String Theory for Pedestrians

CERN academic training
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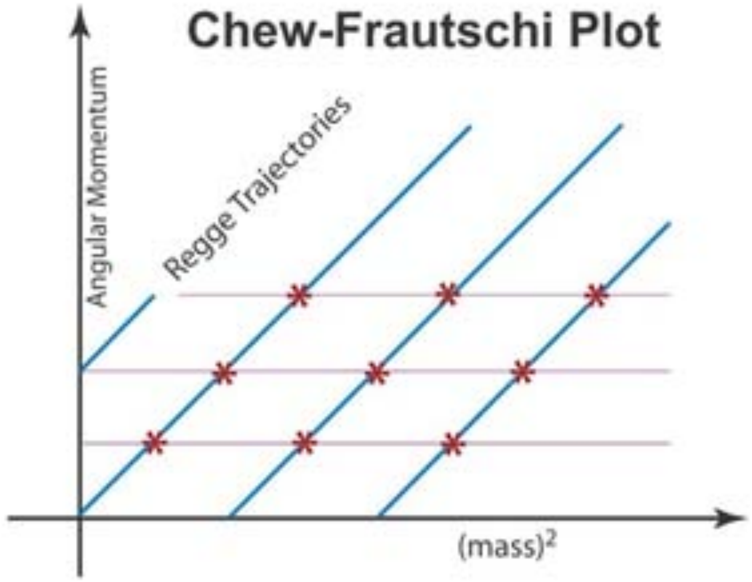
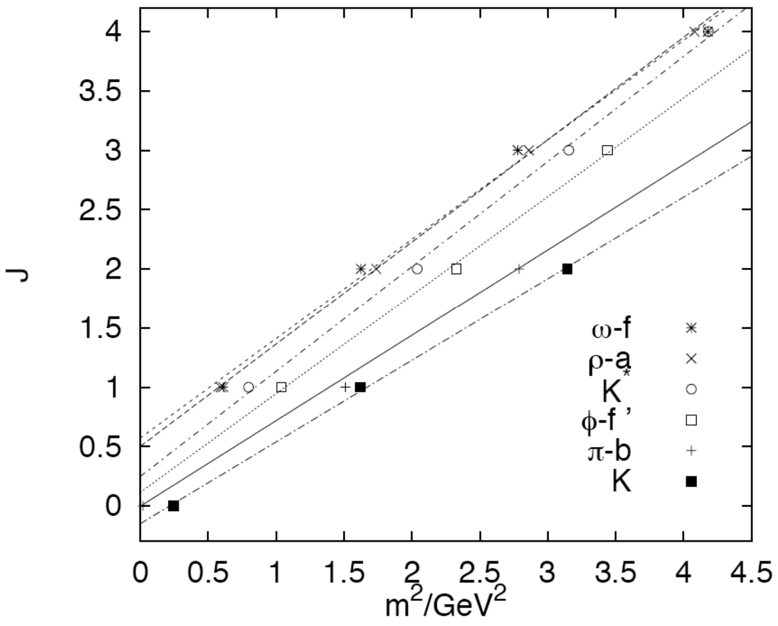
Lecture 1 : The basic notions and tools

The idea that elementary particles may correspond to quantum states of an extended structureless object dates back to 1962, when P.A.M. Dirac tried to model the electron and the muon as different states of a charged membrane.

Some years later it was realized that meson resonances in hadronic collisions could be well described by the excitations of a quantum relativistic string.





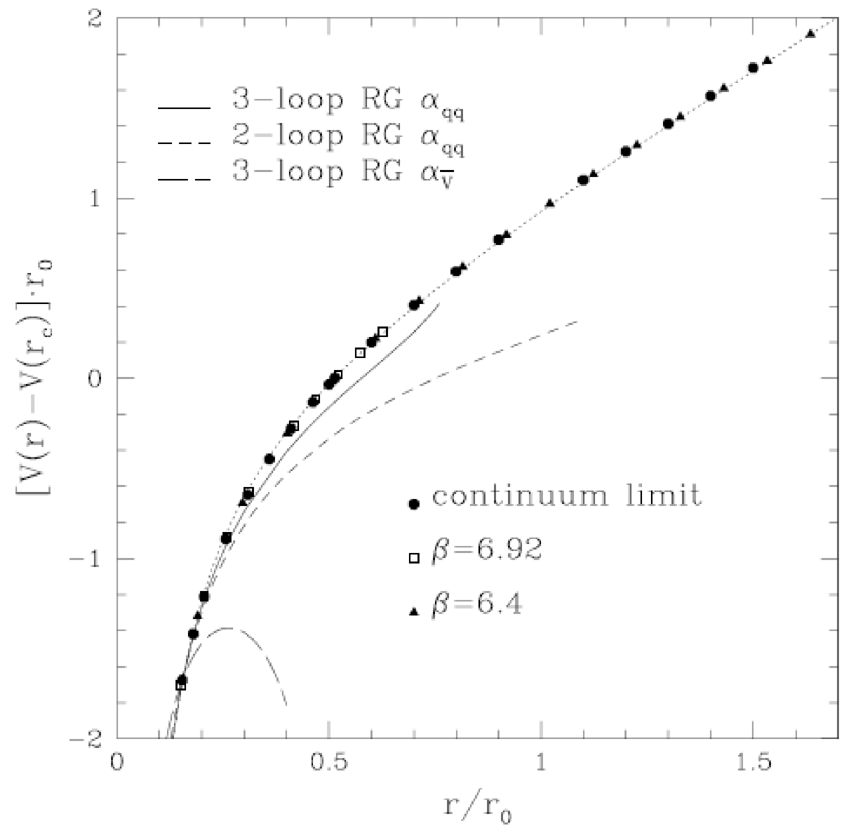
Both ideas are alive today, though in transmuted form.

The quarks, leptons and gauge bosons of the Standard Model are not organized in Regge trajectories [or if they are, they each belong to a separate trajectory]. Yet, because it incorporates quantum gravity, string theory has played a central role in the effort to unify the fundamental forces.

Strong interactions, on the other hand, are described by a beautiful theory: QuantumChromodynamics. Perturbative and lattice QCD calculations are, furthermore, adequate in many contexts.

Thus, if string theory is to play any role, it should be as an analytic tool for accessing the low-E dynamics.

Stretched chromo-magnetic flux tubes do behave like strings, as is clear from lattice calculations of the heavy-quark potential. This explains the qualitative successes of dual (and also the Lund) models.



from Necco+ Sommer, hep-lat/0108008

The modern reincarnation of dual models is the AdS/CFT correspondence. Besides its potential application to QCD, theorists believe that this duality between gauge theories and gravity could have more far-reaching consequences.

To begin, we need to undestand the dynamics of a relativistic string.

This is described by the Nambu-Gotto action, i.e. the (Lorentz- and) reparametrization-invariant area of the string trajectory [or worldsheet] , mutliplied by the string tension.

$X^\mu(\sigma, \tau)$ where

$X^\mu = (X^0, X^1 \cdots X^d)$ and

$X \cdot X = -(X^0)^2 + (X^1)^2 \cdots + (X^d)^2$

$$S_{NG} = T_F \int d\tau d\sigma \sqrt{(\partial_\tau X \cdot \partial_\sigma X)^2 - (\partial_\tau X)^2 (\partial_\sigma X)^2}$$

string tension

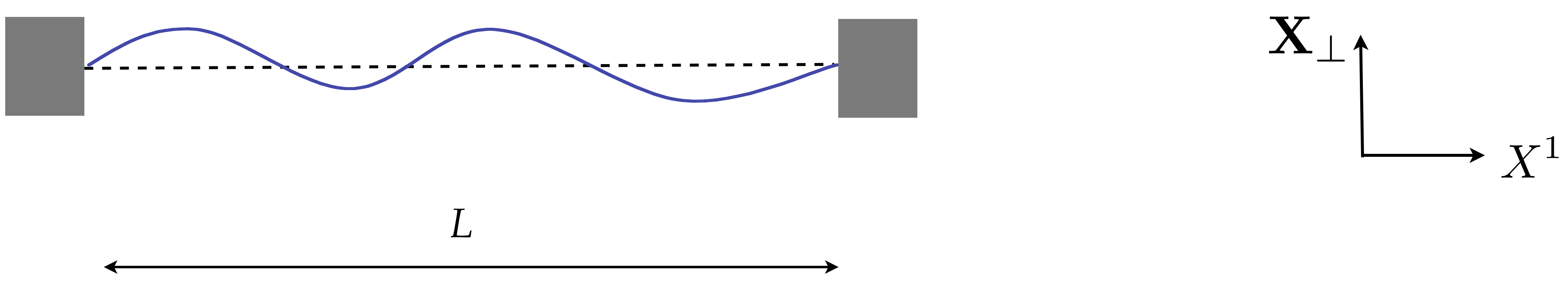
$T_F \equiv \dfrac{1}{2\pi\alpha'}$

$-\det \hat{g}$, where the induced metric is

$\hat{g}_{ab} = \partial_a X \cdot \partial_b X$

NB: This is not the unique invariant action, but it is the lowest-order term in derivatives (higher-order terms involve extrinsic or intrinsic curvature).

When compared, say, to a violin string, relativistic strings have unusual properties:



In the so-called static parametrization ($X^0 = \tau,\ X^1 = \sigma$) the NG action reads:

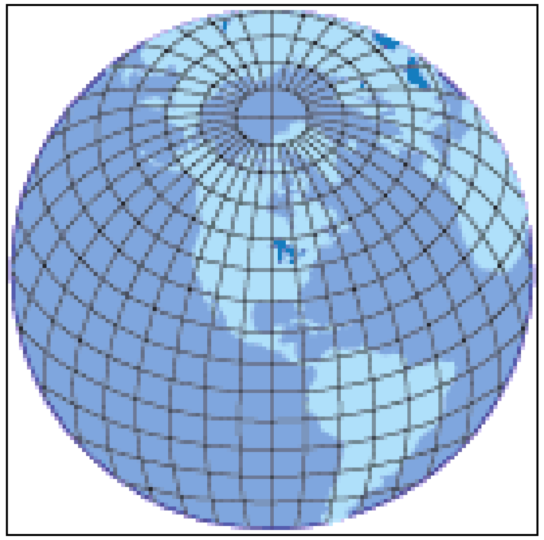
$$S_{NG} = TL\left[1 - \frac{1}{2}(\partial_\tau \mathbf{X}_\perp)^2 + \frac{1}{2}(\partial_\sigma \mathbf{X}_\perp)^2 + \cdots\right]$$

→ mass density = tension , and waves propagate at the speed of light .

A violin string, by contrast, has mass density > tension ; it supports both transverse and longitudinal waves traveling at subluminal speeds.

A more convenient parametrization is by conformal coordinates, in which the tangent vectors are everywhere orthogonal and of equal (up to a sign) length. The induced metric in such a coordinate system is conformally flat:

$$\hat{g}_{ab} = \begin{pmatrix} (\partial_\tau X)^2 & \partial_\tau X \cdot \partial_\sigma X \\ \partial_\tau X \cdot \partial_\sigma X & (\partial_\sigma X)^2 \end{pmatrix} = |\partial_\tau X|^2 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$



The parametrization of the earth by latitude and longitude is orthogonal but NOT conformal. One degree of latitude corresponds roughly to 110 km, while a degree in longitude corresponds to 110 km on the equator, and zero on the poles.

In conformal coordinates the NG equations become the free-wave equations in 2d:

$$(\partial_\tau^2 - \partial_\sigma^2) X^\mu = 0 \implies X^\mu = \frac{\alpha'}{2} p^\mu (\tau - \sigma) + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} a_n^\mu e^{in(\tau - \sigma)} + \frac{\alpha'}{2} p^\mu (\tau + \sigma) + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \tilde{a}_n^\mu e^{in(\tau + \sigma)}$$

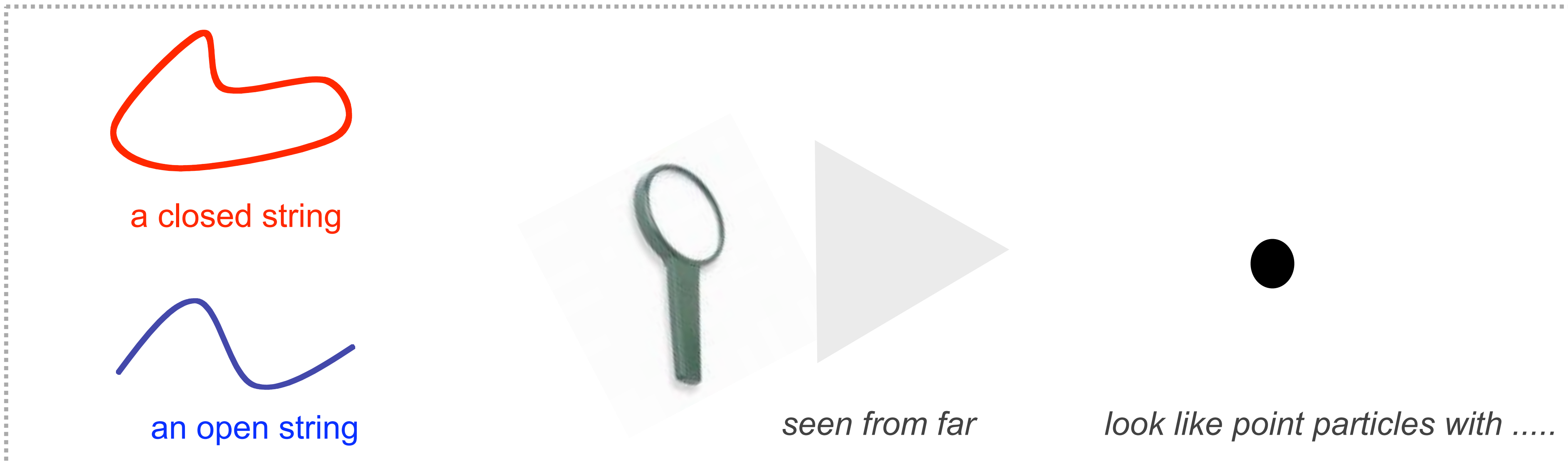
center-of-mass momentum

right-moving excitations

left-moving excitations

We may also write $X^\mu = f^\mu(\tau - \sigma) + \tilde{f}^\mu(\tau + \sigma)$, where f^μ, $\tilde{f}^\mu$ are independent functions in the case of closed strings. Open strings have only standing waves, i.e.

$$f^\mu = \pm \tilde{f}^\mu \quad \text{for} \quad \begin{cases} \text{free-endpoint (Neumann)} \\ \text{fixed-endpoint (Dirichlet)} \end{cases} \quad \text{boundary conditions.}$$



mass m	⟷	vibration energy
spin J		intrinsic angular momentum
charge q		momentum in 5th dimension

To compute the mass, let's look at the conformal-gauge conditions : $f' \cdot f' = \tilde{f}' \cdot \tilde{f}' = 0$.

These are constraints on phase-space, i.e. on the initial data. They can be solved most easily in the light-cone gauge:

$$X^+ = \alpha' p^+ \tau \qquad \text{where}$$

$$X^\pm \equiv X^0 \pm X^1 \quad (\Longrightarrow X \cdot X = -X^+ X^- + |\mathbf{X}^\perp|^2\ )$$

Since $(f^+)' = (\tilde{f}^+)' = \frac{1}{2}\alpha' p^+$ we can solve the gauge conditions for f^- and $\tilde{f}^-$.

Thus, only the D-2 transverse oscillation modes are physical degrees of freedom.

Furthermore the gauge conditions give:

$$m^2 = -p^2 = \frac{4}{\alpha'} \sum_{n=1}^{\infty} |\mathbf{a}_n^\perp|^2 = \frac{4}{\alpha'} \sum_{n=1}^{\infty} |\tilde{\mathbf{a}}_n^\perp|^2 \ .$$

positive definite and continuous mass spectrum

$1/\alpha'$ for open strings

Similarly, one can compute the angular momentum in the c.o.m. rest frame and show that:
$J \leq \alpha' m^2$ (as compared with $J \sim \sqrt{E}$ for a rigid bar); try to prove this!

open strings

Of course the masses and spins of elementary particles must be discrete.

No problem, this is automatic upon quantization:

$$\frac{1}{n}|a_n^j|^2 \to \mathcal{N}_n^j = 0, 1, 2\cdots$$

number of excitations with frequency *n* and polarization *j*

What looks harder is: how to obtain massless particles, like the photon?
The ground state energy comes here to the rescue! e.g. for open strings:

$$\alpha' m_0^2 = (D-2)\sum_1^{\infty}\frac{n}{2} = -\frac{D-2}{24}$$

NB: the divergence is unambiguously subtracted, because the (infinite) energy density must be (by locality) $\propto 1/\epsilon^2$, where ϵ is the short-distance cutoff.

⇒ The ground state $|0\rangle_o$ has imaginary mass, it is a tachyon.

The first excited states $\alpha^j_{-1}|0\rangle_o$ transform as a vector of the transverse rotation group, and have mass $\alpha' m^2 = 1 - \frac{D-2}{24}$. Since only massless particles can have *(D-2)* polarization states, consistency requires

D=26 ← critical dimension

Thus the massless states of an open string correspond to a higher-dimensional photon.
Remaining states have mass $\geq 1/\sqrt{\alpha'}$, and they are organized in Regge trajectories.

Repeating the analysis for the closed strings one finds a tachyon $|0\rangle_c$ at the lowest level, and the following states at zero mass:

$\alpha^j_{-1}\tilde{\alpha}^l_{-1}|0\rangle_c$

→ symmetric traceless: graviton (h^{jl})

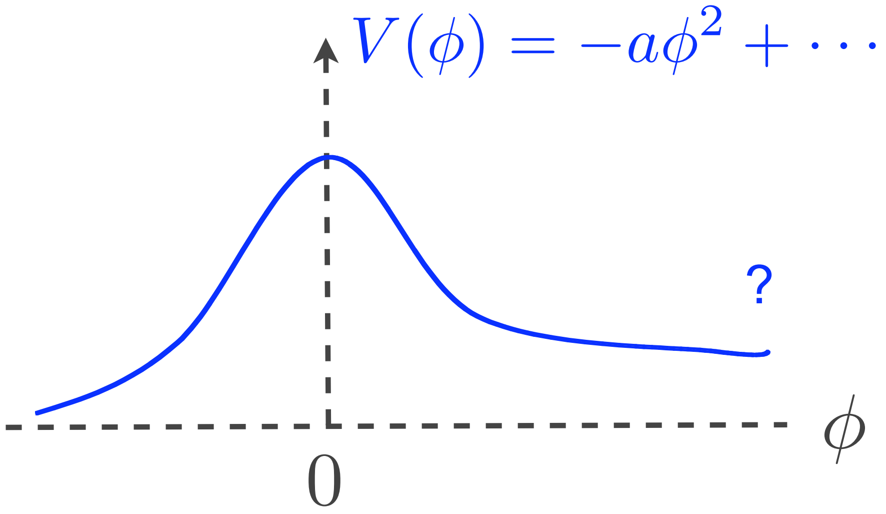
→ anti-symmetric 2-index (B^{jl})

→ trace part: dilaton (Φ)

The fact that the closed-string spectrum includes a massless spin-2 state prompted the reinterpretation of string theory as a theory of quantum gravity.

Scherk, Schwarz; Yoneya '74

What about the problem of the tachyon? In ordinary field theory a scalar tachyon field signals a perturbative instability of the vacuum:



In bosonic string field theory the ultimate fate of *26D* Minkowski spacetime is not well understood [the known stable backgrounds are in *2D*].

There is, however, a remedy for stability: space-time supersymmetry.

There exist several [technically-different but physically-equivalent] descriptions of the superstring. In the so-called NSR formulation, one introduces one anticommuting coordinate for each normal (commuting) coordinate of the string:

NSR super-coordinate: $(X^\mu, \psi^\mu, \tilde{\psi}^\mu)$

left and right real fermions on worldsheet

The critical dimension is now *D=10.*
The rest of the analysis proceeds as before, with two crucial differences:

The fermions can be either periodic (Ramond) or antiperiodic (Neveu-Schwarz), with modes that have either integer or half-integer frequencies [for the open string this corresponds to $\psi^\mu = \tilde{\psi}^\mu$ at one end, and $\psi^\mu = \pm\tilde{\psi}^\mu$ at the other].

In the Ramond sector, there are anti-commuting zero modes acting on the states:

$$\{\psi_0^\mu, \psi_0^\nu\} = \eta^{\mu\nu} \quad \longleftarrow \quad \text{Dirac } \gamma \text{ matrices.}$$

These states transform therefore as space-time spinors!

It is consistent (and necessary) to impose definite world-sheet fermion parity.

This is known as the GSO projection: $(-)^F = -1$.

It projects out the tachyon, and acts as a chirality projection on Ramond states.

Let us consider the spectrum of the open superstring:

$\alpha' m^2$ \ sectors	Neveu-Schwarz	Ramond
$-\frac{1}{2}$	~~$\|0\rangle_{NS}$~~	
0	$\psi^j_{-\frac{1}{2}} \|0\rangle_{NS}$	~~$\|0,+\rangle_R$~~ $\|0,-\rangle_R$

.... plus massive states.

D=10 Weyl Majorana spinor

The massless states are those of the *D=10,* N=1 supersymmetric Maxwell theory.

For closed strings, one must impose separate boundary conditions and GSO projections on the left and right fermions. The massless states are:

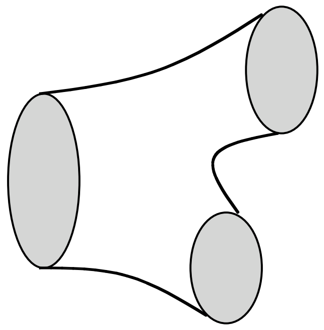
$$\psi^j_{-\frac{1}{2}}\tilde{\psi}^l_{-\frac{1}{2}}|0\rangle_{NS/NS} \longrightarrow$$ graviton, dilaton, NS-NS tensor

$$\psi^j_{-\frac{1}{2}}|0\rangle_{NS/R}\ ,\ \tilde{\psi}^l_{-\frac{1}{2}}|0\rangle_{R/NS} \longrightarrow$$ gravitini with { opposite (IIA) / equal (IIB) } chirality

$$|0\rangle_{R/R} \longrightarrow$$ R-R antisymmetric tensor fields [$\simeq$ bi-spinors]

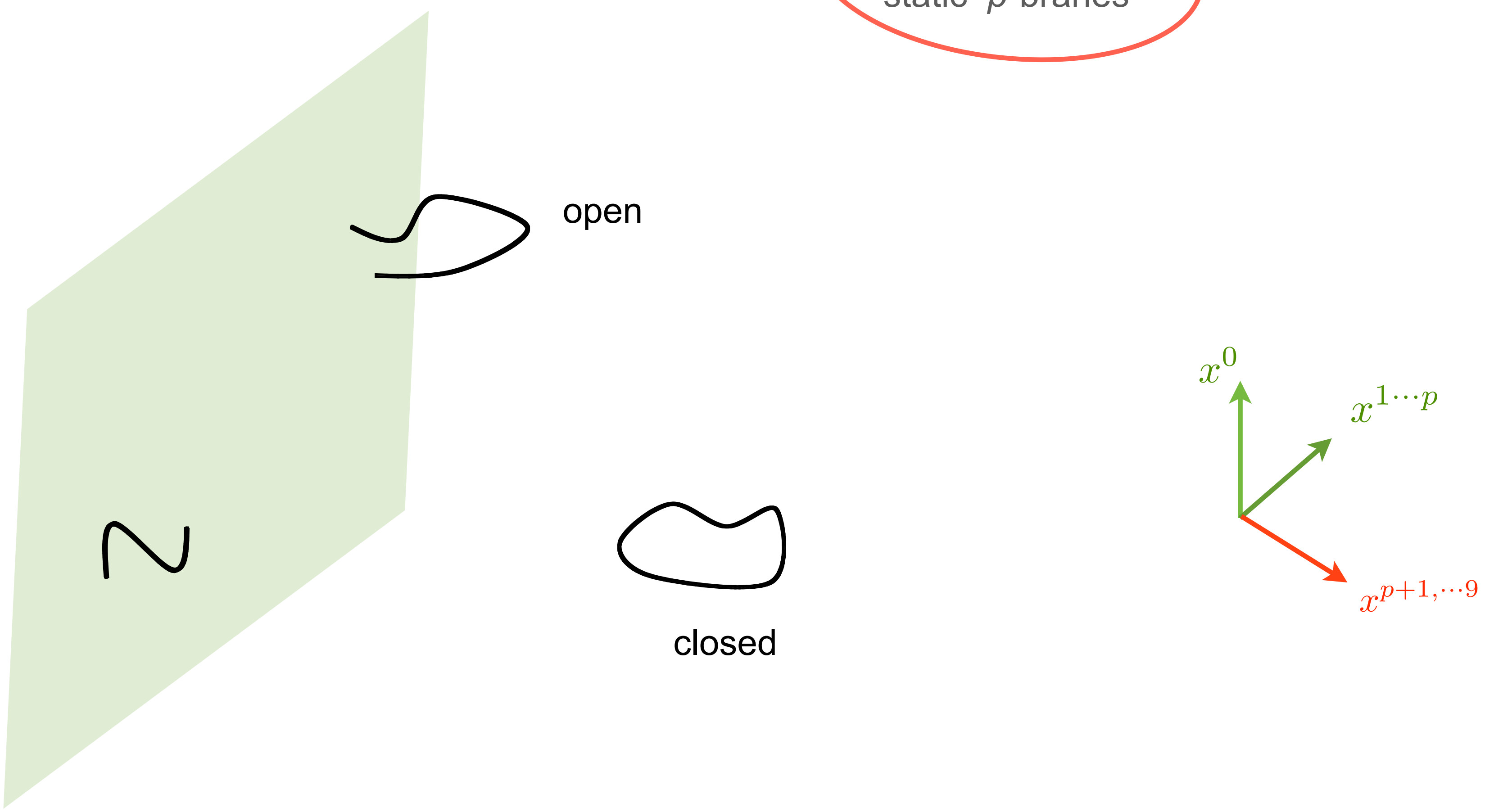
⇨ These states are those of the maximal (N=2) supergravity theories in *D=10* .

By studying the interactions of the massless modes it has been shown that the effective low-E theories are precisely N=2 (IIA or IIB) supergravity in ten dimensions. This is not surprising: maximal two-derivative supergravities are unique, i.e. they are completely fixed by symmetry.



How about the open strings? Recall that each coordinate can obey either Neumann or Dirichlet boundary conditions: open-string endpoints are thus stuck on hyper-planar subspaces, or Dp-branes.

more precisely, worldvolumes of static *p*-branes



D-branes interact with the closed strings [e.g. an open string can emit a closed one]. They have in particular RR-charge and mass density (tension); they are solitonic excitations of type II string theory, analogous to magnetic monopoles.

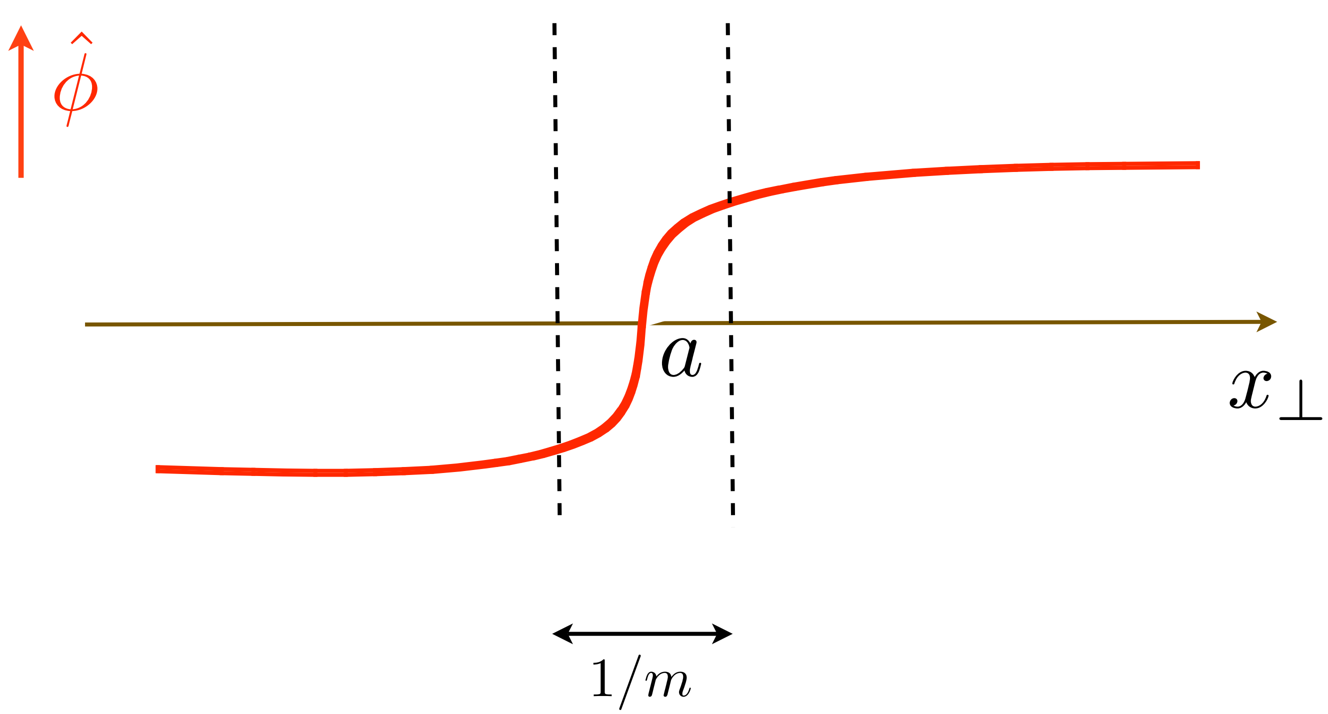
Polchinski '95

The simplest soliton is the kink. This is a domain wall in a scalar-field theory with a double-well potential [in *d* spatial dimensions it is a *p=(d-1)* - brane]. As a toy example, consider the following two-scalar model:

$$\mathcal{L}(\phi,\chi) \;=\; \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 + \frac{g}{8}[\phi^2 + \chi^2 - \frac{m^2}{g}]^2 + \frac{\tilde{g}}{8}\chi^4$$

The two vacua are at $(\phi,\chi) = (\pm\frac{m}{\sqrt{g}}, 0)$, and a kink solution is

$$\hat{\phi} \;=\; \pm\frac{m}{\sqrt{g}}\tanh\left(\frac{m(x_\perp - a)}{2}\right) \;,\; \hat{\chi} = 0 \;.$$



width $\sim 1/m$

tension $\sim m^3/g$

heavy at weak coupling

The low-E excitations in the background of the kink are of two kinds:

(i) those of the field $\chi(x)$, which is massless far from the kink;

(ii) the long-wavelength transverse excitations of the brane, $\phi(x) \sim Y(x_{\|})\partial_a\hat{\phi}(x_{\perp})$.

The corresponding effective action reads:

$$S_{\rm eff} = \int_{\rm bulk} [\frac{1}{2}(\partial\chi)^2 + \frac{g+\tilde{g}}{8}\chi^4] + \int_{\rm brane} [\frac{1}{2}(\partial Y)^2 + V_b(Y,\chi)] + \cdots$$

NB: there is, in fact, also a tachyonic field on the brane, corresponding to the fact that there is a lower-tension, stable domain wall. For simplicity, we have neglect this tachyon in the current discussion.

This can be derived from the initial Lagrangian, by a decomposition of the scalar fields in modes of the reduced transverse-space linearized equations:

$$\Phi := \phi + i\chi = \sum_\lambda \psi_\lambda(x_{\|})\Phi_\lambda(x_\perp)$$

both continuous (bulk), and discrete (localized) modes

Unfortunately, we don't know the Lagrangian of (second-quantized) string field theory.

Nevertheless, we interpret closed and open strings as the bulk and brane-localized modes in the presence of solitons. In particular, the low-E excitations of a D-brane are described by open strings with $m^2 \ll 1/\alpha'$.

For an isolated D*p*-brane, we saw that these strings correspond to a *D=10* Maxwell (super)field, dimensionally reduced to *D= p+1* :

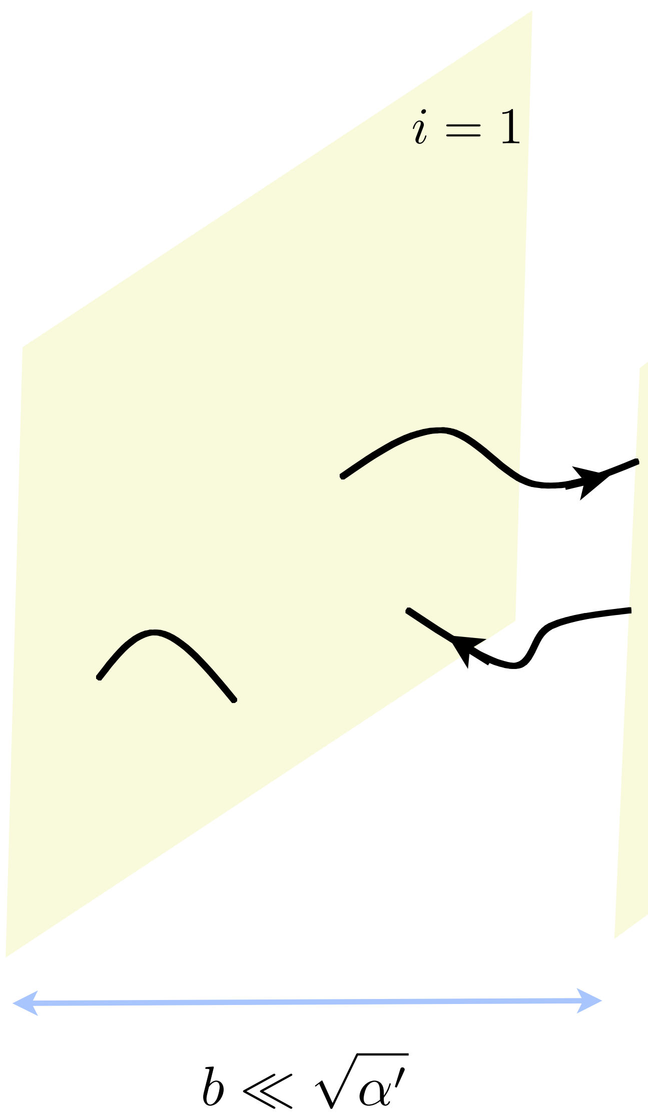
$A^{\alpha=0,\cdots,p}$	photon field
$Y^{J=p+1,\cdots,9}$	scalars (= transverse coordinates)
$\lambda^{a=1\cdots 16}$	gauginos

The effective two-derivative D-brane action is a free, supersymmetric Maxwell theory.

Covariantized by the Dirac-Born-Infeld action:

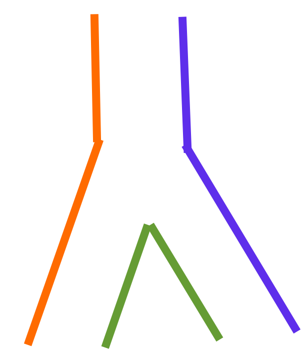
$$\int d^{p+1}\zeta \sqrt{-\det(\partial_\alpha Y \cdot \partial_\beta Y + F_{\alpha\beta})} + \text{fermionic}$$

A surprise comes when one considers two nearby D-branes:



Fields acquire Chan-Paton indices, labeling the branes of the string endpoints:

$$A^{\alpha}_{ij} \;,\; \Phi^{J}_{ij} \;,\; \lambda^{a}_{ij}$$



and they interact as matrix products.

The low-E field theory is supersymmetric Yang-Mills theory with gauge group U(2) ; for *N* D-branes the group is U(N) .

spontaneously-broken to *U(1)*x ... x *U(1)* when branes are separated ("Coulomb phase")

The double role of D-branes, as (1) solitons in a theory of gravity, and (2) habitats of non-abelian gauge theories, is at the core of most recent developments in the subject. More in the upcoming lectures.

Here, I will conclude with one last remark: the type IIA theory has stable supersymmetric D-particles with mass $\sim 1/g_s\sqrt{\alpha'}$. These become light when the coupling is strong. But is it possible to add more light fields to the highly-constrained N=2, *10D* supergravity theory?

The only known extension is maximal supergravity in 11 dimensions [from which the IIA theory is obtained by dimensional reduction].

Cremmer, Julia, Scherk '78

A reasonable conjecture (that passes many tests): strongly-coupled IIA theory has a dual description in terms of *D=11* supergravity compactified on a large circle, and coupled consistently to membranes and five-branes.

Hull, Townsend '94
Witten '95

It is rather unlikely that *D=11* supergravity is a consistent theory at all scales. Its UV completion (if it exists) has been called M-theory; it has no dimensionless parameter, and no sharp definition.

Let us now summarize:

A. Relativistic strings can be consistently quantized. They describe infinite towers of particles, with spin and mass-squared on Regge trajectories. One of these particles is massless and has spin 2: it is the graviton.

B. To solve the problem of the tachyon one needs supersymmetry. Also, consistency requires 10 space time dimensions. Closed strings give a finite theory of quantum gravity whose low-E limit is type II supergravity.

C. The theory has solitonic excitations that can be described by D-branes. These have the surprising property of binding non-abelian Yang Mills theories in their worldvolume.

Further reading: there are many textbooks on string theory.
Here is a (partial) list:

B. Zweibach, *First Course in String Theory*

M. Green, J. Schwarz, E. Witten, *Supertring Theory*

J. Polchinski, *String Theory*

E. Kiritsis, *String Theory in a Nutshell*

K+M. Becker, J. Schwarz, *String Theory and M-theory*