_B B(\Phi_B) \left(\underbrace{\Delta_{t_I,t_0}}_{\text{No radiation}} + \sum_{(jk)} \underbrace{\Delta_{t_I,t} \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}}_{\text{radiation}} \right)$$

- In small p_T region, regain ordinary shower:

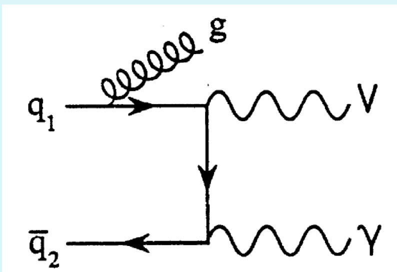
$$\frac{R(\Phi)}{B(\Phi_B)} d\Phi_{\text{rad}} \approx \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}$$

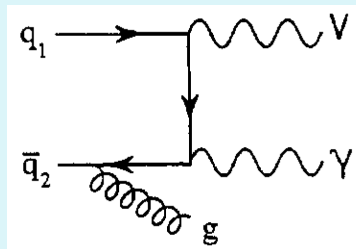
- In large p_T region, $\because \Delta_{t,t'} \to 1$, regain NLO ME:

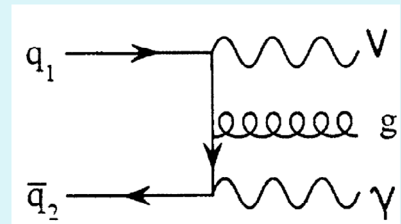
$$d\sigma = \bar{B} \times \frac{R}{B} \approx R \times (1 + \mathcal{O}(\alpha_s)) \qquad \int \delta(\Phi_B - \bar{\Phi}_B) d\sigma = \bar{B}(\bar{\Phi}_B)$$

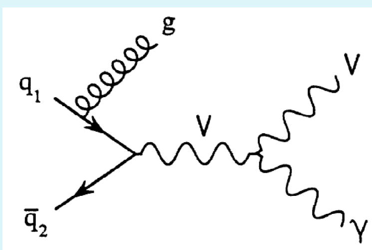
Gluon Radiation

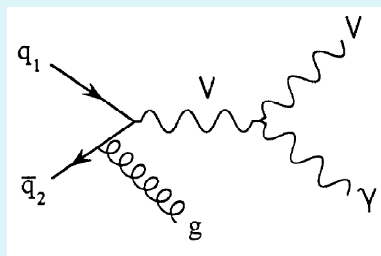
- We try to calculate real radiation term $R(\Phi_1)$ only for gluon radiation first (in fact 8 diagrams)











…

- This calculations can be done numerically in Herwig++
- Calculate cross section factor and Jacobian to get:

$$\frac{R(\Phi_1)}{B(\Phi_B)} = \frac{\left|M^R(\Phi_R,\Phi_B)\right|^2}{\left|M^B(\Phi_B)\right|^2} \cdot \frac{\lambda^{1/2}(\hat{s}^R,s_1,0)\cdot\lambda^{1/2}(s_1,m_V^2,0)\cdot\hat{s}^{B2}}{\lambda^{1/2}(\hat{s}^B,m_V^2,0)\cdot 16\pi\cdot 4\pi^2\cdot\hat{s}^{R2}s_1} \cdot \left|\frac{\partial(\cos\theta_g,s_1)}{\partial(y_g,p_{Tg})}\right|$$

In the Catani-Seymour framework

- Implement Powheg in C-S subtraction framework:
- Born and radiation phase space mapping $\Phi_{n+1} \Rightarrow \overline{\Phi}^{(\alpha)}{}_{B}, \Phi^{(\alpha)}{}_{R}$
- Separate the real radiation into pieces $R^{(\alpha)}(\Phi^{(\alpha)}{}_{R}, \Phi^{(\alpha)}{}_{B})$ with different singular in Catani-Seymour Formalism:

$$R^{(\alpha)} = R \cdot S^{(\alpha)} \qquad \sum_{\alpha} S^{(\alpha)} = 1$$

Where $S^{(\alpha)}$ are calculated with Catani-Seymour dipole function

$$\mathcal{S}_{\alpha} = \frac{\mathcal{D}_{\alpha}}{\sum_{\beta} \mathcal{D}_{\beta}}$$

NLO matrix element

- Gluon radiation & quark radiation

soft and conlinear with incoming partons

collinear with incoming partons and outgoing photon (photon fragmentation)

- We first consider contribution of gluon radiation and IR singularities in virtual loop and PDF
- Real radiation will be added in the codes
- The dipoles part $d\sigma^{A}_{ab}(p_a, p_b)$ to cancel IR divergence in virtual loop $d\sigma^{V}_{ab}(p_a, p_b)$ and PDF counter-term $d\sigma^{C}_{ab}(p_a, p_b, \mu_F^2)$
- Convenient to deal with $\int_{\Phi_R} d\sigma^{A}_{ab}(p_a, p_b) + d\sigma^{C}_{ab}(p_a, p_b, \mu_F^2)$ together

$$\sigma^{\rm NLO}_{ab}(p_a, p_b; \mu_F^2) = \int_{m+1} \left(d\sigma^{R}_{ab}(p_a, p_b) - d\sigma^{A}_{ab}(p_a, p_b) \right)$$

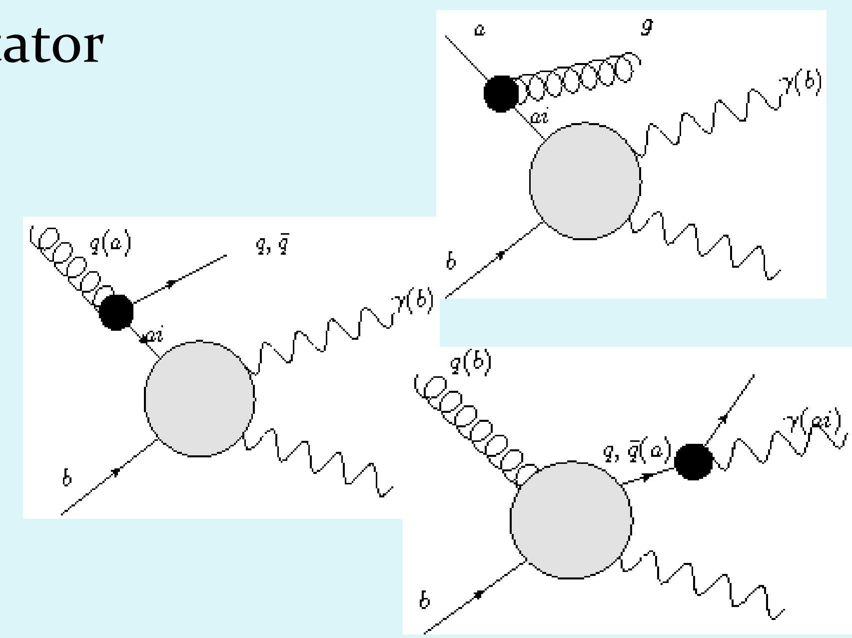
$$+ \left[\int_{m+1} d\sigma^{A}_{ab}(p_a, p_b) + \int_{m} d\sigma^{V}_{ab}(p_a, p_b) + \int_{m} d\sigma^{C}_{ab}(p_a, p_b; \mu_F^2) \right]$$

Catani-Seymour formalism

- Subtraction formalism to single out different IR singularities by various dipole functions
- Mapping Born and radiation phase space smoothly
- Make the real radiations, virtual loop and initial & final collinear remnants finite respectively: compare them

In our case, there are kinds of dipoles: $\mathcal{D}^{ai,b}$

1. Initial $q/\bar{q}$ emits a gluon, with spectator initial $q/\bar{q}$: $\mathcal{D}^{qg,\bar{q}}$ $\mathcal{D}^{\bar{q}g,q}$
2. Initial gluon emits $q/\bar{q}$, with spectator initial $q/\bar{q}$: $\mathcal{D}^{gq,\bar{q}}$ $\mathcal{D}^{g\bar{q},q}$
3. Initial gluon emits $q/\bar{q}$, with spectator, photon: $\mathcal{D}_{\gamma}^{gq}$ $\mathcal{D}_{\gamma}^{g\bar{q}}$
4. Final $q/\bar{q}$ emits photon $\mathcal{D}_{q\gamma}^{b}$ $\mathcal{D}_{\bar{q}\gamma}^{b}$



Dipoles with two initial partons and no identical final state parton

- Dipoles of kind 1. are simpler without $\mathcal{D}_{k}^{ai}$
- Kind 2. & 3. are with different initial parton (PDF) from 1. but the same Born part $\left|M^{B}\right|_{ab}^{2}(p_a, p_b)$, will be considered with photon fragmentation
- Kind 4. associated with photon fragmentation are QED dipoles
- After integrating out the phase space of the radiation parton ai, dipoles part can be separated into pieces depending on dimensional factor ε and faction x

$$
{}_{m+1,ab}\langle 1,\ldots,m+1;a,b||1,\ldots,m+1;a,b\rangle_{m+1,ab} = \sum_{k\neq i}\mathcal{D}_{k}^{ai}(p_1,\ldots,p_{m+1};p_a,p_b) + \mathcal{D}^{ai,b}(p_1,\ldots,p_{m+1};p_a,p_b) + \ldots
$$

Cancelling the IR divergence

- $\int_{\Phi_R} d\sigma^A_{ab}(p_a, p_b) + d\sigma^C_{ab}(p_a, p_b, \mu_F^2)$ is separated into three operators: $I(\varepsilon)$ with up to $\frac{1}{\varepsilon^2}$ divergence exactly cancels that in $d\sigma^V_{ab}(p_a, p_b)$
- and operators $K^{a,a'}(x) + P^{a,a'}(x, \hat{s}, \mu_F^2)$ have already cancelled divergences in initial parton PDFs $d\sigma^C_{ab}(p_a, p_b, \mu_F^2)$, so finite.

$$\int_{m+1} d\sigma^A_{ab}(p_a, p_b) + \int_m d\sigma^C_{ab}(p_a, p_b, \mu_F^2) =$$

$$\int_m \left[d\sigma^B_{ab}(p, \bar{p}) \cdot \boldsymbol{I}(\epsilon) \right] + \sum_{a'} \int_0 dx \int_m \left[\boldsymbol{K}^{a,a'}(x) \cdot d\sigma^B_{a'b}(xp, \bar{p}) \right]$$

$$+ \sum_{a'} \int_0^1 dx \int_m \left[\boldsymbol{P}^{a,a'}(xp, x; \mu_F^2) \cdot d\sigma^B_{a'b}(xp, \bar{p}) \right]$$

$$+ \sum_{b'} \int_0^1 dx \int_m \left[\boldsymbol{K}^{b,b'}(x) \cdot d\sigma^B_{ab'}(p, x\bar{p}) \right]$$

$$+ \sum_{b'} \int_0^1 dx \int_m \left[\boldsymbol{P}^{b,b'}(x\bar{p}, x; \mu_F^2) \cdot d\sigma^B_{ab'}(p, x\bar{p}) \right]$$

Finite Dipoles &Virtual loop

- The finite part of the first term after cancelling IR divergence:

$$\int_{\Phi_R} d\sigma_{ab}^{A}(p_a, p_b) \otimes I(\varepsilon) \xrightarrow{\text{finite}} \frac{\alpha_S}{2\pi} C_F [2(5 - \frac{\pi^2}{3}) d\sigma_0^B + 3 d\sigma_1^B + 2 d\sigma_2^B + O(\varepsilon)]$$

where $d\sigma^B = d\sigma_0^B + \varepsilon \cdot d\sigma_1^B + \varepsilon^2 \cdot d\sigma_2^B$

- The finite parts in virtual loop $d\sigma_{ab}^{V}(p_a, p_b)$ is:

$$\mathcal{N}_{\text{in}} \frac{d\phi_B(p_a + p_b)}{n_{s,a} n_{s,b} \Phi(p_a, p_b)} \frac{G^V(\alpha, \theta_W, \cdots)}{2} \frac{Q_a t + Q_b u}{t + u} [Q_a F^V(t, u, \hat{s}) + Q_b F^V(u, t, \hat{s})]$$

where $G^V(\alpha, \theta_W, \cdots)$ is the coupling square factor for boson V

Initial states collinear remnants

- Thank to the simple color structure and color conservation,
- color dependent matrix element in collinear remnants $d\sigma_{ab}^{C}(p_a, p_b, \mu_F^2)$ is separated into a factor $\times$ $\left|M^B(p_\gamma, p_V, x_a p_a, p_b)\right|^2$ or $\left|M^B(p_\gamma, p_V, p_a, x_b p_b)\right|^2$ $(p_i = z_i \cdot P)$
- Change the integrated order of x and z, x can be integrated out:

$$\int_0^1 dx \int_0^1 dz f(z)\sigma(xz) = \int_0^1 dz \int_z^1 dx f(\frac{z}{x})\frac{\sigma(z)}{x}$$

- I find $(\frac{2}{1-x}\ln\frac{1-x}{x})_+$ in $\overline{K}^{qq}(x)$ should be different from that in [1] to avoid x=0 singularities $-(\frac{2}{1-x})_+\ln x + [\frac{2}{1-x}\ln(1-x)]_+ - \delta(1-x)\frac{\pi^2}{3}$
- So $\sum_{a'}\int dx \int_{\Phi_B} d\sigma_{a'b}^B(xp_a, p_b)\otimes(K+P)^{aa'}(x) + \sum_{b'}\int dy \int_{\Phi_B} d\sigma_{ab'}^B(p_a, yp_b)\otimes(K+P)^{bb'}(y) =$

$$-\frac{\alpha_S}{2\pi}C_F d\sigma_0^B(p_a + p_b, z_a, z_b)\otimes(KP[z_a, f_a] + KP[z_b, f_b])$$

where functionals

$$KP[z,f] = \int_z^1 [\ln\frac{(1-x)^2\hat{s}}{x\mu_F^2}(\frac{2}{1-x} - \frac{1+x^2}{1-x}\frac{f(z/x)}{x\cdot f(z)}) + \frac{2}{1-x}\ln x]dx$$

$$+\int_0^z \frac{2}{1-x}\ln\frac{(1-x)^2\hat{s}}{\mu_F^2}dx - \int_z^1 dx\frac{1-x}{x}\frac{f(z/x)}{f(z)} + (5 - \frac{\pi^2}{3})$$

[1] Catani,Seymour, *Nucl. Phys.* B 485(1997) 291

Current numerical results

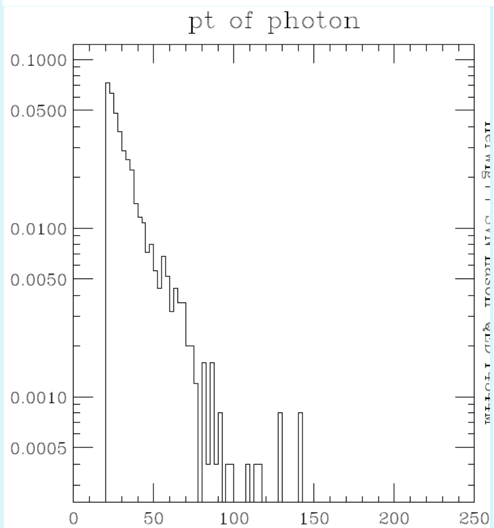
- With photon cuts: $p_T \geq 20\text{GeV}$, $|\eta| \leq 2.7$, multi-particle interaction, hadronization and jet decay turned off, only hardest parton emission
- Run for 1000 events

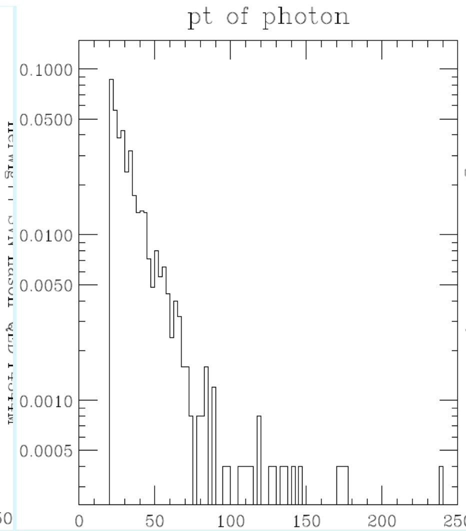
ME+Parton shower	Total cross sections
NLO+gluon Powheg	48(+-2)pb
NLO+no Powheg	49(+-2)pb
LO(NLOweight=1)+gluon Powheg	34(+-1)pb
NLO(only virtual loop)+gluon Powheg	40(+-1)pb

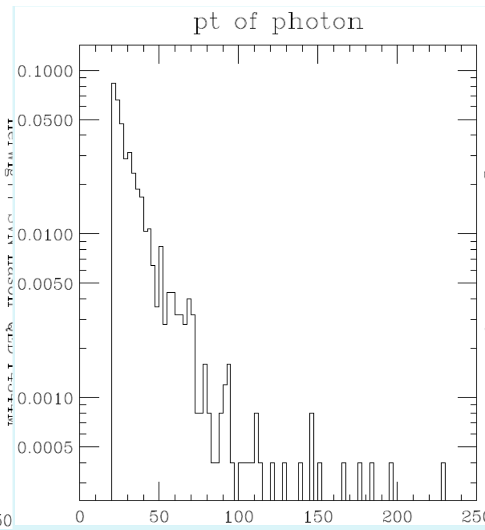
Differential cross sections:

NLO+Powheg	LO+ Powheg	NLO+no Powheg

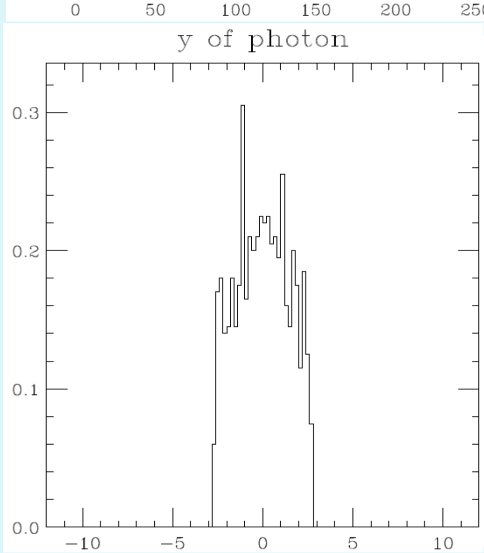
Shorter high p_T tail (smaller p_T) NLO+Powheg< NLO+no Powheg <LO+Powheg

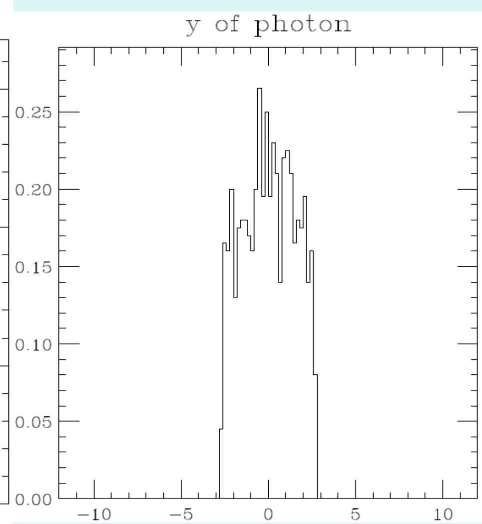


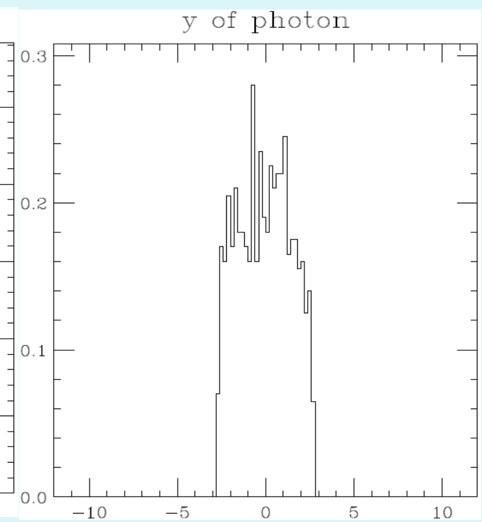




larger rapidity (less transverse) NLO+Powheg> NLO+no Powheg >LO+Powheg







Outlook and Plan

- Adding finite gluon radiation part to NLO matrix element

$$\int_{m+1}\left[\left(d\sigma^{R}_{ab}(p_a,p_b)\right)_{\epsilon=0}-\left(\sum_{\text{dipoles}} d\sigma^{B}_{ab}(p_a,p_b)\otimes\left(dV_{\text{dipole}}+dV'_{\text{dipole}}\right)\right)_{\epsilon=0}\right]$$

- Further calculations for quark radiation collinear remnants of photon fragmentation and their dipoles to matrix element & Powheg: Completed NLO
- How NLO corrects LO
- Anomalous WWγ couplings and beyond SM
- W/Z decay: ask for additional contributions of photon radiates from decayed lepton