CONTENT

Lecture 3 : Lattice design

- ☐ Emittance
- ☐ Design criteria
- ☐ Optimisation

Equilibrium Beam Emittance

- The natural horizontal emittance for an isomagnetic ring i.e. all bending magnets having same bending radius is :

$$\varepsilon_x = \frac{C_q \gamma^2 \langle H \rangle_{dipole}}{J_x \rho}$$

J_x the horizontal damping partition number.

J_x ~1 (zero field gradient in bending magnet)
$J_x < 2$ (vertical focusing in bending magnet) :
(potentially) emittance reduction of a factor two

$C_q = 3.83 \times 10^{-13}$ m and γ is the Lorentz factor.
ρ is the bending radius.

H is the so called lattice invariant or dispersion's emittance or H-function

$$H(s) = \gamma_x(s)\eta^2(s) + 2\alpha_x(s)\eta(s)\eta'(s) + \beta_x(s)\eta'^2(s)$$

$\langle ... \rangle$ average taken only in the part of the circumference where photons are emitted, (BM and IDs)

- In practical units, ε_x is given by :

$$\varepsilon_x[nm.rad] = 1470\, E[GeV]^2 \frac{\langle H \rangle_{dipole}}{\rho J_x}$$

ε_x is completely determined by the energy, bending field and lattice functions.

Equilibrium Beam Emittance

- After calculation of the <H> value, the natural horizontal emittance for an isomagnetic ring i.e. all bending magnets having same bending radius can be expressed as :

$$\varepsilon_x = F\frac{C_q\gamma^2\theta_b^3}{J_x}$$

J_x the horizontal damping partition number.
θ deviation angle of one bending magnet
ρ bending radius
l_b bending magnet length
N number of bending magnets

$C_q = 3.83 \times 10^{-13}$ m and γ is the Lorentz factor.

$$F = \frac{1}{3}\left[\frac{\beta_0}{l_b} - \frac{1}{4}\alpha_0 + \frac{1}{20}\gamma_0 l_b\right]$$

$\beta_0,\alpha_0,\gamma_0$ twiss parameters at the entry of the BM

It is a general lattice property, <u>there is no assumption on the lattice type</u>.

$$\varepsilon_x \propto \frac{1}{N_b^3}$$

⇒ Should use **many short Bending Magnets** to get **low emittance**

Minimum Emittance (with Achromatic Condition)

The minimum equilibrium beam emittance in an isomagnetic ring with an Achromatic Arc Condition, $\eta_0 = \eta'_0 = 0$, at the entrance of the **BM** :

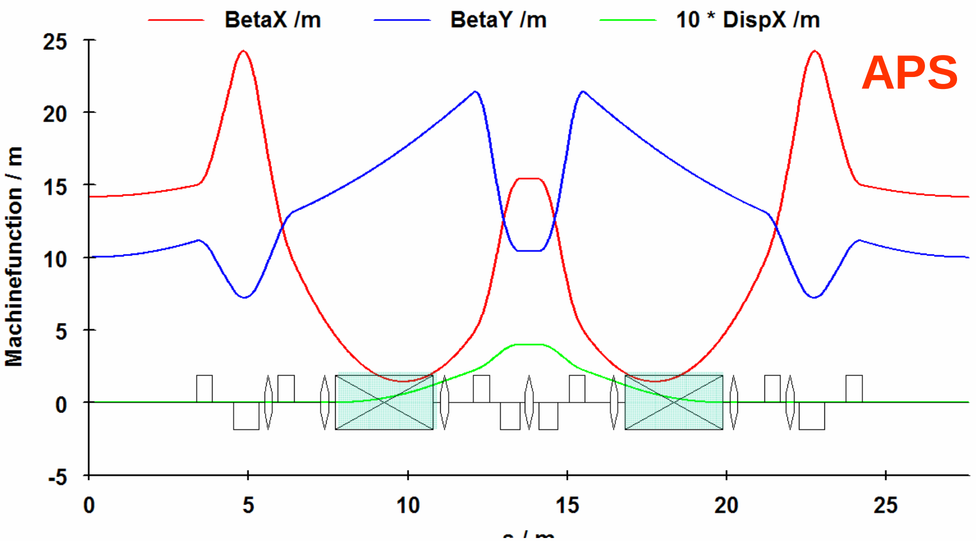
$$\boxed{\varepsilon_{x,\min} = \frac{C_q \gamma^2 \Theta^3}{4\sqrt{15}\, J_x}}$$

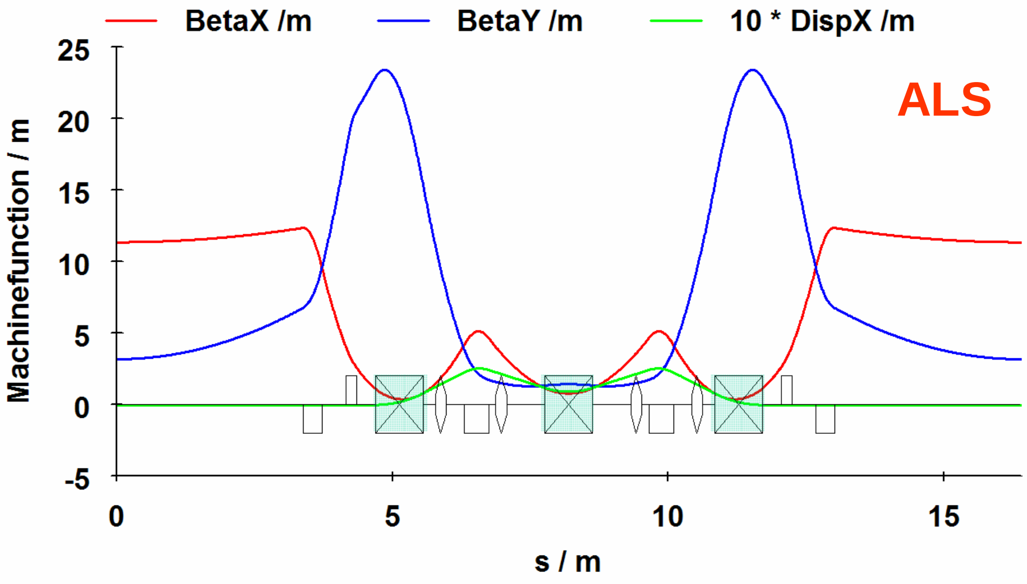
⇒DBA or TBA lattices

(Double/Triple Bend Achromat)

DBA used at:
ESRF,ELETTRA,
APS, SPring8,
Bessy-II,Diamond,
SOLEIL, SPEAR3
...

TBA used at:
ALS, SLS,
PLS,TLS
…

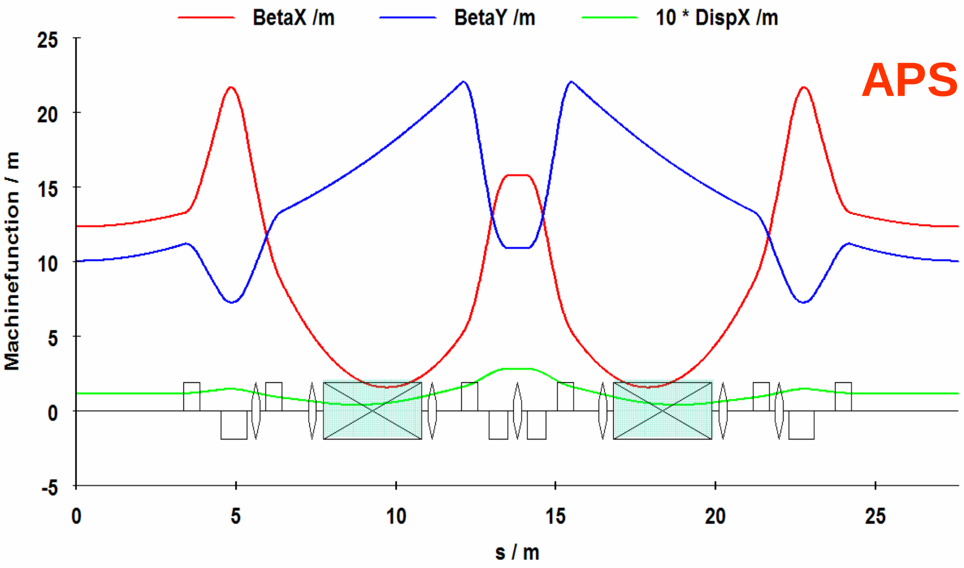




Minimum Emittance (without Achromatic Condition)

By breaking the achromatic condition (non-zero dispersion in straight sections) we can obtain the configuration in which the emittance becomes the smallest.

$$\boxed{\varepsilon_{x,\min} = \frac{C_q \gamma^2 \Theta^3}{12\sqrt{15}\,J_x}}$$



⇒It is smaller **by a factor 3** than in the achromatic arc configuration

Example of Machines which move from Achromatic conditions to non zero dispersion in SS

ESRF	7 nm	→ 3.8 nm
APS	7.5 nm	→ 2.5 nm
SPring8	4.8 nm	→ 3.0 nm
SPEAR3	18.0 nm	→ 9.8 nm
ALS (SB)	10.5 nm	→ 6.7 nm

Emittance Achieved

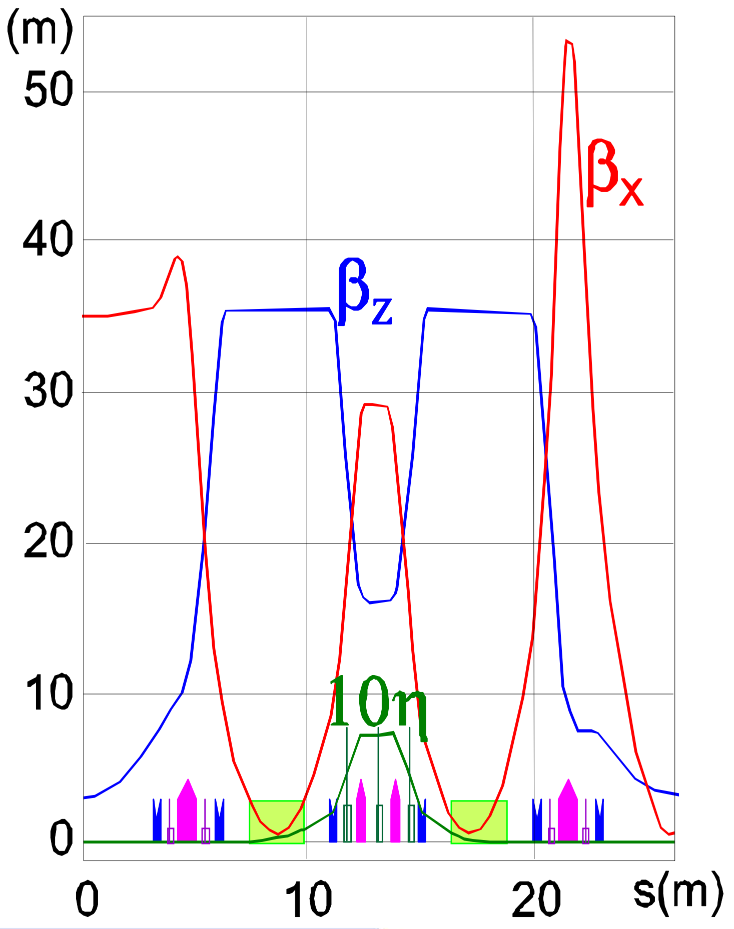
The minimum emittance can’t be easily achieved :

Ideal values for SOLEIL and Design values (without achromatic condition) :

$$\alpha_{0,min} = \sqrt{15} = 3.873 \qquad \alpha_0 = 1.8$$

$$\beta_{0,min} = 2.17m \qquad \beta_0 = 1.5\,m$$

$$\varepsilon_{x0,min} = 1.8\ nm.rad \qquad \varepsilon_{x0} = 3.7\ nm.rad$$



The ideal value $\alpha_{0,min} = \sqrt{15}$

causes the betatron function to reach a sharp minimum inside the **BM** and then to increase from there on to large values in the quadrupoles, leading to extremely high **chromaticity**

Emittance Achieved

⇒Chromaticity has to be corrected for two reasons :

Momentum acceptance : some variation of energy deviation has to be accepted by the storage ring for reasons of **beam lifetime**.

Head tail instability: collective oscillation of electrons in head and tail of the bunch leading to very fast beam loss.=> damped by operating with positive chromaticity

Chromaticity : $\xi \sim \int (- K_Q \beta + m_s \beta \eta) \, ds \geq 0$

quadrupole strength
(introduce negative ξ)

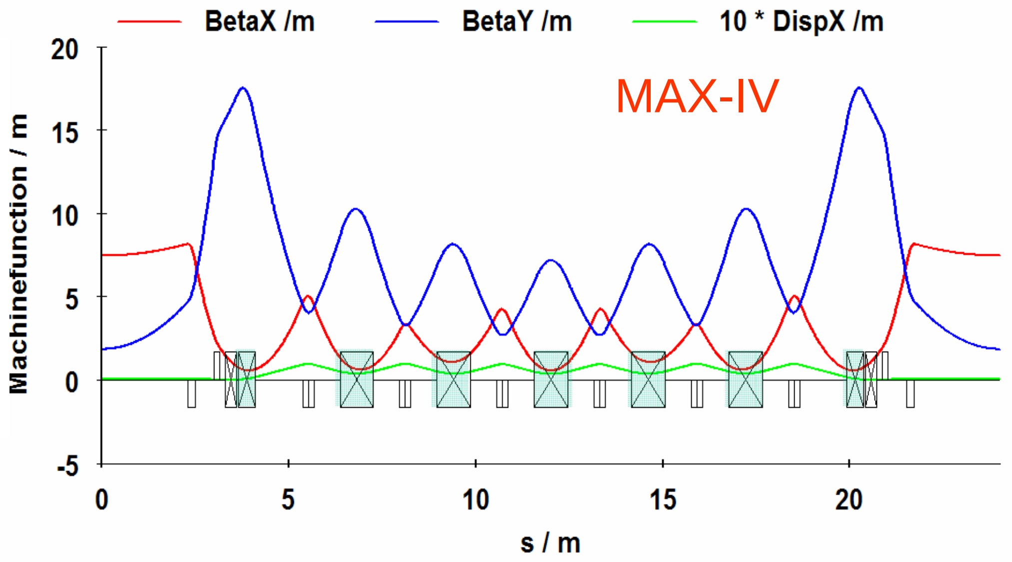
sextupole strength
(correct the ξ)

Strong chromaticity correction sextupoles reduce the dynamic aperture and this negatively impacts on the beam lifetime.

Ultra low emittance Emittance rings

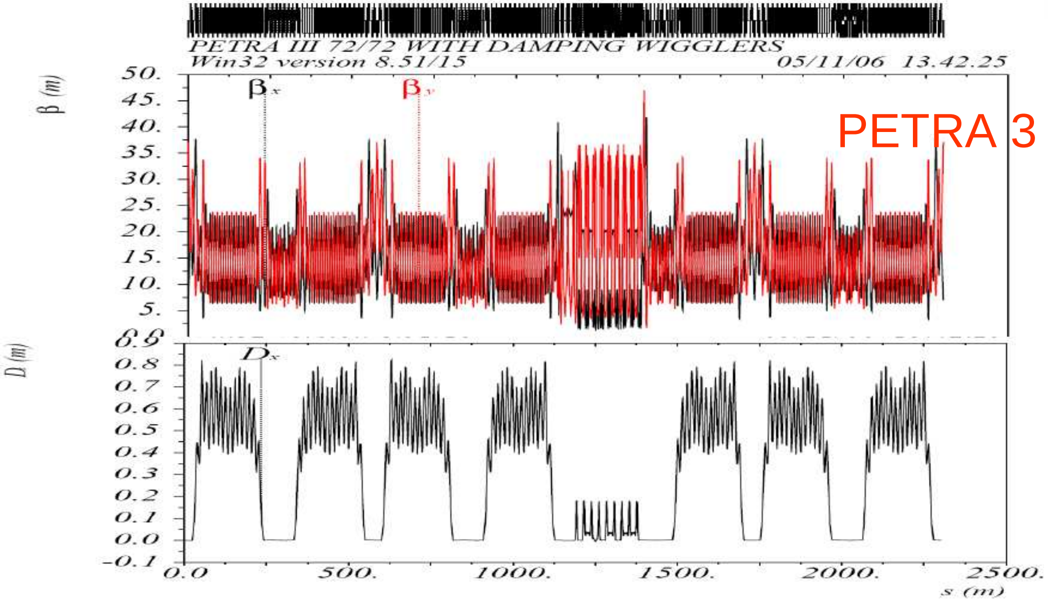
- **MAX-IV (Sweden):**

  7-BA (zero dispersion in SS) 12 cells
  3 GeV
  287 m
  12 SS x 4.6 m = 55 m
  => εx = 0.8 nm.rad
  Project just approved



- **PETRA 3 (DESY) :**

  7 old octants FODO + 1 New octant DB
  6 GeV
  2304 m
  Damping Wigglers
  6 SS for a total of 45 m (14 IDs)
  => εx = 1 nm.rad
  Commissioning soon !

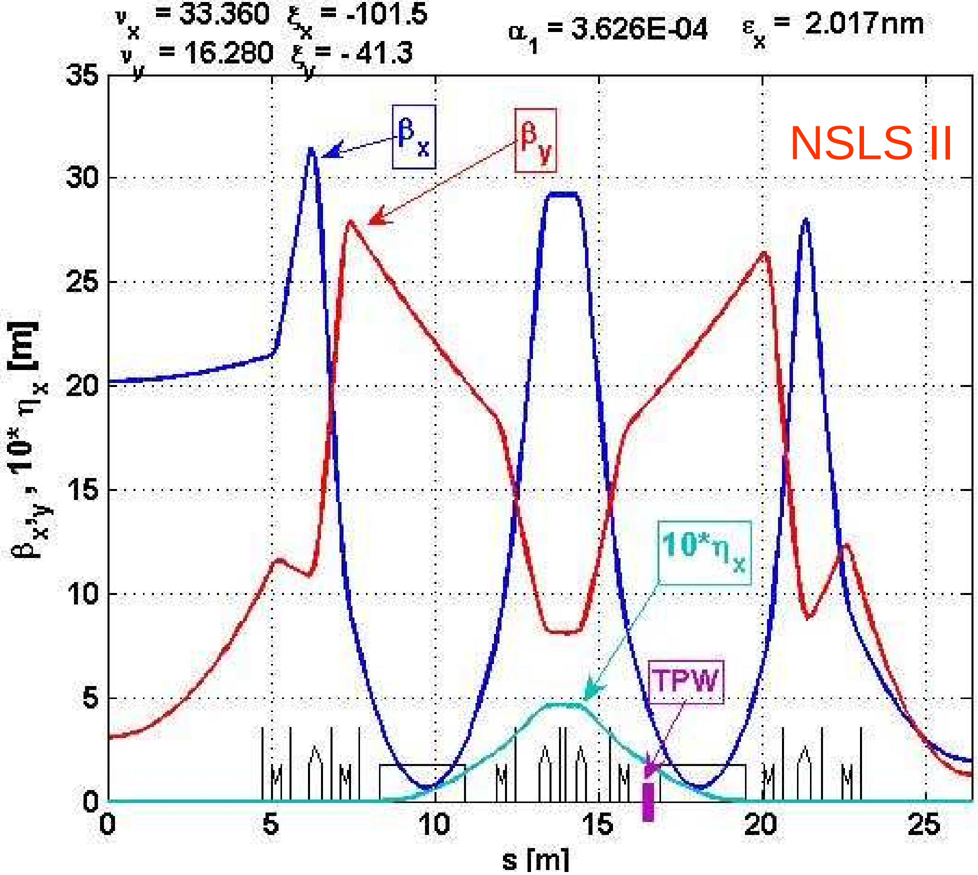


Ultra low emittance Emittance machine

❑ **NSLS II (BNL) :**

DBA (30 cells)
3 GeV
792 m
Damping Wigglers
30 SS for a total of 240 m
=> $\varepsilon x = 0.6$ nm.rad

Construction starts in 2009



No Damping wiggler => ε_0 = 2nm
@ 2 x (theoretical minimum)

Low bend field B = 0.4 T
➔ low radiation loss U_0=286kV

Damping wigglers for small emittance

$$\varepsilon = \varepsilon_0 \times U_0 / (U_0+U_w)$$

=> ε = 0.6 nm

Beam sizes and Effective emittances

Due to the non zero dispersion η_x in the straight section, the horizontal beam size is enlarged by the beam energy spread.This energy spread results from the lattice properties which imposes an equilibrium value. Typically, $\sigma_E = \delta E/E \sim 10^{-3}$.

$$\sigma_x = \sqrt{\varepsilon_x \beta_x + (\eta_x \sigma_E)^2} \qquad \sigma_{x'} = \sqrt{\varepsilon_x / \beta_x} \quad \Rightarrow \quad (\varepsilon_x)_{effective} = \sigma_x . \sigma_{x'}$$

H Size

H Divergence

	BetaX m	EtaX m	SigmaX µm	Sigma XP µrad	Effective Emittance H
Short straight	17,8	0,285	**388**	**14,5**	5,61 nm.rad
Medium straight	4,0	0,133	**182**	**30,5**	5,56 nm.rad
Long straight	10,1	0,200	**281**	**19,2**	5,40 nm.rad
Dipole 4°	0,38	0,021	**43**	**107,0**	

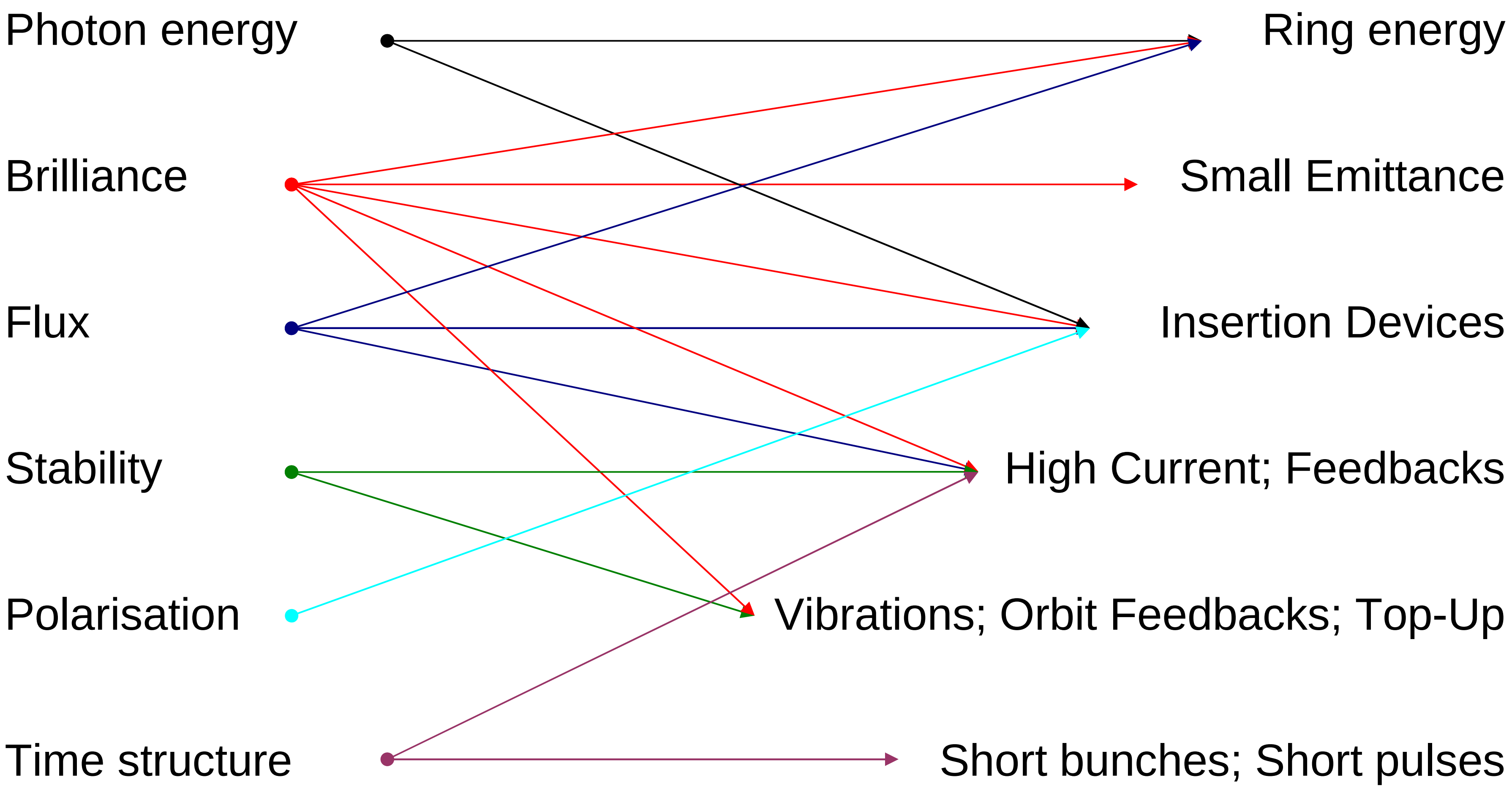
SOLEIL

H emit= 3.7 nm.rad,

$\delta E/E = 1.016\ 10^{-03}$

The effective emittance is still smaller than the emittance achieved in achromatic conditions (~9 nm.rad) !

Accelerator Physics challenges



Photon energy

Brilliance

Flux

Stability

Polarisation

Time structure

Ring energy

Small Emittance

Insertion Devices

High Current; Feedbacks

Vibrations; Orbit Feedbacks; Top-Up

Short bunches; Short pulses

Design Criteria for SOLEIL

- High **Brilliance** and Coherence
- Large beam lifetime and good injection efficiency (**Top-Up**)
- Extensive use of Insertion Devices such as **Undulators** and Wigglers (highest ratio of available straight sections to the circumference). **Variable Polarisation.**
- **Long** straight sections (4)
- **Tunability**: the right photon energy for each experiment (many different ID types)
- **Stability**: intensity (Beam Lifetime), position, size and energy
- Compactness (**Budget**)
- Upgrade potential

Lattice Design Interface

- **Magnet Design**: technological limits, coil space, multipolar errors
- **Vacuum**: impedance, pressure, physical apertures, space
- **Radiofrequency**: Energy acceptance, bunch length, space
- **Diagnostics**: Beam Position Monitors, positioning,…, space
- **Alignment**: Orbit distortions and correction
- **Mechanical Engineering**: Girders, vibrations
- **Design Engineering**: Assembling and feasibility
- **Insertions Devices :** small gap in vacuum undulators, high field wigglers,.

➡ **Space requirements**: Magnet, Vacuum, RF, Diagnostics and Engineering

Linear Optimisation

✓ Reasonable maximum for βx and $\beta z < 30$m (High beta values amplify errors)

✓ Reasonable beta split at the centre of the achromat

✓ Natural chromaticities : $\xi_x < -100$ and $\xi_z < -50$

✓ Dispersion (η_x) at the centre of the achromat > 0.25m

✓ Low βz (~1m) in the centre of undulator straight sections
high brilliance and accomodation of low gap IDs

✓ Minimum Beam Stay Clear for efficient injection ⇨ minimum ratio of $(\beta x)_{max}/(\beta x)_{inj}$ ($\beta xinj > 10$m)

Sextupoles Positions

- ❑ 2 purposes :
  - correction of both chromaticities ξ_x , ξ_z
  - on momentum and off momentum dynamic aperture optimisation

⇨ Phase optimisation to minimise nonlinear effects

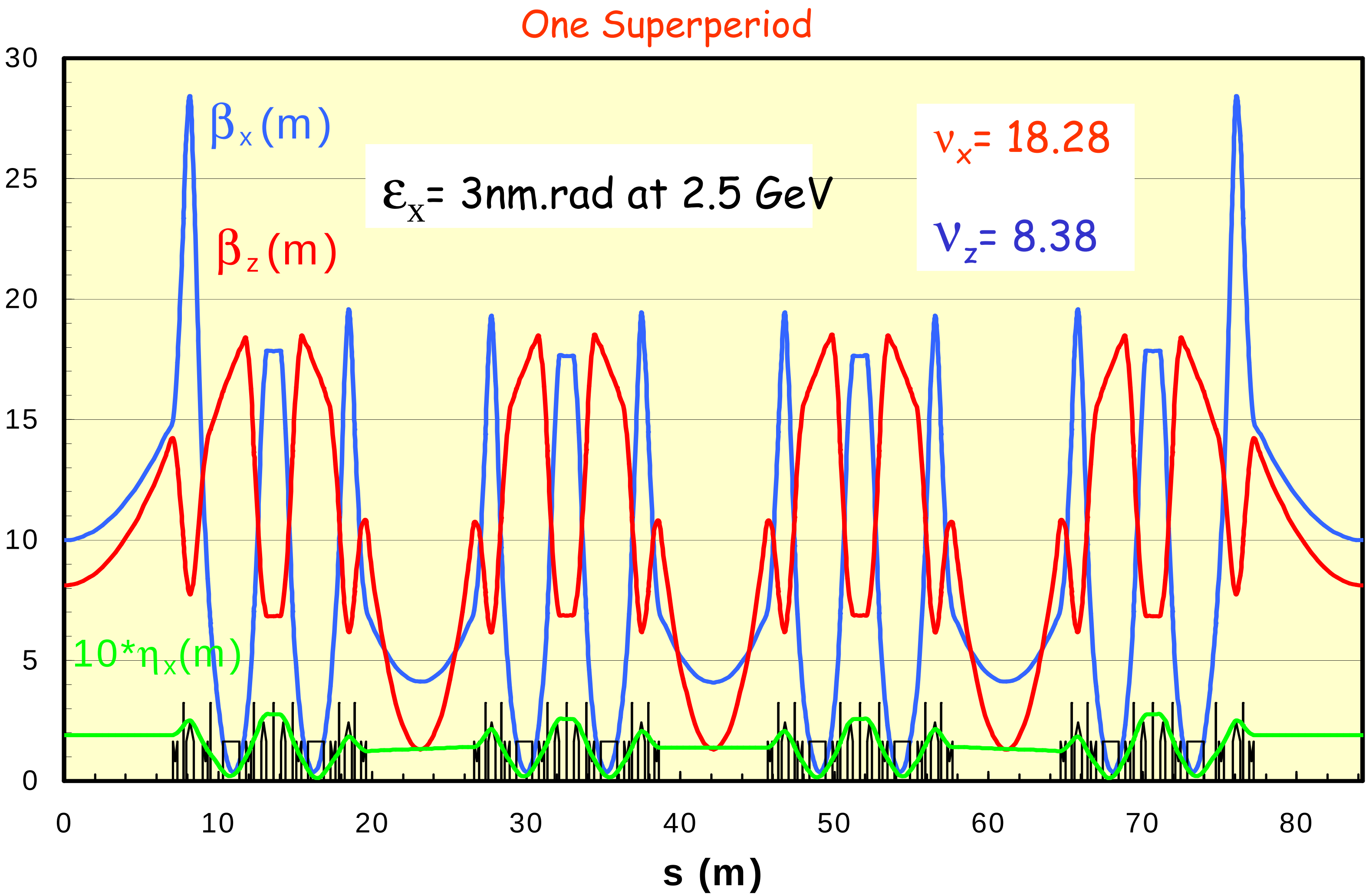
⇨ Large number of sextupole families

⇨ LOW sextupoles strengths

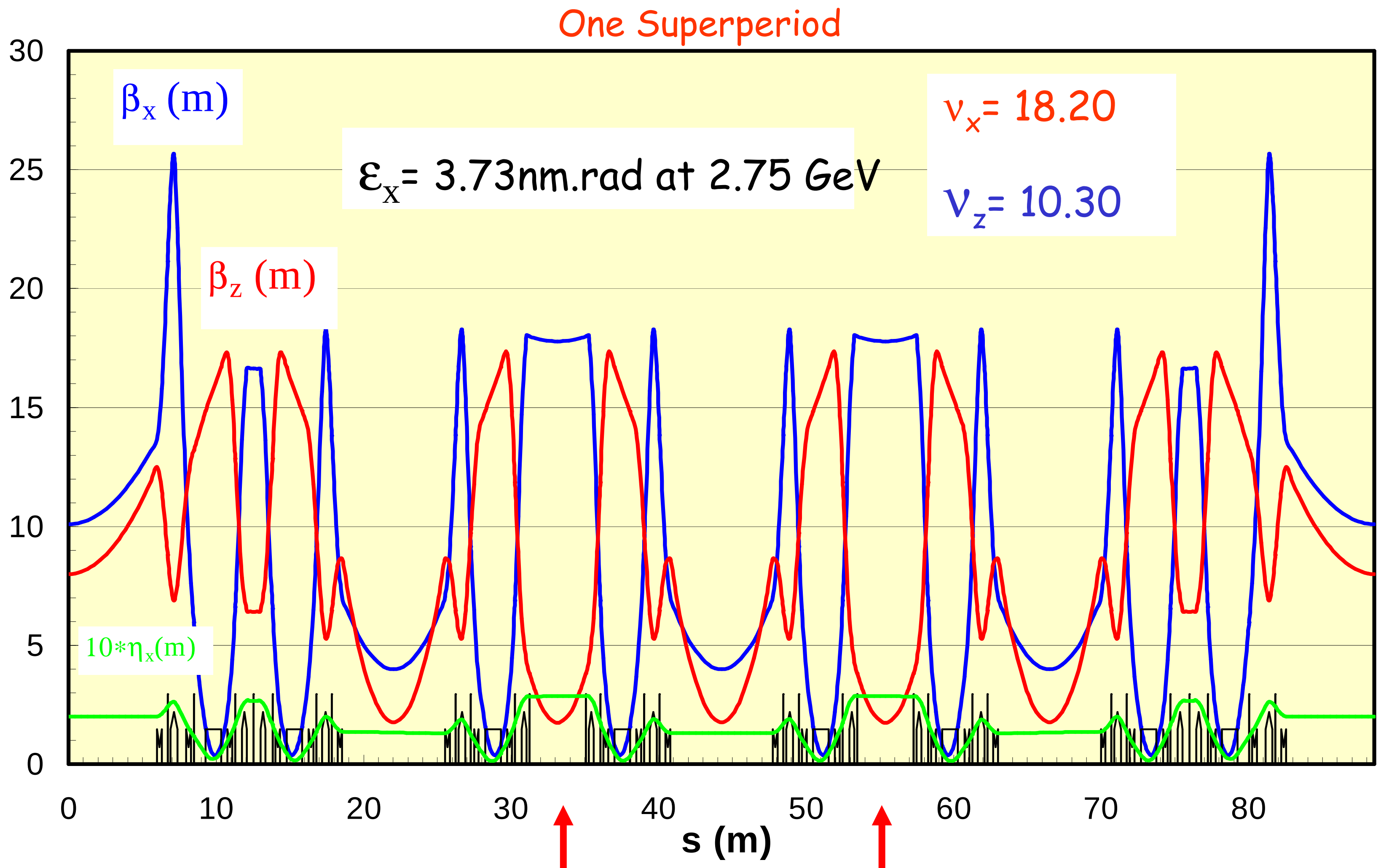
⇨ Positions where $\beta_x << \beta_z$ then $\beta_x >> \beta_z$

⇨ At least 2 such positions where the η_x is large

Optical functions for the SOLEIL initial lattice (APD)



Optical functions for the SOLEIL present lattice



Opening of the achromat to create short SS

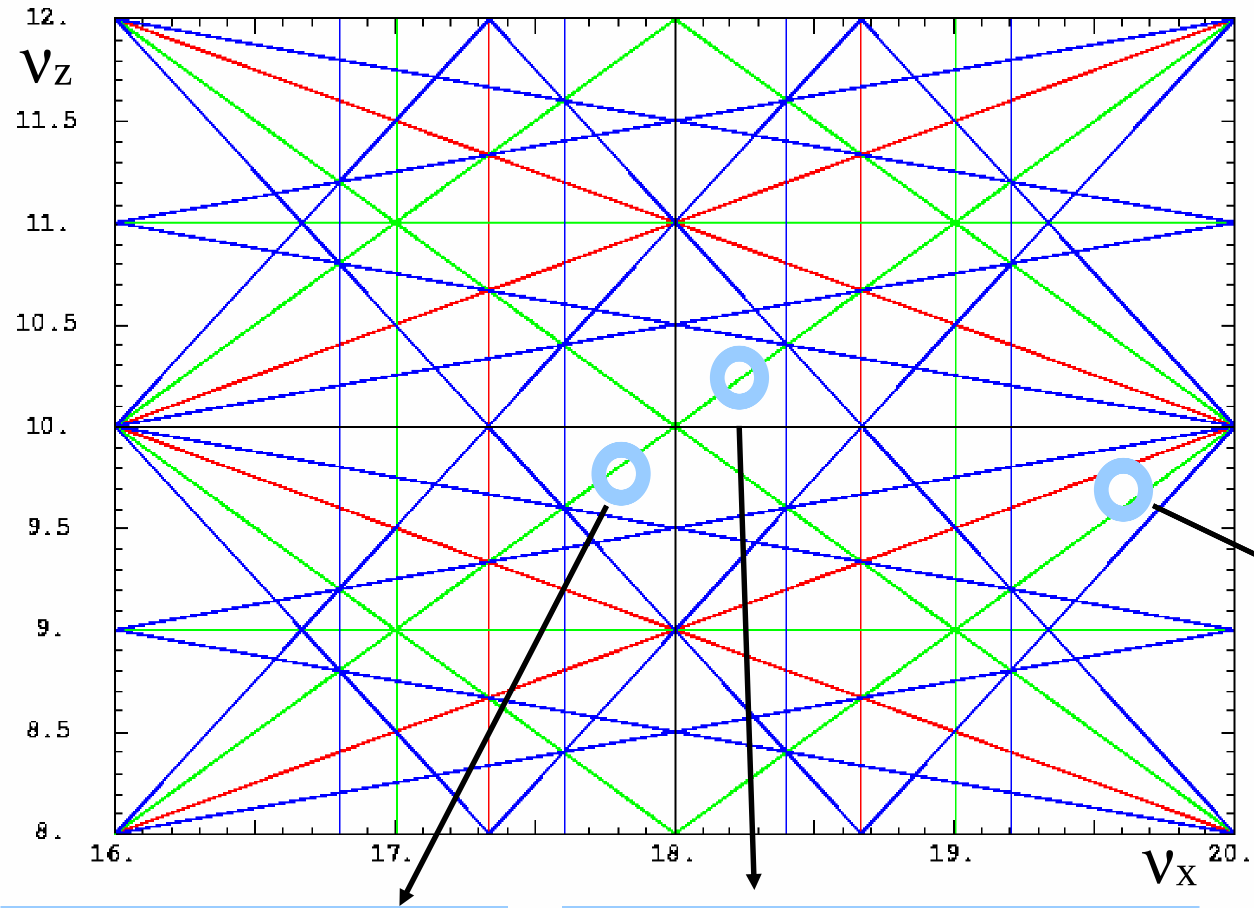
Factor of Merit ?

Source	Energy (GeV)	Θ	C(m)	ΣL_{SS}	ε_{x0} (nm.rad)	F
ALS	1.9	0.1745	197	81	5.6	0.48
BESSYII	1.9	0.1963	240	89	6.4	0.68
ESRF	6	0.09817	844	201.6	4	1.73
DIAMOND	3	0.1309	562	218.2	2.74	2.11
ELETTRA	2	0.2618	258	74.78	7	3.05
SLS	2.4	0.2440	288	63	5	6.13
SOLEIL	2.75	0.1963	354	159.6	3.7	10.66

$$F = 10^5 \times \left(\frac{\sum L_{SS}}{Circumference} \right) / \left(\varepsilon_n \right)^2$$

$$\varepsilon_n = \frac{\varepsilon_{x0}}{\left(Energy \right)^2 \times \left(\Theta \right)^3}$$

Working point : Tune Diagram



Systematic Resonances

2nd order
3rd order
4th order
5th order

m nx+n nz =p

Larger dynamic aperture
Higher emittance

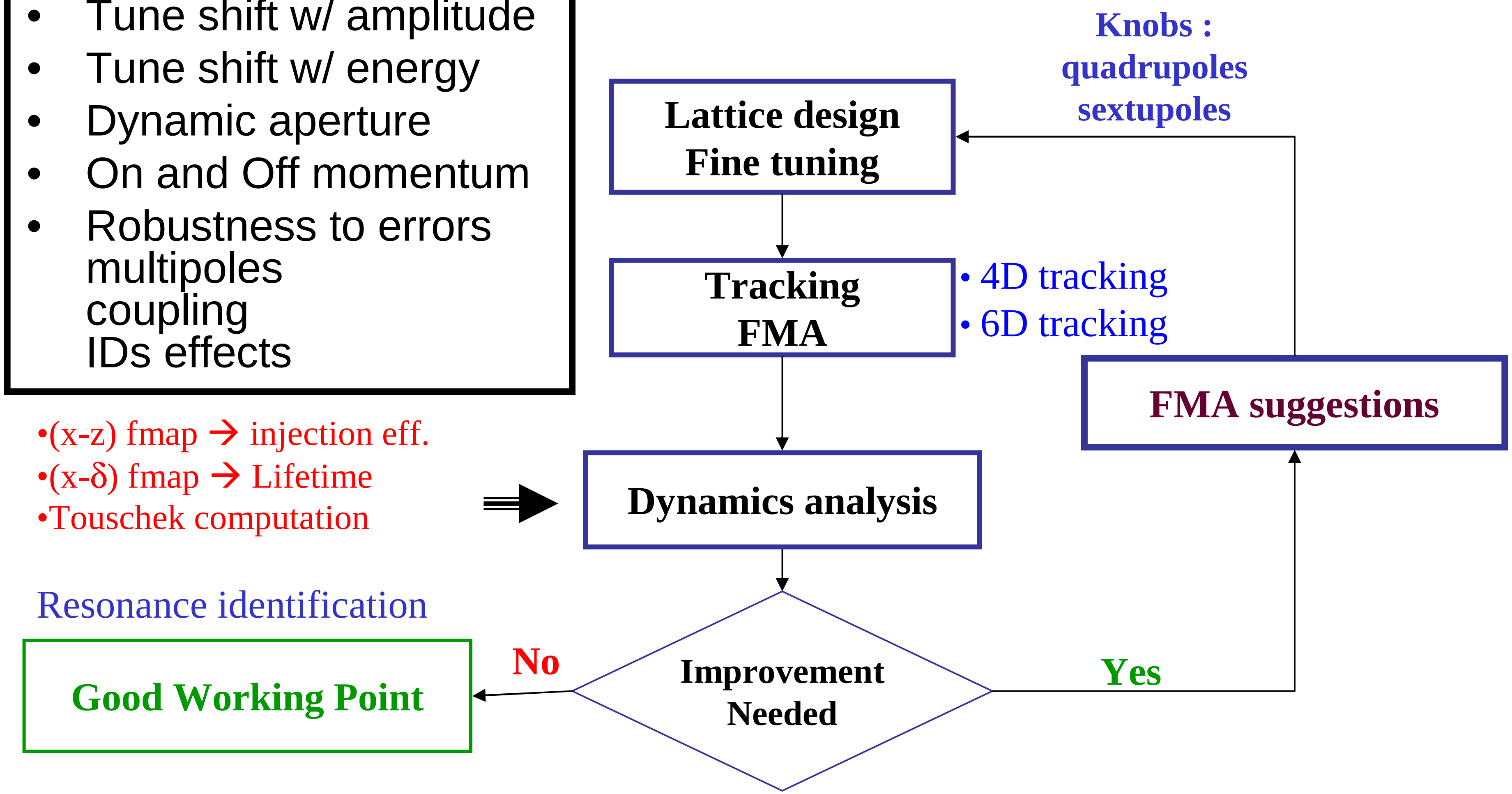
Smaller dynamic aperture
Lowest emittance

Good Compromise between
dynamic aperture and emittance

Non linear Optimization Strategy

Quality factors :

- Tune shift w/ amplitude
- Tune shift w/ energy
- Dynamic aperture
- On and Off momentum
- Robustness to errors
  multipoles
  coupling
  IDs effects



Knobs :
quadrupoles
sextupoles

Lattice design
Fine tuning

Tracking
FMA

- 4D tracking
- 6D tracking

FMA suggestions

- (x-z) fmap → injection eff.
- (x-δ) fmap → Lifetime
- Touschek computation

Dynamics analysis

Improvement Needed

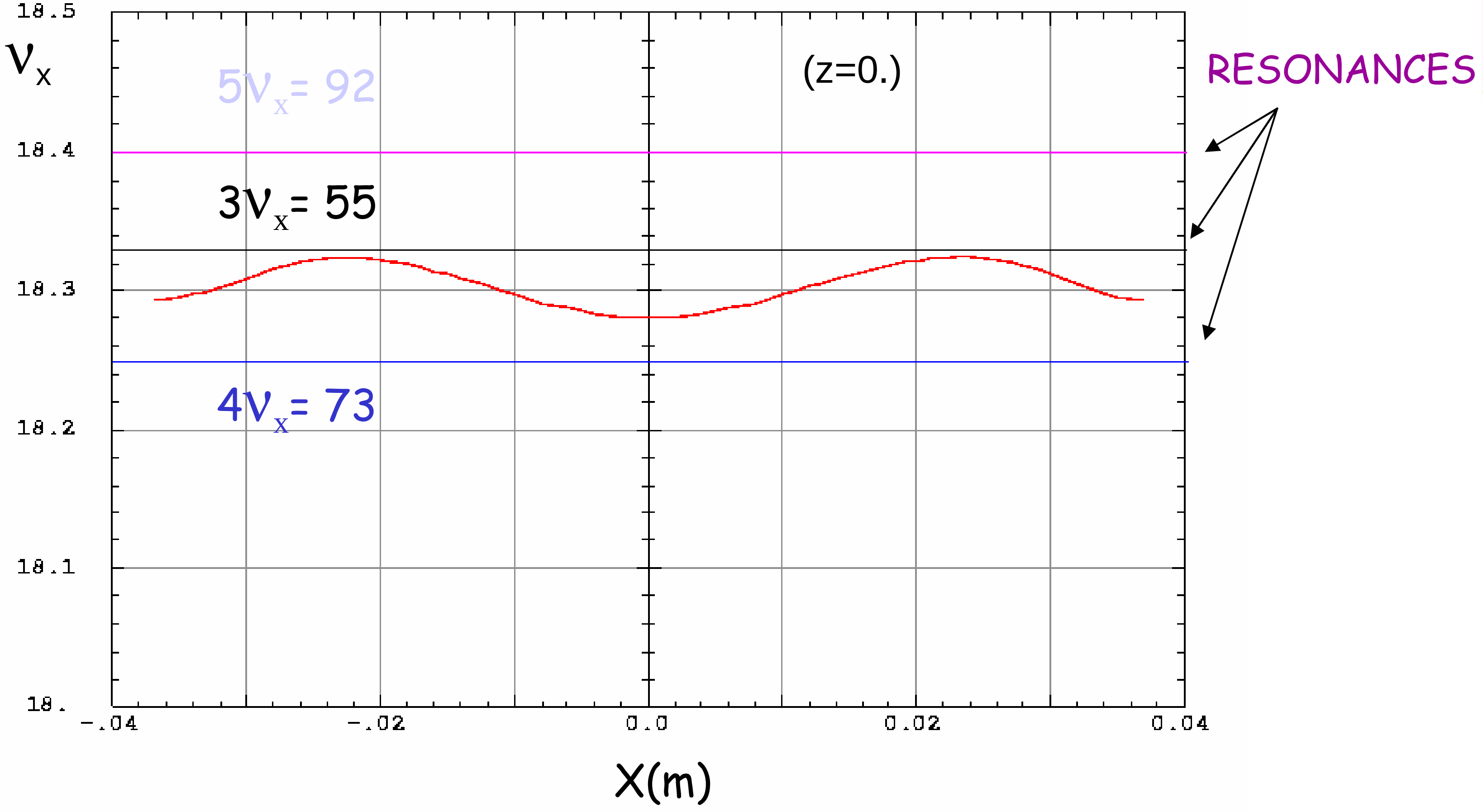
Yes

No

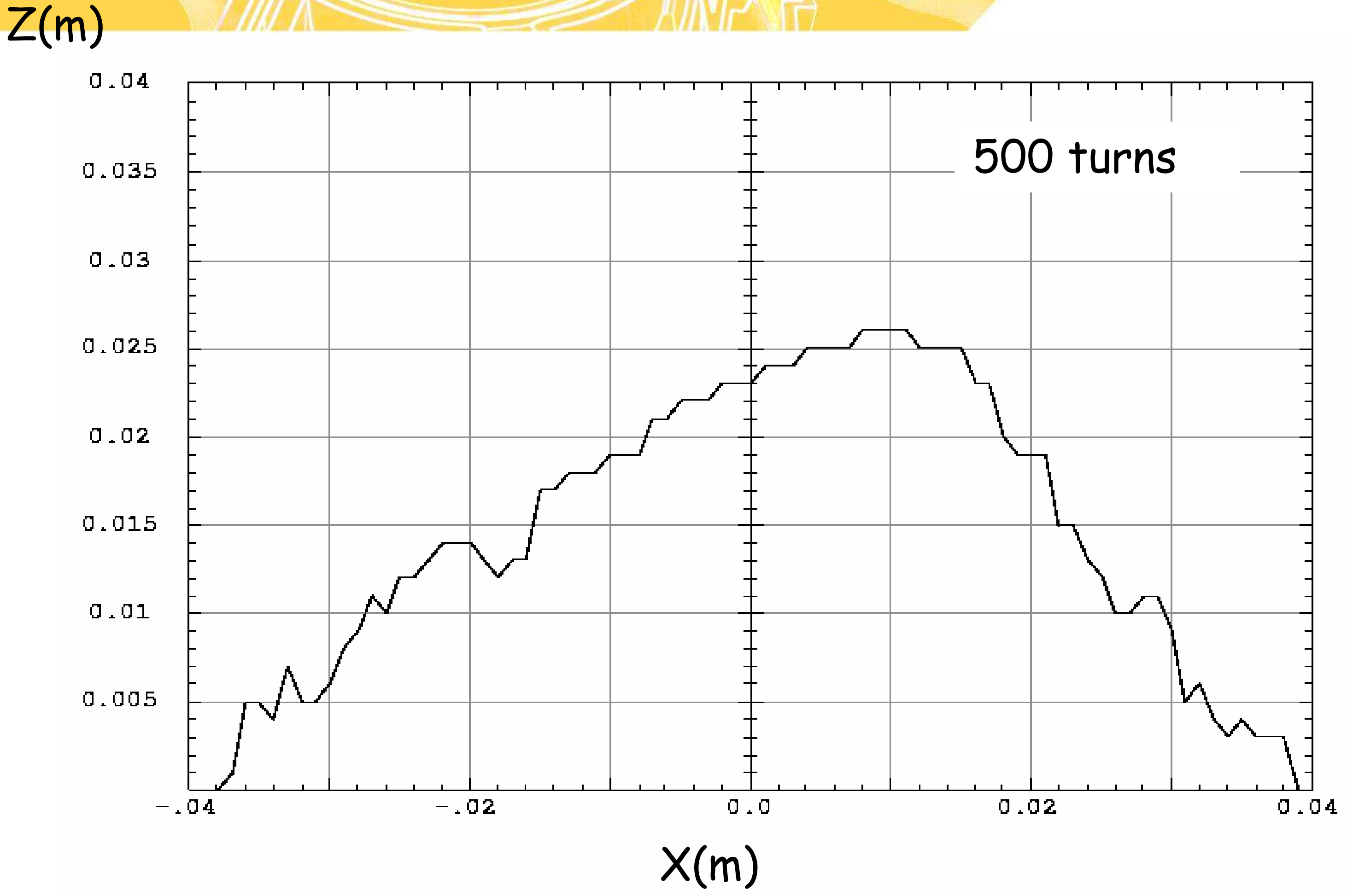
Resonance identification

Good Working Point

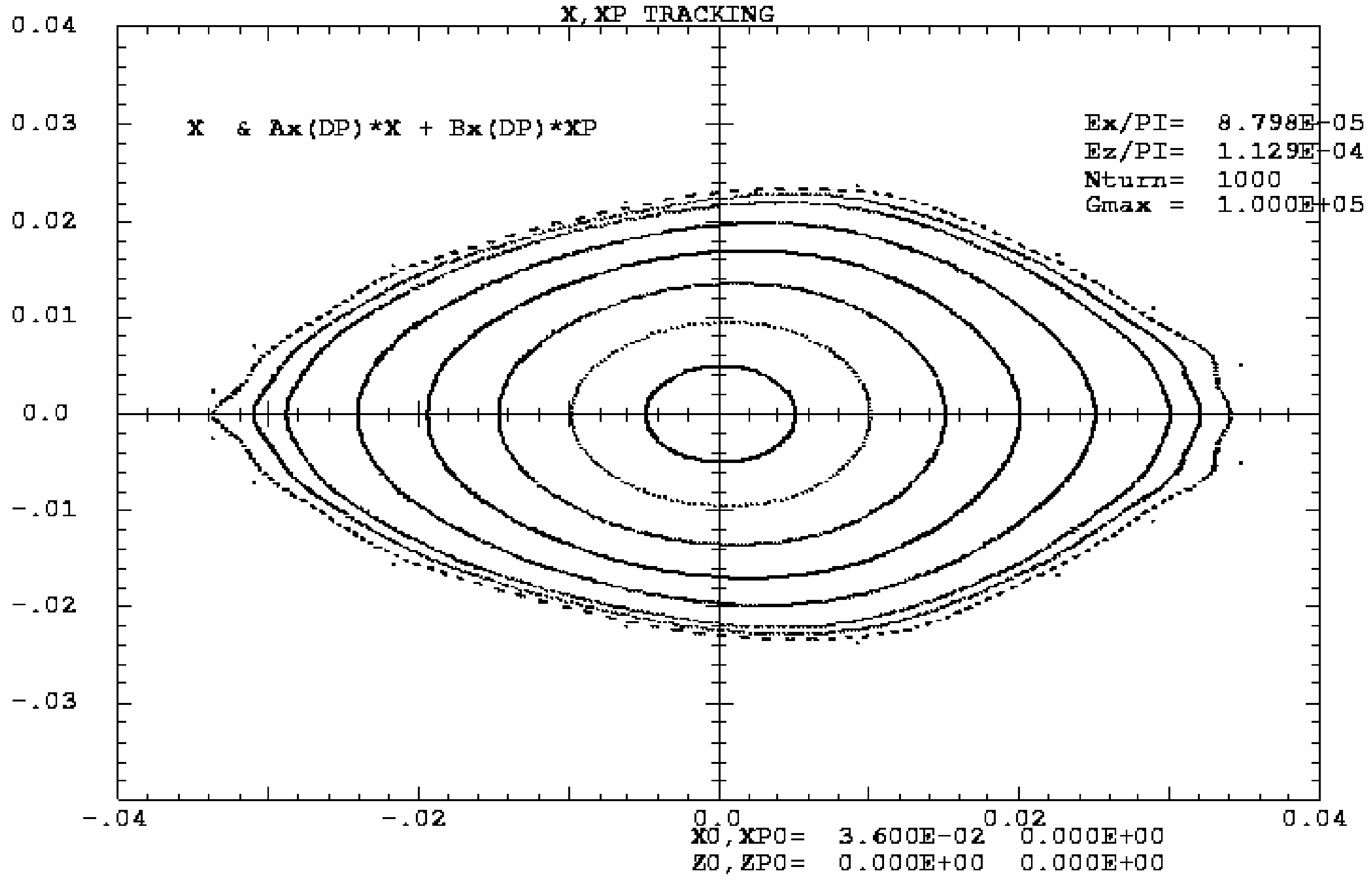
Horizontal tune shift with amplitude



On-Momentum dynamic aperture



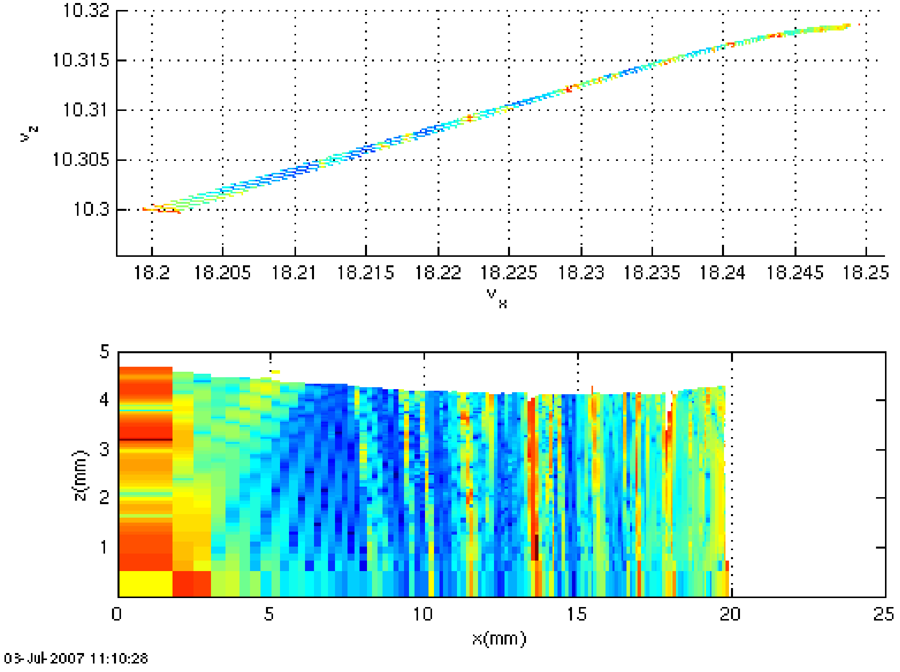
Phase Space diagram



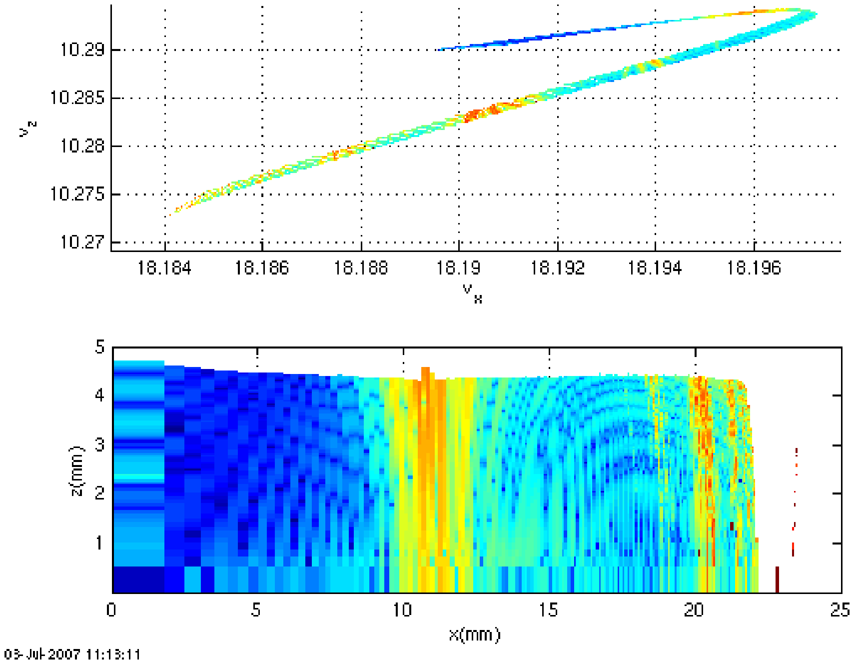
Frequency Map Analysis (NAFF)

Launched particles over a fine X-Z grid plotting
Numerical tunes
Highlighting, nonlinearity (diffusion rate)

SOLEIL

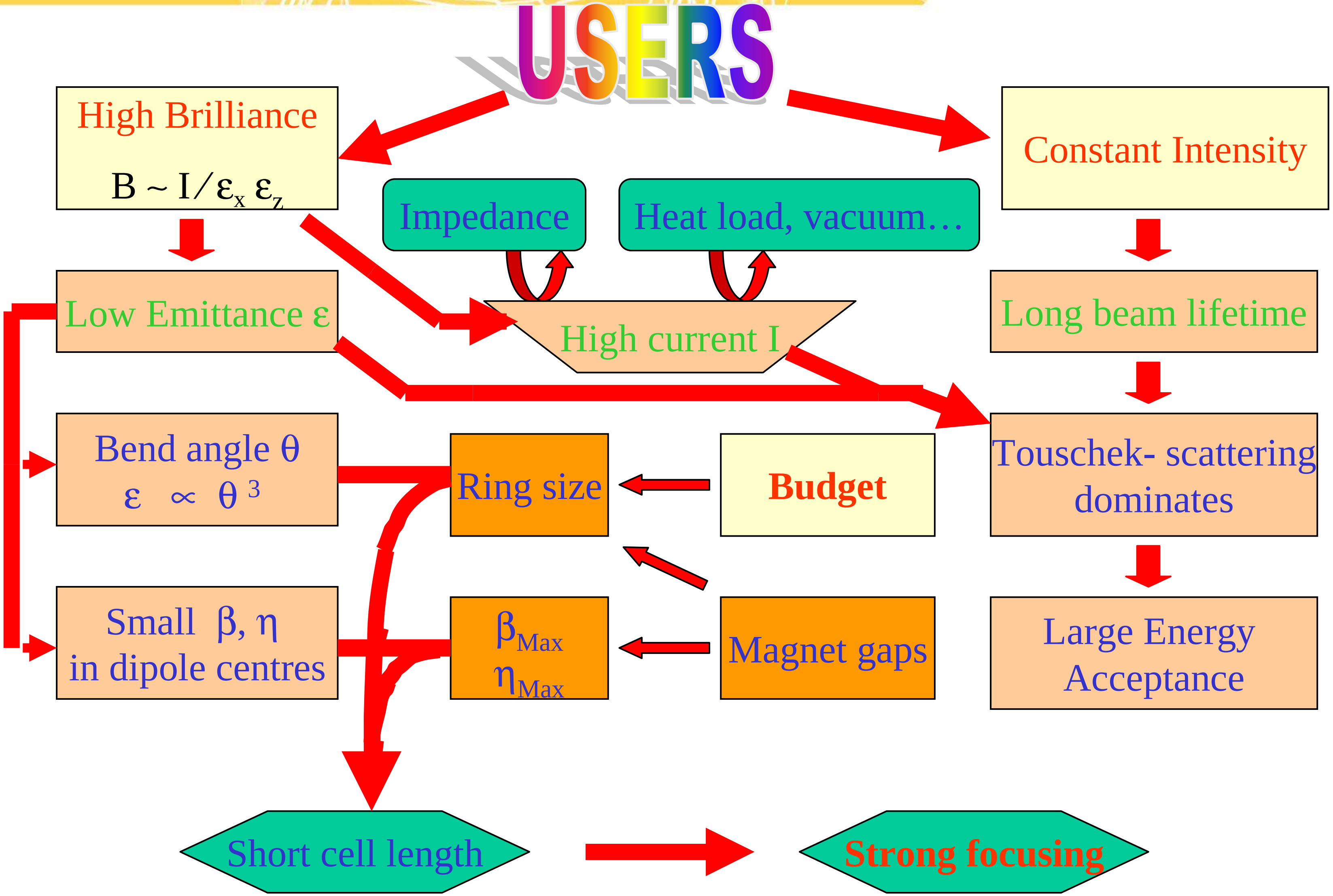


Working point1



Working point 2

Interdependence of key parameters



USERS

High Brilliance

$B \sim I/\varepsilon_x \varepsilon_z$

Constant Intensity

Impedance

Heat load, vacuum…

Low Emittance ε

High current I

Long beam lifetime

Bend angle θ

$\varepsilon \propto \theta^3$

Ring size

Budget

Touschek- scattering dominates

Small β, η in dipole centres

β_{Max}

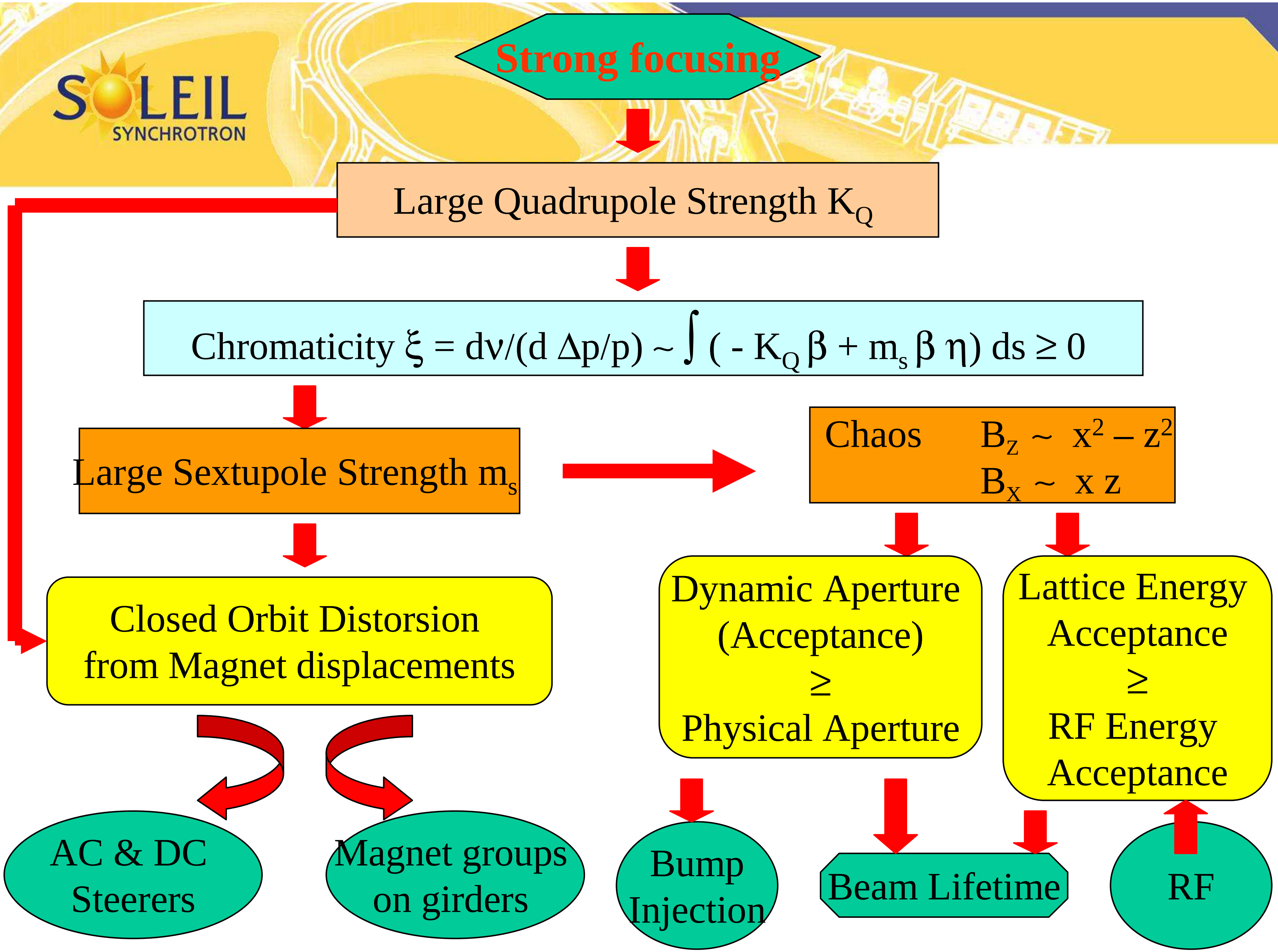
η_{Max}

Magnet gaps

Large Energy Acceptance

Short cell length

Strong focusing



Strong focusing

Large Quadrupole Strength K_Q

Chromaticity $\xi = d\nu/(d\,\Delta p/p) \sim \int (- K_Q \beta + m_s \beta\, \eta)\, ds \geq 0$

Large Sextupole Strength m_s

Chaos $B_Z \sim x^2 - z^2$
$B_X \sim x\, z$

Closed Orbit Distorsion from Magnet displacements

Dynamic Aperture (Acceptance) $\geq$ Physical Aperture

Lattice Energy Acceptance $\geq$ RF Energy Acceptance

AC & DC Steerers

Magnet groups on girders

Bump Injection

Beam Lifetime

RF