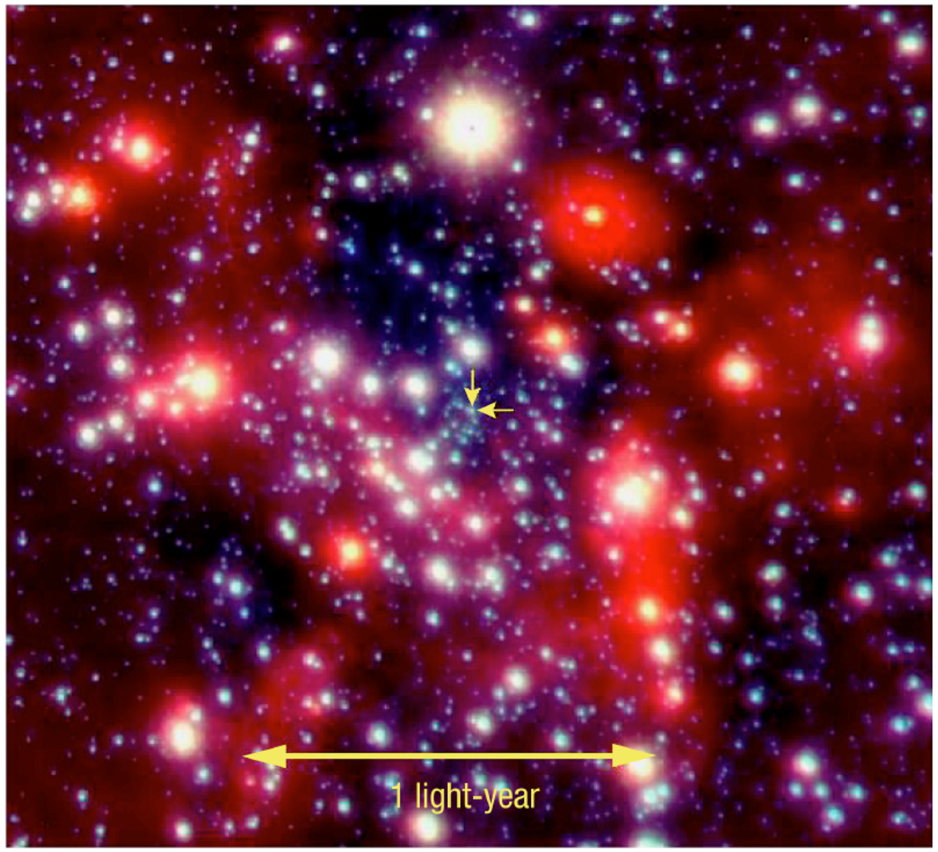
String Theory for Pedestrians

CERN academic training
February 23-25, 2009

C. Bachas - ENS, Paris

Lecture 3 : Black Holes and the AdS/CFT correspondence.

Black holes are fascinating astrophysical objects.



The Centre of the Milky Way
(VLT YEPUN + NACO)
ESO PR Photo 23a/02 (9 October 2002)

The one in the center of our galaxy has a mass a few million times the solar mass, or

$$M_{\rm BH} \sim 10^{44}\ M_{\rm Planck}\quad ,$$

within a radius of ten billion kilometers. It is a supermassive black hole, as classical an object as it gets in our Universe.

The defining feature of a classical BH is that it is surrounded by an event horizon. This is a surface of no escape even for light, so that everything that happens behind a horizon is, for ever, hidden to outside observers.

[Time-reversed] Rhine as a rowers' black hole:

asymptotic

horizon

singularity

velocity field →

→

turbulent !*?#

> top speed of Olympic champ

Passing the horizon seems very innocent while it is happening. It's like being in a rowboat above Niagara Falls. If you accidentally pass the point where the current is moving faster than you can row, you are doomed. But there is no sign—DANGER! POINT OF NO RETURN—to warn you. Maybe on the river there are signs but not at the horizon of a black hole.

(Lenny Susskind, CA Literary Review)

The geometry of a charged BH is described by the Reissner-Nordström metric:

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2^2 \ , \qquad \text{where}$$

$$f(r) = (1 - \frac{r_+}{r})(1 - \frac{r_-}{r}) \qquad \text{with} \qquad r_\pm = G_N M \pm \sqrt{G_N^2 M^2 - Q^2}$$

charge in units where Coulomb's constant =1 .

The (outer) horizon is at $r = r_+$, and $G_N M \geq Q$ by the cosmic-censorship hypothesis (no naked singularities). The Schwarzschild BH is found for *Q=0* , while the extremal BH is obtained when the inequality is saturated .

NB: Astrophysical black holes have zero charge; but in our later discussion we will focus on near-extremal BHs, so the charge is essential .

Though the horizon of a large BH is (for almost all purposes) a smooth classical region, there is a tiny quantum effect happening there: Hawking radiation.

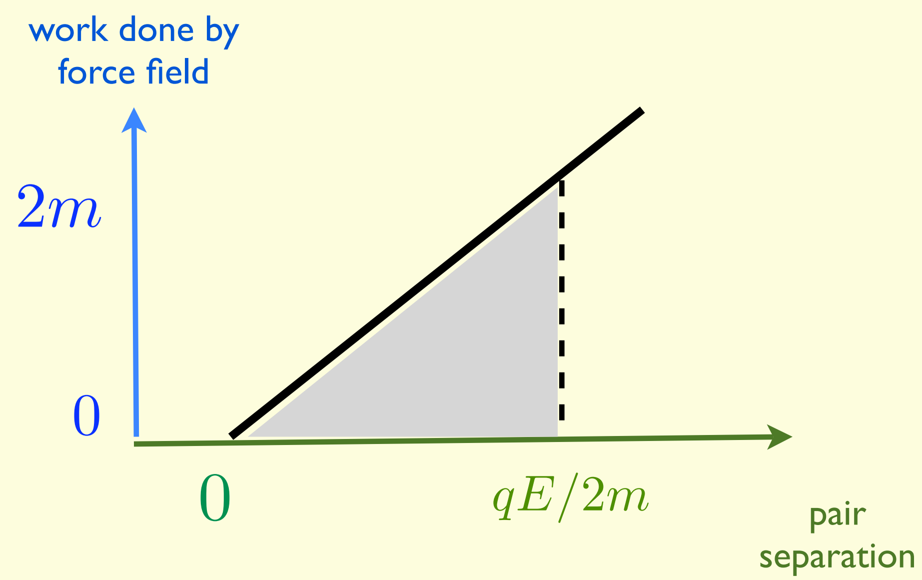
hand-waiving argument:

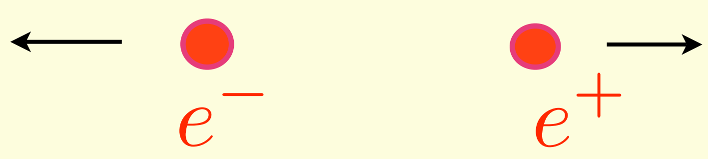
Recall Schwinger's pair creation in an electric field,

rate / unit volume $\sim e^{-\# m^2/qE\hbar}$

numerical factor

this is a quantum-tunneling probability: $\propto \exp(\text{barrier} \times \text{distance}/\hbar)$





In Schwarzschild BH replace the force field $qE \to G_N Mm/r_+^2 \sim m/G_N M$, to find

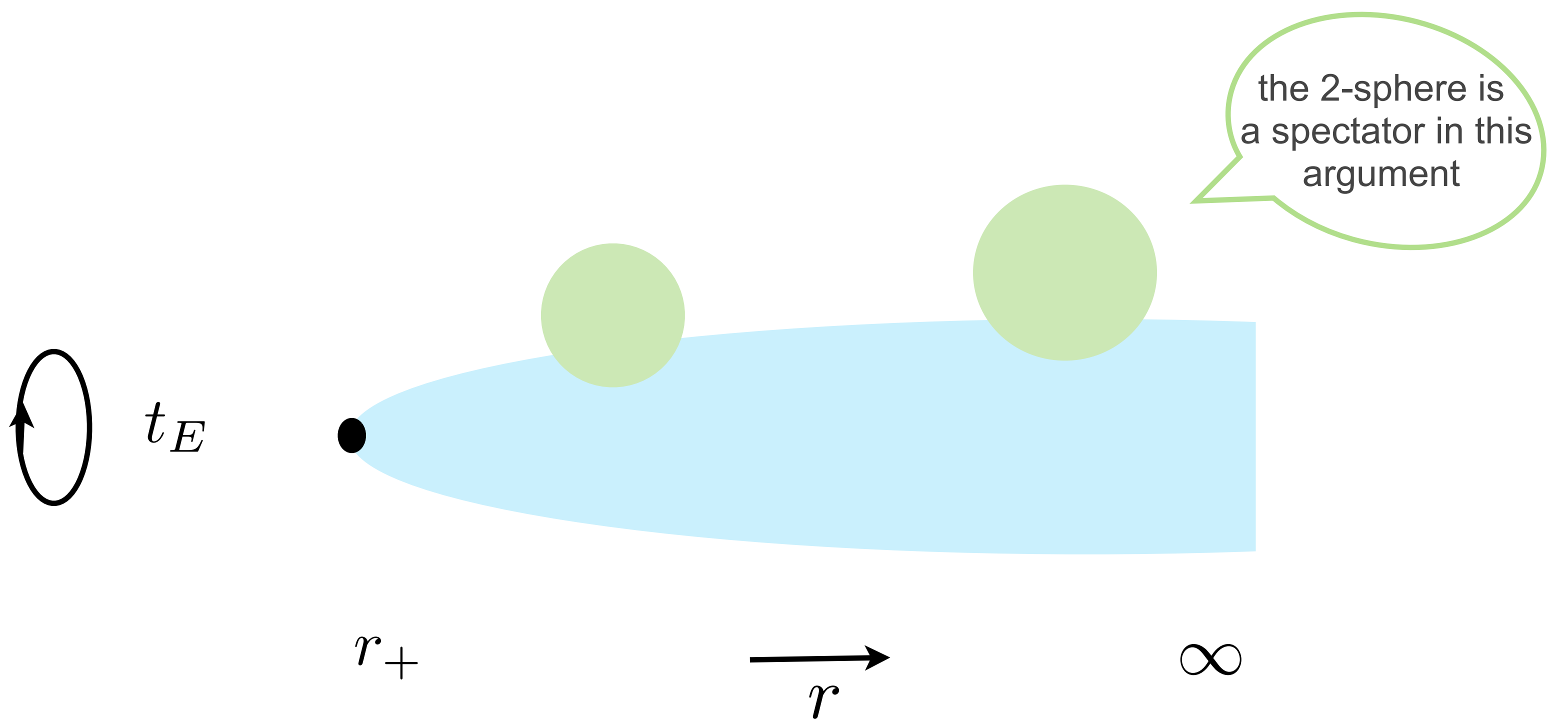
rate / horizon area $\sim e^{-\# \, m G_N M/\hbar}$

thermal !

exact argument:

A way to put a quantum field theory at finite temperature T, is to compactify Euclidean time: $t_E \equiv t_E + \hbar/k_B T$.

Compactifying the asymptotic time of the Reissner-Nordstrom BH, gives a cigar geometry with a conical singularity at the tip:



To analyze this conical singularity, change radial coordinate:

$$r - r_+ = \left(\frac{r_+ - r_-}{4r_+^2} \right) \rho^2 \quad \Longrightarrow \quad ds^2 \simeq \underbrace{d\rho^2 + \rho^2 (\frac{2\pi T_H}{\hbar} dt_E)^2}_{\text{Tip of cigar}} + r_+^2 d\Omega_2^2 \ ,$$

where the Hawking temperature is $T_H \ := \ \hbar \frac{r_+ - r_-}{4\pi r_+^2} \ = \begin{cases} \frac{\hbar}{8\pi G_N M} & \text{Schwarzschild} \\ 0 & \text{extremal} \end{cases}$

Choosing the periodicity of the time coordinate so that $T = T_H$ results in a non-singular geometry. This allows the definition of a KMS state [defined by functional integral] thereby showing that the BH is at equilibrium with the asymptotic heat bath.

invariant under imaginary time translations

⇒ Black Holes must have a non-zero temperature !

NB: putative "BHs" produced at the LHC have $T_H \sim M_{QG} \left(\frac{M_{QG}}{M} \right)^{\frac{1}{D-3}} \geq TeV$ it is unlikely that they will sit around for too long.

Giddings, Mangano '08
Ellis, Giudice, Mangano, Tkachev , Wiedemann '08

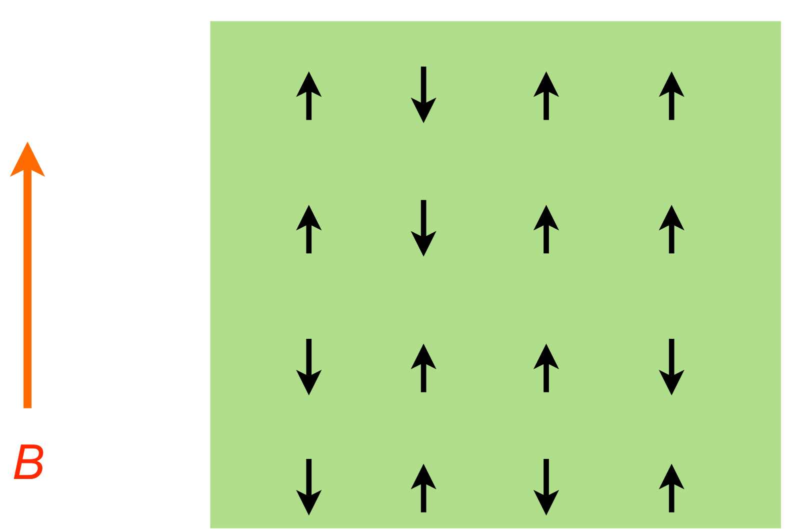
If BHs have a temperature, then from the first law of thermodynamics we deduce that they must also have an **entropy**:

$$dM = T_H \, dS_{BH} \implies S_{BH} = \frac{4\pi r_+^2}{4 G_N \hbar} \, .$$

horizon area

Bekenstein-Hawking

Entropy is a derived quantity, given some microscopic description of the system.



Think e.g. of a paramagnet in a magnetic field *B.* Suppose that it is described at the microscopic level by *N* non-interacting spins:

$$E := N\epsilon = -\sum 2\mu_B \vec{\sigma} \cdot \vec{B}$$
$$= \mu_B B (N_+ - N_-)$$

Bohr magneton

number of up spins

Calculating the entropy is a simple counting problem:

$$S = \log \begin{pmatrix} N \\ N_+ \end{pmatrix} = -N(y_+ \log y_+ + y_- \log y_-) \quad , \text{ where } \quad y_\pm := \frac{1 \pm \epsilon/\mu_B B}{2}$$

For weakly-interacting spins it isn't that simple, but this is only a question of technical prowess. At strong coupling, on the other hand, thinking of the spins as the microscopic degrees of freedom may be altogether misleading.

Can one do a similar calculation for the black holes ?

Note that the semiclassical derivation gave already the (universal) equation of state of the BH, so this would be a consistency check of the quantum-gravity theory.

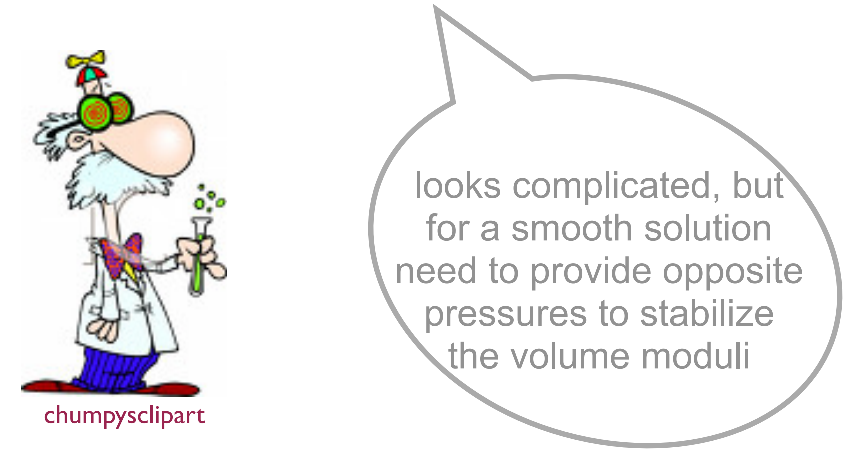
One may be also interested to know what are the degrees of freedom "that hide behind the horizon", and how to compute small-*M* corrections.

Now we have seen in lecture 1 that D-branes are string-theory solitons, whose low-E weak-coupling dynamics are given by an effective field theory of open strings. Computing the entropy in this regime should be straightforward. But are these solitons Black Holes?

Focus on a specific example, type-IIB theory compactified on $T^4 \times S^1$. *Strominger, Vafa '96*

Consider a configuration made of :

N_5	D5-branes wrapping	$T^4 \times S^1$
N_1	D-strings wrapping	S^1
N_p	units of KK momentum along	S^1



From a distance, this will look like a particle in 4+1 non-compact dimensions, carrying three different types of charge.

We want to compare with the corresponding BH solution of the effective supergravity theory in 5D. As in lecture 2, the normalization of the various charges is completely fixed by string theory.

The corresponding extremal solution is:

$$ds^2 = -f^{-2/3}dt^2 + f^{1/3}(dr^2 + r^2 d\Omega_3^2) \ , \quad \text{where} \quad f(r) = H_1(r)H_5(r)H_p(r)$$

with $\quad H_i = 1 + \frac{r_i^2}{r^2} \quad$ and

$$r_1^2 = \frac{(2\pi)^4 \alpha'^{\,3} g_s}{V_4} N_1$$
$$r_5^2 = g_s \alpha' N_5$$
$$r_p^2 = \frac{(2\pi)^6 \alpha'^{\,4} g_s^2}{V_4 R} N_p \ .$$

} charge normalization

radius of S1 and volume of T4

The horizon is at r=0 , and its area is $2\pi^2 r_1 r_5 r_p$.

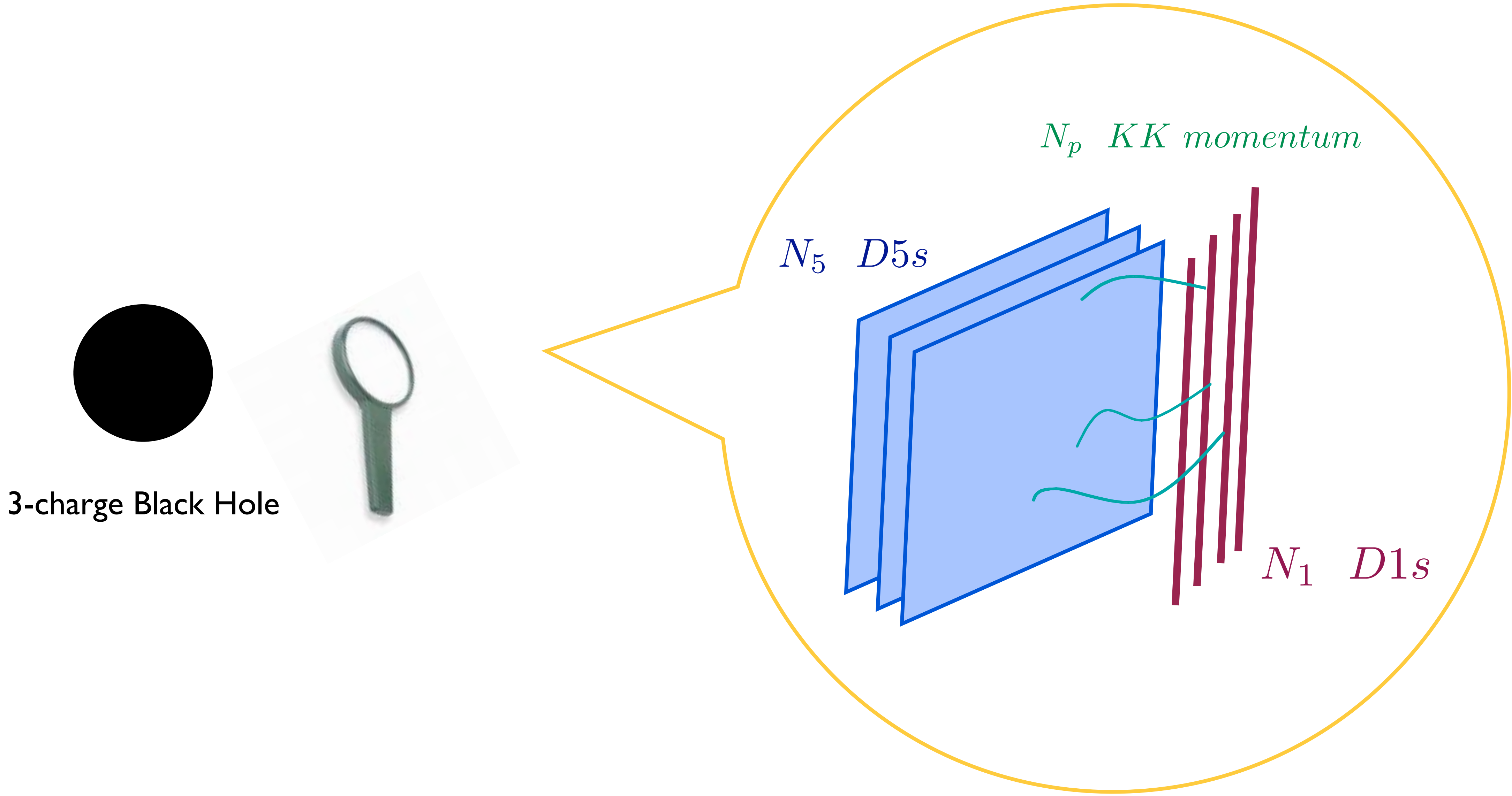
Using the value of the 5D Newton's constant, $\frac{1}{16\pi G_N} = \frac{RV_4}{(2\pi)^7 \alpha'^{\,4} g_s^2}$

our convention here for gs differs from that of lecture 2

leads to the BH entropy:

$$S_{BH} = \frac{A}{4G_N} = 2\pi\sqrt{N_1 N_5 N_p}$$

Can we find the same result from the corresponding microscopic description?



The effective low-E theory on the D-branes [neglecting string excitations and the KK modes on T4] is a $\frac{1}{2}N_{max}$ supersymmetric $U(N_1) \times U(N_5)$ gauge theory, with $N_1^2 + N_5^2 + N_1 N_5$ hypermultiplets. Its details are a little complicated, but it can be analyzed with standard techniques.

for a review, see e.g. David, Mandal, Wadia hep-th/0203048

What we need is to count the number of states with total momentum N_p and the lowest possible energy. This boils down to counting the # of ways of distributing the total momentum among the $N_1 N_5$ massless hypermultiplets, which is a simple combinatorial problem:

$$\left(\prod_{n=1}^{\infty} \frac{(1+q^n)}{(1-q^n)} \right)^{4N_1N_5} := \sum_{N_p} q^{N_p} e^{S(N_1N_5, N_p)} \qquad \Longrightarrow S \simeq 2\pi \sqrt{N_1 N_5 N_p} \ .$$

quantum-statistical generating function

for large N_p

The two results agree ! But how come, given that the two calculations have a priori very different regions of validity ?

The gravity approximation requires e.g. that $r_1, r_5, (V_4)^{1/4} \gg \sqrt{\alpha'}$, $G_N^{1/3}$, which imply $N_1 g_s$, $N_5 g_s \gg 1$, so that the D-brane theory is strongly-coupled.

The day is saved thanks to supersymmetry: what we are counting are the supersymmetric ground states in a given charge sector [1/8-BPS black holes] ; modulo a mild assumption, this number is a topological index which does not change as we vary the continuous parameters of the theory.

Computing protected quantities in two different ways is interesting [and checks the theory's consistency] but as such of limited scope. A more far-reaching story is, however, at work here: holographic duality or AdS/CFT correspondence.

This is easier to discuss in a simpler system, which will also bring us closer to particle theory: a stack of N non-compact D3-branes.

of dyonic nature, so the dilaton is stabilized and the solution smooth

Gravity solution:

$$ds^2 = H^{-1/2}(-dt^2 + d\vec{x}\cdot d\vec{x}) + H^{1/2}(dr^2 + r^2 d\Omega_5^2)$$

where $$H = 1 + \frac{L^4}{r^4}\ , \quad L^4 := 4\pi g_s \alpha'^{\,2} N\ .$$

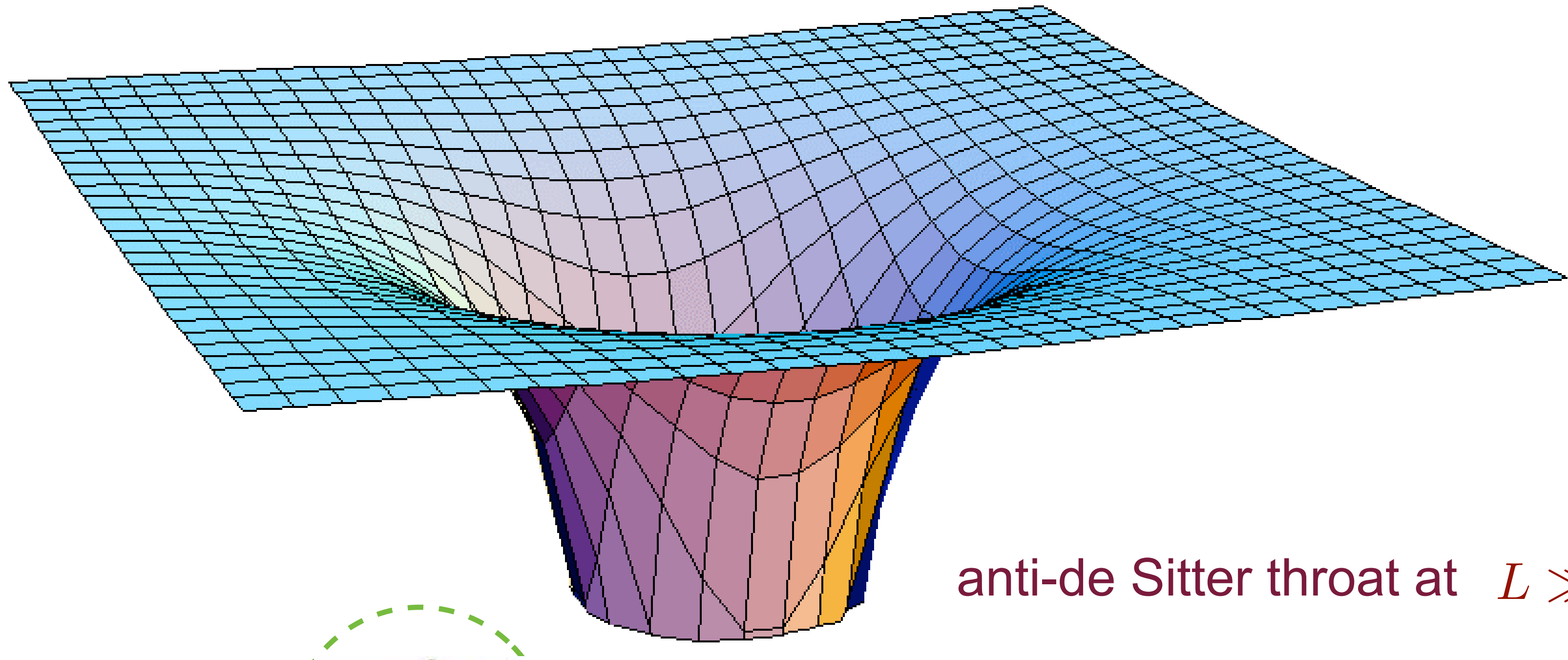
to be trusted if $L \gg \sqrt{\alpha'}\ ,\ G_N^{1/8}$ or equivalently:

$$4\pi g_s N \gg 1\ , \quad \text{and} \quad N \gg 1\ .$$

the D3-brane geometry:

(drawing from talk by T. Klose)

asymptotically-flat region



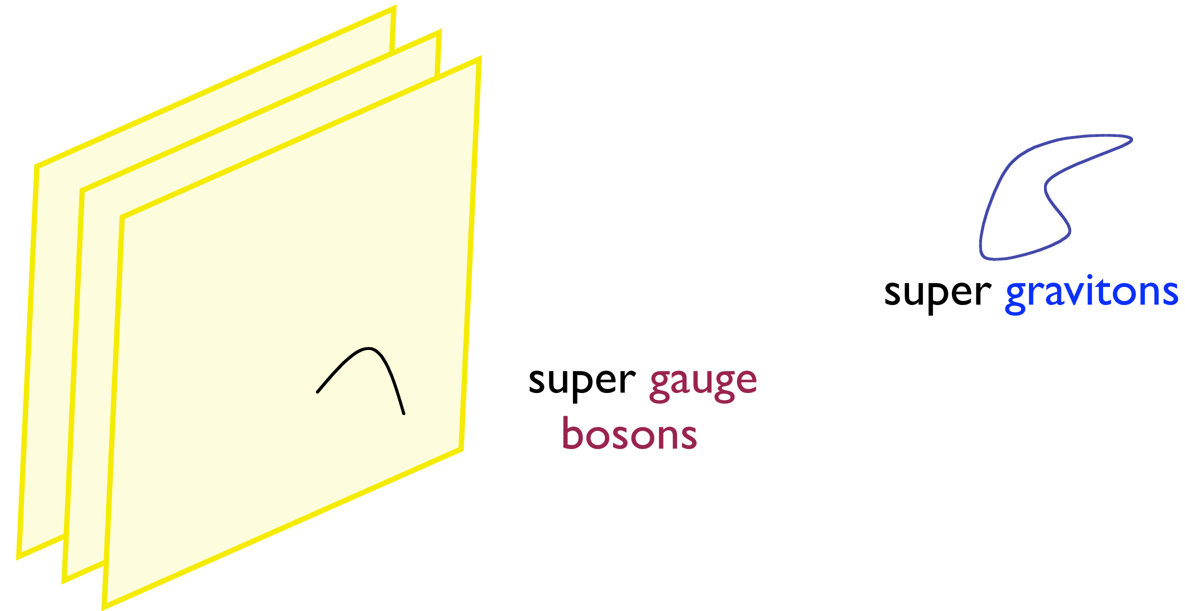
anti-de Sitter throat at $L \gg r := \frac{L^2}{u}$

$$ds^2 \simeq L^2 \underbrace{\frac{dx_\mu dx^\mu + du^2}{u^2}}_{AdS_5} + \underbrace{L^2\, d\Omega_5^2}_{\times S_5}$$

low-E excitations: (1) <u>all</u> possible excitations in the throat [very large redshift] , plus

(2) decoupled long-wavelength (super-)gravitons in the bulk .

the same configuration at
weak coupling and low E:



effective theory of D3-branes (no string effects else no decoupling)

(1) $\mathcal{N}=4$ super Yang-Mills theory in 4D , plus
(2) decoupled low-E (super-)gravitons in the bulk .

Since $A+B=A+C \implies B=C$, the natural conjecture is:

$\mathcal{N}=4$ super Yang-Mills in 4D = type IIB string theory in $AdS_5 \times S^5$

Maldacena '97
Gubser, Klebanov, Polyakov '98
Witten '98

$\mathcal{N} = 4$ super Yang-Mills is a very special theory:

$$S_{\mathcal{N}=4} = \frac{1}{g^2_{YM}} \int d^4x \, \mathrm{tr} \left\{ \frac{1}{4} F^2 + \frac{1}{2} (D\Phi)^2 - \frac{1}{4} [\Phi, \Phi]^2 + \Psi D^\mu \gamma_\mu \Psi - \frac{1}{2} \Psi [\Phi^a \Gamma_a, \Psi] \right\}$$

reduction of 10D super YM

6 scalars

4 gauginos

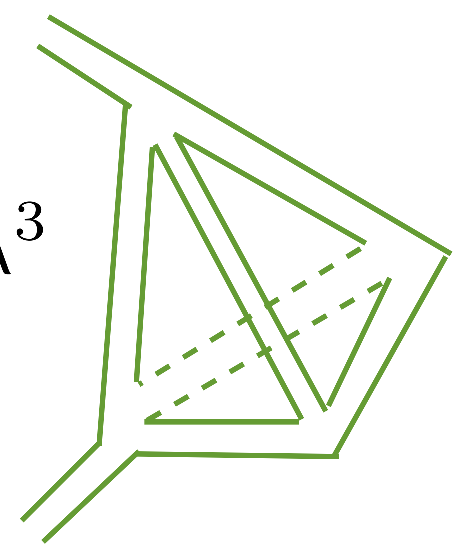
It has vanishing β- function and no confinement: it is a conformal theory for all g_{YM}

As for any theory with matrix-like interactions, the Feynman diagrams can be organized in a [double-line] topological expansion:

λ^4

disk

$N^{-2} \lambda^3$



disk with handle

$\lambda := N g^2_{YM}$ 't Hooft coupling

$N =$ # of colors

't Hooft '74

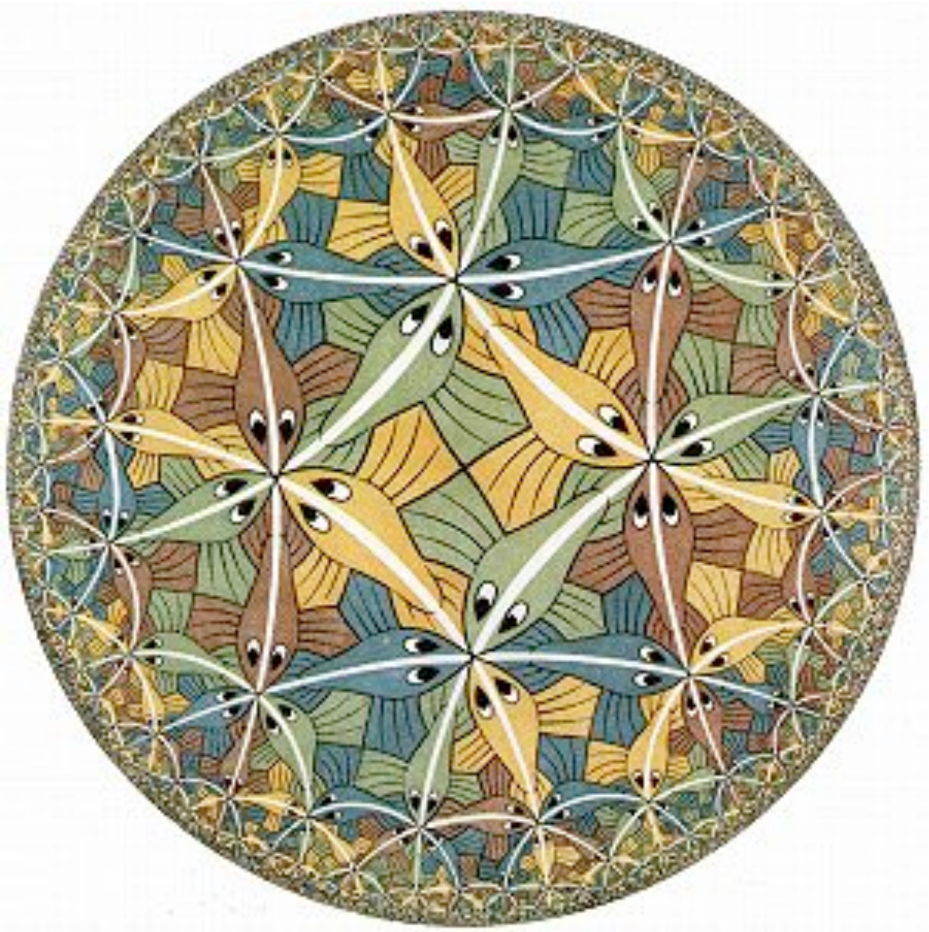
Theorists have long suspected that the large-N limit of gauge theory is a weakly-coupled string theory. AdS/CFT has made this hypothesis very sharp.
Here is a [partial] holographic dictionary:

$\mathcal{N}=4$ super Yang-Mills	IIB strings in $AdS_5 \times S^5$.
't Hooft coupling λ	radius in string units $L^4 M_s^4$
# of colors N	radius in Planck units $\sim L^4 M_{Pl}^4$
single-trace (gauge-invariant) operators	*single*-string states
dilatation operator (scaling dimensions)	Hamiltonian in global time (energies)
RG flow under relevant perturbations	[domain-wall] extrapolating solutions
heavy external quark	string stretching to the boundary

strong/weak-coupling duality

planar limit = free strings

see next page



M.C. Escher, *Circle Limit III*, 1959.
strictly-speaking this is EAdS2

[Global] AdS is a funny space time

Because of the infinite blue-shift it acts like a trap: this is why the energy [scaling dimension] spectrum is discrete

It is also unusually stable: the volume of a large ball grows like its surface area. Thus bubbles of “true-vacuum” don’t always nucleate.

Moving towards the interior is like changing the energy scale: there are no new degrees of freedom, all information is captured (holographically) by those degrees of freedom living in the UV.

Fascinating! But what is AdS/CFT really good for ?

Solve the strong-coupling, large-N limit of certain gauge theories;

Combine with integrability to solve the large-N limit $\forall\ \lambda$!

for field theorists this would be a major achievement

... but QCD has $N=3$, six flavors of quarks, and (most importantly) both a strong- and a weak-coupling energy regime ...

infrared slavery

asymptotic freedom

Lattice gauge theory of little use for out-of-equilibrium and real-time

... still a long way to go, but ....

.... in some situations, e.g. for the quark-gluon plasma, even a qualitative model could be of much help

Finally, don't forget where this came from:

much progress though last word has not been said

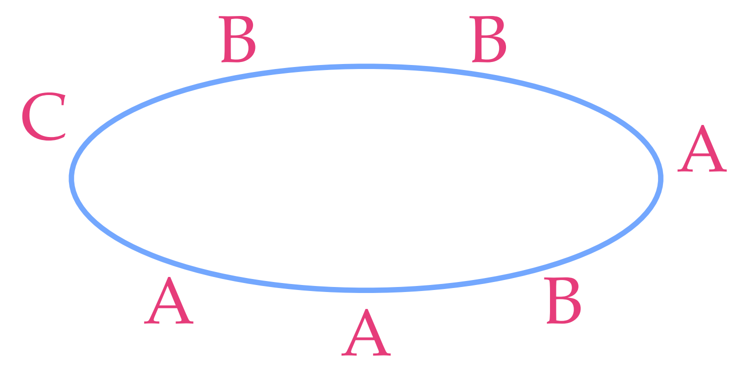
Solve the (apparent?) paradoxes of BH evaporation; understand the physics of the Big-Bang (initial singularity).

????

A word on integrability:

single-trace, gauge-invariant operators ⇨ spin chains

words with letters in $\{\Phi, \Psi, \cdots\}$



The Hamiltonian is (almost surely) integrable, but the range of the interaction grows with the power of λ .

Lipatov '00 ;
Minahan, Zarembo '02
Bena, Roiban, Polchinski '02
....

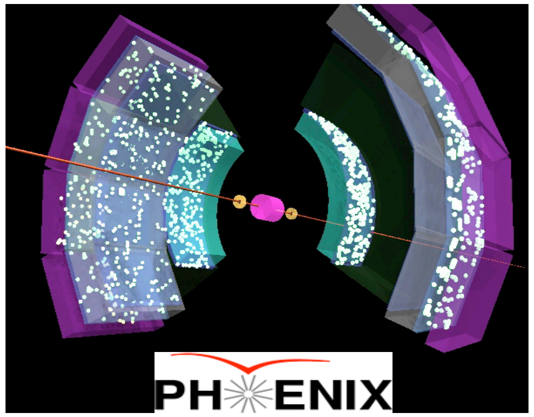
For infinite chains, spectrum determined by symmetry and a S-matrix phase;
A closed-form bootstrap equation for this phase is known, so anomalous dimensions of long operators computable in principle $\forall\lambda$.

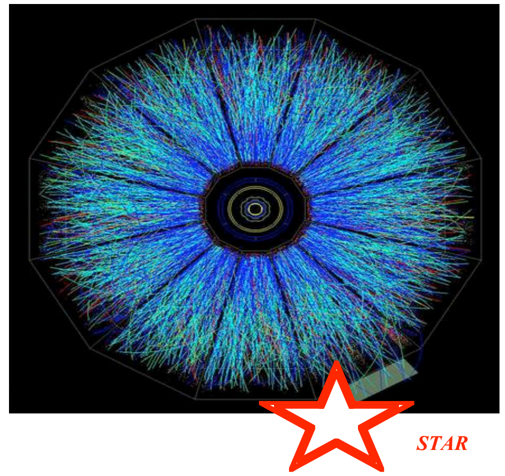
cusp anomalous dimension, $f(\lambda) = \dfrac{\Delta - S}{\log S}$ for large S

Beisert, Eden, Staudacher '06

Impressive agreement with perturbative calculations; story progresses fast.

A word on the quark-gluon plasma:





RHIC: Gold + Gold

$\sqrt{s} \simeq 200\ GeV/(\text{nucleon pair})$

E-density $\simeq 5\ GeV/fm^3$;

$T \simeq 300\ MeV$; $\tau \simeq 10^{-23} sec$

too short-lived to use external probes ! study indirectly

Data (pattern of elliptic particle flow; jet quenching) suggests a droplet of low-viscosity ideal fluid !

unexpectedly large: $\sim$ 20% of high-E jets as compared to Deuteron + Gold

AdS/CFT gives better fit to data than perturbative QCD, e.g.

	(expmt)	(pert)	(AdS/CFT)
shear viscosity/entropy density	0.1 - 0.2	≥ 1	$1/4\pi \simeq 0.08$
jet quenching parameter in GeV^2/fm	5 - 15	$\simeq 1$	$\simeq 4.5$

damping of thermal horizon fluctuations

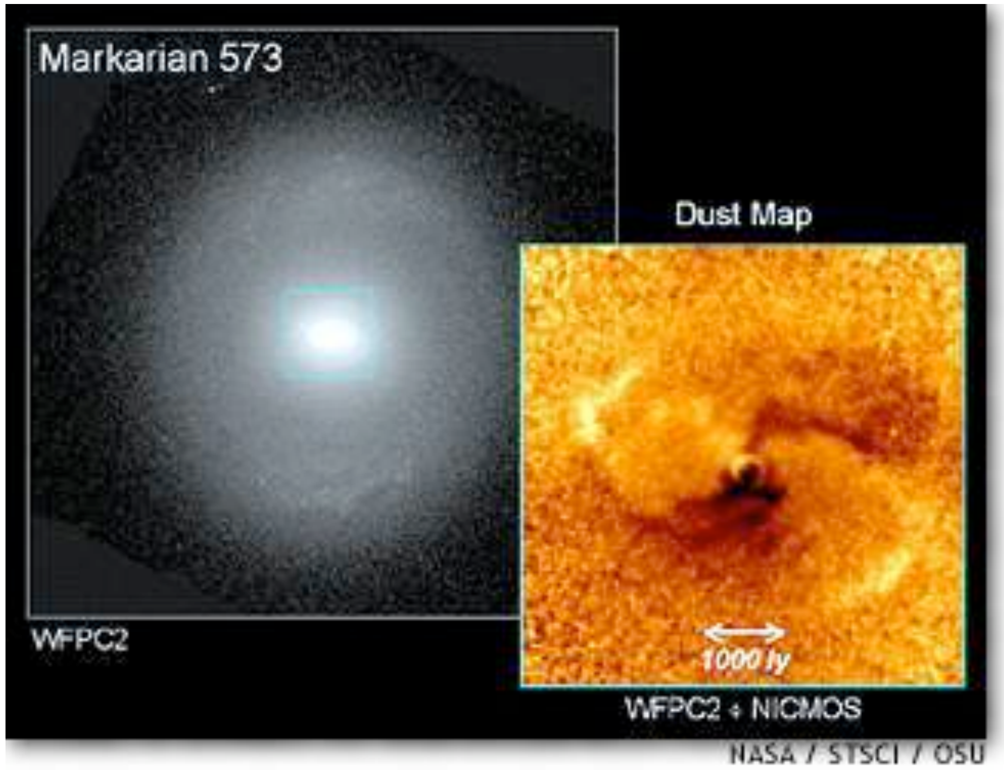
string diffusion + drag

Policastro, Son, Starinets '01 ; ....

for reviews and refereneces see: *Shuryak, arXiv:0807.3033*

Wiedemann, Nucl.Phys.A805:274-282,2008

The End



The orangish picture focuses on the very center of the active galaxy Markarian 573.
The image, which combines readings taken in visible and near-infrared light, traces a spiral of galactic dust with what scientists believe is a supermassive black hole at its center.

... and for fun:
(reconstructed) throat of an
astrophysical Black Hole