Physics of Glue

30 years
of the Lund String

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Lund
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Gösta Fest

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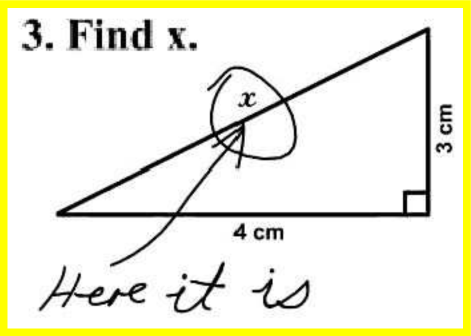

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Gösta Gustafson is a happy man whose quiver is packed with such arrows

prelude

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Understanding the interface — *metamorphosis* of coloured quarks into "white" hadrons — remains the main, most difficult, quest and headache.

Jet as a "string" of hadrons

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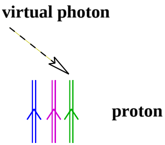

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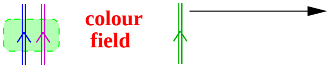

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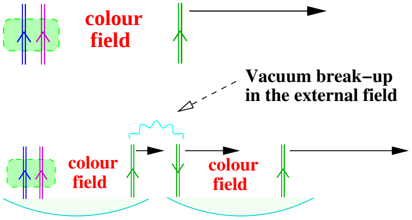

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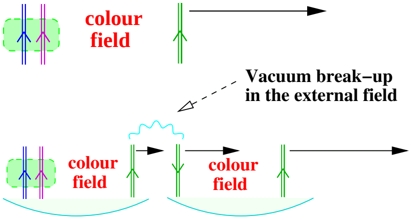
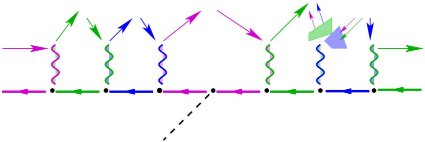


Feynman Hadron Plateau: "one" hadron per unit $\Delta\omega/\omega$

Colorful Lund wisdom



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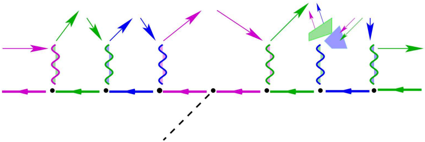

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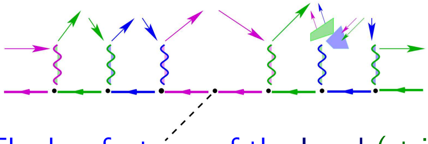

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The key features of the Lund (string) hadronization picture:

- Uniformity in *rapidity*: $dN_h = \text{const} \times d\omega_h/\omega_h$
- Limited $k_\perp$ of hadrons
- Quark combinatorics at work:
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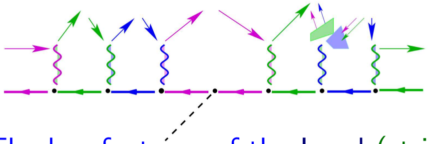

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(JETSET, Ariadne, Fritiof, Linked Dipole Chain, . . .)

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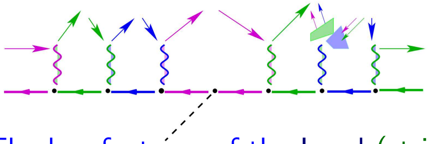

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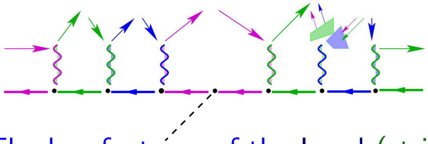

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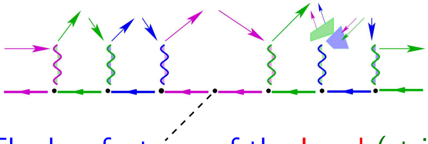

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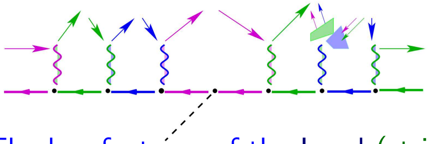

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A Semiclassical Model for Quark Jet Fragmentation.
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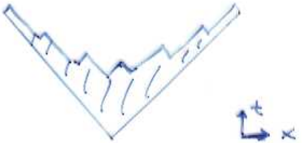

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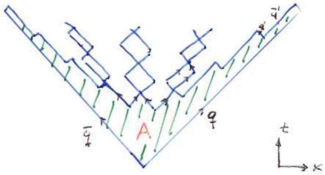

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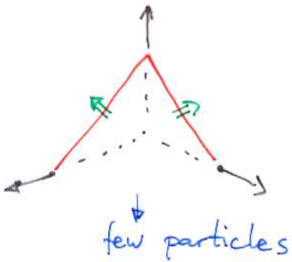

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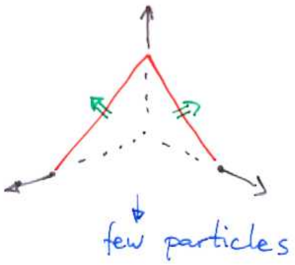
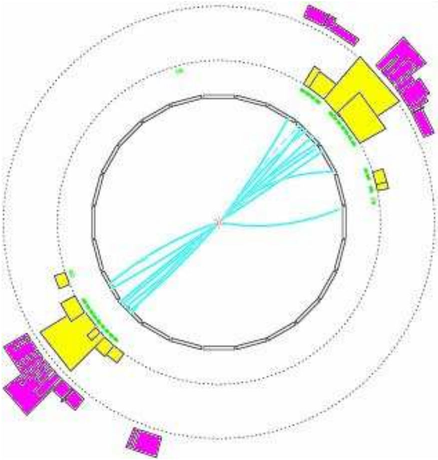


The crucial step:

Stressing the rôle of *colour topology* in multiple hadroproduction

Hadrons between Jets

$q\bar{q} \longrightarrow$ hadrons



Near 'perfect' 2-jet event

2 well collimated jets of particles.

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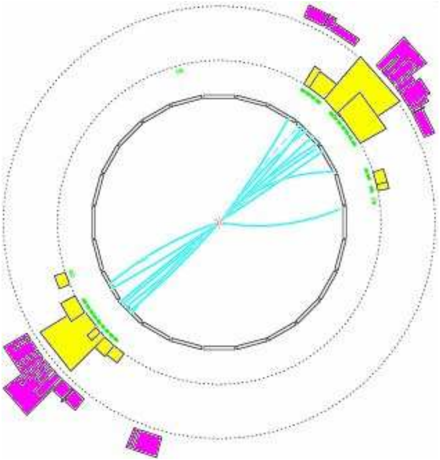

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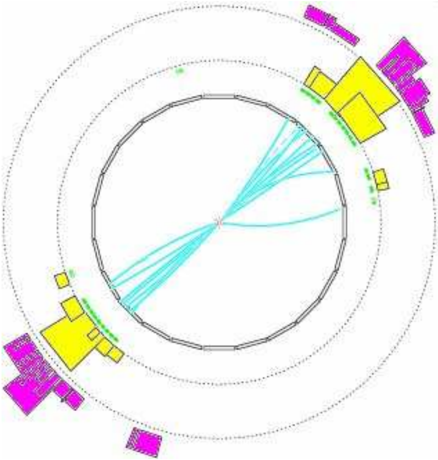

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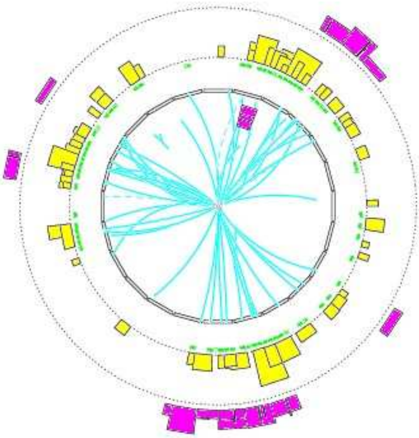
Moreover,

In 10% of e^+e^- annihilation events
— striking fluctuations !

Third jet



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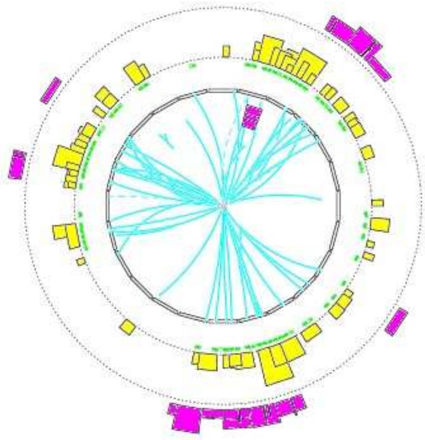

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Hard gluon bremsstrahlung off the $q\bar{q}$ pair *may be expected* to give rise to 3-jet events . . .

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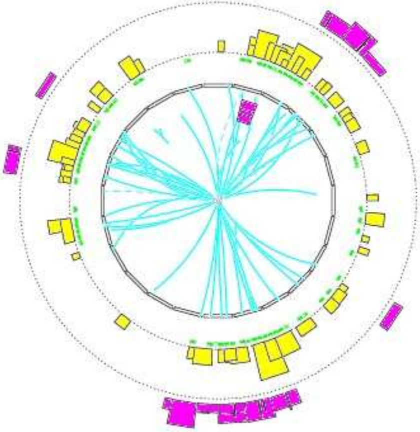

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The first QCD analysis was done by J.Ellis, M.Gaillard & G.Ross (1976)

- Planar events with large $k_\perp$;
- How to measure gluon spin ;
- Gluon jet – softer, more populated.

How does gluon hadronize?

QCD possesses $N_c^2 - 1$ gauge fields — vector gluons g.

At large distances, they are supposed to "glue" quarks together.

At small distances (space-time intervals) g is as legitimate a parton as q is.

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Gluon $\simeq$ quark-antiquark pair:

$$3 \otimes \bar{3} = N_c^2 = 9 \quad \simeq \quad 8 = N_c^2 - 1.$$

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Lund model interpretation of a *gluon* —

Gluon – a "*kink*" on the "string" (colour tube)
that connects the quark with the antiquark

Colour "drag"

Look at hadrons produced in a $q\bar{q}$ + photon e^+e^- annihilation event (recall Tornbjörn's)

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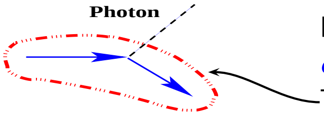

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The hot-dog of hadrons that was "*cylindric*" in the cms, is now *lopsided* [boosted string]

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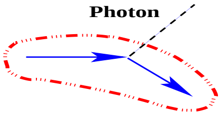

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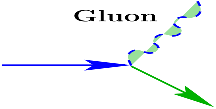

Now substitute a gluon for the photon in the same kinematics.

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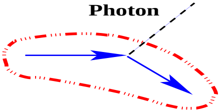

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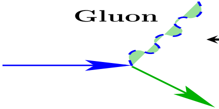
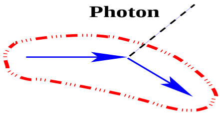


The gluon carries "double" colour charge; quark pair is *repainted* into octet colour state.

Colour "drag"



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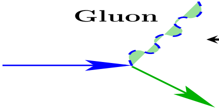

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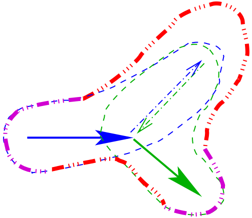

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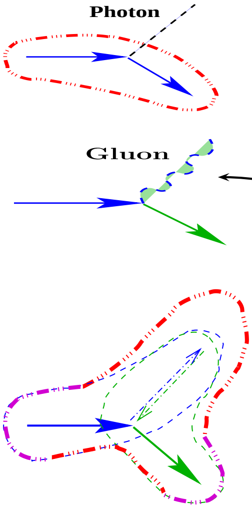

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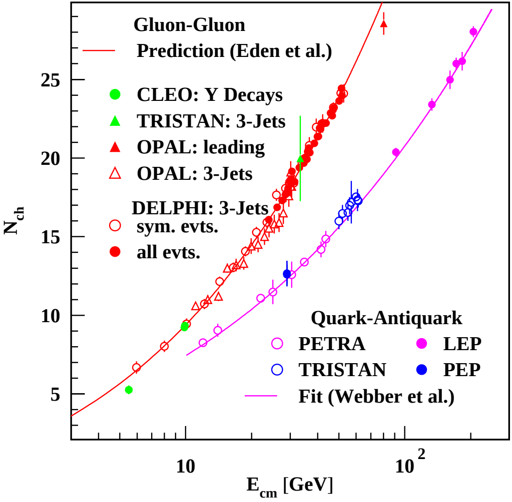
The first immediate consequence :

Double Multiplicity of hadrons
in fragmentation of the *gluon*

Comparing hadron multiplicities



Look at experimental findings

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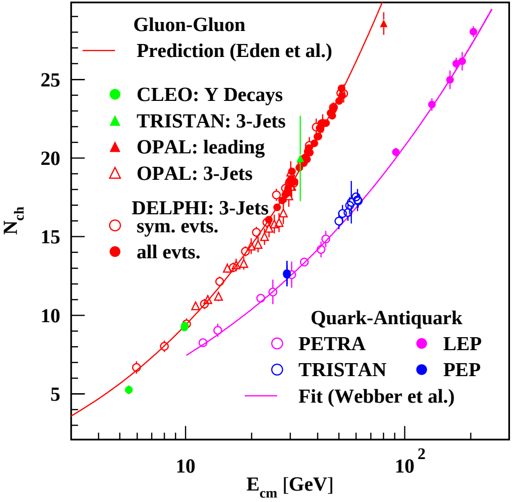

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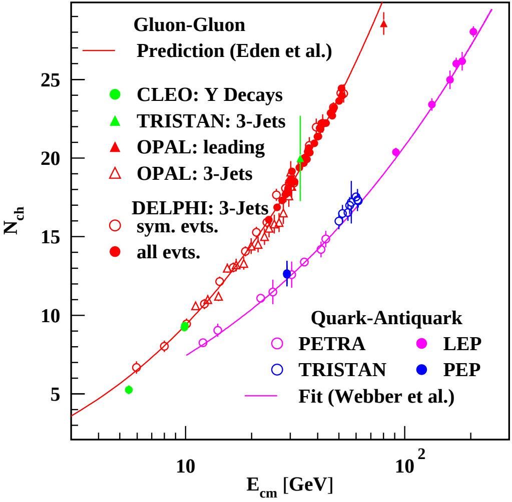

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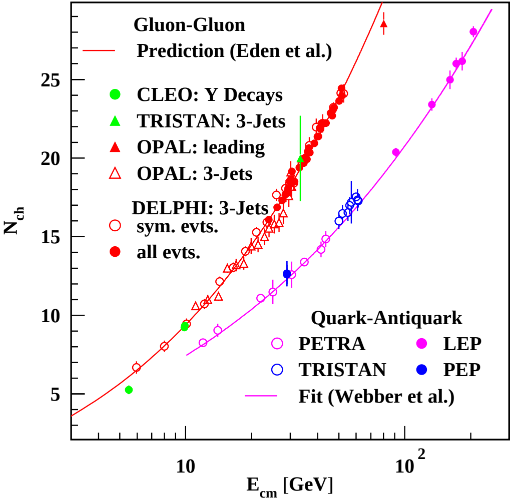

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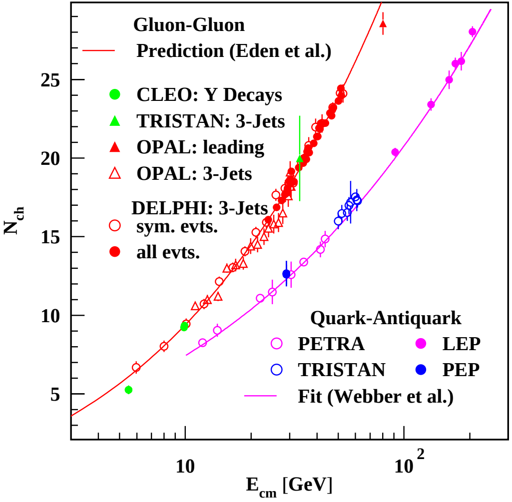

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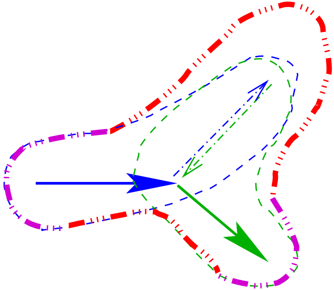
Now let's look at a more subtle consequence of Lund wisdom

intERjet QCD radiation



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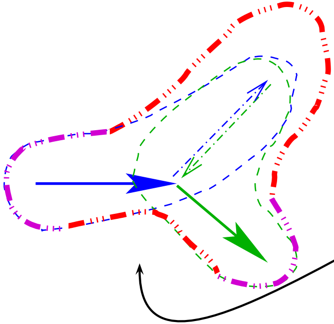

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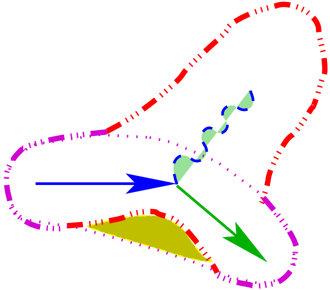

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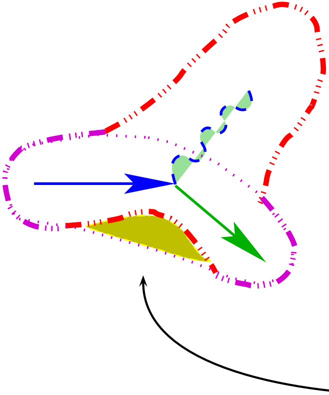

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Destructive interference
from the QCD point of view

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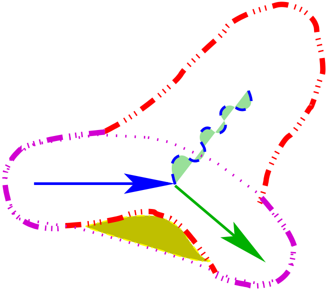

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QCD prediction :

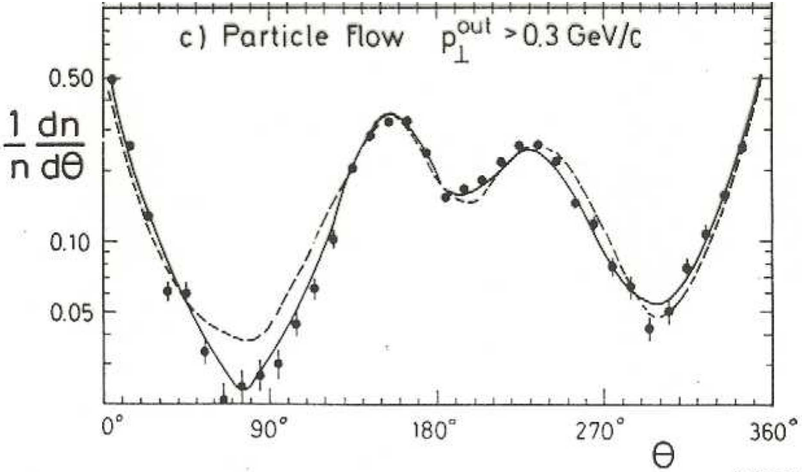
$$\frac{dN_{q\bar{q}}^{(q\bar{q}\gamma)}}{dN_{q\bar{q}}^{(q\bar{q}g)}} \simeq \frac{2(N_c^2-1)}{N_c^2-2} = \frac{16}{7}$$

(experiment: 2.3 ± 0.2)

Ratios of hadron flows between jets in various multi-jet processes — example of non-trivial CIS (collinear-and-infrared-safe) QCD observable

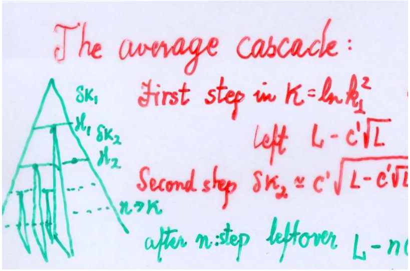
String Effect

Prediction: 1978 (Lund) Measurement: 1981 (JADE)



Gluon multiplication

Gösta's *Origami*



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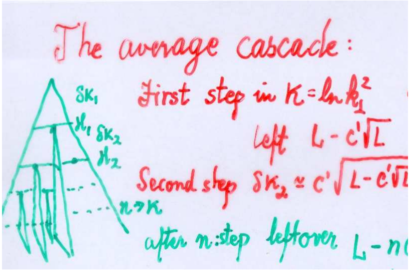

- *Fractal structure* of parton cascades
- Multiplicity *anomalous dimension*
- Fragmentation functions

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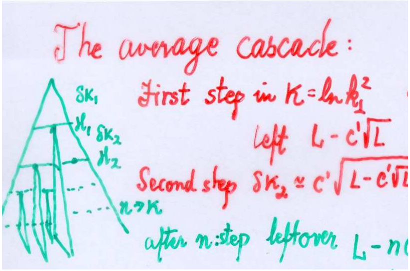

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radiation of a gluon $\quad\equiv\quad$ dipole $\longrightarrow$ two dipoles

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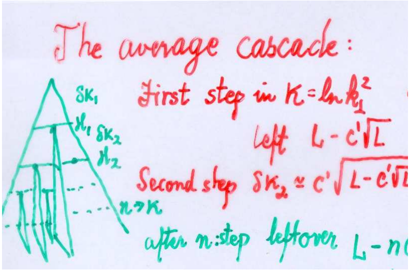

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The base for the *Ariadne* Monte Carlo generator

gluons in-between-jets

2- and 3-prong colour antennae are sort of "trivial": coherence being taken care of, the answers turned out to be essentially additive.

The case of $2 \to 2$ hard parton scattering is more involved (4 emitters), especially so for *gluon–gluon* scattering.

The difficult quest of sorting out large angle gluon radiation in all orders in $(\alpha_s \log Q)^n$ was set up and solved by George Sterman and collaborators.

Here one encounters 6 (5 for $SU(3)$) colour channels that mix with each other under soft gluon radiation, and the classical picture of gluon (or dipole) multiplication is likely to fail.

A recent (2005) addition to the problem (G.Marchesini & YLD) made one think of a *hidden simplicity* ...

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$$\frac{\partial}{\partial \ln Q} M \propto \left\{ -N_c \ln\left(\frac{t\,u}{s^2}\right) \cdot \hat{\Gamma} \right\} \cdot M, \qquad \hat{\Gamma} V_i = E_i V_i.$$

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Three "ain't-so-simple" ones were found to satisfy the cubic equation:

$$\left[E_i - \frac{4}{3}\right]^3 - \frac{(1+3b^2)(1+3x^2)}{3}\left[E_i - \frac{4}{3}\right] - \frac{2(1-9b^2)(1-9x^2)}{27} = 0,$$

where

$$x = \frac{1}{N_c}, \qquad b \equiv \frac{\ln(t/s) - \ln(u/s)}{\ln(t/s) + \ln(u/s)}$$

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Mark the *mysterious symmetry* w.r.t. to $x \to b$: interchanging internal (group rank) and external (scattering angle) variables of the problem ...

more hints

Some news concerning

apparent complexity/hidden simplicity

of gluon dynamics

more hints

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... continuing Andrjey's string of *puzzles*

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Have a look at the *simplest* element of the parton multiplication Hamiltonian (non-singlet anomalous dimension) in three loops, α_s^3

3rd loop non-singlet a.d.

$$P^{(2)+}_{\mathrm{ns}}(x) = 16 C_A C_F n_f \Bigg(\frac{1}{6} p_{\mathrm{qq}}(x) \Bigg[\frac{10}{3}\zeta_2 - \frac{209}{36} - 9\zeta_3 - \frac{167}{18}\mathrm{H}_0 + 2\mathrm{H}_0\zeta_2 - 7\mathrm{H}_0$$

$$+3\mathrm{H}_{1,0,0} - \mathrm{H}_3 \Bigg] + \frac{1}{3} p_{\mathrm{qq}}(-x) \Bigg[\frac{3}{2}\zeta_3 - \frac{5}{3}\zeta_2 - \mathrm{H}_{-2,0} - 2\mathrm{H}_{-1}\zeta_2 - \frac{10}{3}\mathrm{H}_{-1,0} - \mathrm{H}_-$$

$$+2\mathrm{H}_{-1,2} + \frac{1}{2}\mathrm{H}_0\zeta_2 + \frac{5}{3}\mathrm{H}_{0,0} + \mathrm{H}_{0,0,0} - \mathrm{H}_3 \Bigg] + (1-x)\Bigg[\frac{1}{6}\zeta_2 - \frac{257}{54} - \frac{43}{18}\mathrm{H}_0 - \frac{1}{6}$$

$$-(1+x)\Bigg[\frac{2}{3}\mathrm{H}_{-1,0} + \frac{1}{2}\mathrm{H}_2 \Bigg] + \frac{1}{3}\zeta_2 + \mathrm{H}_0 + \frac{1}{6}\mathrm{H}_{0,0} + \delta(1-x)\Bigg[\frac{5}{4} - \frac{167}{54}\zeta_2 + \frac{1}{20}\zeta_2$$

$$+16\, C_A {C_F}^2 \Bigg(p_{\mathrm{qq}}(x) \Bigg[\frac{5}{6}\zeta_3 - \frac{69}{20}{\zeta_2}^2 - \mathrm{H}_{-3,0} - 3\mathrm{H}_{-2}\zeta_2 - 14\mathrm{H}_{-2,-1,0} + 3\mathrm{H}_{-2,0}$$

$$-4\mathrm{H}_{-2,2} - \frac{151}{48}\mathrm{H}_0 + \frac{41}{12}\mathrm{H}_0\zeta_2 - \frac{17}{2}\mathrm{H}_0\zeta_3 - \frac{13}{4}\mathrm{H}_{0,0} - 4\mathrm{H}_{0,0}\zeta_2 - \frac{23}{12}\mathrm{H}_{0,0,0} + 5\mathrm{H}$$

$$-24\mathrm{H}_1\zeta_3 - 16\mathrm{H}_{1,-2,0} + \frac{67}{9}\mathrm{H}_{1,0} - 2\mathrm{H}_{1,0}\zeta_2 + \frac{31}{3}\mathrm{H}_{1,0,0} + 11\mathrm{H}_{1,0,0,0} + 8\mathrm{H}_{1,1,0,0}$$

3rd loop, more

$$+\frac{67}{9}\mathrm{H}_2 - 2\mathrm{H}_2\zeta_2 + \frac{11}{3}\mathrm{H}_{2,0} + 5\mathrm{H}_{2,0,0} + \mathrm{H}_{3,0}\Big] + p_{\mathrm{qq}}(-x)\Big[\frac{1}{4}\zeta_2{}^2 - \frac{67}{9}\zeta_2 + \frac{31}{4}\zeta_3$$

$$-32\mathrm{H}_{-2}\zeta_2 - 4\mathrm{H}_{-2,-1,0} - \frac{31}{6}\mathrm{H}_{-2,0} + 21\mathrm{H}_{-2,0,0} + 30\mathrm{H}_{-2,2} - \frac{31}{3}\mathrm{H}_{-1}\zeta_2 - 42\mathrm{H}$$

$$-4\mathrm{H}_{-1,-2,0} + 56\mathrm{H}_{-1,-1}\zeta_2 - 36\mathrm{H}_{-1,-1,0,0} - 56\mathrm{H}_{-1,-1,2} - \frac{134}{9}\mathrm{H}_{-1,0} - 42\mathrm{H}_{-1}$$

$$+32\mathrm{H}_{-1,3} - \frac{31}{6}\mathrm{H}_{-1,0,0} + 17\mathrm{H}_{-1,0,0,0} + \frac{31}{3}\mathrm{H}_{-1,2} + 2\mathrm{H}_{-1,2,0} + \frac{13}{12}\mathrm{H}_0\zeta_2 + \frac{29}{2}\mathrm{H}$$

$$+13\mathrm{H}_{0,0}\zeta_2 + \frac{89}{12}\mathrm{H}_{0,0,0} - 5\mathrm{H}_{0,0,0,0} - 7\mathrm{H}_2\zeta_2 - \frac{31}{6}\mathrm{H}_3 - 10\mathrm{H}_4\Big] + (1-x)\Big[\frac{133}{36} +$$

$$-\frac{167}{4}\zeta_3 - 2\mathrm{H}_0\zeta_3 - 2\mathrm{H}_{-3,0} + \mathrm{H}_{-2}\zeta_2 + 2\mathrm{H}_{-2,-1,0} - 3\mathrm{H}_{-2,0,0} + \frac{77}{4}\mathrm{H}_{0,0,0} - \frac{209}{6}$$

$$+4\mathrm{H}_{1,0,0} + \frac{14}{3}\mathrm{H}_{1,0}\Big] + (1+x)\Big[\frac{43}{2}\zeta_2 - 3\zeta_2{}^2 + \frac{25}{2}\mathrm{H}_{-2,0} - 31\mathrm{H}_{-1}\zeta_2 - 14\mathrm{H}_{-1,-}$$

$$+24\mathrm{H}_{-1,2} + 23\mathrm{H}_{-1,0,0} + \frac{55}{2}\mathrm{H}_0\zeta_2 + 5\mathrm{H}_{0,0}\zeta_2 + \frac{1457}{48}\mathrm{H}_0 - \frac{1025}{36}\mathrm{H}_{0,0} - \frac{155}{6}\mathrm{H}_2$$

3rd loop, and more

$$+2\mathrm{H}_{2,0,0} - 3\mathrm{H}_4\Big] - 5\zeta_2 - \frac{1}{2}\zeta_2{}^2 + 50\zeta_3 - 2\mathrm{H}_{-3,0} - 7\mathrm{H}_{-2,0} - \mathrm{H}_0\zeta_3 - \frac{37}{2}\mathrm{H}_0\zeta_2$$

$$-2\mathrm{H}_{0,0}\zeta_2 + \frac{185}{6}\mathrm{H}_{0,0} - 22\mathrm{H}_{0,0,0} - 4\mathrm{H}_{0,0,0,0} + \frac{28}{3}\mathrm{H}_2 + 6\mathrm{H}_3 + \delta(1-x)\Big[\frac{151}{64} +$$

$$-\frac{247}{60}\zeta_2{}^2 + \frac{211}{12}\zeta_3 + \frac{15}{2}\zeta_5\Big]\Big) + 16\, C_A{}^2 C_F\Big(p_{\mathrm{qq}}(x)\Big[\frac{245}{48} - \frac{67}{18}\zeta_2 + \frac{12}{5}\zeta_2{}^2 + \frac{1}{2}$$

$$+\mathrm{H}_{-3,0} + 4\mathrm{H}_{-2,-1,0} - \frac{3}{2}\mathrm{H}_{-2,0} - \mathrm{H}_{-2,0,0} + 2\mathrm{H}_{-2,2} - \frac{31}{12}\mathrm{H}_0\zeta_2 + 4\mathrm{H}_0\zeta_3 + \frac{389}{72}$$

$$-\mathrm{H}_{0,0,0,0} + 9\mathrm{H}_1\zeta_3 + 6\mathrm{H}_{1,-2,0} - \mathrm{H}_{1,0}\zeta_2 - \frac{11}{4}\mathrm{H}_{1,0,0} - 3\mathrm{H}_{1,0,0,0} - 4\mathrm{H}_{1,1,0,0} + 4$$

$$+\frac{11}{12}\mathrm{H}_3 + \mathrm{H}_4\Big] + p_{\mathrm{qq}}(-x)\Big[\frac{67}{18}\zeta_2 - \zeta_2{}^2 - \frac{11}{4}\zeta_3 - \mathrm{H}_{-3,0} + 8\mathrm{H}_{-2}\zeta_2 + \frac{11}{6}\mathrm{H}_{-2,0}$$

$$-3\mathrm{H}_{-1,0,0,0} + \frac{11}{3}\mathrm{H}_{-1}\zeta_2 + 12\mathrm{H}_{-1}\zeta_3 - 16\mathrm{H}_{-1,-1}\zeta_2 + 8\mathrm{H}_{-1,-1,0,0} + 16\mathrm{H}_{-1,-1,2}$$

$$-8\mathrm{H}_{-2,2} + 11\mathrm{H}_{-1,0}\zeta_2 + \frac{11}{6}\mathrm{H}_{-1,0,0} - \frac{11}{3}\mathrm{H}_{-1,2} - 8\mathrm{H}_{-1,3} - \frac{3}{4}\mathrm{H}_0 - \frac{1}{6}\mathrm{H}_0\zeta_2 - 4$$

3rd loop, and again

$$-3\mathrm{H}_{0,0}\zeta_2 - \frac{31}{12}\mathrm{H}_{0,0,0} + \mathrm{H}_{0,0,0,0} + 2\mathrm{H}_2\zeta_2 + \frac{11}{6}\mathrm{H}_3 + 2\mathrm{H}_4\Big] + (1-x)\Big[\frac{1883}{108} - \frac{1}{2}$$

$$-\mathrm{H}_{-2,-1,0} + \frac{1}{2}\mathrm{H}_{-3,0} - \frac{1}{2}\mathrm{H}_{-2}\zeta_2 + \frac{1}{2}\mathrm{H}_{-2,0,0} + \frac{523}{36}\mathrm{H}_0 + \mathrm{H}_0\zeta_3 - \frac{13}{3}\mathrm{H}_{0,0} - \frac{5}{2}\mathrm{H}$$

$$-2\mathrm{H}_{1,0,0}\Big] + (1+x)\Big[8\mathrm{H}_{-1}\zeta_2 + 4\mathrm{H}_{-1,-1,0} + \frac{8}{3}\mathrm{H}_{-1,0} - 5\mathrm{H}_{-1,0,0} - 6\mathrm{H}_{-1,2} - \frac{13}{3}$$

$$-\frac{43}{4}\zeta_3 - \frac{5}{2}\mathrm{H}_{-2,0} - \frac{11}{2}\mathrm{H}_0\zeta_2 - \frac{1}{2}\mathrm{H}_2\zeta_2 - \frac{5}{4}\mathrm{H}_{0,0}\zeta_2 + 7\mathrm{H}_2 - \frac{1}{4}\mathrm{H}_{2,0,0} + 3\mathrm{H}_3 + \frac{3}{4}$$

$$+\frac{1}{4}\zeta_2{}^2 - \frac{8}{3}\zeta_2 + \frac{17}{2}\zeta_3 + \mathrm{H}_{-2,0} - \frac{19}{2}\mathrm{H}_0 + \frac{5}{2}\mathrm{H}_0\zeta_2 - \mathrm{H}_0\zeta_3 + \frac{13}{3}\mathrm{H}_{0,0} + \frac{5}{2}\mathrm{H}_{0,0,0}$$

$$-\delta(1-x)\Big[\frac{1657}{576} - \frac{281}{27}\zeta_2 + \frac{1}{8}\zeta_2{}^2 + \frac{97}{9}\zeta_3 - \frac{5}{2}\zeta_5\Big]\Big) + 16\, C_F n_f^2\Big(\frac{1}{18}p_{\mathrm{qq}}(x)\Big[\mathrm{H}_{0,}$$

$$+(1-x)\Big[\frac{13}{54} + \frac{1}{9}\mathrm{H}_0\Big] - \delta(1-x)\Big[\frac{17}{144} - \frac{5}{27}\zeta_2 + \frac{1}{9}\zeta_3\Big]\Big) + 16\, C_F{}^2 n_f\Big(\frac{1}{3}p_{\mathrm{qq}}(x)\Big[$$

3rd loop, and still some more

$$-\frac{55}{16}+\frac{5}{8}\mathrm{H}_0+\mathrm{H}_0\zeta_2+\frac{3}{2}\mathrm{H}_{0,0}-\mathrm{H}_{0,0,0}-\frac{10}{3}\mathrm{H}_{1,0}-\frac{10}{3}\mathrm{H}_2-2\mathrm{H}_{2,0}-2\mathrm{H}_3\Big]+\frac{2}{3}$$

$$-\frac{3}{2}\zeta_3+\mathrm{H}_{-2,0}+2\mathrm{H}_{-1}\zeta_2+\frac{10}{3}\mathrm{H}_{-1,0}+\mathrm{H}_{-1,0,0}-2\mathrm{H}_{-1,2}-\frac{1}{2}\mathrm{H}_0\zeta_2-\frac{5}{3}\mathrm{H}_{0,0}-$$

$$-(1-x)\Big[\frac{10}{9}+\frac{19}{18}\mathrm{H}_{0,0}-\frac{4}{3}\mathrm{H}_1+\frac{2}{3}\mathrm{H}_{1,0}+\frac{4}{3}\mathrm{H}_2\Big]+(1+x)\Big[\frac{4}{3}\mathrm{H}_{-1,0}-\frac{25}{24}\mathrm{H}_0+$$

$$+\frac{7}{9}\mathrm{H}_{0,0}+\frac{4}{3}\mathrm{H}_2-\delta(1-x)\Big[\frac{23}{16}-\frac{5}{12}\zeta_2-\frac{29}{30}\zeta_2{}^2+\frac{17}{6}\zeta_3\Big]\Big)+16\,{C_F}^3\Big(p_{\mathrm{qq}}(x)\Big[$$

$$+6\mathrm{H}_{-2}\zeta_2+12\mathrm{H}_{-2,-1,0}-6\mathrm{H}_{-2,0,0}-\frac{3}{16}\mathrm{H}_0-\frac{3}{2}\mathrm{H}_0\zeta_2+\mathrm{H}_0\zeta_3+\frac{13}{8}\mathrm{H}_{0,0}-2\mathrm{H}_0$$

$$+12\mathrm{H}_1\zeta_3+8\mathrm{H}_{1,-2,0}-6\mathrm{H}_{1,0,0}-4\mathrm{H}_{1,0,0,0}+4\mathrm{H}_{1,2,0}-3\mathrm{H}_{2,0}+2\mathrm{H}_{2,0,0}+4\mathrm{H}_{2,1}$$

$$+4\mathrm{H}_{3,0}+4\mathrm{H}_{3,1}+2\mathrm{H}_4\Big]+p_{\mathrm{qq}}(-x)\Big[\frac{7}{2}\zeta_2{}^2-\frac{9}{2}\zeta_3-6\mathrm{H}_{-3,0}+32\mathrm{H}_{-2}\zeta_2+8\mathrm{H}_{-2}$$

$$-26\mathrm{H}_{-2,0,0}-28\mathrm{H}_{-2,2}+6\mathrm{H}_{-1}\zeta_2+36\mathrm{H}_{-1}\zeta_3+8\mathrm{H}_{-1,-2,0}-48\mathrm{H}_{-1,-1}\zeta_2+40\mathrm{H}$$

3rd loop, and UFF

$$+48\mathrm{H}_{-1,-1,2} + 40\mathrm{H}_{-1,0}\zeta_2 + 3\mathrm{H}_{-1,0,0} - 22\mathrm{H}_{-1,0,0,0} - 6\mathrm{H}_{-1,2} - 4\mathrm{H}_{-1,2,0} - 32$$

$$-\frac{3}{2}\mathrm{H}_0\zeta_2 - 13\mathrm{H}_0\zeta_3 - 14\mathrm{H}_{0,0}\zeta_2 - \frac{9}{2}\mathrm{H}_{0,0,0} + 6\mathrm{H}_{0,0,0,0} + 6\mathrm{H}_2\zeta_2 + 3\mathrm{H}_3 + 2\mathrm{H}_{3,0}$$

$$+(1-x)\Big[2\mathrm{H}_{-3,0} - \frac{31}{8} + 4\mathrm{H}_{-2,0,0} + \mathrm{H}_{0,0}\zeta_2 - 3\mathrm{H}_{0,0,0,0} + 35\mathrm{H}_1 + 6\mathrm{H}_1\zeta_2 - \mathrm{H}_{1,}$$

$$+(1+x)\Big[\frac{37}{10}{\zeta_2}^2 - \frac{93}{4}\zeta_2 - \frac{81}{2}\zeta_3 - 15\mathrm{H}_{-2,0} + 30\mathrm{H}_{-1}\zeta_2 + 12\mathrm{H}_{-1,-1,0} - 2\mathrm{H}_{-1,0}$$

$$-24\mathrm{H}_{-1,2} - \frac{539}{16}\mathrm{H}_0 - 28\mathrm{H}_0\zeta_2 + \frac{191}{8}\mathrm{H}_{0,0} + 20\mathrm{H}_{0,0,0} + \frac{85}{4}\mathrm{H}_2 - 3\mathrm{H}_{2,0,0} - 2\mathrm{H}_3$$

$$-\mathrm{H}_4\Big] + 4\zeta_2 + 33\zeta_3 + 4\mathrm{H}_{-3,0} + 10\mathrm{H}_{-2,0} + \frac{67}{2}\mathrm{H}_0 + 6\mathrm{H}_0\zeta_3 + 19\mathrm{H}_0\zeta_2 - 25\mathrm{H}_{0,0}$$

$$-2\mathrm{H}_2 - \mathrm{H}_{2,0} - 4\mathrm{H}_3 + \delta(1-x)\Big[\frac{29}{32} - 2\zeta_2\zeta_3 + \frac{9}{8}\zeta_2 + \frac{18}{5}{\zeta_2}^2 + \frac{17}{4}\zeta_3 - 15\zeta_5\Big]\Big)$$

music of the spheres

2×2 anomalous dimension matrix occupies

1 st loop: $1/10$ page

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3 rd loop: 100 pages (200 K asci)

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[waterfall of results launched
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$$V \sim \begin{cases} 10^{\frac{N(N-1)}{2}-1} \\ 10^{2^{N-1}-2} \end{cases}$$

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not too encouraging a trend ...

Fighting complexity

How to reduce complexity ?

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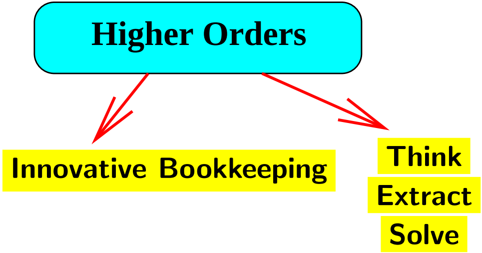
Guidelines



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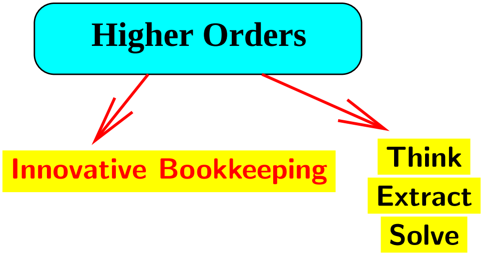

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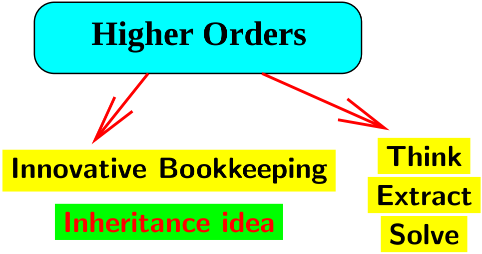

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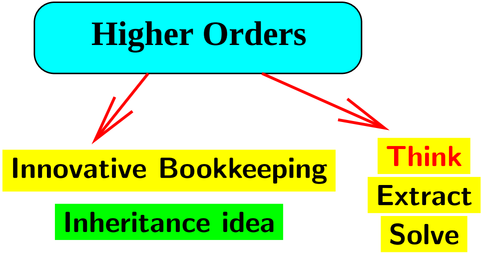

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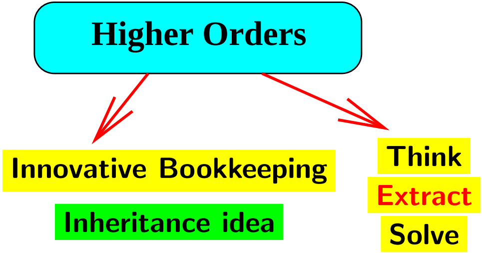

An essential part of gluon dynamics is Classical. (F.Low)

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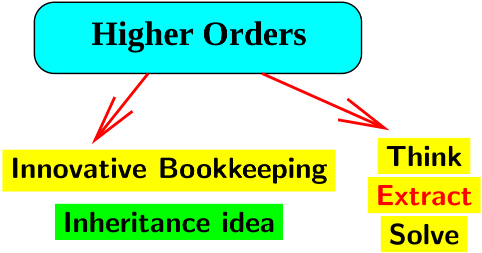

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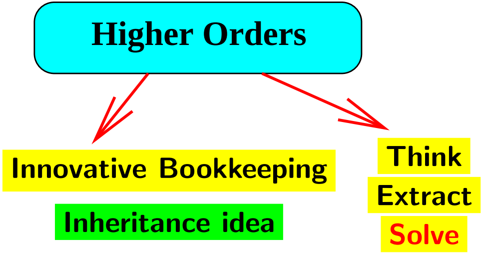

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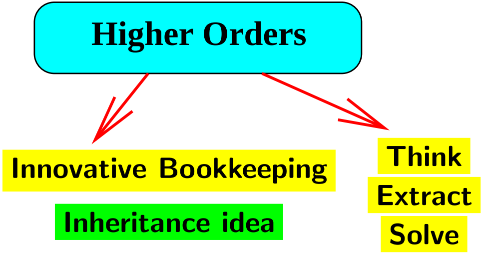

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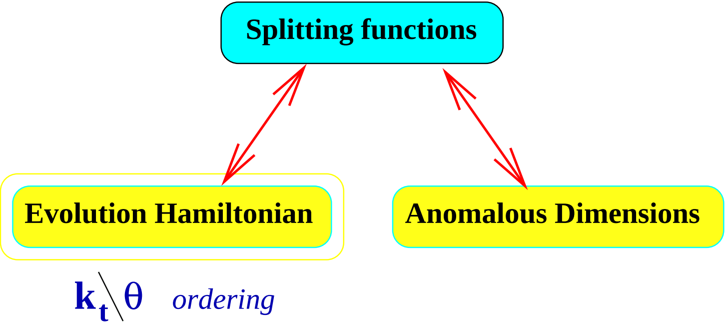
➥ A playing ground for theoretical theory: SUSY, AdS/CFT, . . .

Innovative Bookkeeping

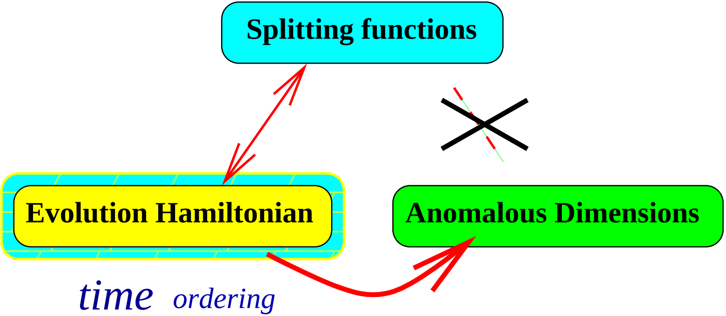
In the standard approach,



- parton splitting functions are equated with anomalous dimensions;
- they are different for DIS and e^+e^- evolution;
- "clever evolution variables" are different too

Innovative Bookkeeping

In the new approach,



- splitting functions are disconnected from the anomalous dimensions;
- the evolution kernel is identical for space- and time-like cascades (Gribov–Lipatov reciprocity relation true in all orders);
- unique evolution variable — parton fluctuation time

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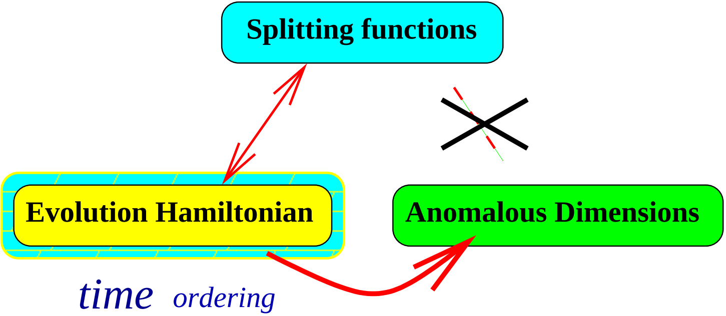

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old new evolution — Innovative Bookkeeping

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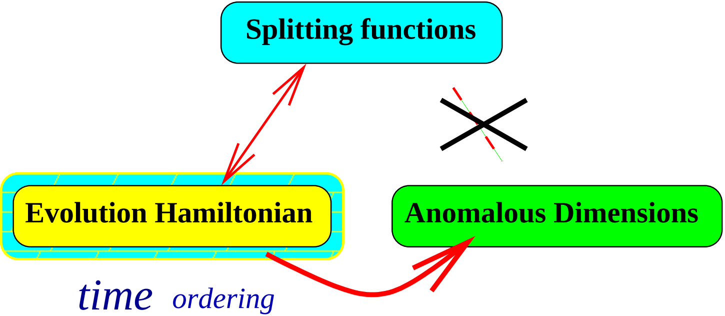

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inherited from previous loops !

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D-r, Marchesini & Salam (2005)

was verified at 3 loops for the nonsinglet channel, $(\gamma^{(T)} - \gamma^{(S)}) = \mathrm{OK}$

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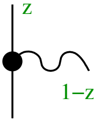
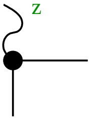
Recall an old hint from QCD ...

Relating parton splittings



$$= C_F \cdot \frac{1+z^2}{1-z}$$



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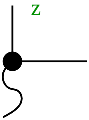
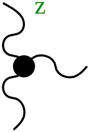


$$= T_R \cdot \left[\, z^2 + (1-z)^2 \,\right]$$



$$= N_c \cdot \frac{1+z^4+(1-z)^4}{z(1-z)}$$

Four "parton splitting functions"

$${}_q^{q[g]}(z)\,, \qquad {}_q^{g[q]}(z)\,, \qquad {}_g^{q[\bar{q}]}(z)\,, \qquad {}_g^{g[g]}(z)$$

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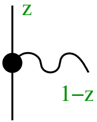

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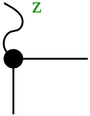

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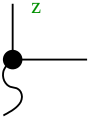

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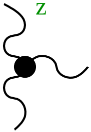
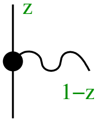


$$= N_c \cdot \frac{1+z^4+(1-z)^4}{z(1-z)}$$

- Exchange the decay products : $z \to 1-z$

$^{q[g]}_{q}(z) \quad ^{g[q]}_{q}(z) \qquad ^{q[\bar{q}]}_{g}(z) \qquad ^{g[g]}_{g}(z)$

Relating parton splittings



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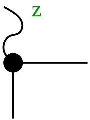

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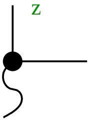

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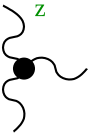

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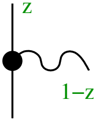

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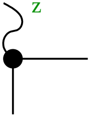

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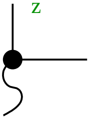

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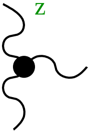

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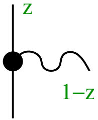
Three (QED) "kernels" are inter-related; gluon self-interaction stays put :

$${}^{q[g]}_{q}(z)\,, \quad {}^{g[q]}_{q}(z)\,, \quad {}^{q[\bar{q}]}_{g}(z) \quad ; \quad {}^{g[g]}_{g}(z)$$

Relating parton splittings



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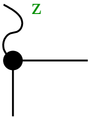

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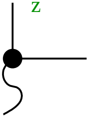

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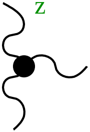

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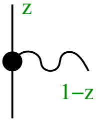

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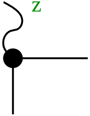

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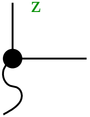

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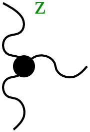

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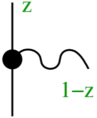

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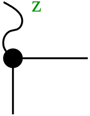

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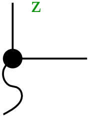

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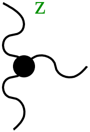

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from Bookkeeping to Solving

The integrability feature manifests itself already in *certain sectors* of QCD, in specific problems where one can *identify* QCD with SUSY-QCD :

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WHY and **WHAT FOR** ?

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And here we arrive at the second — Divide and Conquer — issue

LBK wisdom

Recall the diagonal first loop anomalous dimensions:

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Let us look at the rôles these animals play on the QCD stage

Gluenatomy

Clagons :

- ✗ Classical Field
- ✓ infrared singular, $d\omega/\omega$
- ✓ define the physical coupling
- ✓ responsible for
  - ➥ DL radiative effects,
  - ➥ reggeization,
  - ➥ QCD/Lund string (gluers)
- ✓ play the major rôle in evolution

Quagons :

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In addition,

- ✗ Tree multi-clagon (Parke–Taylor) amplitudes are *known exactly*
- ✗ It is clagons which dominate in all the *integrability cases*

N=4 SUSY Yang–Mills

Maximally super-symmetric YM field model:
Matter content = 4 Majorana fermions, 6 scalars;
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- $\beta(\alpha)\equiv 0$ in all orders ! $\Longrightarrow$ $\gamma\Rightarrow\dfrac{x}{1-x}$ + **no quagons !**

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QCD and SUSY-QCD share the gluon sector.

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Employ $\mathcal{N}=4$ SYM to simplify the major part of the QCD dynamics !

to conclude

- A steady progress in high order perturbative QCD calculations is worth accompanying by reflections upon the origin and the structure of higher loop correction effects
- Reformulation of parton cascades in terms of Gribov–Lipatov reciprocity respecting evolution equations (RREE)
  - reduces complexity by (at leat) an order of magnitude
  - improves perturbative series (less singular, better "convergent")
  - links interesting phenomena in the DIS and e^+e^- annihilation channels
- The Low theorem should be part of theor.phys. curriculum, worldwide
- Complete solution of the $\mathcal{N}=4$ SYM QFT should provide us with a *one-line-all-orders* description of the major part of QCD dynamics
- Physics of Glue whose exploration was pioneered by Gösta and Bo thirty years ago remains too rich and promising a field to retire !

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The Lund Dipole