String Theory for Pedestrians

CERN academic training
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C. Bachas - ENS, Paris

Lecture 2 : String Unification of particles and
of their fundamental interactions

The most compelling hypothesis for physics beyond the Standard Model is arguably that of Supersymmetric Grand Unification. Let's start with a brief review of why this is so. For [*guides to more*] references see for instance:

S. Raby, in Particle Data Group report, and hep-ph/0608183 .

[also textbooks by G. Ross, and by H. Georgi ; J. Ellis in "Les Houches" 1981; S. Dimopoulos, S. Raby, F. Wilczek, Physics Today - October 1991.]

$\Rightarrow$ The SM particles fit nicely in mutliplets of SU(5), SO(10) or E6 ; in particular electric charge is quantized .

This is automatic for the spin-1 gauge bosons $(gluons, W^{\pm}, Z, \gamma)$ but not for the families of quarks and leptons. Most convincingly, their hyper-charge assignments fit like a glove!

A basis for the spinor of SO(2n) is given by a choice $\gamma^j\gamma^{j+1} = \pm i.$

One complete family of quarks and leptons fits in a single 16-dimensional spinor representation of

$$SO(10) \supset SU(5) \times U(1)_X$$

$$\supset \underbrace{SU(3)_c \times SU(2)_w \times U(1)_Y}_{\textit{Standard Model}} \times U(1)_X$$

Here *SU(5)* is the minimal GUT group, and the extra generator *X* is related to baryon and lepton number,

$$X = \frac{1}{2}\sum(signs) = Y - \frac{5}{2}(B-L)$$

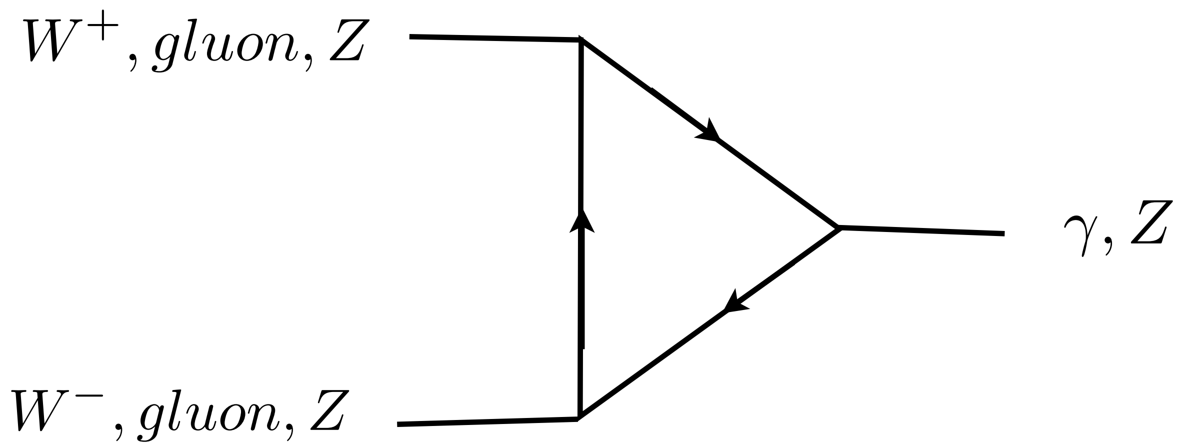
(normalized so that $\mathrm{tr}_{\mathrm{F}} X^2 = 10$.)

u	+	-	-	+	-
u	-	+	-	+	-
u	-	-	+	+	-
d	+	-	-	-	+
d	-	+	-	-	+
d	-	-	+	-	+
u^c	-	+	+	-	-
u^c	+	-	+	-	-
u^c	+	+	-	-	-
d^c	-	+	+	+	+
d^c	+	-	+	+	+
d^c	+	+	-	+	+
ν	+	+	+	+	-
e	+	+	+	-	+
e^c	-	-	-	+	+
N	-	-	-	-	-
	$\underbrace{}_{SU(3)_c}$			$\underbrace{}_{SU(2)_w}$	

This "success" must be somewhat moderated, because hyper-charge assignments are strongly constrained by the requirement of anomaly cancellation.

The decoupling of the [spurious] longitudinal gauge-boson polarizations in the triangle diagram implies:

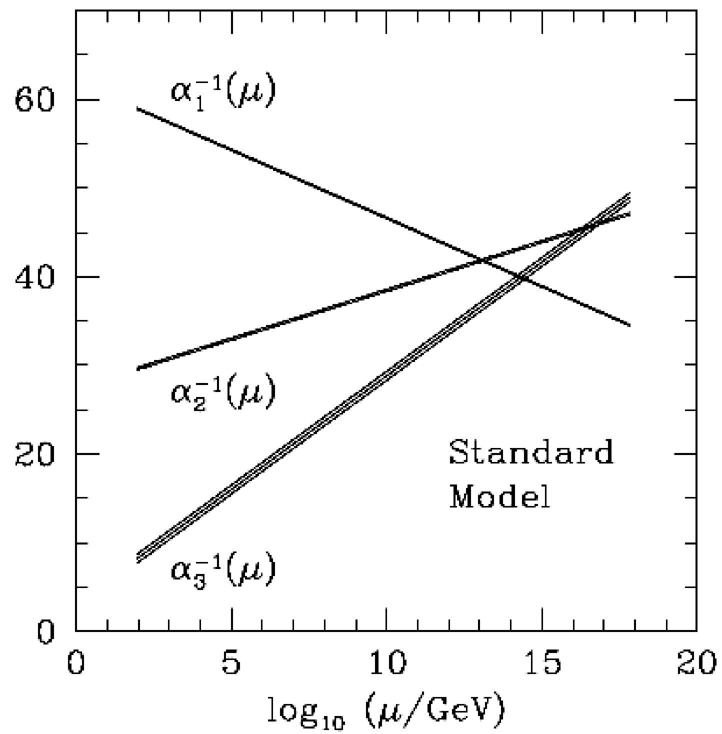
$$\sum Y^3 = \sum_{doublets} Y = \sum_{triplets} Y = 0$$

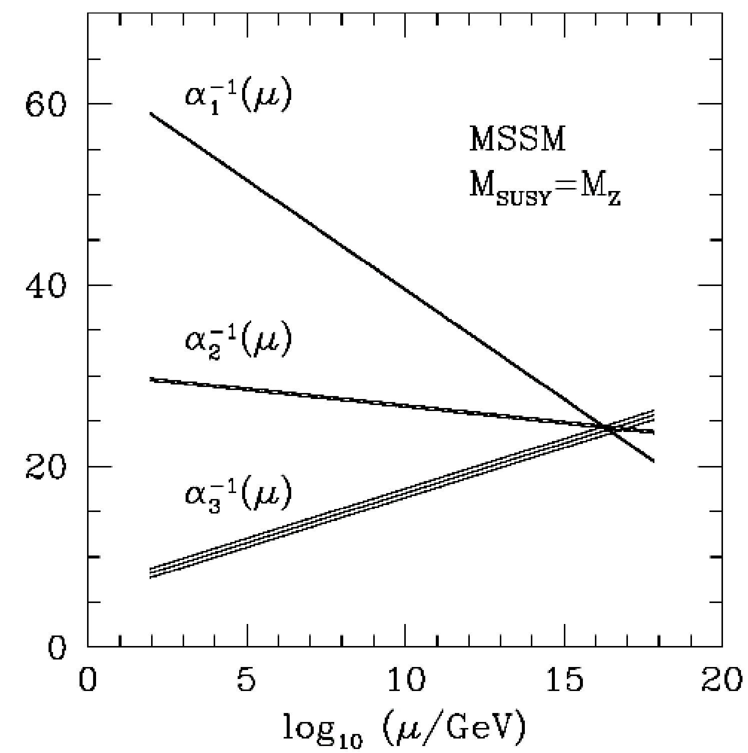


Not an option: gauge anomalies imply that the theory is sick.

Even if family replication is taken for granted, there are 3 equations for 4 ratios of hyper-charges: the fourth ratio did not have to fit.

Gauge coupling unification: assuming the MSSM particle content, the extrapolated gauge couplings all meet.





You have surely seen these famous plots: the one-loop evolution equations for the three gauge couplings reads

$$\alpha_i^{-1}(M_Z) = \alpha_{\mathrm{GUT}}^{-1}(M_U) \ + \ \frac{b_i}{2\pi}\log(M_Z/M_U) + \delta_i$$

Assuming unification

threshold corrections $\mathcal{O}(1)$

$$\text{where} \quad (b_1, b_2, b_3) = \begin{cases} (\frac{41}{10}, -\frac{19}{6}, -7) & \text{SM} \\ (\frac{33}{5}, 1, -3) & \text{MSSM} \end{cases}$$

Two input parameters for 3 couplings can "predict" the value of α_3 .

The experimental values at the Z mass are:

$$\begin{aligned} \alpha_3 &= 0.1187 \pm 0.0020 \\ \alpha_{EM}^{-1} &= 127.906 \pm 0.019 \\ \sin^2\theta_W &= 0.2312 \pm 0.0002 \end{aligned}$$

where $\alpha_1 = \frac{5}{3}\alpha_Y = \frac{5}{3}\frac{\alpha_{EM}}{\cos^2\theta_W}$, and $\alpha_2 = \frac{\alpha_{EM}}{\sin^2\theta_W}$.

The fit is excellent for the MSSM, with $\alpha_{GUT}^{-1}(M_U) \simeq 25 \ \text{and} \ M_U \simeq 3 \times 10^{16} GeV$.

The largest uncertainty comes from the model-dependent threshold corrections, which depend on heavy particles near the unification scale.

The value of M_U in the susy case ensures that the half-time of the universal, gauge-boson-mediated, proton-decay mode $p \to e^+\pi^0$ is $\tau_p \simeq 10^{34-38}$ years, beyond [but close to] the current super-Kamiokande limit: $\tau_p > 5 \times 10^{33}$ years.

GUTs predict correctly the order of magnitude of neutrino masses.

Majorana neutrino masses correspond to dimension-5 operators in the SM :

$$(h^{ij})(\phi^\dagger L_i)^T \underbrace{C^{-1}}_{\text{charge conjugation}} (\phi^\dagger L_j) \;\rightarrow\; h^{ij} \underbrace{v^2}_{\text{Higgs vev}} \nu_i^T C^{-1} \nu_j \;.$$

To fit the available data one needs $(h^{ij})^{-1} \sim 10^{13-15} GeV$

This can arise naturally in the SO(10) GUT through the seesaw mechanism:

The right-handed singlets N_i can have Majorana masses at the GUT scale, and standard Yukawa couplings λ_{ij} to the ν_j .

The mass matrix for one generation, $\begin{pmatrix} 0 & \lambda v \\ \lambda v & M_N \end{pmatrix}$

has a small eigenvalue $\simeq \dfrac{(\lambda v)^2}{M_N}$ which is in the right ball-park.

For a review and references see R.N. Mohapatra et al, hep-ph/0510213 .

Many of the detailed predictions of GUTs depend on the little-known details of the symmetry-breaking sector: what scalar-field representations, with what potential and what Yukawa couplings? Many specific models can be excluded because of wrong mass relations, or too fast proton decay.

e.g. via color-triplet higgsino exchange in sGUTS, leading to

$p \rightarrow K^+ + \bar{\nu}_\tau$

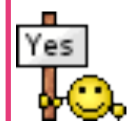
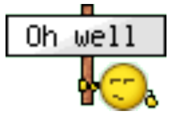
Besides triangle-anomaly cancellation [automatic in the case of *SO(10)*] there is no theory principle to guide us through this unknown territory. Furthermore:

the hierarchy $M_Z \ll M_U$ is stable but not explained, and gravity is not part of the game.

Nevertheless, supersymmetric GUTs capture many non-trivial features of our low-energy world [and have also nice implications for cosmology: baryogenesis and dark matter].

They are thus the theorists' best current bet for physics bSM.

How has string theory changed the story?

Included quantum gravity: it can be done ! 	and coupling unification falls nicely in place .... 
Modified nature of gauge-hierarchy problem: (dynamical) vacuum stability 	... but did not solve it 
Changed the model-building rules: extra dimensions, not <u>any</u> representations, branes and fluxes 	helps a little, but has not narrowed down the possibilities 
Numerous “experimental” consequences: susy, proton decay, gravity modifications, cosmic strings, KK and Regge states, axions .... 	at what scale? any smoking gun? 

$\exists$ two classes of weakly-coupled semi-realistic string vacua :

Heterotic and type I [or type II orientifolds]

They have different properties, so I will discuss them in turn.

Heterotic strings are closed and oriented: they have the degrees of freedom of the bosonic/supersymmetric string in the right/left - moving sector :

$$X^{\mu=0,\cdots 9} \quad \text{and} \quad \left[\begin{array}{ll} \psi^{\mu=0,\cdots 9} & \text{right} \\ \tilde{\eta}^{a=1,\cdots 32} & \text{left} \end{array} \right.$$

fermionized left current algebra

This is consistent, because left- and right-moving excitations do not talk.

Gross, Harvey, Martinec, Rohm '84

We can construct the spectrum as in lecture 1. The low-lying states are:

tachyon

$$\cancel{\tilde{\eta}^a_{-1/2}}|0\rangle_{NS}$$

massless

$$\boxed{\begin{array}{c} \psi^\mu_{-\frac{1}{2}}\,\tilde{a}^\nu_{-1}|0\rangle_{NS} \\ \cancel{\tilde{a}^\nu_{-1}|0,-\rangle_R} \\ \tilde{a}^\nu_{-1}|0,+\rangle_R \end{array}} \qquad \boxed{\begin{array}{c} \psi^\mu_{-\frac{1}{2}}\,\tilde{\eta}^a_{-\frac{1}{2}}\,\tilde{\eta}^b_{-\frac{1}{2}}|0\rangle_{NS} \\ \cancel{\tilde{\eta}^a_{-\frac{1}{2}}\,\tilde{\eta}^b_{-\frac{1}{2}}|0,-\rangle_R} \\ \tilde{\eta}^a_{-\frac{1}{2}}\,\tilde{\eta}^b_{-\frac{1}{2}}|0,+\rangle_R \end{array}}$$

wrong GSO

graviton +
susy company

SO(32) gauge field
+ susy company

The effective D=10 theory of these massless modes is N=1 supergravity coupled to N=1 super-Yang-Mills with gauge group SO(32) or E8 x E8.

requires separate
fermion parities
for left-movers

These are the two non-anomalous possibilities in ten dimensions.

Green, Schwarz '84

The effective Lagrangian in 10d is $$\int \mathcal{L}_{(10)} = \frac{M_s^8}{g_s^2}\int \mathcal{R}_{(10)} + \frac{M_s^6}{8g_s^2}\int \mathrm{tr}_v\, F^2 + \cdots$$

where g_s is the (appropriately-normalized) string-coupling constant, and $M_s \equiv \frac{1}{\sqrt{\alpha'}}$ the string scale.

the first string excitation has mass = 2Ms

After compactification to 4d one finds:

$$\int \mathcal{L}_{(4)} = \underbrace{\frac{M_s^8 V^{(6)}}{g_s^2}}_{=\frac{1}{2\kappa^2}} \int \mathcal{R}_{(4)} + \underbrace{\frac{k M_s^6 V^{(6)}}{8g_s^2}}_{=\frac{1}{2g_{YM}^2}} \int \mathrm{tr}_v\, F^2 + \cdots$$

where $V^{(6)}$ is the volume, and k (the integer) embedding level.

This implies the universal relation: $$\boxed{\kappa^{-2} = \frac{M_s^2}{\alpha_{YM}}\left(\frac{2}{k\pi}\right)}$$

Notice that the compactification volume drops out, because gauge bosons and gravitons both live in 10 space-time dimensions.

This universal relation stays valid even when the string and compactification scales are comparable (so that a two-stage reduction is not justified).

Ginsparg '87

A minimal hypothesis is that $M_U \sim M_s$ i.e. that the string and unification scales coincide. If so, one can use the SM data to compute the Planck length:

$$\kappa^{-1} \simeq 10^{17} GeV$$ theory

to compare with $$\kappa^{-1} \simeq 2.4 \times 10^{18} GeV$$ experiment

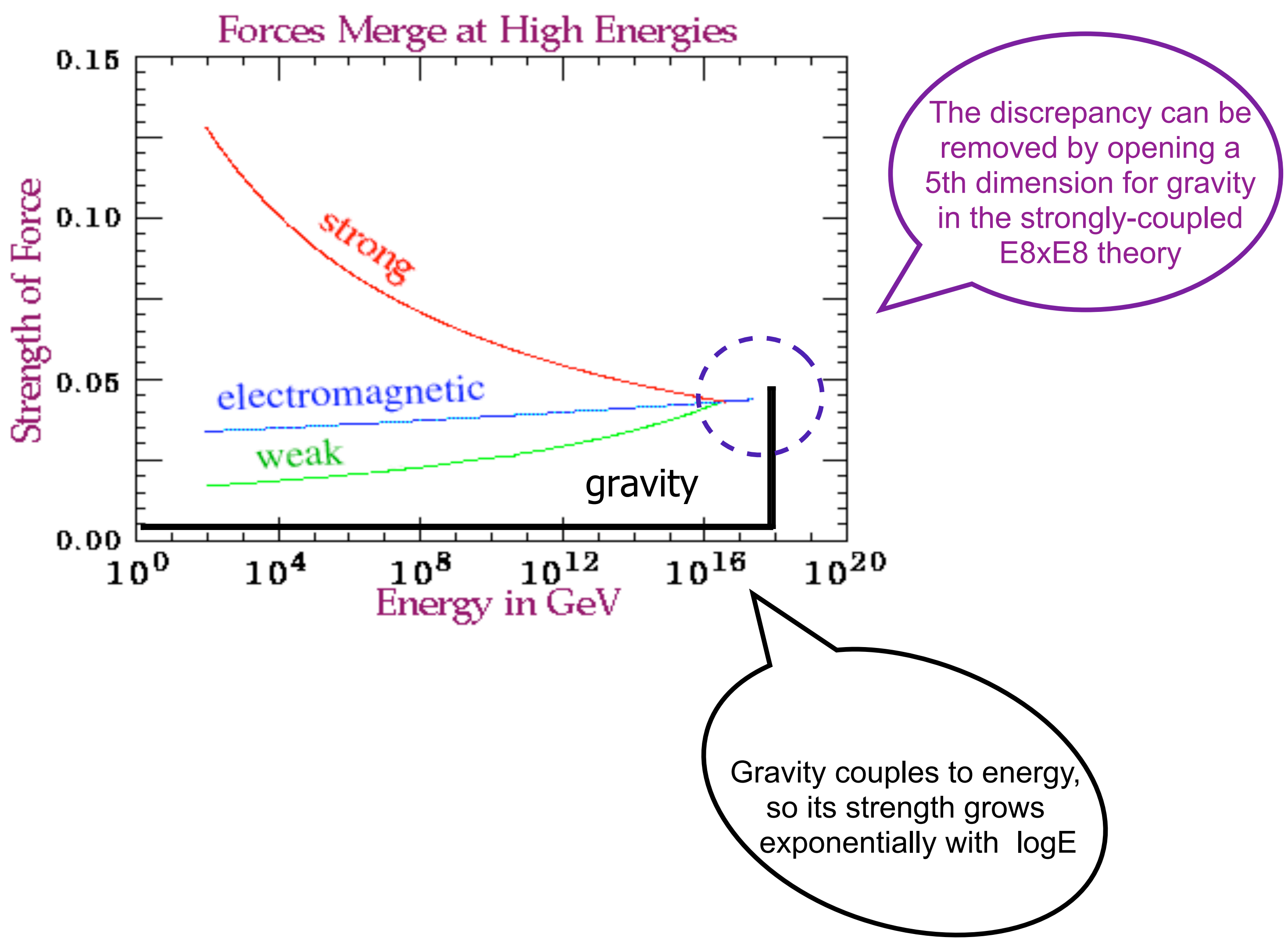
Although the agreement is not perfect, the error is only few percent on a logarithmic scale.

Since SM data "need not have known" about Newton's constant, this is a successful prediction of the minimal heterotic unification.

NB: The relation between κ^{-1} and M_s is classical, and the above discrepancy could be conceivably removed by threshold corrections. These were computed in various models but don't seem to help. Proposed modifications of the minimal scenario involve small scale hierarchies, or extra matter. Many are reasonable but none is compelling.

see e.g. K. Dienes, hep-th/9602045

<u>Minimal Heterotic Unification</u>: 2 input parameters for 4 coupling constants



Witten '96

<u>NB</u>: Furthermore, because the GUT breaking can have higher-dimensional origin, problematic features of GUTs (e.g. doublet-triplet splitting) can be improved.

Orientifold models have both closed and open superstrings. They are obtained from the type II theories in two steps:

(1) Keep only states invariant under orientation reversal & a space reflection

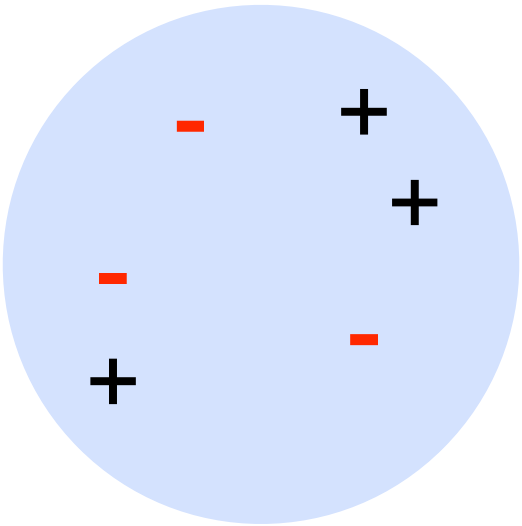
$$\boxed{(X^1, \cdots X^p, X^{p+1} \cdots X^9) \to (X^1, \cdots X^p, -X^{p+1} \cdots - X^9) \qquad \mathcal{O}p \quad \text{orientifold}}$$

Sagnotti '87

(2) Introduce, if necessary, Dp-branes in order to cancel RR charge.

see lecture 1

By Poincaré invariance the D-branes and orientifold fixed loci must fill the three non-compact space dimensions; in the compact 6d space we should then have as many sources as sinks of RR flux. Failure to ensure this leads to effective field theories with anomalies.



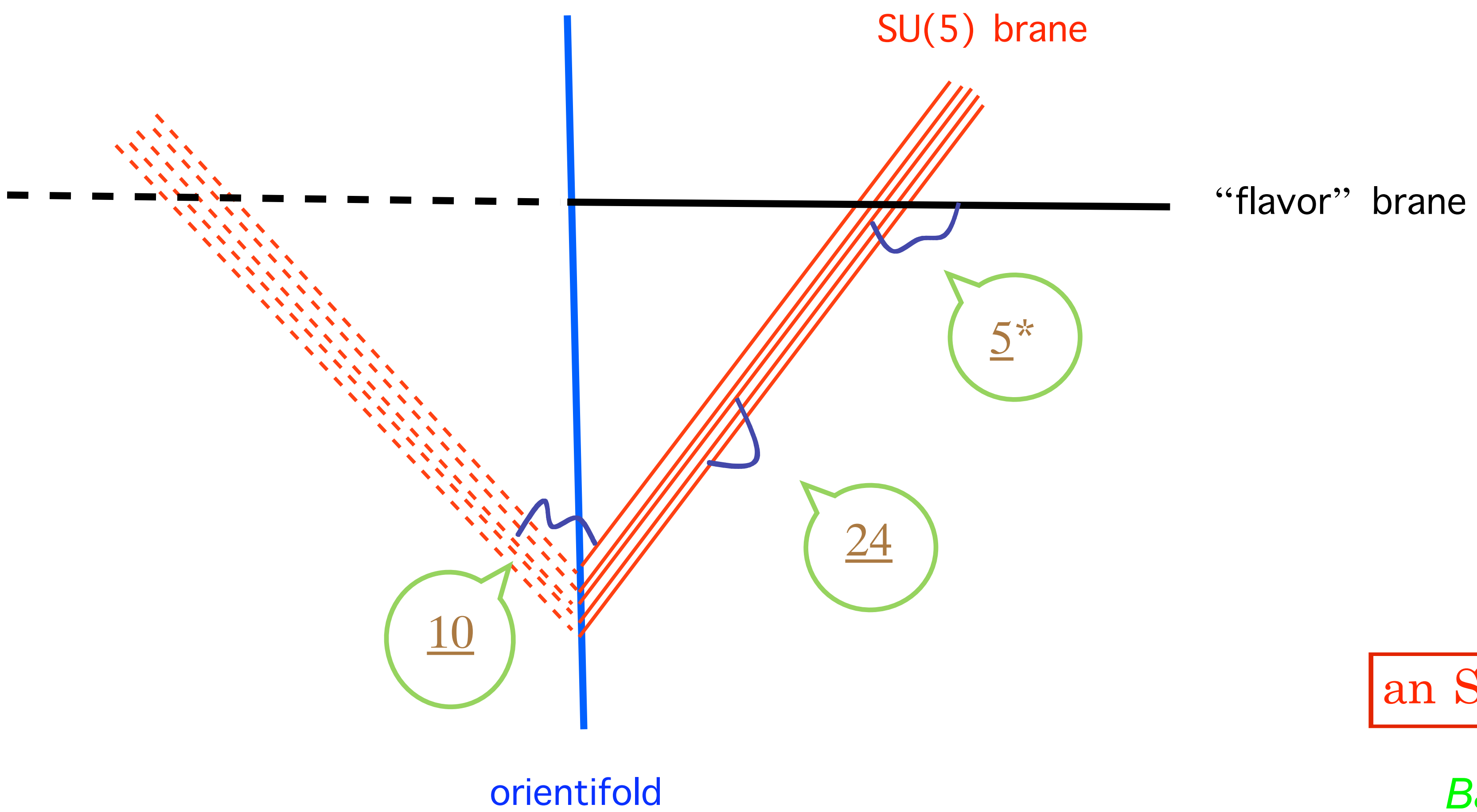
e.g. point-particles on a 3-sphere: the total charge must be zero, since electric-flux lines have nowhere to escape to.

A class of such string vacua is known as intersecting (or magnetized) D-brane models.

T-dual

(Unbroken) gauge theories live on identical, coincident D-branes, while chiral matter resides on intersections with other D-branes, or with their mirror images.

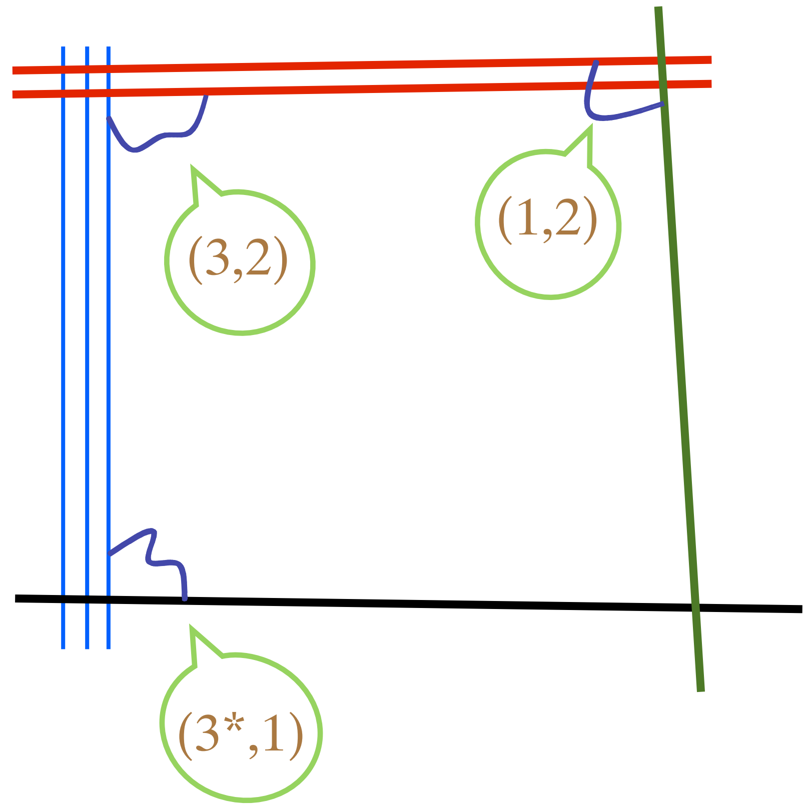
Two examples:



an SU(5)-GUT model

Bachas '95

Blumenhagen, Kors, Lüst, Ott '01



intersecting D6-brane (S)SM

Ibanez, Marchesano, Rabadan '01
Cvetic, Shiu, Uranga '01 *(susy)*

The number of (chiral) families is given by the D-brane intersection number.

A generic feature are extra U(1)s , some of which obtain a mass through anomalous couplings to axion fields.

$$A_\mu \partial^\mu a + aF \wedge F$$

Such constructions and their detailed properties have been studied extensively in recent years. For a recent review, consult for instance:

Blumenhagen, Cvetic, Langacker, Shiu, hep-th/0502005,

and references therein.

The effective action in 4 dimensions reads:

volume of wrapped cycle

$$\int \mathcal{L}_{(4)} = \underbrace{\frac{M_s^8 V^{(6)}}{g_s^2}}_{=\,\frac{1}{2\kappa^2}} \int \mathcal{R}_{(4)} + \sum_a \underbrace{\frac{M_s^{p-3} V^{(p-3)}}{\mathcal{N}_p \, g_s}}_{=\,\frac{1}{2g_{YM}^2}} \int \mathrm{tr}_v \, F_a^2 + \cdots$$

numerical factor

Contrary to the heterotic string, gauge couplings need not unify in orientifold models, and there is no universal relation tying the string scale to the Planck scale.

This latter depends, in particular, very sensitively on the volume of the compact space transverse to the “Standard Model brane”,

$$V_\perp \equiv \frac{V^{(6)}}{V^{(p-3)}} \; .$$

Orientifold vacua are thus less restrictive (or less predictive) than those of the weakly-coupled heterotic string; they allow for some “exotic” possibilities:

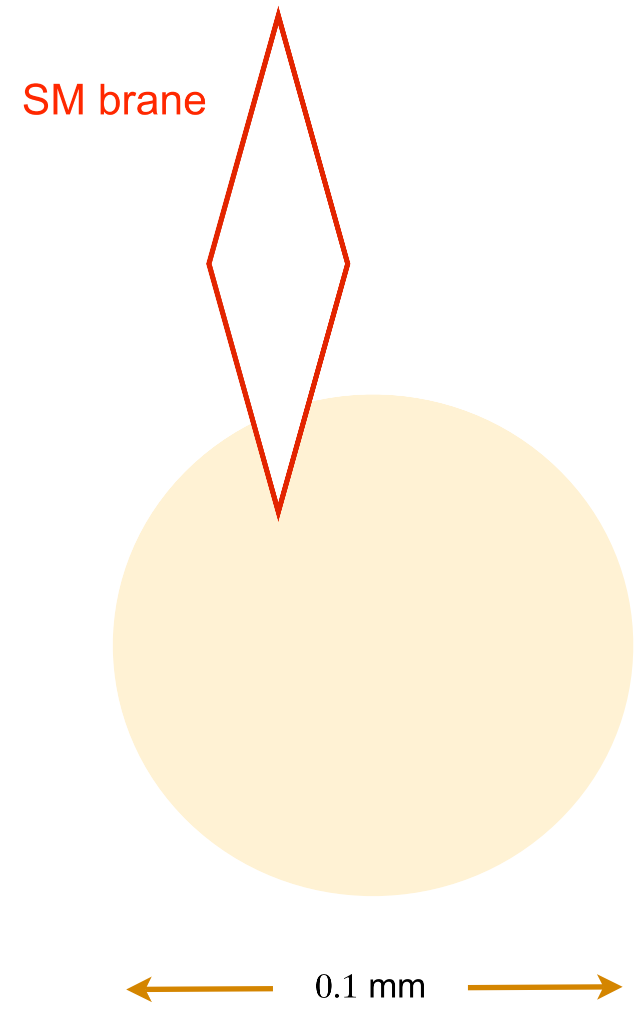
If, for example, the SM "lives" on a 7-brane, one finds $M_{\text{Planck}} \sim M_s \sqrt{\frac{V_\perp M_s^2}{g_s}}$

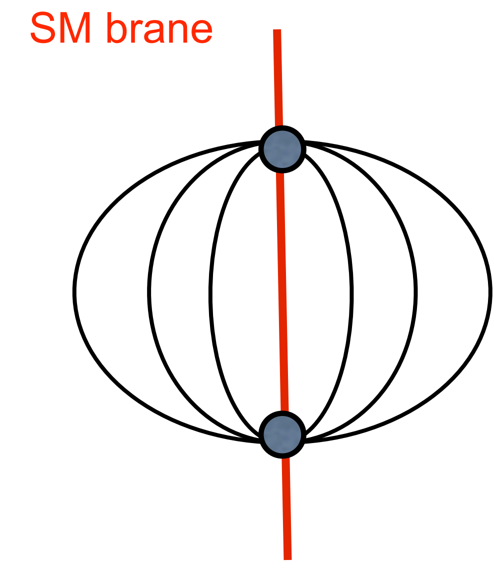
An extreme possibility is then $g_s \sim \mathcal{O}(1)$,

$$V_\perp \sim (100\,\mu m)^2 \;, \quad \text{and} \;\; M_s \sim \mathcal{O}(TeV) \;.$$

The weakness of gravity in this model is due to the spreading of flux in the extra dimensions.

Arkani-Hamed, Dimopoulos, Dvali '98
Antoniadis + ADD '98





The problem of the gauge hierarchy is here recast [but not solved] as the question:
Why is the volume of the transverse dimensions so large ?

This brings us to the more general question, common to both the heterotic and the type I strings, the question of vacuum selection and stability.

The shape and size of the compact manifold may vary from place to place in the 4D world. In the effective 4D theory some of these characteristics are described by light scalar fields. Those that can be varied continuously in the vacuum [such as the radii and angles of tori] have vanishing potential; they are the compactification "moduli."

Consider, as an example, a 6D theory compactified to 4D on a constant-curvature space,

$$ds^2_{(6)} = e^{\phi/2} ds^2_{(4)} + e^{-\phi/2} \hat{g}_{mn} dz^m dz^n \ .$$

volume field

The effective action [in Einstein frame] reads:

$$\frac{1}{2\kappa^2} \int [\mathcal{R}_{(4)} - \frac{1}{4}(\partial\phi)^2 + k e^{\phi}] + \int e^{a\phi} \mathcal{L}_{\text{matter}}$$

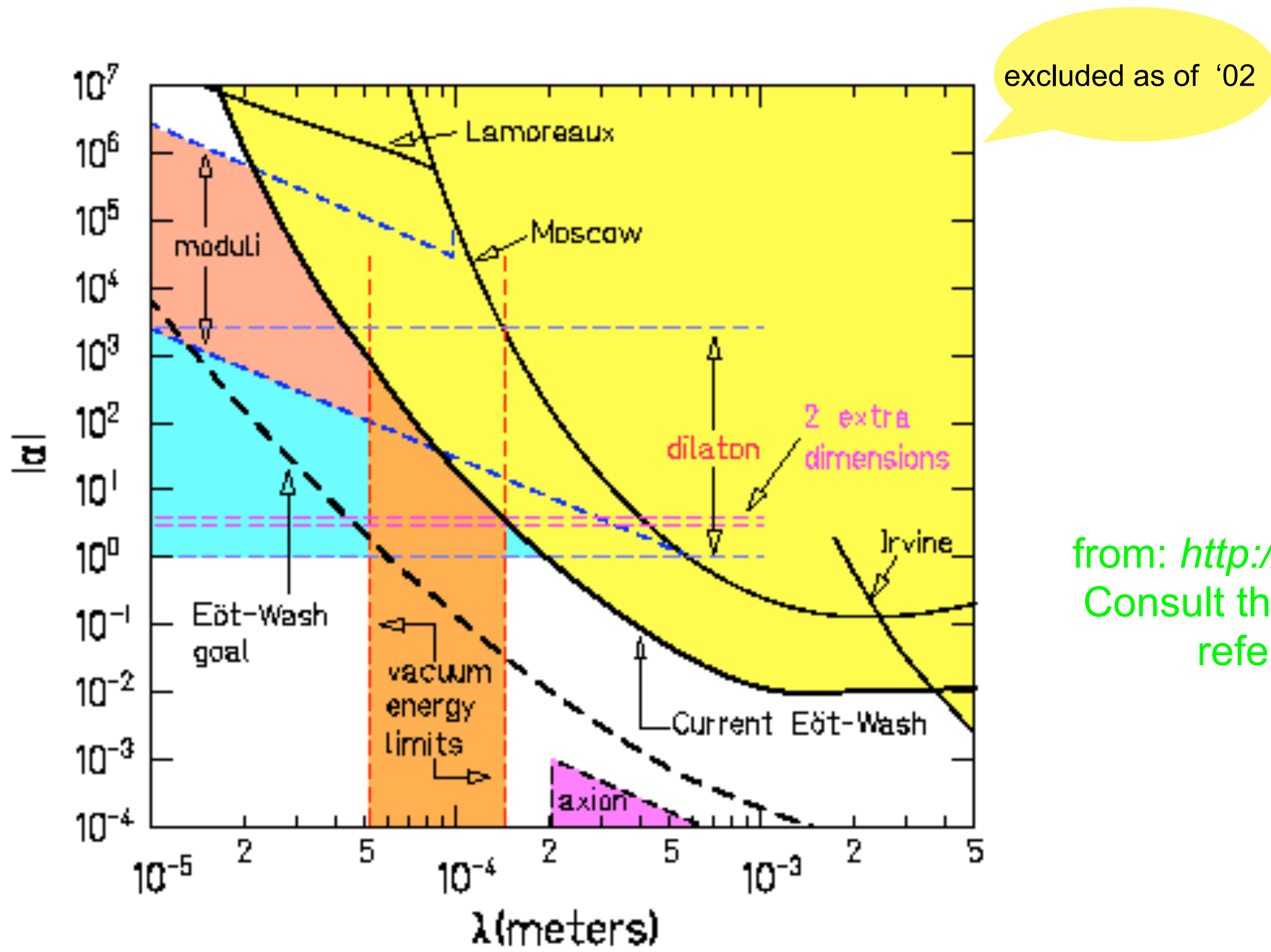
1, 0 or -1

the power depends on the type of matter

The volume field has flat potential for *k=0*, and couples to matter with gravitational strength.

Such massless fields are ruled out by short-range gravity experiments:

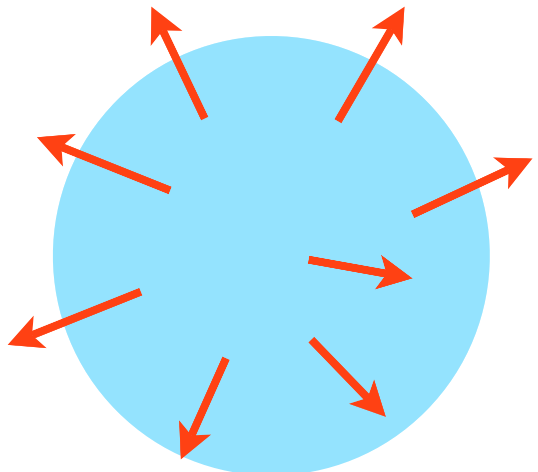
Strength relative to gravity of new force



from: *http://www.npl.washington.edu/eotwash/*
Consult the site for more recent data, and for references to other experiments.

Amusing remark: required differential-acceleration sensitivities of 10^{-13} cm/s^2. If an object, initially at rest, had maintained that acceleration since the time of Pericles, it would now be moving as fast as the end of the minute hand on a standard wall clock.

A simple mechanism to stabilize moduli is by turning-on non-zero flux(es) of the antisymmetric tensor fields. This can be illustrated with a Maxwell field in our 6D example:



Consider the sphere compactification, with a monopole field threading through it,

$$F_{\theta\varphi} = \frac{n}{2q}\,\sin\theta$$

where q is the unit charge, and n integer.

The potential for the radius field , $V(\phi) = -\dfrac{1}{2\kappa^2}\, e^{\phi} + \dfrac{\pi n^2}{2q^2}\, e^{3\phi/2}$, has a stable minimum with negative cosmological constant.

the curvature term tries to shrink the 2-sphere ...

while the flux tries to expand it. Flux cannot relax, because it is quantized.

Flux compactifications have been systematically studied in the past few years, and examples with all the moduli stabilized are known [e.g. the $AdS_4 \times S_7$ “Freund-Rubin” compactification of M-theory].

for a review see: Graña, hep-th/0509003

There are, however, two important difficulties:

(1) Known vacua are supersymmetric and have negative (or zero) energy density, while in our universe supersymmetry is broken and the cosmological constant is tiny but positive [$\Lambda \sim (10^{-3} eV)^4$].

The two problems are related. There are some ideas on how to address them; not, however, yet compelling and calculable.

choice of discrete fluxes,
compact-space topology etc

(2) The number of supersymmetric stable vacua is huge; if only a small fraction of them can be “lifted” to positive Λ , how do we choose?

Cosmological? Anthropic? a useful calculational approach has yet to emerge.

Gravity keeps (most of) its secrets; particle physics is intimately related to them.

.... and in String Theory these facts are most clearly exposed.