

Geometric Z' Boson as Dark Matter and Its Implications for Black Hole Formation

Beyhan Puliçe, Durmuş Ali Demir Hocam ile

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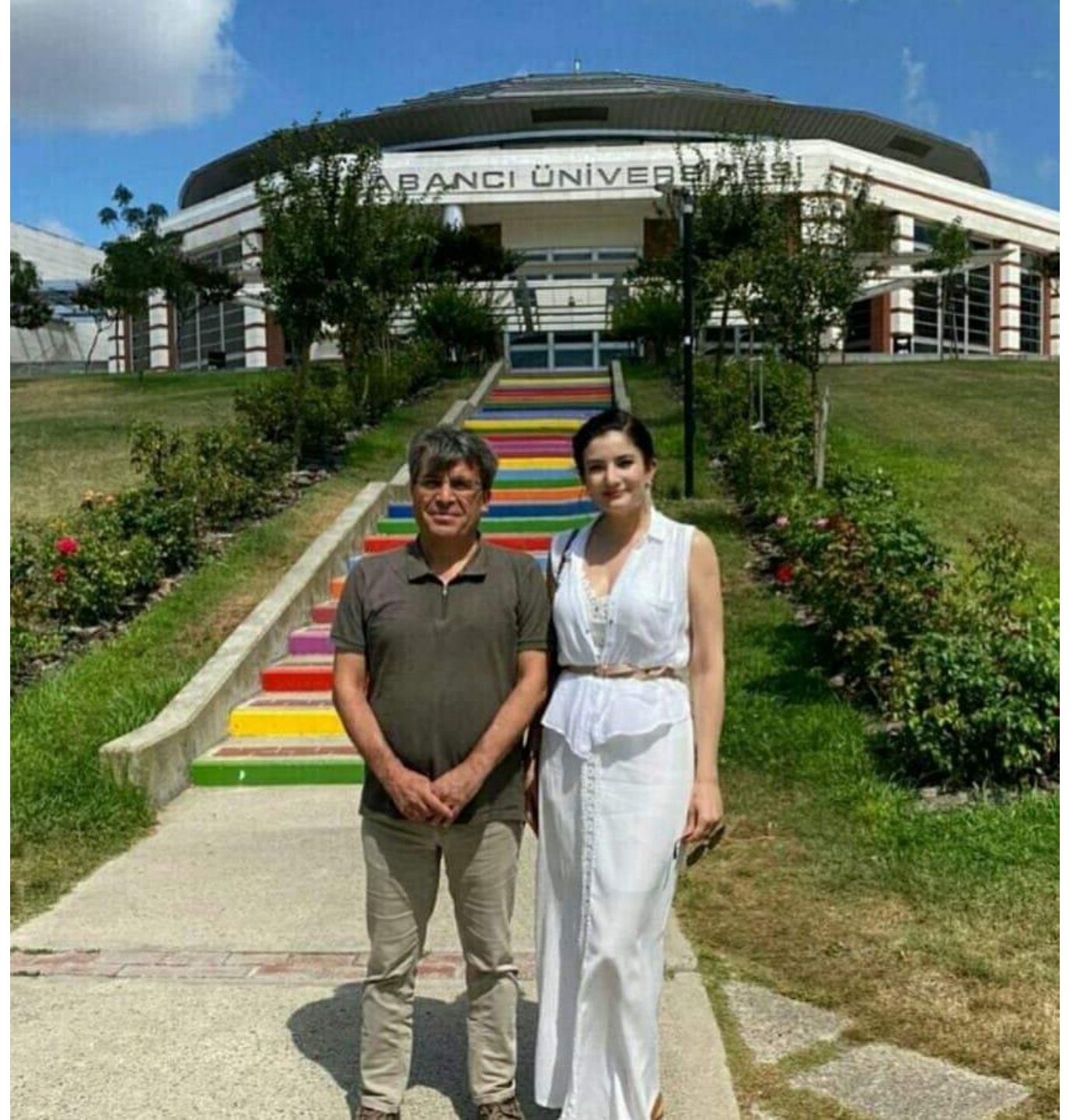


İstanbul Yüksek Enerji Fiziği Çalıştayı @İstanbul Üniversitesi (18 Mayıs, 2024)

Prof. Durmuş Ali Demir



A great physicist and a candid friend..





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Recent works with Hocam

- Ghorani, E.; Matri, S., Rayimbaev, J., **P. B.**, Atamurotov, F.; Abdujabbarov, A.; **Demir, D.**, (under review), “[Constraints on Einstein--Geometric Proca--AdS Black Hole from QPO Data](#)”, (Q1), *European Physical Journal C*
- **P. B.**; Pantig, R. C.; Övgün, A. and **Demir, D.**, (2024), “[Asymptotically-Flat Black Hole Solutions in Symmergent Gravity](#)”, (Q1) *Fortschritte der Physik*
- **P. B.**; Pantig, R. C.; Övgün, A. and **Demir, D.**, (2023),” [Constraints on charged Symmergent black hole from shadow and lensing](#) “, (Q1) *Classical and Quantum Gravity*, 40 195003
- Ghorani, E.; **P. B.**; Atamurotov, F.; Rayimbaev, J.; Abdujabbarov, A.; **Demir, D.**, (2023), “[Probing Geometric Proca with Black Hole Shadow and Photon Motion](#)”, (Q1) *European Physical Journal C*, 83, 318 (2023)
- **Demir, D.** and **P. B.** (2022), “[Geometric Proca with matter in metric-Palatini gravity](#)”, (Q1) *European Physical Journal C*, 82 (2022) 11, 996
- **P. B.** (2021), “[A Family-nonuniversal U \(1\) Model for Excited Beryllium Decays](#)”, (Q1) *Chinese Journal of Physics*, 71 (2021) 506-517
- **Demir, D.** and **P. B.** (2020), “[Geometric Dark Matter](#)”, (Q1) *Journal of Cosmology and Astroparticle Physics*, 04 (2020) 051

Palatini gravity

$$S[g, \Gamma] = \int d^4x \sqrt{-g} \left\{ \frac{M_{Pl}^2}{2} g^{\mu\nu} \mathbb{R}_{\mu\nu}(\Gamma) + \mathcal{L}(g, \Gamma, \Psi_m) \right\}$$

affine connection

$$\Gamma_{\mu\nu}^{\lambda} = \Gamma_{\nu\mu}^{\lambda}$$

affine Ricci curvature tensor

Levi-Civita connection of the metric $g_{\mu\nu}$

$${}^g\Gamma_{\mu\nu}^{\lambda} = \frac{1}{2} g^{\lambda\rho} (\partial_{\mu} g_{\nu\rho} + \partial_{\nu} g_{\rho\mu} - \partial_{\rho} g_{\mu\nu})$$

Palatini gravity

$$\frac{\partial S[g, \Gamma]}{\partial \Gamma_{\mu\nu}^{\lambda}} = 0 \longrightarrow \Gamma \nabla_{\lambda} g_{\mu\nu} = 0 \longrightarrow \Gamma_{\mu\nu}^{\lambda} = {}^g\Gamma_{\mu\nu}^{\lambda}$$

$$\frac{\partial S[g, \Gamma]}{\partial g^{\mu\nu}} = 0 \longrightarrow \mathbb{R}_{\mu\nu}(\Gamma) - \frac{1}{2} g^{\alpha\beta} \mathbb{R}_{\alpha\beta}(\Gamma) g_{\mu\nu} = T_{\mu\nu}$$

- Einstein field equations in GR arise dynamically:

$$R_{\mu\nu}({}^g\Gamma) - \frac{1}{2} g^{\alpha\beta} R_{\alpha\beta}({}^g\Gamma) g_{\mu\nu} = T_{\mu\nu}$$

- In metrical theory, it is not possible to get the Einstein field equations without adding extrinsic curvature to cancel out the surface terms.

[J. York, Phys. Rev. Lett. 28 \(1972\) 1082 G.](#)

[G. Gibbons & S. Hawking, Phys. Rev. D15 \(1977\)](#)

Experiment:

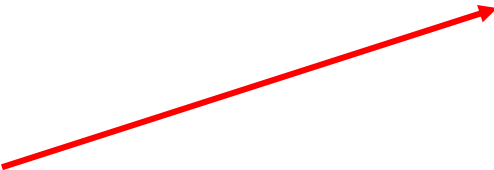
- Heavy Z' bosons seem to weigh very heavy (CMS and ATLAS $M_{Z'} \geq 5 \text{ TeV}$).
- Light Z' bosons might exist with mass near the WISP domain (Atomki: $M_{Z'} \approx 17 \text{ MeV}$) .

[A. J. Krasznahorkay et al. Phys.Rev.Lett. 116 \(2016\) 4](#)

Theory:

- Z' bosons are often associated with some $U(1)'$ symmetry beyond the SM.
- Z' bosons can arise also from beyond-the-GR geometry (non-metricity vector in Palatini gravity).

Extended Palatini gravity

$$S_{EP} [g, \Gamma] = \int d^4x \sqrt{-g} \left\{ \frac{M_{Pl}^2}{2} g^{\mu\nu} \mathbb{R}_{\mu\nu}(\Gamma) - \frac{\xi}{4} \bar{\mathbb{R}}_{\mu\nu}(\Gamma) \bar{\mathbb{R}}^{\mu\nu}(\Gamma) + \mathcal{L}(^g\Gamma, \Psi_m) \right\}$$


- antisymmetric part of the affine Ricci curvature tensor

$$\bar{\mathbb{R}}_{\mu\nu}(\Gamma) = \partial_\mu \Gamma_{\lambda\nu}^\lambda - \partial_\nu \Gamma_{\lambda\mu}^\lambda$$

- antisymmetric affine Ricci acts like field strength tensor of an Abelian vector field:

$$V_\mu = \Gamma_{\lambda\mu}^\lambda \text{ and } \bar{\mathbb{R}}_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$$

- V_μ is the source of **geometric Z'** and can be related to non-metricity for a symmetric affine connection

[V. Vitagliano et al., Phys. Rev. D 82 \(2010\) 084007](#)

[D. Demir, BP, JCAP 04 \(2020\) 51](#)

Extended Palatini gravity with Matter

$$S_{EPm} [g, \Gamma] = \int d^4x \sqrt{-g} \left\{ \frac{M_{Pl}^2}{2} g^{\mu\nu} \mathbb{R}_{\mu\nu}(\Gamma) - \frac{\xi}{4} \overline{\mathbb{R}}_{\mu\nu}(\Gamma) \overline{\mathbb{R}}^{\mu\nu}(\Gamma) + \mathcal{L}(\Gamma, \Psi_m) \right\}$$

matter sector involves affine connection $\Gamma_{\mu\nu}^\lambda$ (not ${}^g\Gamma_{\mu\nu}^\lambda$)

[D. Demir, BP, JCAP 04 \(2020\) 51](#)

[D. Demir, BP, Eur. Phys. J. C 82 \(2022\) 996](#)

➤ affine connection appears in the fermion kinetic term:

$${}^\Gamma\nabla_\mu\psi = (\nabla_\mu + \frac{1}{2}Q_\mu)\psi$$

[L. Fatibene et al., gr-qc/9608003 \(1996\)](#)

[M. Adak, T. Dereli, L. Ryder, Int. J. Mod. Phys. D 12 \(2003\) 145](#)

Extended Palatini gravity with Matter

- In general affine connection is decomposed as

$$\Gamma_{\mu\nu}^{\lambda} = {}^g\Gamma_{\mu\nu}^{\lambda} + \Delta_{\mu\nu}^{\lambda}$$

symmetric tensor field

$$\Delta_{\mu\nu}^{\lambda} = \Delta_{\nu\mu}^{\lambda}$$

- which leads to affine Ricci curvatures:

$$\mathbb{R}_{\mu\nu}(\Gamma) = \mathbb{R}_{\mu\nu}({}^g\Gamma) + \nabla_{\lambda}\Delta_{\mu\nu}^{\lambda} - \nabla_{\nu}\Delta_{\lambda\mu}^{\lambda} + \Delta_{\rho\lambda}^{\rho}\Delta_{\mu\nu}^{\lambda} - \Delta_{\nu\lambda}^{\rho}\Delta_{\rho\mu}^{\lambda}$$

$$\bar{\mathbb{R}}_{\mu\nu}(\Gamma) = \partial_{\mu}\Delta_{\lambda\nu}^{\lambda} - \partial_{\nu}\Delta_{\lambda\mu}^{\lambda}$$

[D. Demir, BP, JCAP 04 \(2020\) 51](#)

[D. Demir, BP, Eur. Phys. J. C 82 \(2022\) 996](#)

Extended Palatini gravity with Matter

- Action after decomposition:

$$S[g, \Delta, \Psi_m] = \int d^4x \sqrt{-g} \left\{ \frac{M_{Pl}^2}{2} g^{\mu\nu} \mathbb{R}_{\mu\nu}(^g\Gamma) - \frac{1}{4} \xi g^{\mu\alpha} g^{\nu\beta} (\partial_\mu \Delta_{\lambda\nu}^\lambda - \partial_\nu \Delta_{\lambda\mu}^\lambda) (\partial_\alpha \Delta_{\rho\beta}^\rho - \partial_\beta \Delta_{\rho\alpha}^\rho) \right. \\ \left. + \frac{M_{Pl}^2}{2} g^{\mu\nu} (\Delta_{\rho\lambda}^\rho \Delta_{\mu\nu}^\lambda - \Delta_{\nu\lambda}^\rho \Delta_{\rho\mu}^\lambda) + \mathcal{L}(g, ^g\Gamma, \Delta, \Psi_m) \right\}$$

Quadratic term involves all components of $\Delta_{\mu\nu}^\lambda$

Kinetic term involves only the vector field $\Delta_{\lambda\nu}^\lambda$

[D. Demir, BP, JCAP 04 \(2020\) 51](#)

[D. Demir, BP, Eur. Phys. J. C 82 \(2022\) 996](#)

Extended Palatini gravity with Matter

- One can express $\Delta_{\mu\nu}^{\lambda}$ in terms of nonmetricity tensors

$$\Delta_{\mu\nu}^{\lambda} = \frac{1}{2}g^{\lambda\rho}(Q_{\mu\nu\rho} + Q_{\nu\mu\rho} - Q_{\rho\mu\nu})$$

nonmetricity tensor

$$Q_{\lambda\mu\nu} = -{}^{\Gamma}\nabla_{\lambda}g_{\mu\nu}$$

- For an action of a consistent vector field theory, tensor field enjoys the form

$$\Delta_{\mu\nu}^{\lambda} = -3Q^{\lambda}g_{\mu\nu} + Q_{\nu}\delta_{\mu}^{\lambda} + Q_{\mu}\delta_{\nu}^{\lambda}$$

[D. Demir, BP, JCAP 04 \(2020\) 51](#)

[D. Demir, BP, Eur. Phys. J. C 82 \(2022\) 996](#)

Geometric-Z' Field with Matter:

$$S[g, Y, \Psi_m] = \int d^4x \sqrt{-g} \left\{ \frac{M_{Pl}^2}{2} R({}^g\Gamma) - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} - \frac{1}{2} M_Y^2 Y_\mu Y^\mu + g_Y \bar{f} \gamma^\mu f Y_\mu + \bar{\mathcal{L}}(g, {}^g\Gamma, \Psi_m) \right\}$$

canonical geometric Z' :

$$Y_\mu \equiv 2\sqrt{\xi} Q_\mu$$

❖ geometric Z' mass:

$$M_Y^2 = \frac{3}{2\xi} M_{Pl}^2$$

❖ geometric Z' couples to fermions universally:

$$g_Y = \frac{1}{4\sqrt{\xi}}$$

❖ quarks and leptons (not the Higgs and gauge bosons) couple to the geometrical Z' directly, universally and in an Abelian gauge field fashion.

matter sector
without geometric Z'

[D. Demir, BP, JCAP 04 \(2020\) 51](#)

[D. Demir, BP, Eur. Phys. J. C 82 \(2022\) 996](#)

Geometric-Z' Field -> Geometric-Z' Boson:

- In the flat metric limit $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$ quantum field theory is effective hence geometric-Z' changes to the field operator

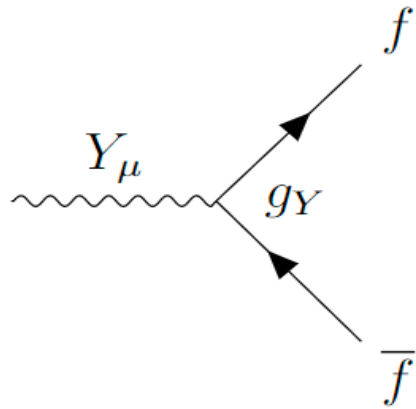
$$\hat{Y}^\mu(x) = \sum_{\lambda=0}^3 \int \frac{d^3\vec{p}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega(\vec{p})}} \left\{ \hat{a}(\vec{p}, \lambda) \epsilon^\mu(\vec{p}, \lambda) e^{-ip \cdot x} + \hat{a}^\dagger(\vec{p}, \lambda) \epsilon^{\mu*}(\vec{p}, \lambda) e^{ip \cdot x} \right\}$$

- with the commutation relation: $[\hat{a}(\vec{p}, \lambda), \hat{a}^\dagger(\vec{p}', \lambda')] = i\delta^3(\vec{p} - \vec{p}') \delta_{\lambda\lambda'}$

- and the polarization sum: $\sum_{\lambda=1}^3 \epsilon^\mu(\vec{p}, \lambda) \epsilon^{\nu*}(\vec{p}, \lambda) = \eta^{\mu\nu} - \frac{p^\mu p^\nu}{M_Y^2}$

Geometric Dark Matter

- First thing to check is **the geometric Z'** lifetime, hence we find its **decay rate**



$$\Gamma (Y \rightarrow f\bar{f}) = \frac{N_c^f}{8\pi} \left(\frac{3}{2\xi} \right)^{\frac{3}{2}} \left(1 + \frac{4\xi m_f^2}{3M_{Pl}^2} \right) \left(1 - \frac{8\xi m_f^2}{3M_{Pl}^2} \right)^{\frac{1}{2}} M_{Pl}$$

- For the summation over fermions, **the lifetime** of the Y_μ becomes

$$\tau_Y = \frac{1}{\Gamma_{\text{tot}}} = \frac{4\pi}{5} \left(\frac{2}{3} \right)^{3/2} \frac{\xi^{3/2}}{M_{Pl}}$$

- $\tau_Y > t_U = 13.8 \times 10^9$ years if $\xi > 1.1 \times 10^{40}$
- decay rate stays physical if $\xi < 1 \times 10^{41}$

Geometric Dark Matter

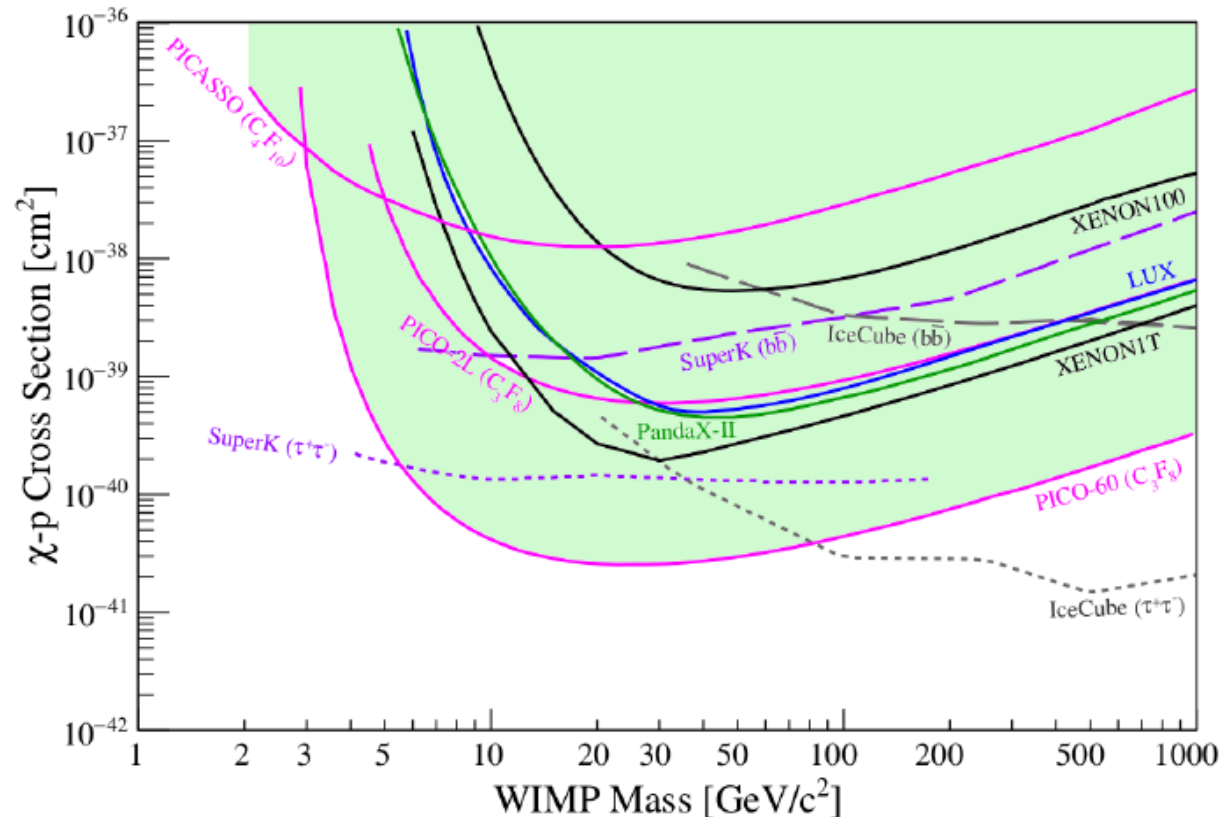
- ❖ Bounds on the quadratic term parameter lead to the allowed mass range:

$$9.4 \text{ MeV} < M_Y < 28.4 \text{ MeV}$$

- ❖ and its lifetime ranges from $4.4 \times 10^{17} \text{ s}$ to $1.2 \times 10^{19} \text{ s}$

Direct search experiments

- ❑ Direct search experiments put stringent upper limits on the cross section for scattering of dark matter off the SM particles.
- ❑ Present status of a WIMP-proton cross section:

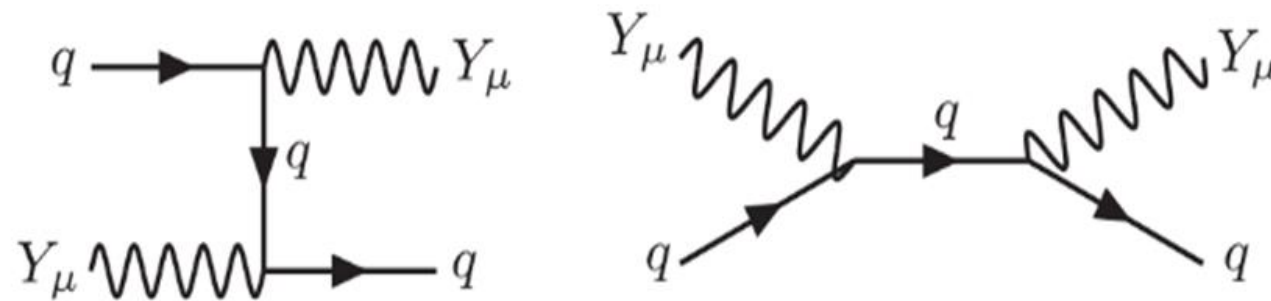


Most stringent limit is around:

$$\sigma_p^{SD} \sim \mathcal{O}(10^{-41}) \text{ cm}^2$$

Geometric Dark Matter

- It is required to compute the scattering rate of the geometric Z' to check its detection in direct searches
- relevant diagrams:

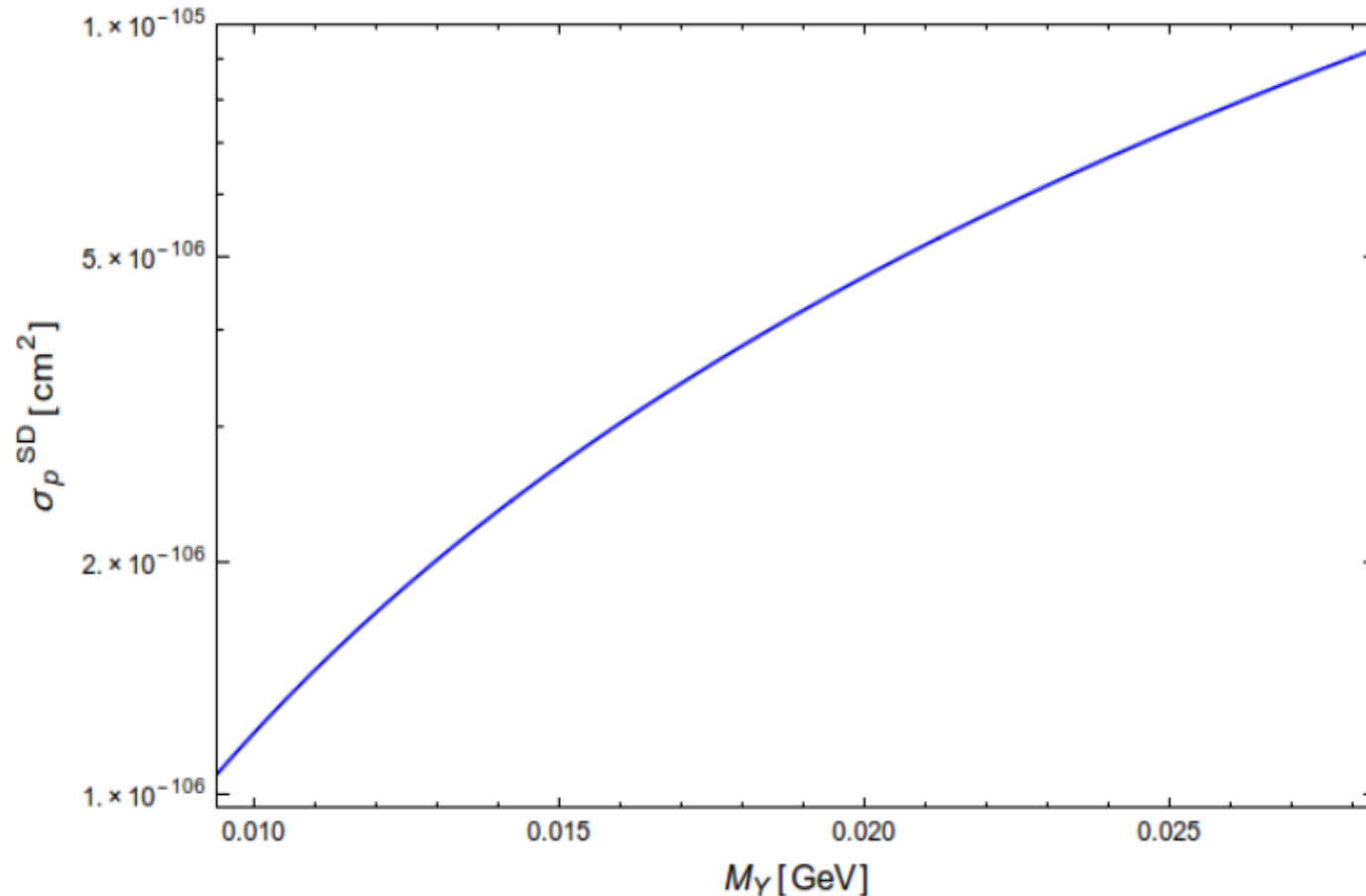


$Y_\mu q \rightarrow Y_\mu q$ scattering

$$\mathcal{M} = -i \frac{9}{4\xi} \bar{u}(k') \left(\gamma^\nu \frac{\not{k} - \not{p}' + m_q}{(k - p')^2 - m_q^2} \gamma^\mu + \gamma^\mu \frac{\not{k} + \not{p} + m_q}{(k + p)^2 - m_q^2} \gamma^\nu \right) u(k) \epsilon_\mu^*(p') \epsilon_\nu(p)$$

Geometric Dark Matter

Y_μ - proton cross section:

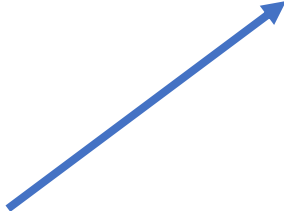


- Geometric Z' is undetectable within today's experimental precision limits
- Cross section is too small to be measurable by any of the current experiments.

← $\mathcal{O}(10^{-106}) \text{ cm}^2$

This might explain the current dark matter conundrum.

Extended Metric-Palatini Gravity augmented with metrical curvature

$$S[g, \Gamma, \Psi_m] = \int d^4x \sqrt{-g} \left\{ \underbrace{\frac{M^2}{2} g^{\mu\nu} R_{\mu\nu}({}^g\Gamma) + \frac{\bar{M}^2}{2} g^{\mu\nu} \mathbb{R}_{\mu\nu}(\Gamma)}_{\text{Einstein-Hilbert term}} - \frac{\xi}{4} \bar{\mathbb{R}}_{\mu\nu}(\Gamma) \bar{\mathbb{R}}^{\mu\nu}(\Gamma) + \mathcal{L}(g, \Gamma, \Psi_m) \right\}$$


- Einstein-Hilbert term will only partially be generated by the affine curvature

Einstein-geometric Proca theory (EGP)

$$S[g, Y, \Psi_m] = \int d^4x \sqrt{-g} \left\{ \frac{M_{Pl}^2}{2} R({}^g\Gamma) - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} - \frac{1}{2} M_Y^2 Y_\mu Y^\mu + g_Y \bar{f} \gamma^\mu f Y_\mu + \mathcal{L}_{rest}(g, {}^g\Gamma, \Psi_m) \right\}$$

➤ Planck scale is composed of the two masses:

$$M_{Pl}^2 = M^2 + \overline{M}^2$$

➤ Geometric-Z' – fermion coupling:

$$g_Y = \frac{1}{4\sqrt{\xi}}$$

➤ Geometric-Z' mass mass:

$$M_Y^2 = \frac{3\overline{M}^2}{2\xi}$$

Spherically-Symmetric Static EGP, with Perfect Fluid

$$S[g, Y] = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa} R({}^g\Gamma) - \underbrace{\frac{1}{4} Y_{\mu\nu} Y^{\mu\nu}}_{\text{red}} - \frac{1}{2} M_Y^2 Y_\mu Y^\mu + \underbrace{g_Y Y_\mu J^\mu}_{\text{blue}} + \underbrace{\mathcal{L}_{rest}}_{\text{green}} \right\}$$

➤ Einstein field equations:

$$R_{\mu\nu}({}^g\Gamma) - \frac{1}{2} R(g) g_{\mu\nu} = \kappa (T_{\mu\nu}^Y + T_{\mu\nu}^{rest})$$

➤ Motion equation for the geometric field:

$$\nabla_\mu Y^{\mu\nu} - M_Y^2 Y^\nu = -g_Y J^\nu$$

Spherically-Symmetric Static EGP, with Perfect Fluid

- Metric ansatz:

$$g_{\mu\nu} = \text{diag}(-f^2(r), \frac{g^2(r)}{f^2(r)}, r^2, r^2 \sin^2 \theta)$$

- Geometric field can be taken as a purely time-like field

$$Y_\mu = \frac{u(r)}{r} \delta_\mu^0$$

- Energy-momentum tensor can be taken to be a perfect fluid

$$T_{\mu\nu}^{rest} = (\rho_M + p_M)v_\mu v_\nu + p_M g_{\mu\nu}$$

- Source of the geometric field

$$J_\mu = \rho_C v_\mu$$

Spherically-Symmetric Static EGP, with Perfect Fluid

$(\mu\nu = 11)$ and $(\mu\nu = 00)$ components of Einstein field equations lead to the ordinary differential equations:

$$2r(f^2)' + 2(f^2 - g^2) = \kappa \left[\frac{M_Y^2 g^2}{f^2} u^2 - \left(u' - \frac{u}{r}\right)^2 + 2p_M r^2 g^2 + \frac{2g_Y \rho_C r g^2}{f} u \right]$$

$$r(g^2)' = \kappa \left[\frac{M_Y^2 g^4}{f^4} u^2 + \frac{r^2 g^4}{f^2} (\rho_M + p_M) + \frac{2g_Y \rho_C g^4}{f^3} u \right]$$

Equation of motion of the geometric field:

$$u'' = \frac{M_Y^2 g^2}{f^2} u + \frac{g'}{g} \left(u' - \frac{u}{r}\right) + g_Y \frac{\rho_C r}{f}$$

Spherically-Symmetric Static EGP, with Perfect Fluid

- We carry the equations into gravitational units by

$$\hat{r} := \kappa^{-1/2} r, \quad \hat{M}_Y^2 := \kappa M_Y^2, \quad \hat{p}_M := \kappa^2 p_M, \quad \hat{\rho}_M := \kappa^2 \rho_M, \\ \hat{\rho}_C := \kappa^{3/2} \rho_C$$

- System of differential equations takes the form:

$$2\left(\hat{r} \frac{df^2}{d\hat{r}} + f^2 - g^2\right) = \hat{M}_Y^2 \frac{g^2}{f^2} u^2 - \left(\frac{du}{d\hat{r}} - \frac{u}{\hat{r}}\right)^2 + 2\hat{p}_M \hat{r}^2 g^2 + \frac{2g_Y \hat{\rho}_C \hat{r} g^2}{f} u, \\ \hat{r} \frac{dg^2}{d\hat{r}} = \hat{M}_Y^2 \frac{g^4}{f^4} u^2 + \frac{\hat{r}^2 g^4}{f^2} (\hat{\rho}_M + \hat{p}_M) + \frac{2g_Y \hat{\rho}_C \hat{r} g^4}{f^3} u, \\ \frac{d^2 u}{d\hat{r}^2} = \hat{M}_Y^2 \frac{g^2}{f^2} u + \frac{1}{2g^2} \frac{dg^2}{d\hat{r}} \left(\frac{du}{d\hat{r}} - \frac{u}{\hat{r}}\right) + g_Y \frac{\hat{\rho}_C \hat{r}}{f}$$

Dusty Black Hole Solutions

- We take for the dust:

$$\hat{p}_M = 0,$$

$$\hat{\rho}_M = \frac{\hat{M}_D}{\hat{r}^3},$$

$$\hat{\rho}_C = \frac{Q_D}{\hat{r}^3}$$

to find the asymptotic solutions for the large radial coordinate ($\hat{r} \rightarrow \infty$)

$$f_\infty^2 \rightarrow c_0^2 \left(1 - \frac{2\hat{M}_B}{\hat{r}_\infty} + \frac{Q_\infty^2}{2c_0^2 \hat{r}_\infty^2} - g_Y \frac{Q_\infty Q_D}{\hat{r}_\infty^2} \right),$$

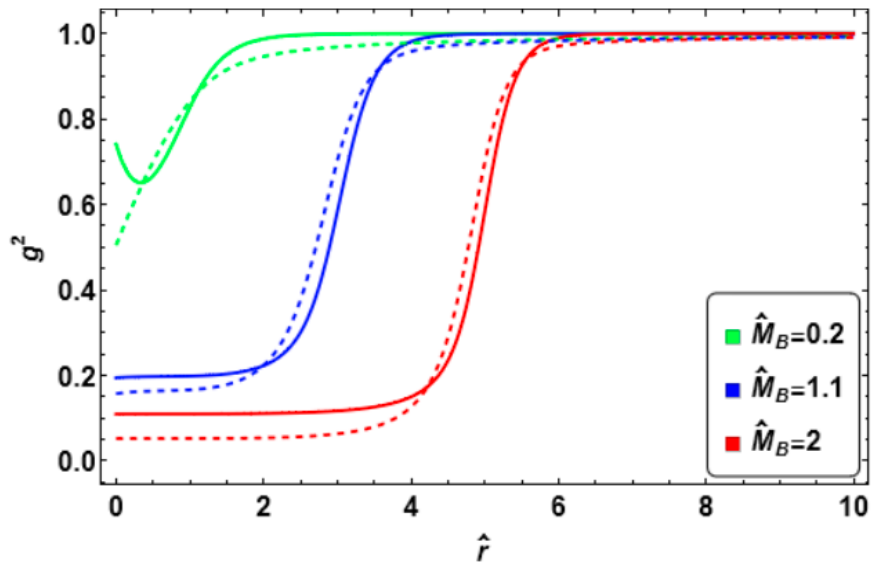
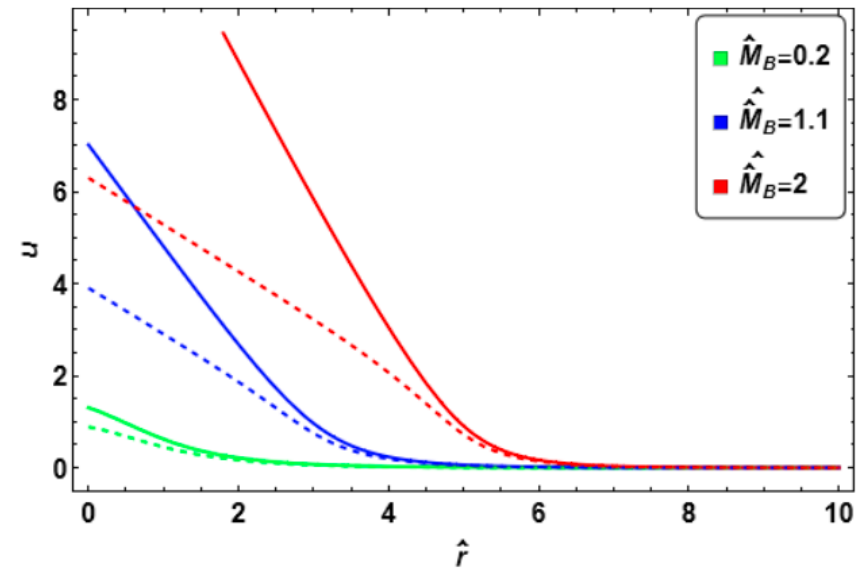
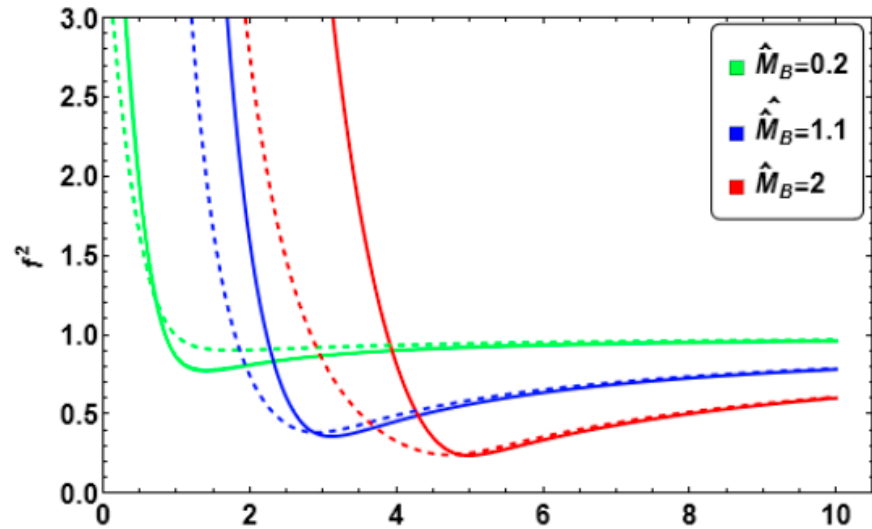
$$g_\infty^2 \rightarrow c_0^2,$$

$$u_\infty \rightarrow Q_\infty e^{-\hat{M}_Y \hat{r}_\infty} - \frac{g_Y Q_D}{2} \left(e^{\hat{M}_Y \hat{r}_\infty} \text{Ei}(-\hat{M}_Y \hat{r}_\infty) + e^{-\hat{M}_Y \hat{r}_\infty} \text{Ei}(\hat{M}_Y \hat{r}_\infty) \right),$$

$$u'_\infty \rightarrow -Q_\infty \hat{M}_Y e^{-\hat{M}_Y \hat{r}_\infty} - g_Y Q_D \left[\frac{1}{\hat{r}_\infty} + \frac{1}{2} \hat{M}_Y \left(e^{\hat{M}_Y \hat{r}_\infty} \text{Ei}(-\hat{M}_Y \hat{r}_\infty) - e^{-\hat{M}_Y \hat{r}_\infty} \text{Ei}(\hat{M}_Y \hat{r}_\infty) \right) \right]$$

Dusty Black Hole Solutions

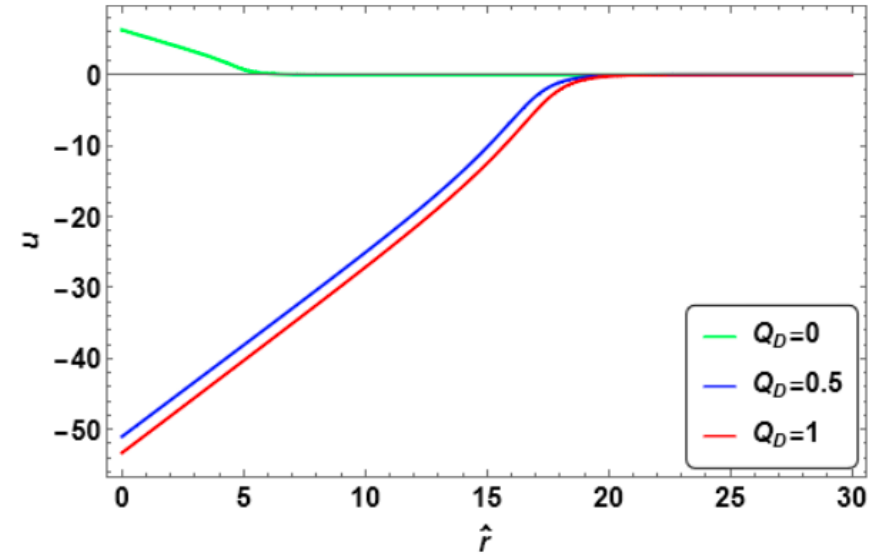
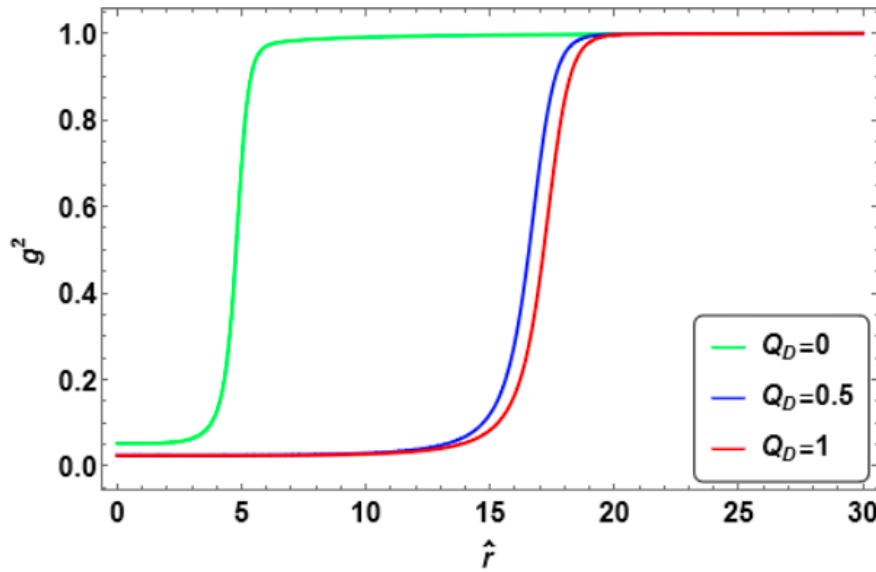
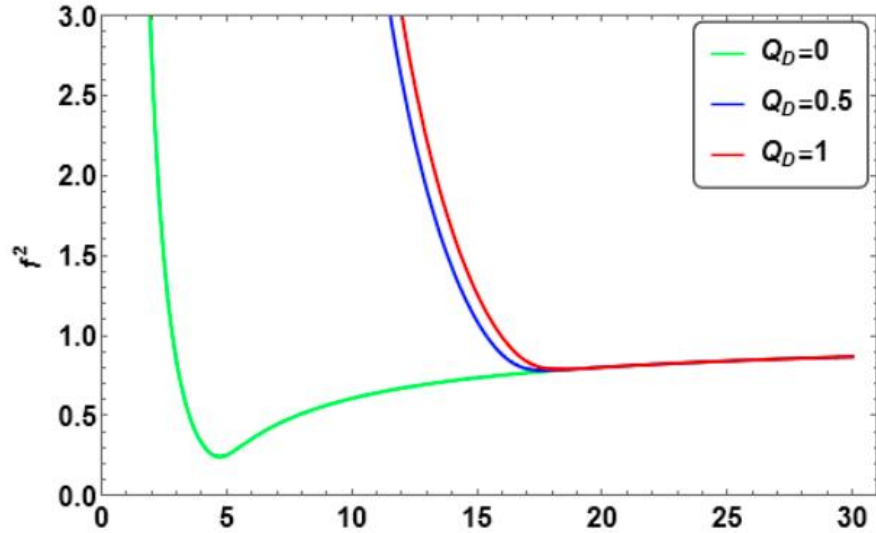
Geometrically - Neutral Dust ($Q_D = 0$)



➤ Effects of geometrically - neutral dust get pronounced at low \hat{r} and high \hat{M}_B values

Dusty Black Hole Solutions

Geometrically - Charged Dust ($Q_D \neq 0$)



- Particular solution gets abruptly shifted from the homogeneous solution by the presence of the geometrically-charged dust distribution
- Charged dust causes push f^2 away from the zero-axis and diminishes therefore possibility of developing a horizon

Extended Metric-Palatini Gravity in AdS background

- EMPG model action:

$$S[g, \Gamma] = \int d^4x \sqrt{-g} \left\{ \frac{M^2}{2} R(g) + \frac{\bar{M}^2}{2} \mathbb{R}(g, \Gamma) + \xi \bar{\mathbb{R}}_{\mu\nu}(\Gamma) \bar{\mathbb{R}}^{\mu\nu}(\Gamma) - V_0 + \mathcal{L}_m({}^g\Gamma, \psi) \right\}$$



$$S[g, Y] = \int d^4x \sqrt{-g} \left\{ \frac{1}{16\pi G_N} R(g) - V_0 - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} - \frac{1}{2} M_Y^2 Y_\mu Y^\mu + \mathcal{L}_m({}^g\Gamma, \psi) \right\}$$

Newton's constant

$$G_N = \frac{1}{8\pi(M^2 + \bar{M}^2)}$$

geometric field mass

$$M_Y^2 = \frac{3\bar{M}^2}{2\xi}$$

canonical geometric field

$$Y_\mu = 2\sqrt{\xi} Q_\mu$$

Extended Metric-Palatini Gravity in AdS background

We can bring the action in a more compact form:

$$S[g, Y] = \int d^4x \sqrt{-g} \frac{1}{2\kappa} \left\{ R(g) - 2\Lambda - M_Y^2 \hat{Y}_\mu \hat{Y}^\mu - \frac{1}{2} \hat{Y}_{\mu\nu} \hat{Y}^{\mu\nu} \right\}$$

cosmological constant

$$\Lambda = \kappa V_0$$

canonical dimensionless
geometric field

$$\hat{Y}_\mu \equiv \sqrt{\kappa} Y_\mu$$

- Einstein field equations

$$R_{\mu\nu} - \Lambda g_{\mu\nu} - \hat{Y}_{\alpha\mu} \hat{Y}^\alpha{}_\nu + \frac{1}{4} \hat{Y}_{\alpha\beta} \hat{Y}^{\alpha\beta} g_{\mu\nu} - M_Y^2 \hat{Y}_\mu \hat{Y}_\nu = 0$$

- Proca equation

$$\nabla_\mu \hat{Y}^{\mu\nu} - M_Y^2 \hat{Y}^\nu = 0$$

[E. Ghorani, BP, F. Atamurotov, J.Rayimbaev, A. Abduljabarov, D. Demir, EPJC 318 \(2023\)](#)

Black Hole Solutions in the Einstein-Geometric Proca Model

- Metric ansatz

$$g_{\mu\nu} = \text{diag}(-h(r), \frac{1}{f(r)}, r^2, r^2 \sin^2 \theta)$$

- and purely time-like field

$$\hat{Y}_\mu = \hat{\phi}(r)\delta_\mu^0$$

- lead to system of equations:

$$\hat{\phi}^2 = \frac{1}{\hat{M}_Y^2 \hat{r}} (fh' - f'h)$$
$$1 - f - \frac{\hat{r}(hf)'}{2h} - \Lambda \hat{r}^2 - \frac{f\hat{r}^2}{2h} \hat{\phi}'^2 = 0$$
$$\frac{\sqrt{hf}}{\hat{r}^2} \left(\hat{r}^2 \sqrt{\frac{f}{h}} \hat{\phi}' \right)' - \hat{M}_Y^2 \hat{\phi} = 0$$

- with dimensionless quantities:

$$\hat{r} := \kappa^{-1/2} r$$

$$\hat{M}_Y^2 := \kappa M_Y^2$$

Black Hole Solutions in the Einstein-Geometric Proca AdS Model

➤ Large r solutions of the Proca field and the metric functions

$$\hat{\phi}(\hat{r}) = \frac{q_1}{\hat{r}^{\frac{1-\sigma}{2}}} + \frac{q_2}{\hat{r}^{\frac{1+\sigma}{2}}}$$

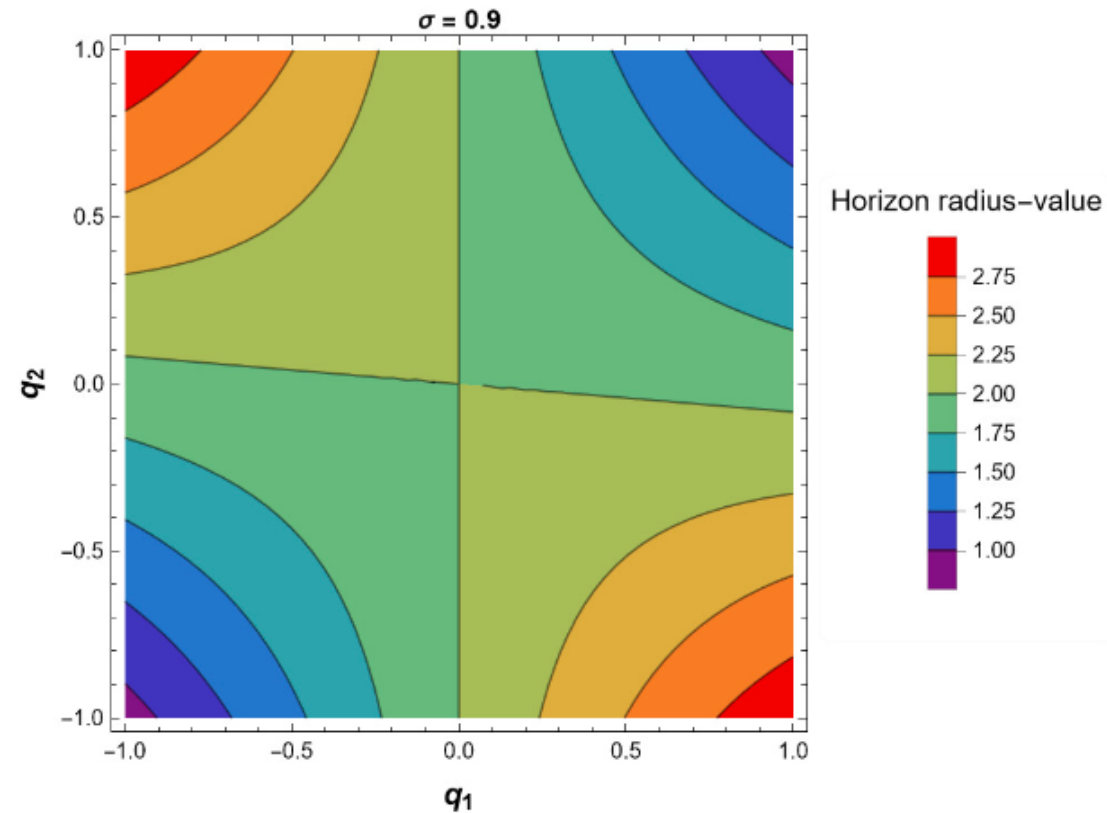
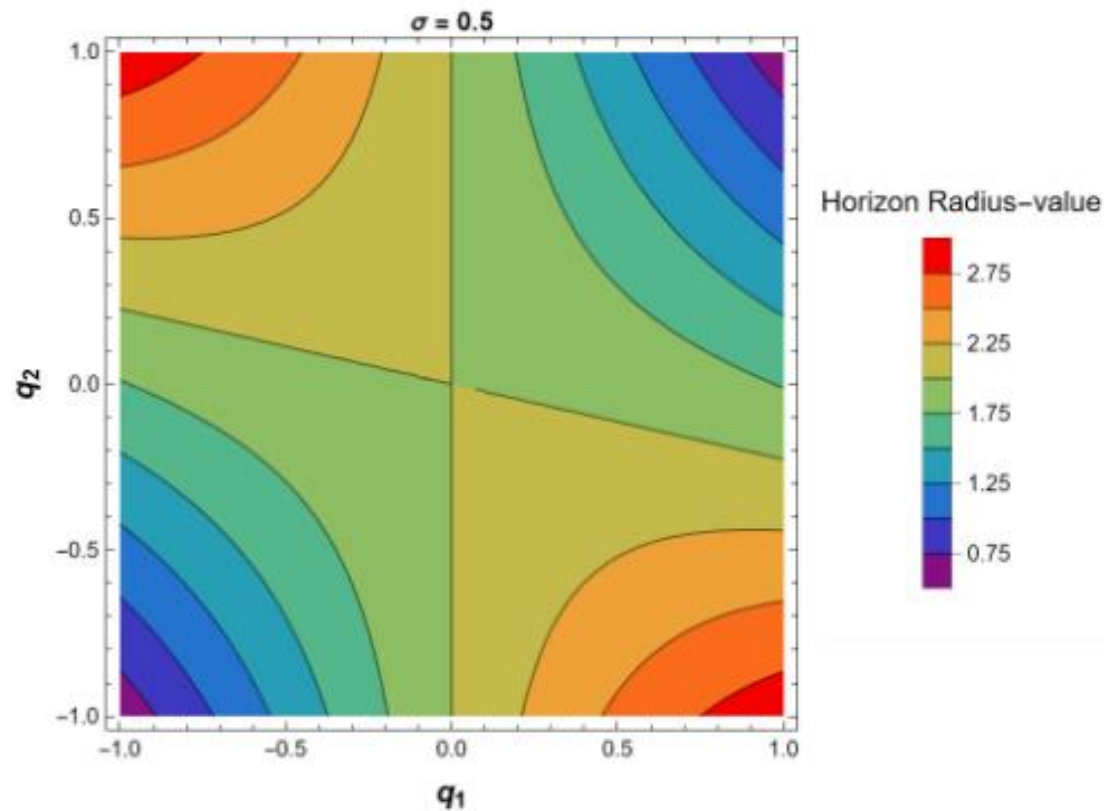
$$f(\hat{r}) = \hat{r}^2 l^{-2} + 1 + \frac{n_1}{\hat{r}^{1-\sigma}} + \frac{n_2}{\hat{r}}$$
$$h(\hat{r}) = \hat{r}^2 l^{-2} + 1 + \frac{m_1}{\hat{r}^{1-\sigma}} + \frac{m_2}{\hat{r}}$$

with

$$\sigma = \sqrt{1 + 4\hat{M}_Y^2 l^2}$$
$$\Lambda = -\frac{3}{l^2}$$

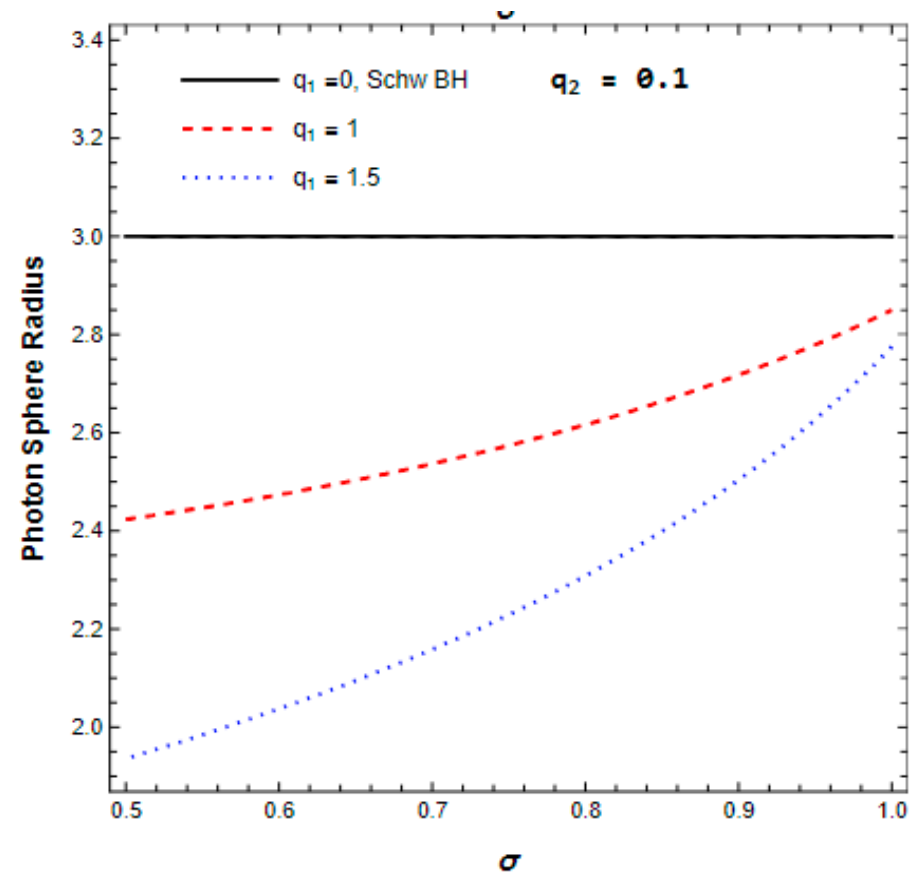
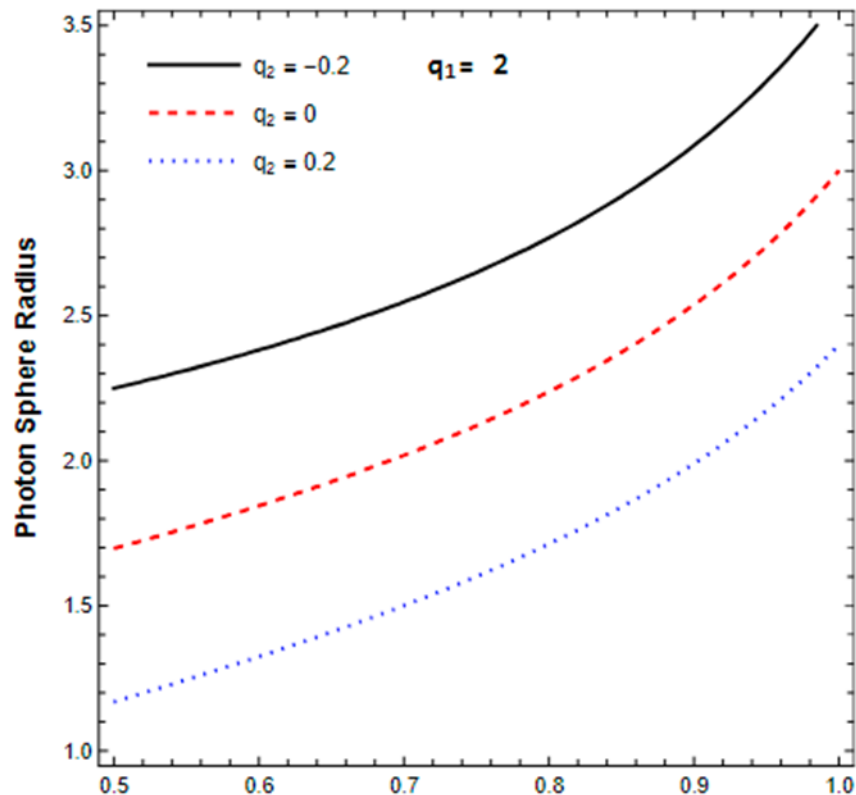
Black Hole Solutions in the Einstein-Geometric Proca AdS Model

- Horizon radius:



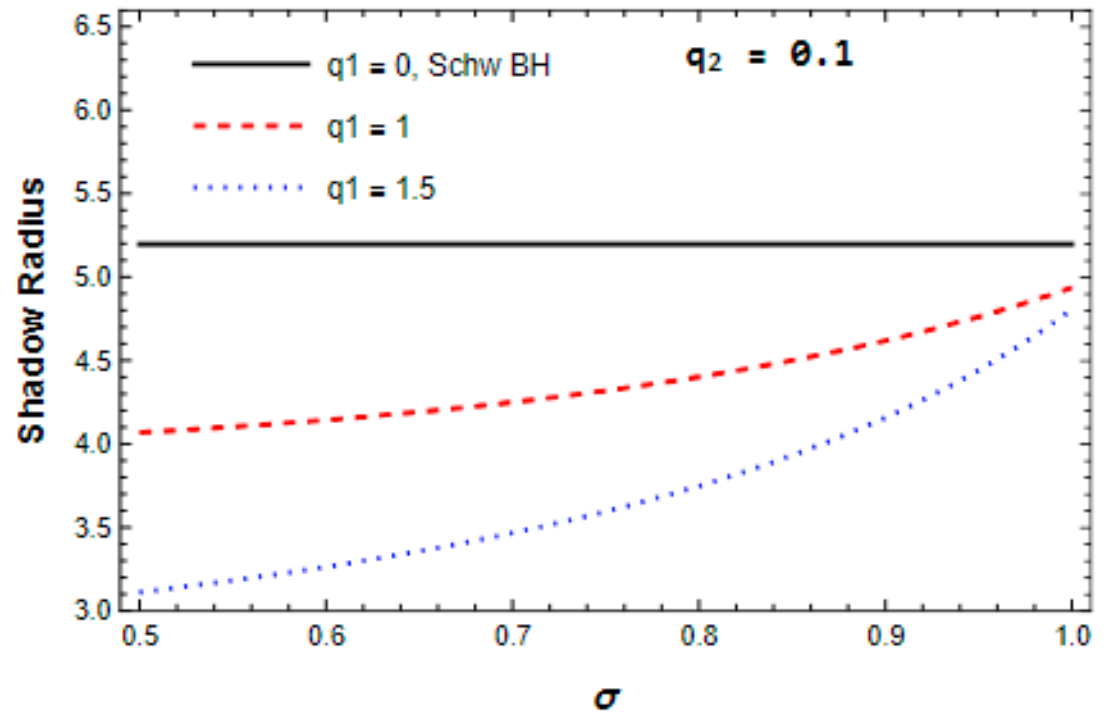
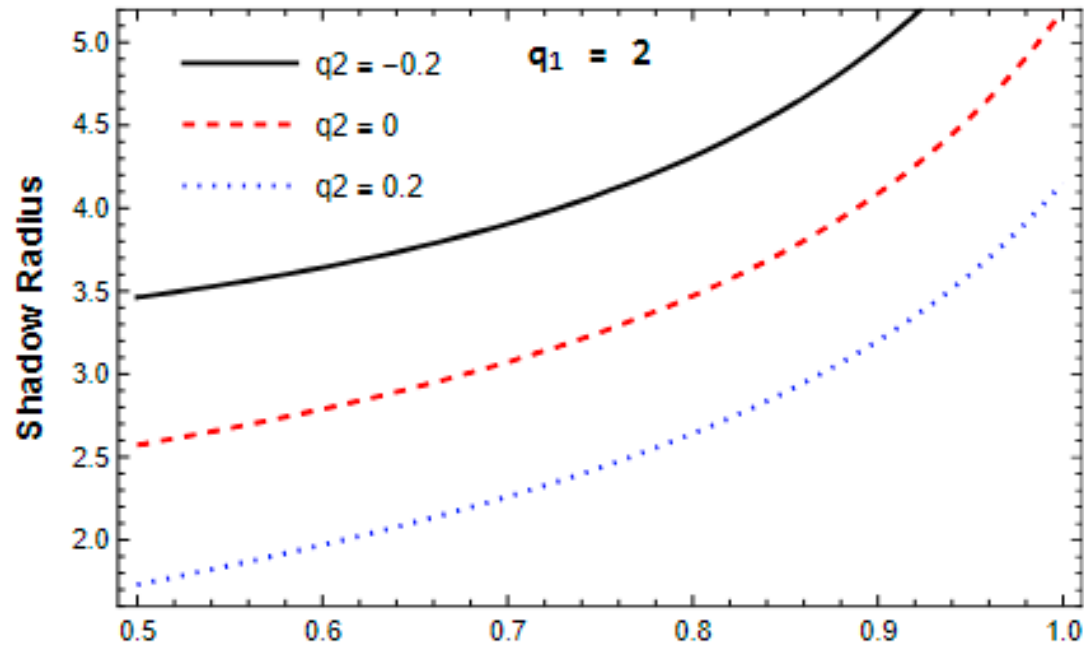
Black Hole Solutions in the Einstein-Geometric Proca AdS Model

- Photon sphere radius:



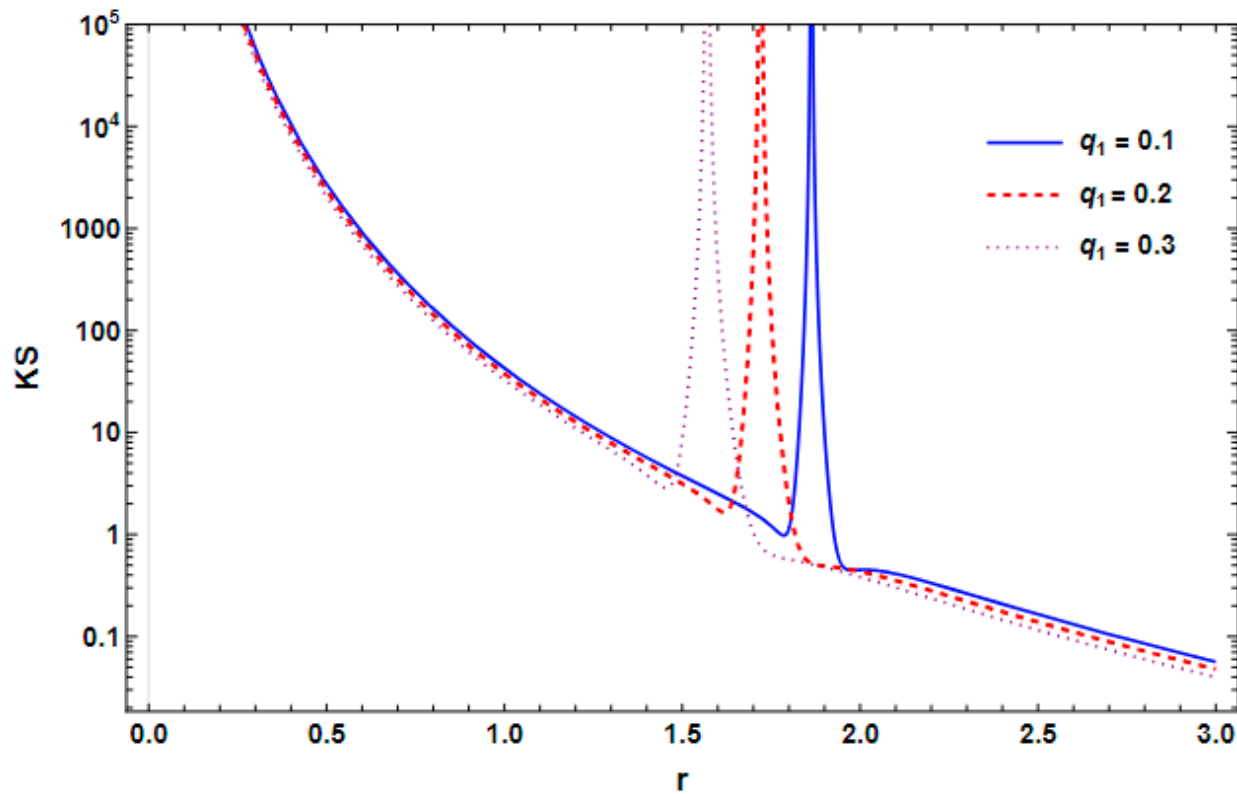
Black Hole Solutions in the Einstein-Geometric Proca AdS Model

- Shadow radius:



Black Hole Solutions in the Einstein-Geometric Proca AdS Model

- Kretschmann scalar:



Implication of Geometric Z' boson as a Fifth Force

- Atomki signal indicates the existence of a new light, neutral particle in excited beryllium-8 decays with a high significance.

[A. J. Krasznahorkay et al. Phys.Rev.Lett. 116 \(2016\) 4](#)

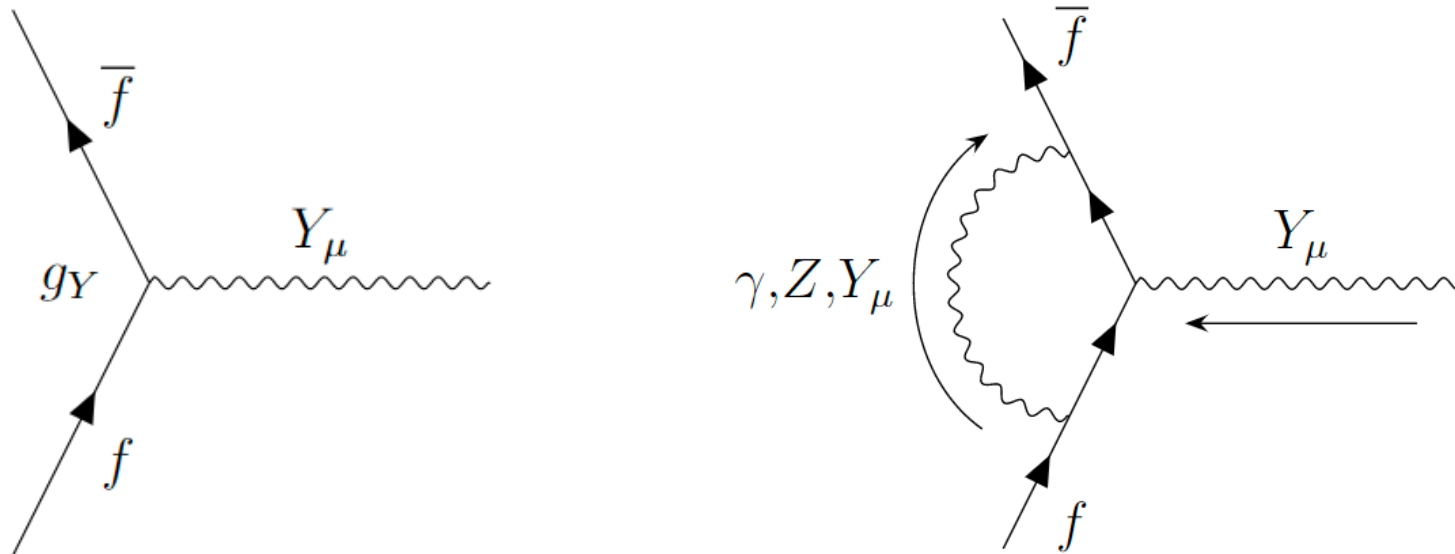
- We proposed a Z' field of a $U(1)'$ symmetry for the Atomki experiment.

[BP, Chin. J. Phys. 71 \(2021\) 506](#)

- **Geometric Z'** can play a role in the recent **fifth force** data.

Implication of Geometric Z' boson as a Fifth Force

- The forms of couplings of the geometric Z' to the fermions will be determined
- The tree-level and loop-induced diagrams will be determined for each Y_μ - fermion coupling
- Hierarchy between the couplings of the fermions will be provided



Charged Black Hole Solutions in Symmergent gravity

Symmergent gravity is described by the action

$$S[g] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left(f(R) - 2\Lambda - \frac{1}{2} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right)$$

$$f(R) = R + \beta R^2$$
$$\beta = -\pi G c_0$$

$$\Lambda = 8\pi G V_0$$

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu$$

$$\hat{A}_\mu = A_\mu / \sqrt{8\pi G}$$

total number of bosonic and fermionic degrees of freedom

$$c_0 = \frac{n_B - n_F}{128\pi^2}$$

Charged Black Hole Solutions in Symmergent gravity

Einstein field equations

$$E_{\mu\nu} \equiv R_{\mu\nu}F(R) - \frac{1}{2}g_{\mu\nu}f(R) + g_{\mu\nu}\Lambda + (g_{\mu\nu}\square - \nabla_{\mu}\nabla_{\nu})F(R) - \hat{T}_{\mu\nu} = 0$$

Maxwell field equations

$$\partial_{\mu}(\sqrt{-g}\hat{F}^{\mu\nu}) = 0$$

For a static spherically-symmetric solution, we propose the metric

$$ds^2 = -h(r)dt^2 + \frac{1}{h(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

and the electromagnetic scalar potential

$$\hat{A}_0 = \hat{q}(r)$$

which leads to the Ricci curvature scalar

$$R = -h'' - \frac{4}{r}h' - \frac{2}{r^2}(h - 1)$$

Field equations in the Symmergent gravity

$$E_0^0 = \Lambda + \frac{h'(r)}{r} + \frac{h(r)}{r^2} - \frac{1}{r^2} + \frac{1}{2}\hat{q}'(r)^2$$

$$+ \beta \left(-2h''''(r)h(r) - h'''(r)h'(r) + \frac{1}{2}h''(r)^2 - \frac{12h'''(r)h(r)}{r} - \frac{2h'(r)h''(r)}{r} - \frac{4h(r)h''(r)}{r^2} \right.$$

$$\left. + \frac{2h'(r)^2}{r^2} + \frac{8h(r)h'(r)}{r^3} - \frac{10h(r)^2}{r^4} + \frac{12h(r)}{r^4} - \frac{2}{r^4} \right),$$

$$E_1^1 = \Lambda + \frac{h'(r)}{r} + \frac{h(r)}{r^2} - \frac{1}{r^2} + \frac{1}{2}\hat{q}'(r)^2$$

$$+ \beta \left(-h'''(r)h'(r) + \frac{1}{2}h''(r)^2 - \frac{4h'''(r)h(r)}{r} - \frac{2h'(r)h''(r)}{r} - \frac{16h(r)h''(r)}{r^2} + \frac{2h'(r)^2}{r^2} \right.$$

$$\left. + \frac{8h(r)h'(r)}{r^3} + \frac{14h(r)^2}{r^4} - \frac{12h(r)}{r^4} - \frac{2}{r^4} \right),$$

$$E_2^2 = \Lambda + \frac{h'(r)}{r} + \frac{h''(r)}{2} - \frac{1}{2}\hat{q}'(r)^2$$

$$+ \beta \left(-2h''''(r)h(r) - 2h'''(r)h'(r) - \frac{1}{2}h''(r)^2 - \frac{10h'''(r)h(r)}{r} - \frac{10h'(r)h''(r)}{r} + \frac{4h'(r)^2}{r^2} \right.$$

$$\left. + \frac{4h(r)h''(r)}{r^2} + \frac{16h(r)h'(r)}{r^3} - \frac{12h'(r)}{r^3} - \frac{14h(r)^2}{r^4} + \frac{12h(r)}{r^4} + \frac{2}{r^4} \right),$$

$$r^2\hat{q}''(r) + 2r\hat{q}'(r) = 0 \quad \text{with} \quad \hat{q}(r) = \frac{Q}{r}$$

Charged Black Hole Solutions in Symmergent gravity

The metric function solution takes the form

$$h(r) = 1 - \frac{2MG}{r} + \frac{1}{(1 + 8\beta\Lambda)} \frac{Q^2}{2r^2} - \frac{\Lambda r^2}{3}$$

Using Symmergent gravity parameters

$$h(r) = 1 - \frac{2MG}{r} + \frac{Q^2}{2\hat{\alpha}r^2} - \frac{(1 - \hat{\alpha})}{24\pi Gc_0} r^2$$

$$\hat{Q}^2 = \frac{Q^2}{2\hat{\alpha}}$$
$$\hat{\Lambda} = \frac{(1 - \hat{\alpha})}{8\pi Gc_0}$$

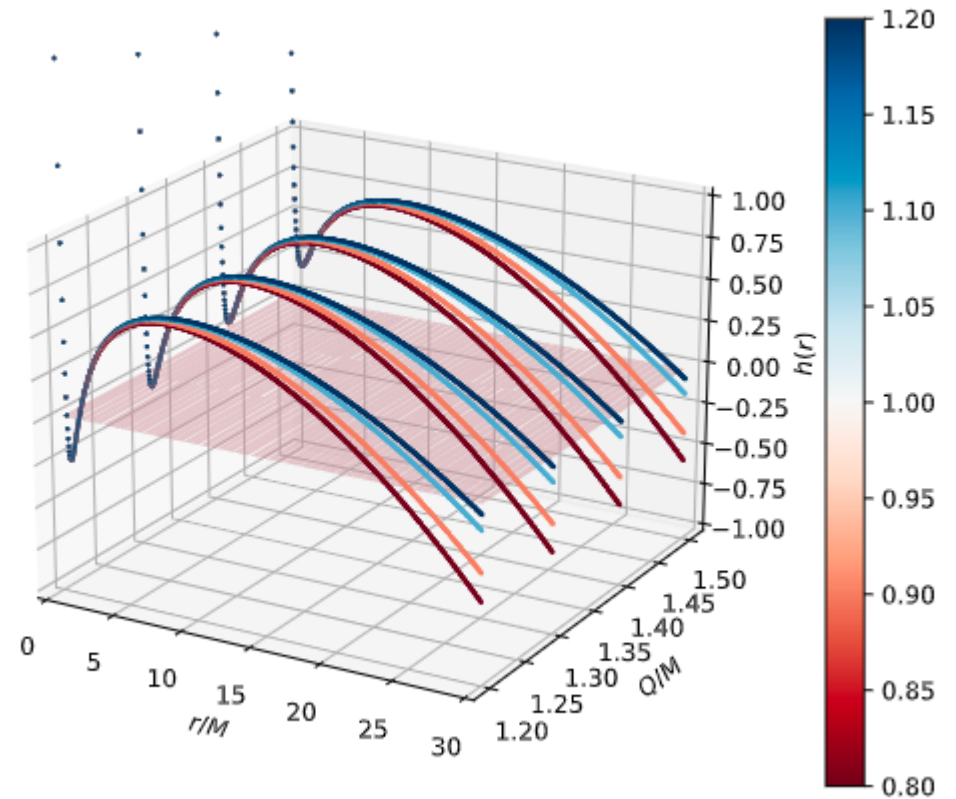
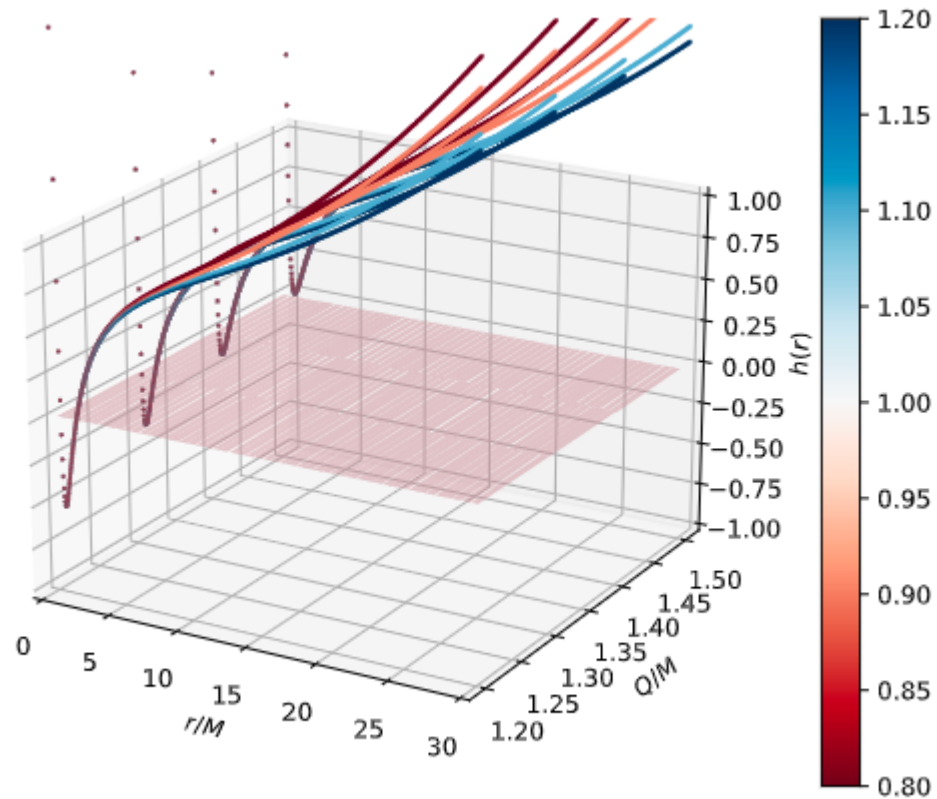
RN – AdS/dS black hole

$\hat{\alpha} > 1$ corresponds to fermion dominance and AdS space

$\hat{\alpha} < 1$ corresponds to boson dominance and dS space

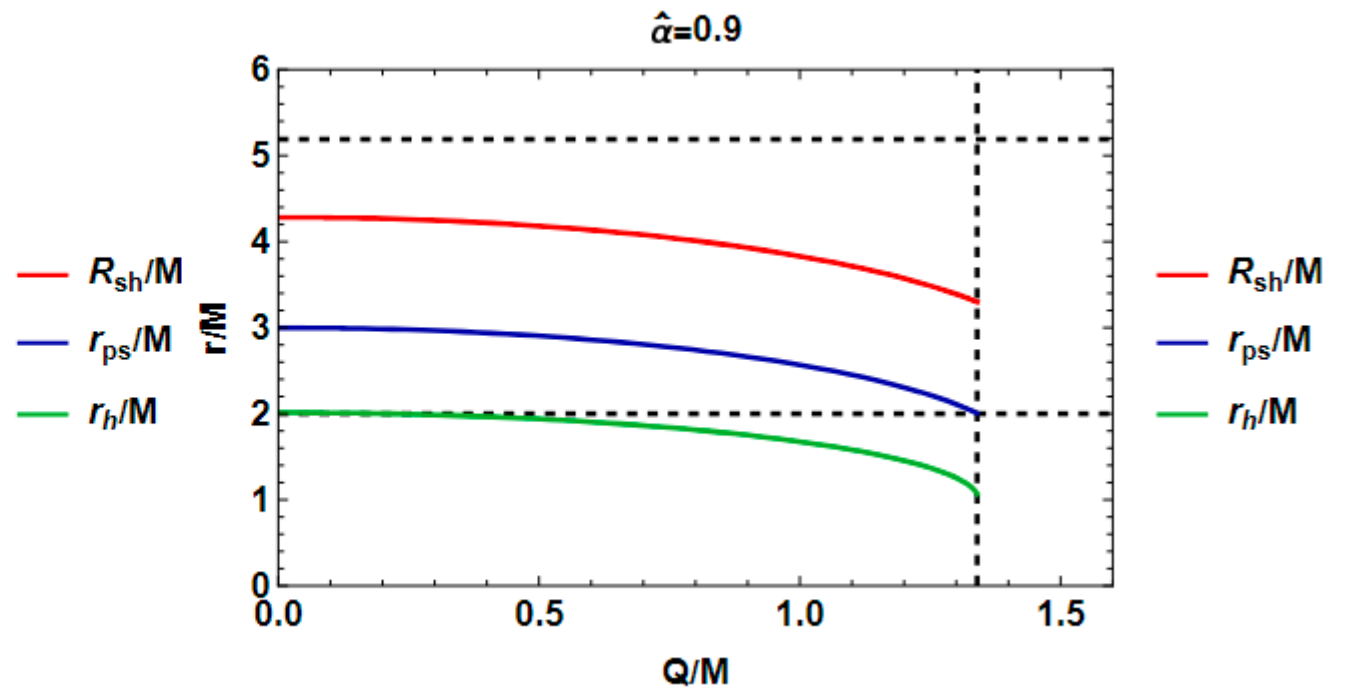
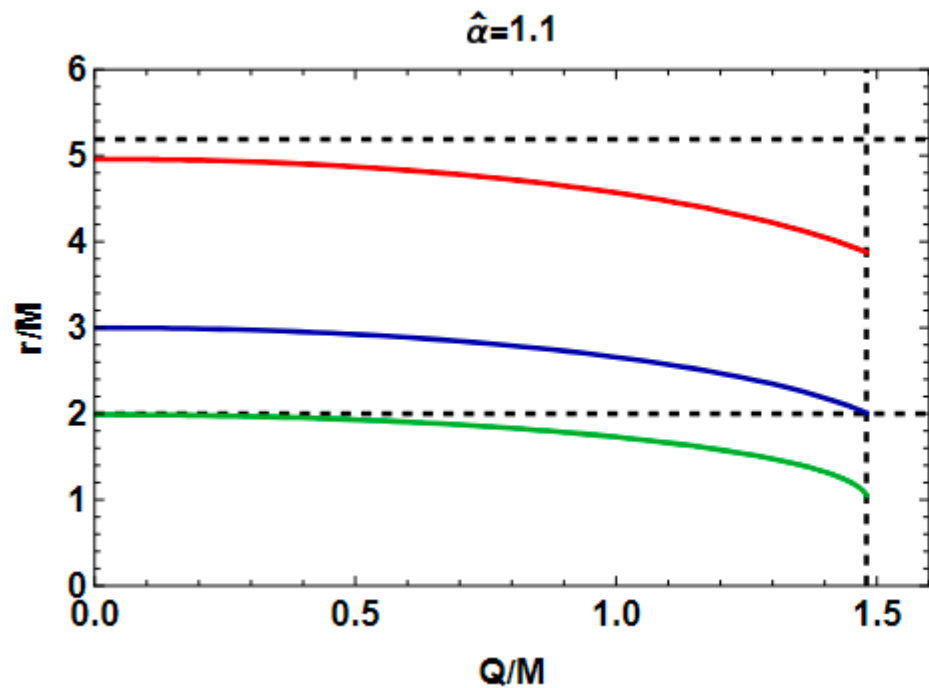
Charged Black Hole Solutions in Symmergent gravity

Metric functions



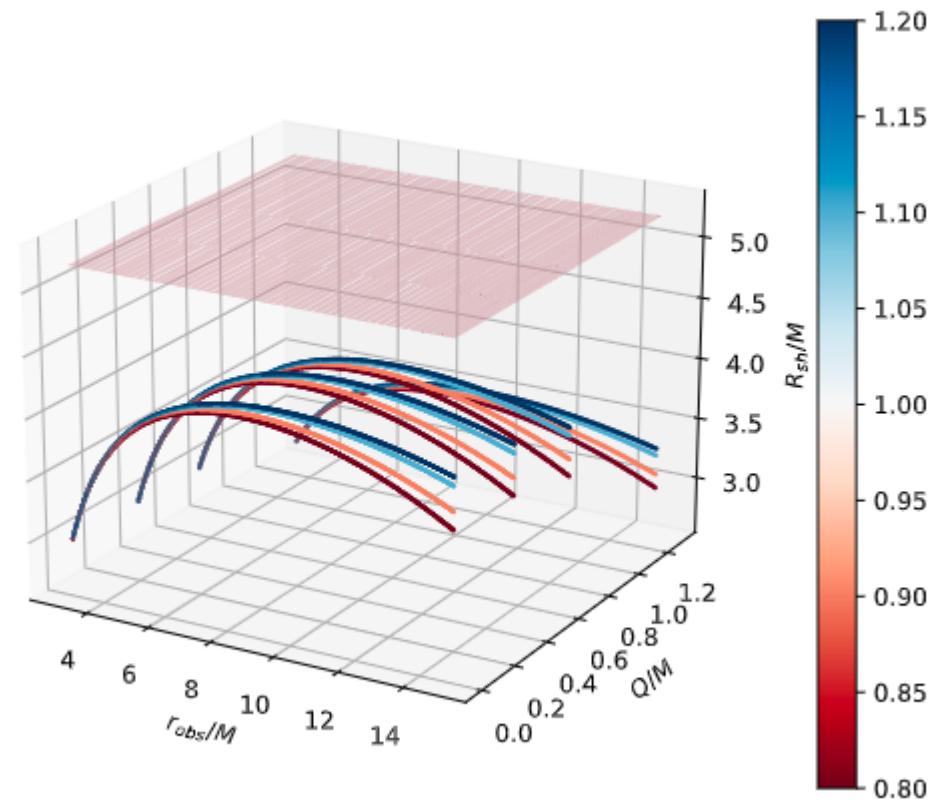
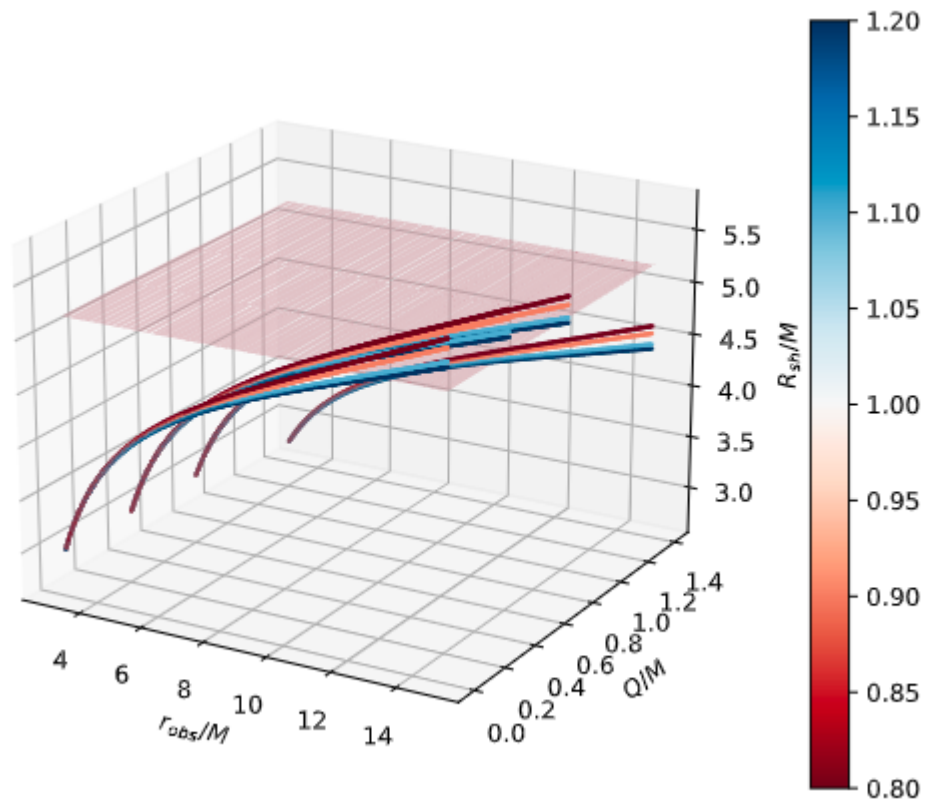
Charged Black Hole Solutions in Symmergent gravity

Shadow radius, photon sphere radius, horizon radius



Charged Black Hole Solutions in Symmergent gravity


Shadow radius



Asymptotically-Flat Black Hole Solutions in Symmergent Gravity

Symmergent gravity is described by the action

$$S = -\frac{c_O}{16} \int d^4x \sqrt{-g} (R^2 + 6\gamma G_N^{-1} R)$$


$$\gamma = -\frac{1}{6\pi c_O} = -\frac{64\pi}{3(n_B - n_F)}$$

Einstein field equations

$$(R + 3\gamma G_N^{-1})R_{\mu\nu} - \frac{1}{4}(R + 6\gamma G_N^{-1})Rg_{\mu\nu} - (\nabla_\mu \nabla_\nu - \square g_{\mu\nu})R = 0$$

Asymptotically-Flat Black Hole Solutions in Symmergent Gravity

Einstein field equations possess static, spherically-symmetric, asymptotically-flat solutions with variable scalar curvature ($R = \text{variable}$). Einstein field equations are satisfied by metric

$$ds^2 = -(1 - \varphi(r))\Psi(r)dt^2 + \frac{dr^2}{(1 + \varphi(r))\Psi(r)} + r^2 d\Omega^2$$

at the linear order in $\varphi(r)$

$$(r^2\Psi(r)\varphi'(r))' = \gamma r^2\varphi(r)$$

Schwarzschild lapse function

$$\Psi(r) = 1 - \frac{2M}{r}$$

Asymptotically-Flat Black Hole Solutions in Symmergent Gravity

Singularity structure of the spacetime is revealed by the Kretschmann scalar

$$R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} = \frac{48M^2}{r^6} (1 + 2\varphi(r))$$

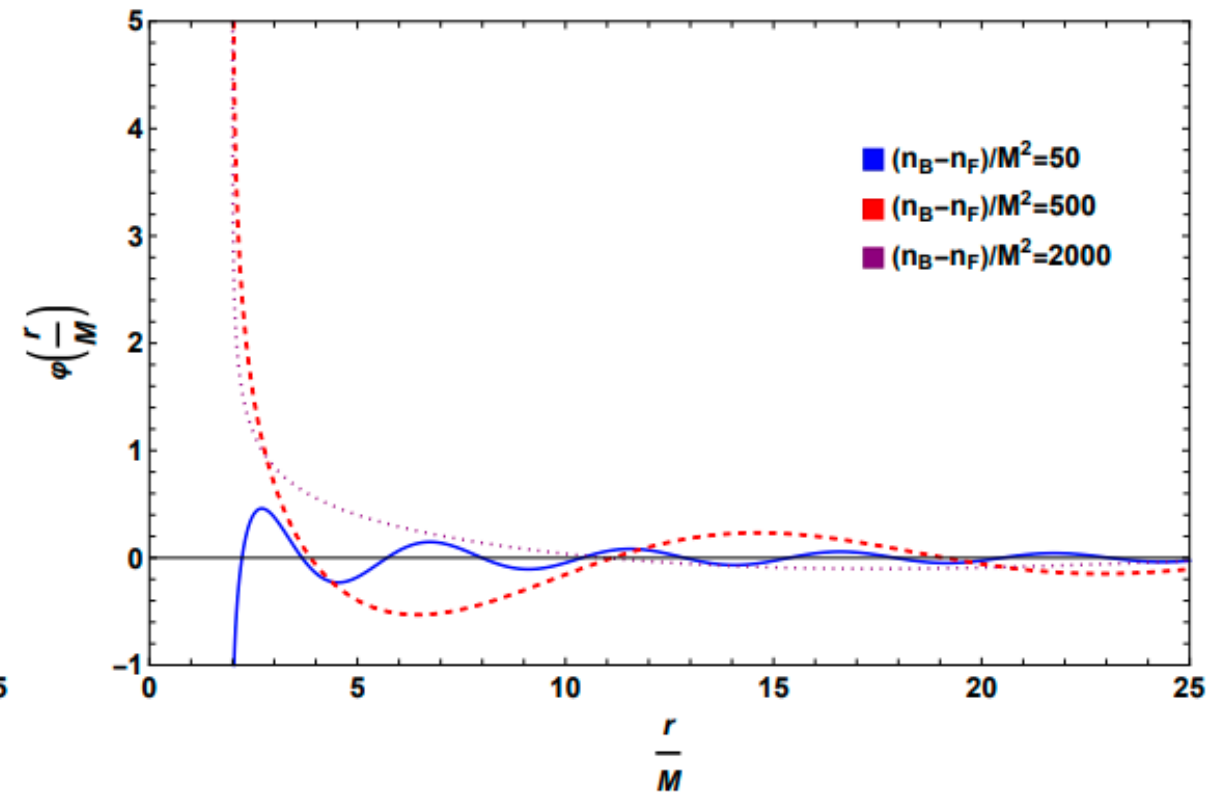
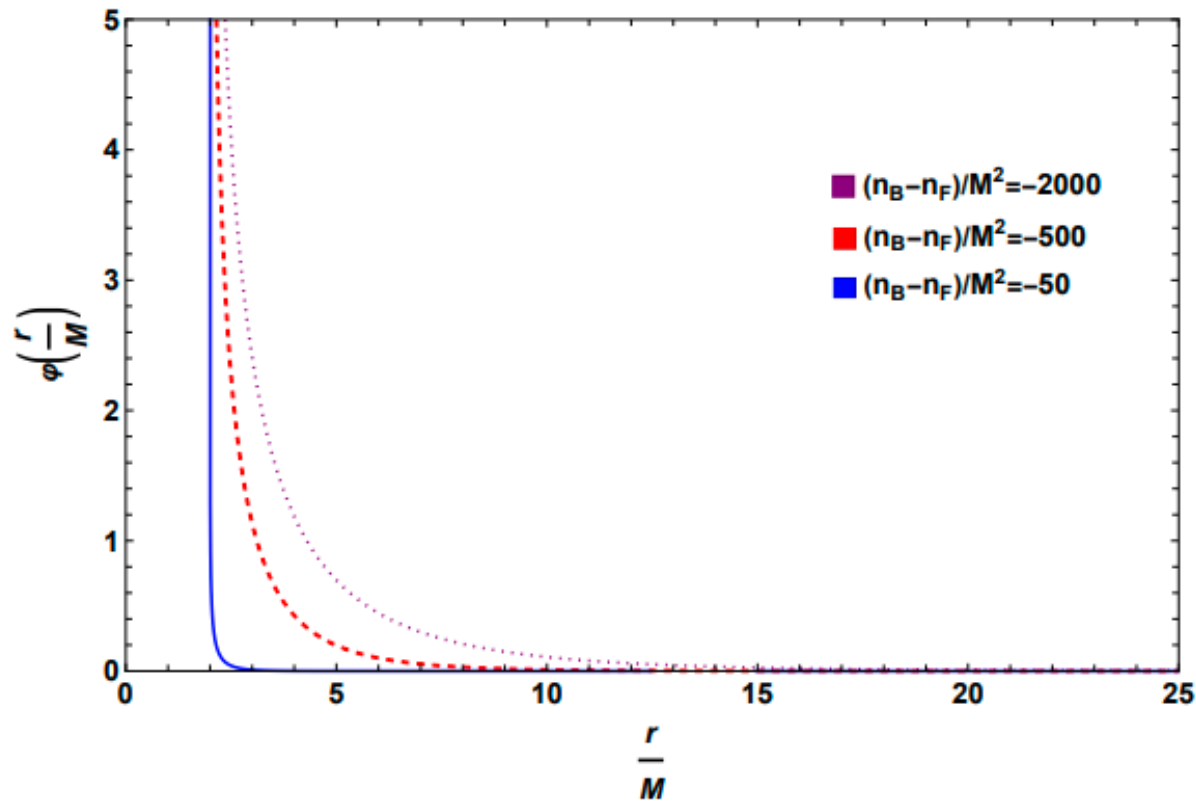
Large- r behavior of φ

$$\varphi(r) = \begin{cases} \frac{e^{-\sqrt{\gamma}r}}{\sqrt{\gamma}r} & \gamma > 0 \\ \frac{\cos(\sqrt{|\gamma|r})}{\sqrt{|\gamma|r}} & \gamma < 0 \end{cases}$$

in the limit $r \gg 2M$ so that $\Psi(r) \approx 1$

Asymptotically-Flat Black Hole Solutions in Symmergent Gravity

Solution of φ



Asymptotically-Flat Black Hole Solutions in Symmergent Gravity

In view of the asymptotic solutions, the metric can be put into

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + C(r) (d\theta^2 + \sin^2 \theta d\phi^2)$$

with the metric potentials

$$\gamma > 0$$

$$A(r) = \left(1 - \frac{2M}{r}\right) \left(1 - \frac{e^{-r\sqrt{\gamma}}}{r\sqrt{\gamma}}\right),$$

$$B(r) = \left(1 - \frac{2M}{r}\right) \left(1 + \frac{e^{-r\sqrt{\gamma}}}{r\sqrt{\gamma}}\right),$$

$$C(r) = r^2 \left(1 - \frac{e^{-r\sqrt{\gamma}}}{r\sqrt{\gamma}}\right).$$

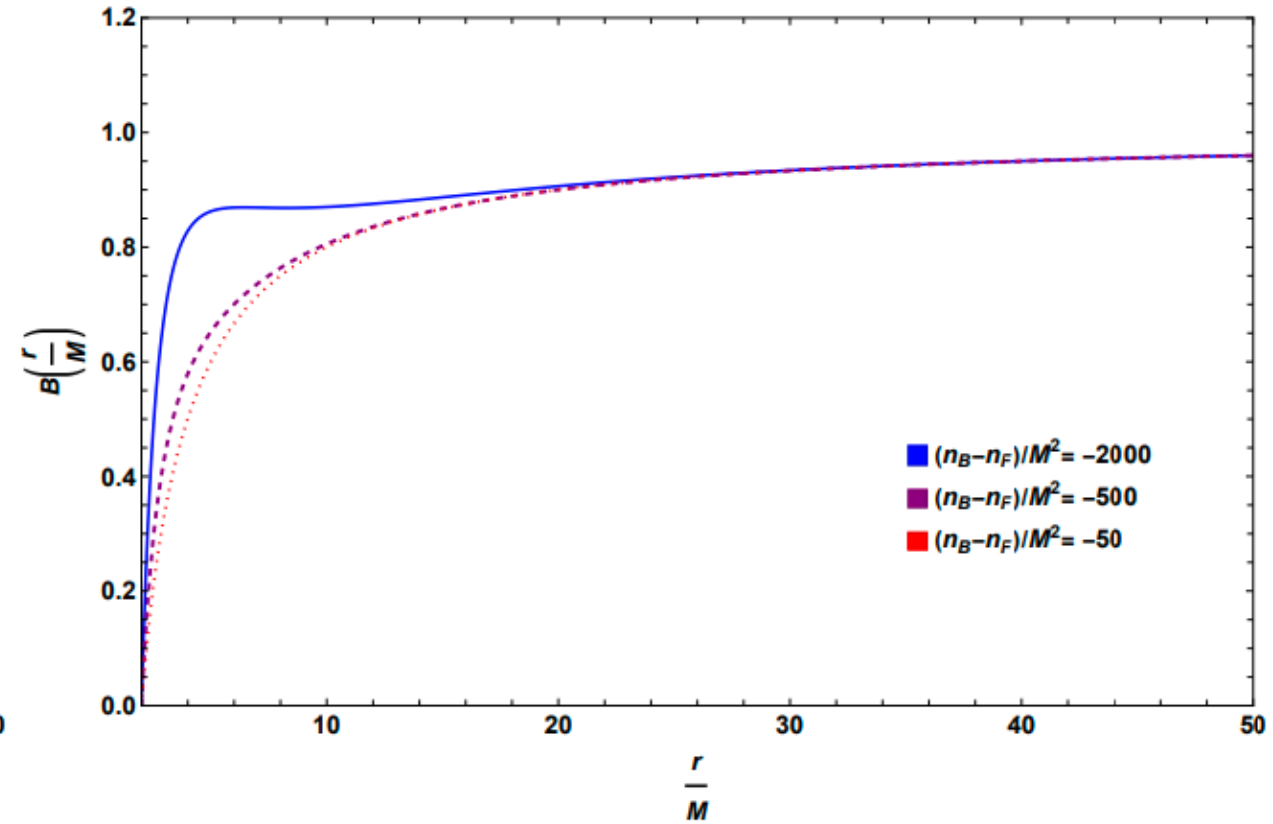
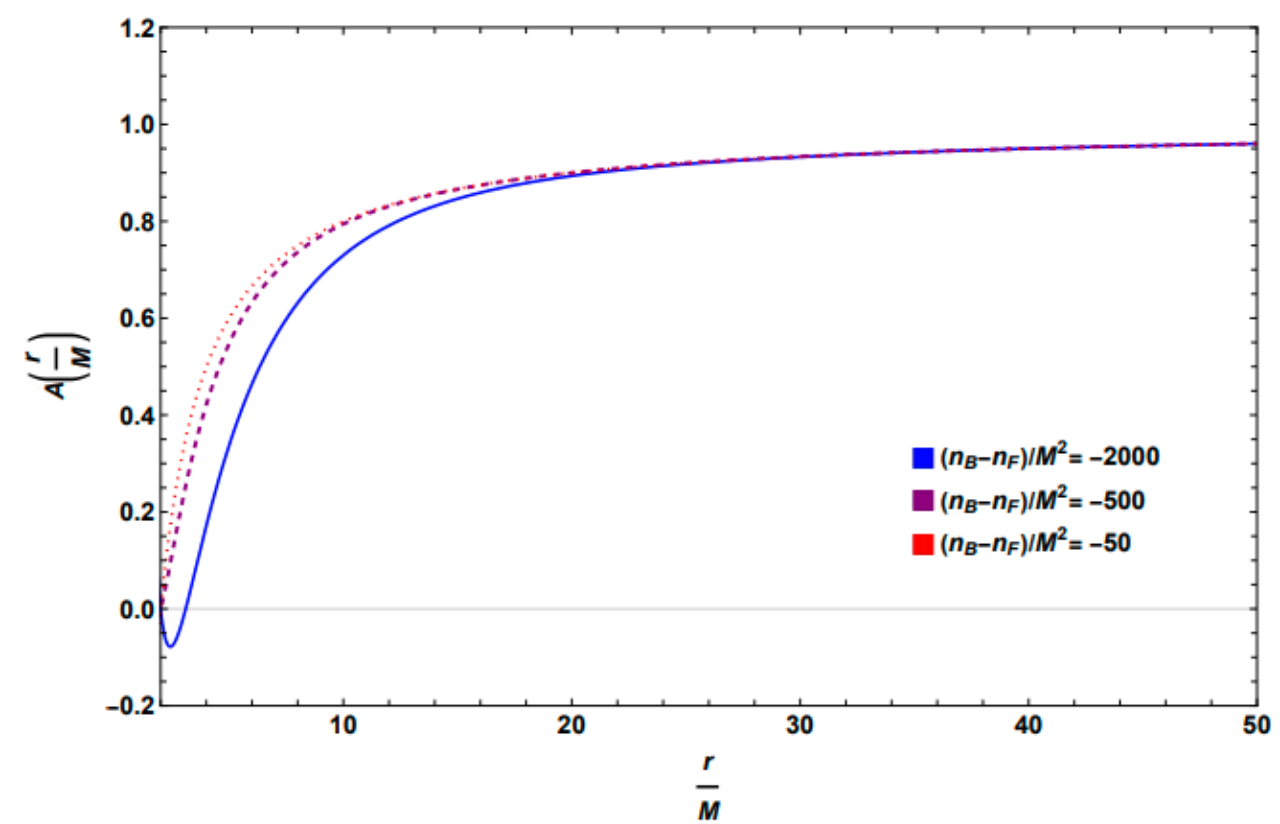
$$\gamma < 0$$

$$A(r) = \left(1 - \frac{2M}{r}\right) \left(1 - \frac{\cos(\sqrt{|\gamma|r})}{r\sqrt{|\gamma|}}\right),$$

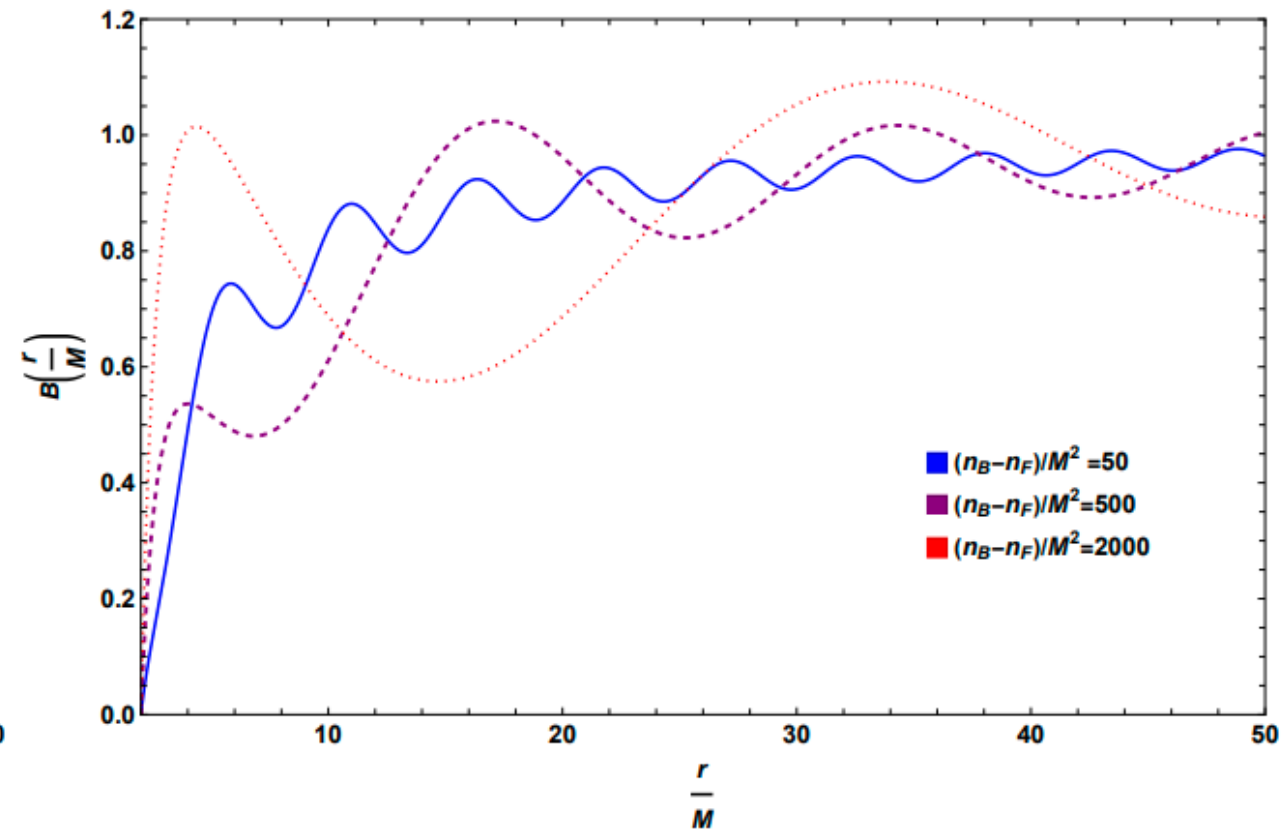
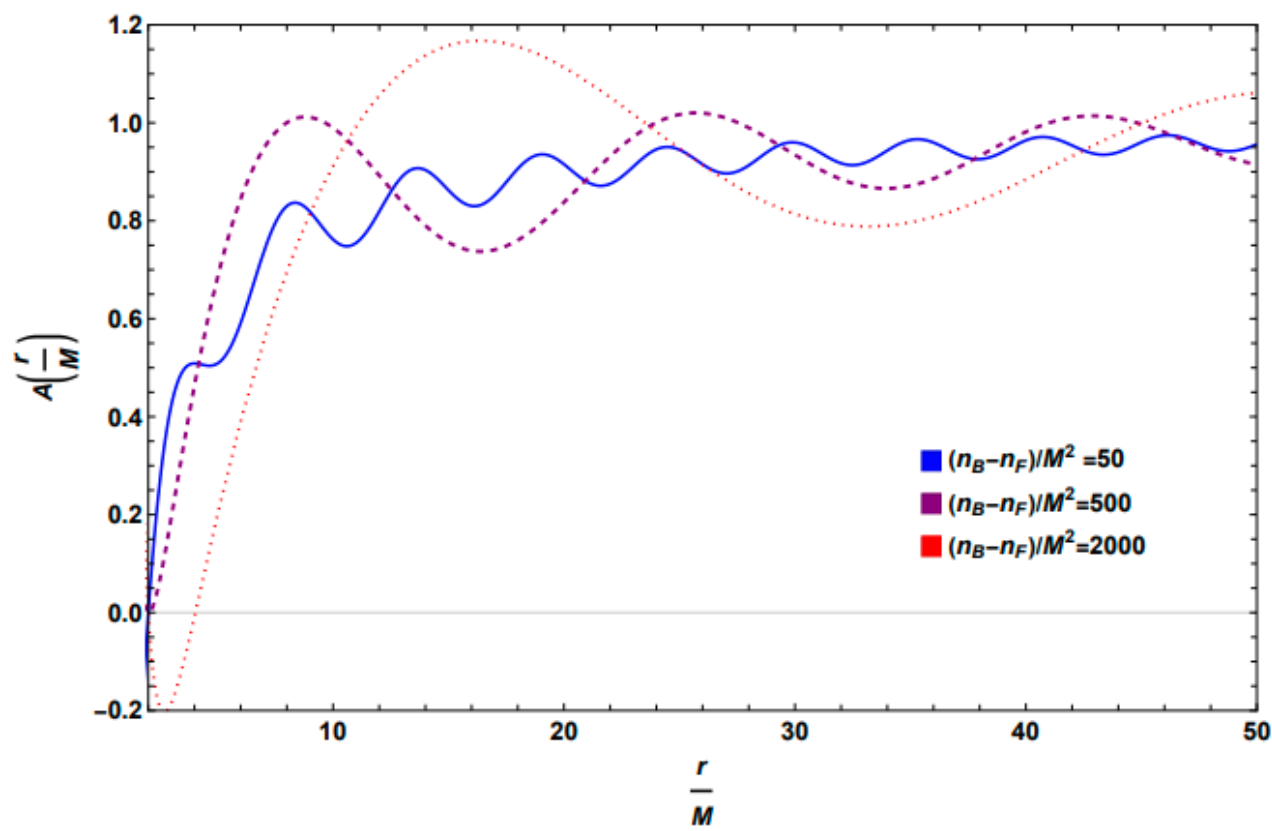
$$B(r) = \left(1 - \frac{2M}{r}\right) \left(1 + \frac{\cos(\sqrt{|\gamma|r})}{r\sqrt{|\gamma|}}\right),$$

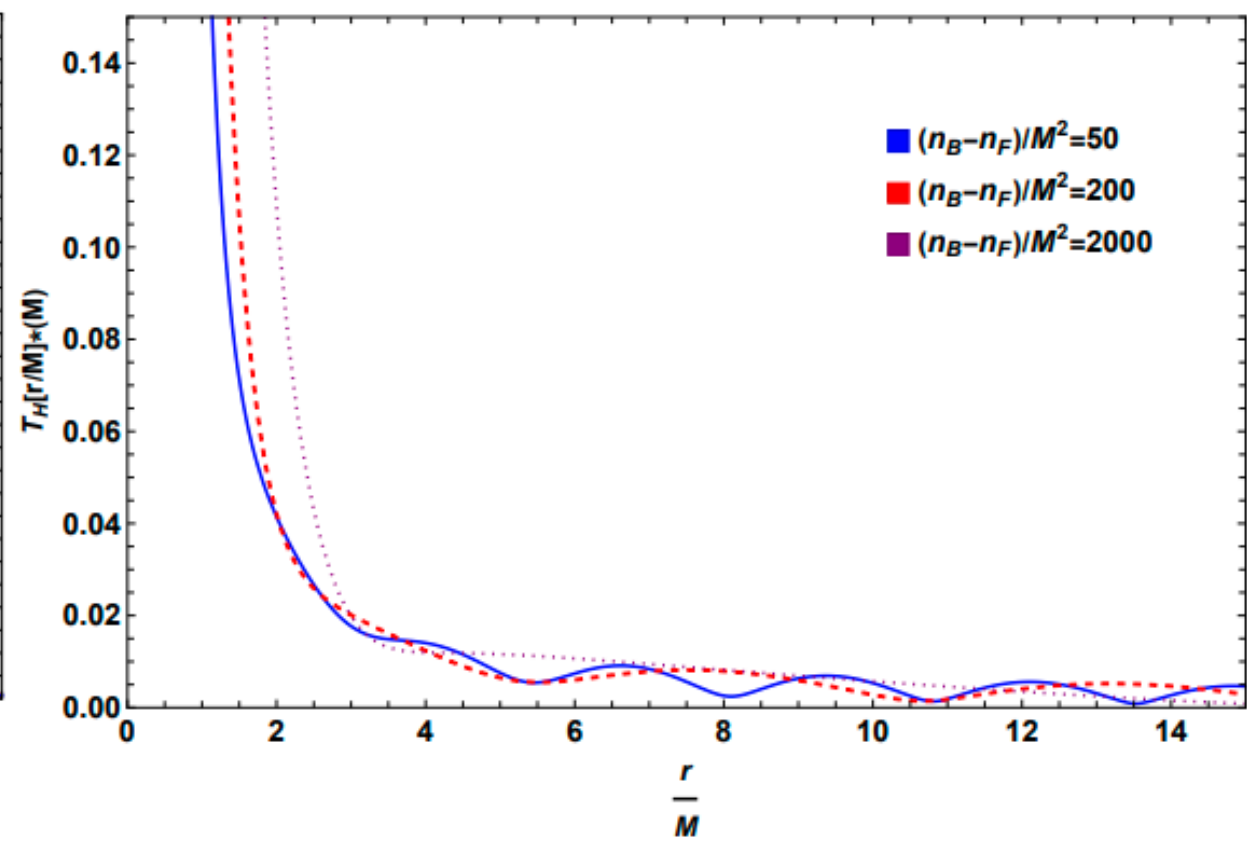
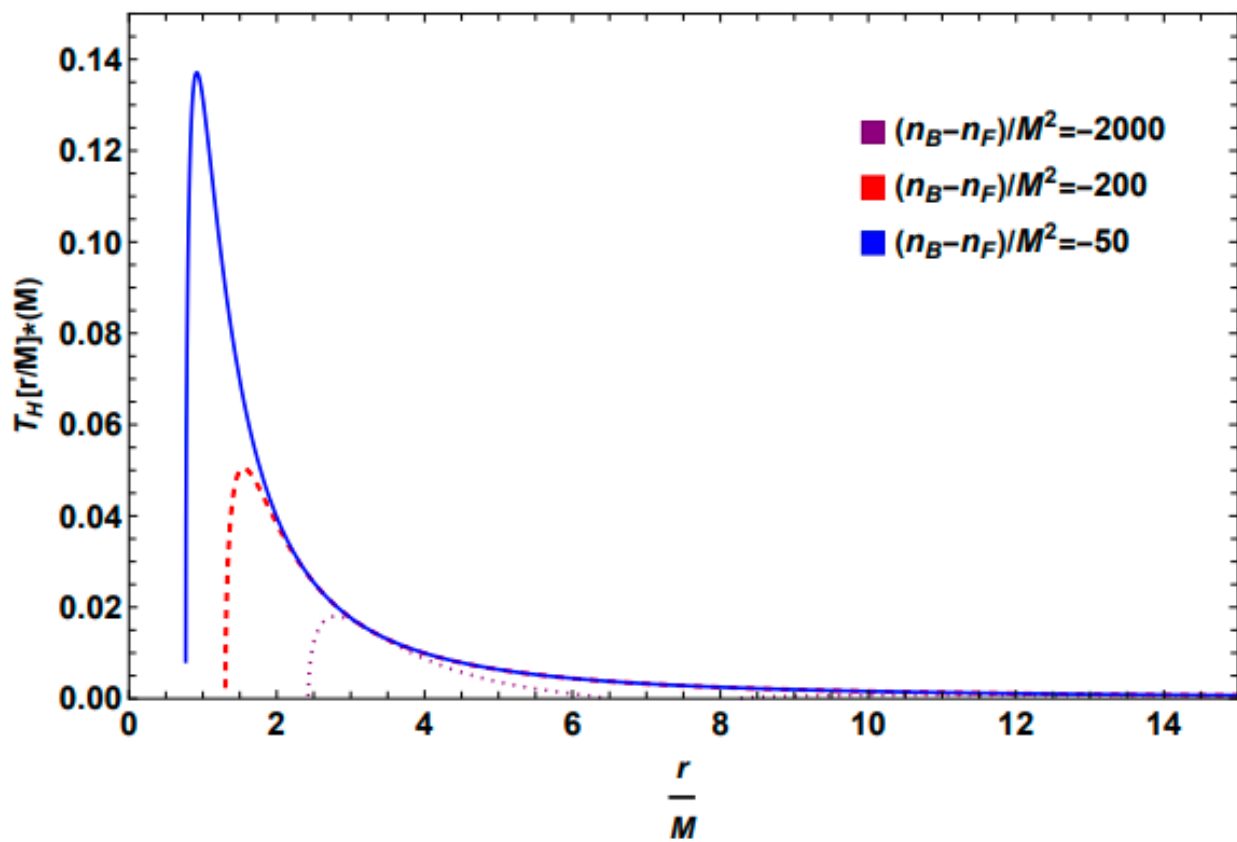
$$C(r) = r^2 \left(1 - \frac{\cos(\sqrt{|\gamma|r})}{r\sqrt{|\gamma|}}\right)$$

Asymptotically-Flat Black Hole Solutions in Symmergent Gravity

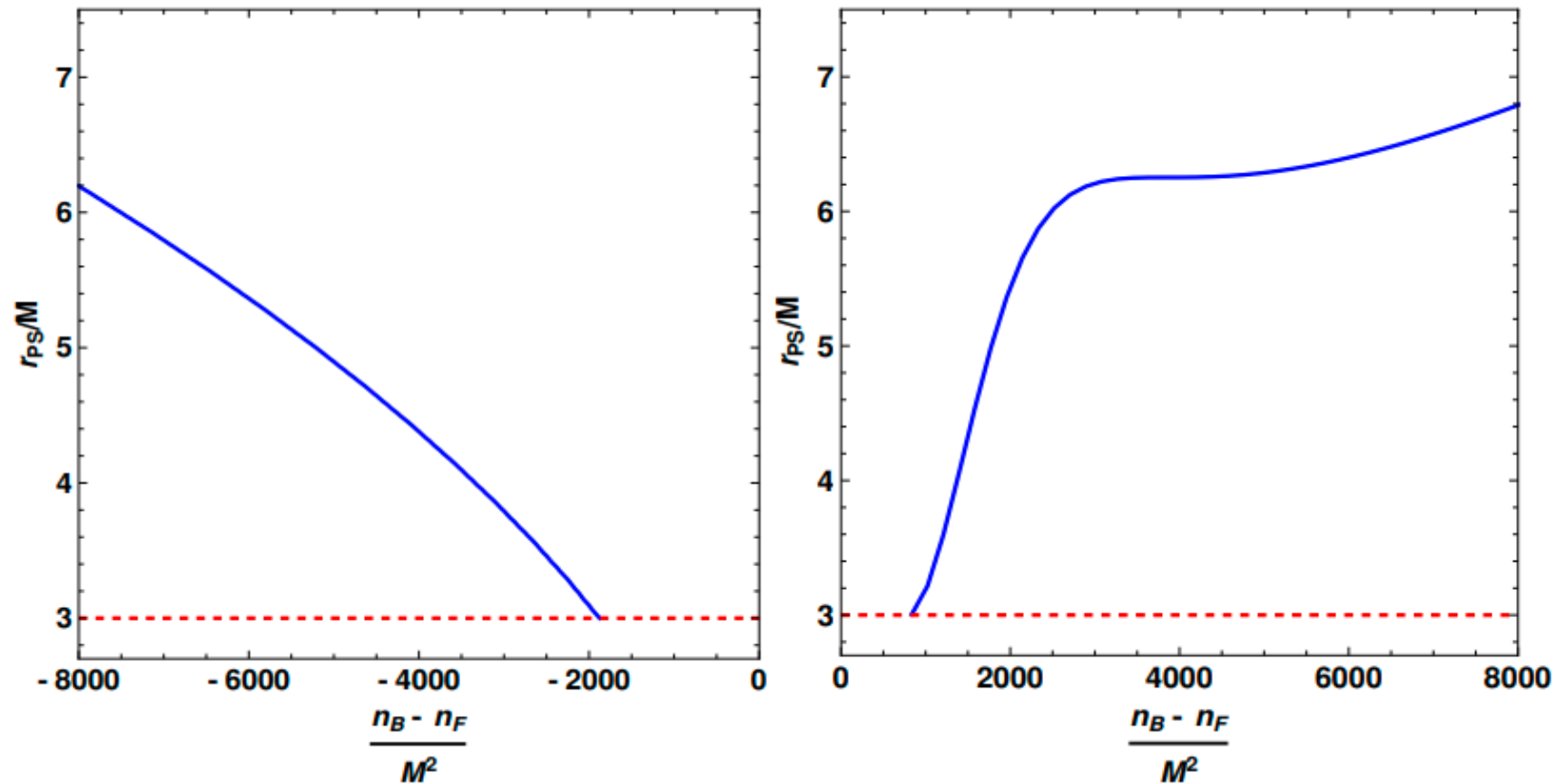


Asymptotically-Flat Black Hole Solutions in Symmergent Gravity

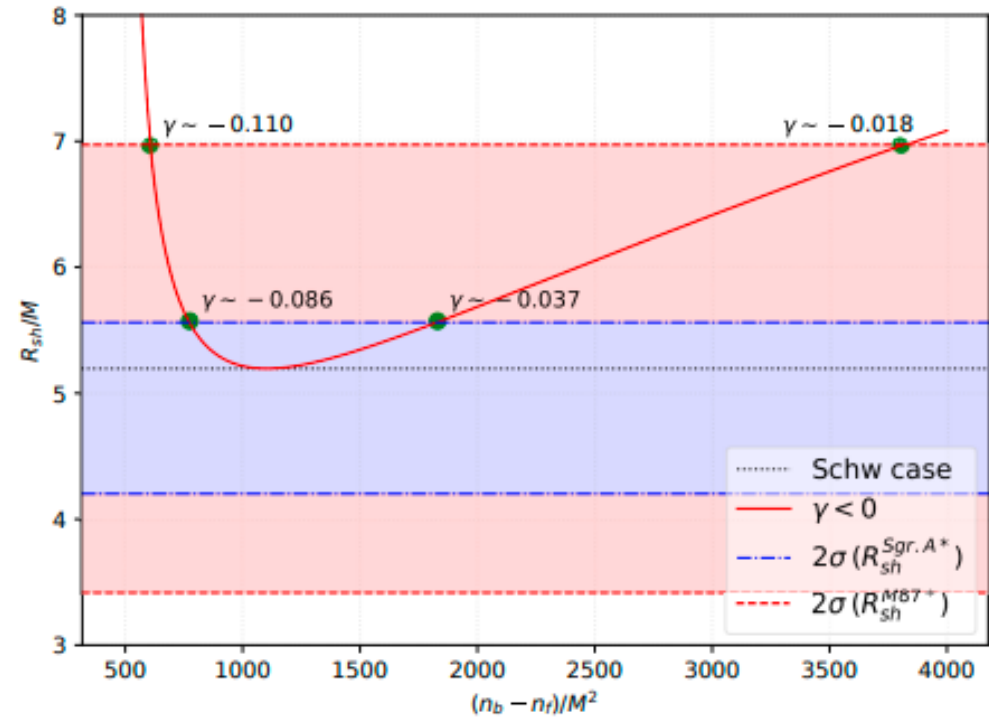
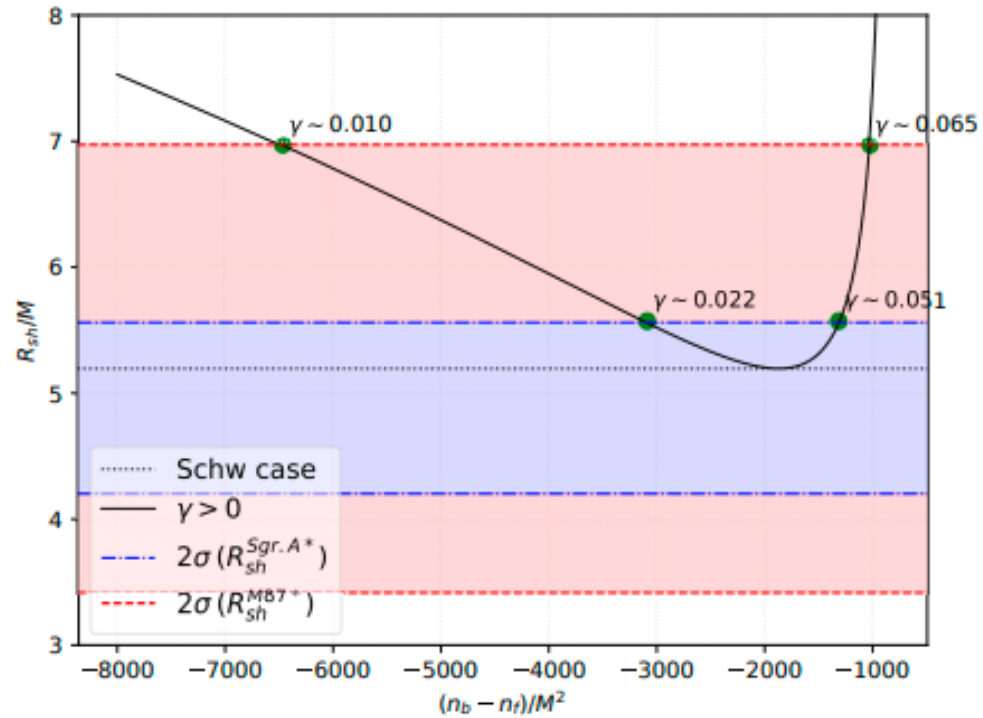




Asymptotically-Flat Black Hole Solutions in Symmergent Gravity

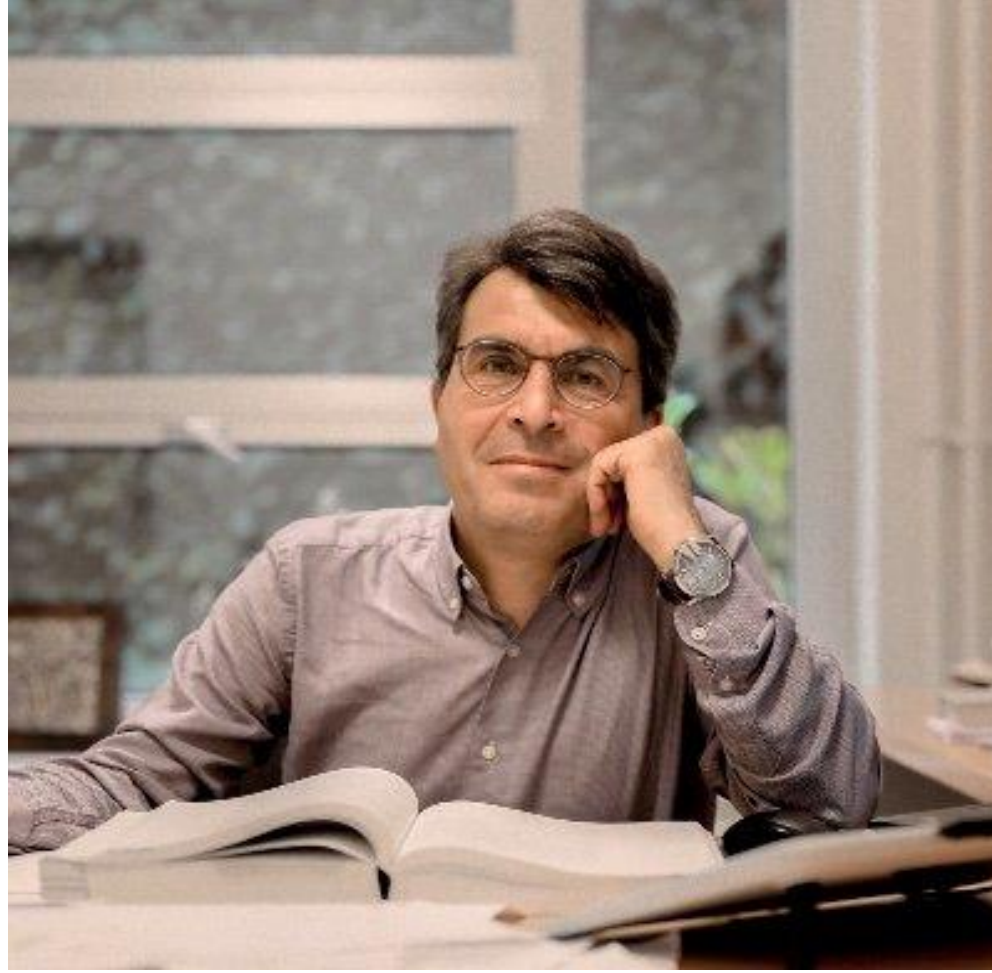


Asymptotically-Flat Black Hole Solutions in Symmergent Gravity



- Metric-Palatini gravity theory reduces to the GR plus a geometrical massive vector theory, which we call **the Einstein-Geometric Proca (EGP) theory**
- EGP model differs from similar models in the literature by its explicit involvement of **the direct coupling between the geometric field and the fermions** in the theory
- We propose a fundamentally different **vector dark matter candidate** from all the other vector dark matter candidates in the literature, mainly due to its geometrical origin
- Our candidate particle, a genuinely geometrical field provided by the extended Palatini gravity, is **a viable dark matter candidate**, and explains the current conundrum by its exceedingly small scattering cross section from nucleon
- **Static spherically symmetric solutions** in the EGP system develop **no horizon** and show the (im)possibility of black hole type solutions in the presence of dust with or without geometrical charge

- Formation of black holes is possible in **the EGP theory in the AdS background**. Photon motion and shadow of the EGP AdS black hole are analyzed.
- **Geometric Z'** can play a role also in the recent **fifth force** data
- Metric-Palatini action in **the Symmergent gravity** reduces to the metrical gravity theory
- We report on **exact charged black hole solutions** in Symmergent gravity with Maxwell field
- Our results provide new insights into the behavior of charged black holes in the context of Symmergent gravity
- In our recent work, we construct **asymptotically-flat symmergent black holes** with variable scalar curvature and use its evaporation and shadow to constrain the symmergent gravity parameters



Sonsuz sevgi ve saygıyla..

SYMMERGENT GRAVITY

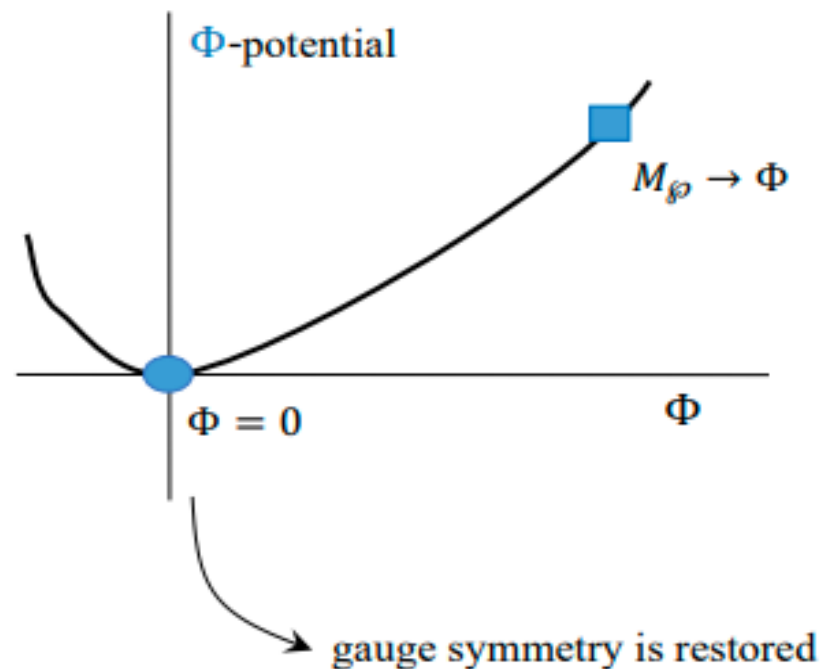
Translation-respecting UV Cutoff M_ϕ

$$M_\phi^2 \text{Tr}[V_\mu \eta^{\mu\nu} V_\nu] \quad \Phi^\dagger V_\mu \eta^{\mu\nu} V_\nu \Phi \quad \eta^{\mu\nu} (D_\mu \Phi)^\dagger (D_\nu \Phi)$$

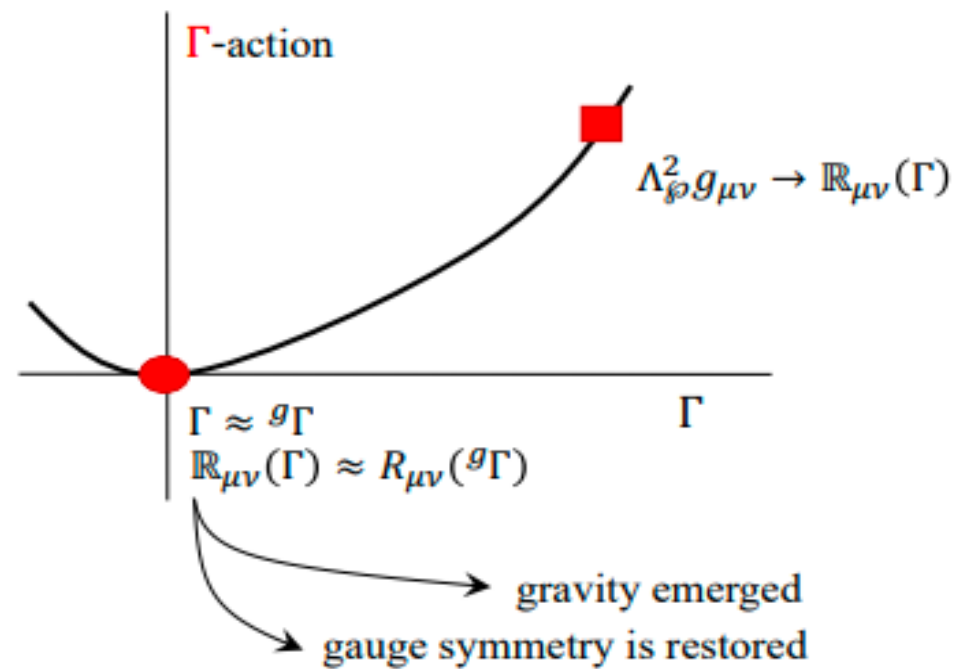
Translation-breaking UV Cutoff Λ_ϕ

$$\Lambda_\phi^2 V_\mu \eta^{\mu\nu} V_\nu \quad V_\mu \mathbb{R}^{\mu\nu}(\bar{\Gamma}) V_\nu \quad V_\mu (\mathbb{R}^{\mu\nu}(\Gamma) - R^{\mu\nu}(g\Gamma)) V_\nu$$

Higgs mechanism:



Symmergence mechanism:



EFFECTIVE QFT: Power-Law Action to Metric-Palatini

$$\delta S_{pow} = \int d^4x \sqrt{-\eta} \left\{ -c_O \Lambda_{\phi}^4 - \sum_m c_m m^2 \Lambda_{\phi}^2 - c_{\phi} \Lambda_{\phi}^2 \phi^{\dagger} \phi + c_V \Lambda_{\phi}^2 \text{tr}[V_{\mu} V^{\mu}] \right\}$$

Higgs-like field: $\Lambda_{\phi}^2 \eta_{\mu\nu} \rightarrow \mathbb{R}_{\mu\nu}(\Gamma)$: Ricci curvature of an affine connection

General covariance: $\eta_{\mu\nu} \rightarrow g_{\mu\nu}$: From flat spacetime to curved spacetime
 $\partial_{\mu} \rightarrow \nabla_{\mu}$

Metric-Palatini gravity:

$$\delta S_{pow} = \int d^4x \sqrt{-g} \left\{ -\frac{c_O}{16} \left(g^{\mu\nu} \mathbb{R}_{\mu\nu}(\Gamma) \right)^2 - \frac{1}{4} \sum_m c_m m^2 g^{\mu\nu} \mathbb{R}_{\mu\nu}(\Gamma) - \frac{c_{\phi}}{4} g^{\mu\nu} \mathbb{R}_{\mu\nu}(\Gamma) \phi^{\dagger} \phi + c_V (\mathbb{R}_{\mu\nu}(\Gamma) - R_{\mu\nu}(g\Gamma)) V_{\mu} V_{\nu} \right\}$$

EFFECTIVE QFT: Quantum Field Theory + Emergent General Relativity:

$$S_{QFT+GR} = \underbrace{S(g, \psi) + \delta S(g, \psi)}_{\text{QFT sector}} + \int d^4x \sqrt{-g} \left\{ \underbrace{-\frac{R(g)}{16\pi G_N} - \frac{c_O}{16} R(g)^2 - \frac{c_{\phi}}{4} R(g) \phi^{\dagger} \phi + \mathcal{O}(G_N)}_{R + R^2 \text{ gravity sector}} \right\}$$

QFT sector

$R + R^2$ gravity sector

symmetry-restoring emergent gravity = "symmergent gravity"