Axion-benzeri karanlık madde için yeni açılımlar

Novel approaches for ALP dark matter

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CE, R. Sato, G. Servant, P.Sørensen, *JCAP* **10** (2022) 053 [2206.14269]
CE, Servant, *JCAP* **01** (2023) 009 [2207.10111]
A. Chatrchyan, **CE**, M. Koschnitzke, G. Servant, *JCAP* **10** (2023) [2305.03756]

Bu çalışmanın bir bölümü TÜBİTAK tarafından “2236 - Uluslararası Deneyimli Araştırmacı Dolaşımı Destek Programı” kapsamında desteklenmiştir. (Project No: 121C404).

Strong CP Problem

The QCD sector of the Standard Model contains a CP violating $\bar{\theta}$-term:

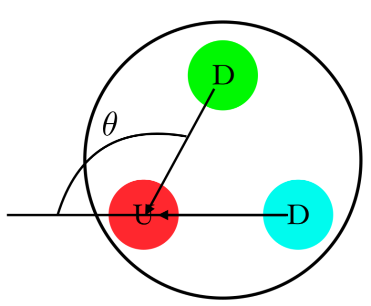
$$\mathcal{L}_{\mathrm{QCD}} \supset \frac{\bar{\theta} g_s^2}{32\pi^2} \,\mathrm{Tr}\, G_{\mu\nu} \widetilde{G}^{\mu\nu}$$

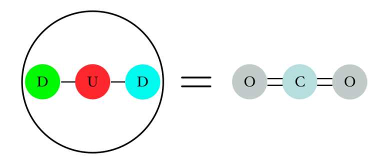
Due to this term, neutrons gets a non-zero electric dipole moment:

$$d_n \approx 3.6 \times 10^{-16}\, \bar{\theta}\, e\,\mathrm{cm}$$

However, this has not been observed in experiments:

$$|d_n| < 2.9 \times 10^{-26}\, e\,\mathrm{cm} \quad \Rightarrow \quad \bar{\theta} \lesssim 10^{-10}\,!$$





Anson Hook, 1812.02669

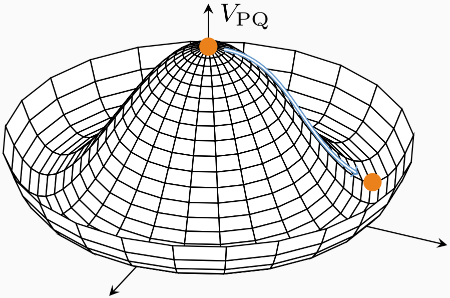
Add a spontaneously broken, and anomalous under QCD global U(1) symmetry to the Standard Model:

$$\mathcal{G}(\mathrm{SM}) \otimes \mathrm{U_{PQ}}(1) \nrightarrow \mathcal{G}(\mathrm{SM}) \quad @f_\phi$$

Spontaneous symmetry breaking creates a Nambu-Goldstone boson named axion.

$$\varphi_{\mathrm{PQ}} = \chi \exp\left\{ i \frac{\phi}{f_\phi} \right\} \equiv \chi \exp\{i\theta\}$$

$\chi \equiv$ radial mode , $\phi \equiv$ axion



- the radial mode corresponds to a heavy particle with mass $m_\chi \sim f_\phi$.
- axion is the degree of freedom of the angular motion which is massless after the symmetry breaking.

Dynamical θ parameter

The Standard Model Lagrangian including the axion takes the form

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{\bar{\theta} g_s^2}{32\pi^2} \text{Tr}\, G_{\mu\nu} \widetilde{G}^{\mu\nu} + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \mathcal{L}_{\text{int}}\left[\frac{\partial_\mu \phi}{f_\phi}\right] + \frac{\phi}{f_\phi}\frac{g_s^2}{32\pi^2} \text{Tr}\, G_{\mu\nu} \widetilde{G}^{\mu\nu}$$

The shift symmetry of the axion allows us to absorb $\bar{\theta}$ into ϕ/f_ϕ by defining the physical axion angle θ:

$$\theta \equiv \bar{\theta} + \frac{\phi}{f_\phi}.$$

After the QCD phase transition, the $\text{Tr}\, G_{\mu\nu}\tilde{G}^{\mu\nu}$ (instanton) terms create an effective potential for the axion which gets minimized at a CP-conserving value: Vafa ve Witten, PRL 53, 535

$$\langle \theta \rangle = 0.$$

The Peccei-Quinn mechanism turns the constant $\bar{\theta}$-parameter into a dynamical one, and $\theta = 0$ is explained by system dynamics.

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Axion potential and the axion mass

The axion potential generated by the instanton contributions take different forms at temperatures above and below the QCD phase transitions:

- Above the phase transition: $T \gg \Lambda_{\rm QCD} \simeq 150\,\text{MeV}$: Borsanyi et al. 1606.07494

$$V(\theta) \approx m_\phi^2(T) f_\phi^2 [1 - \cos(\theta)] \propto T^{-8.16}.$$

- Below the phase transition: $T \ll \Lambda_{\rm QCD}$ Luzio et al. 2003.01100

$$V(\theta) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{\theta}{2}\right)}.$$

- This allows us to obtain a relation between the axion mass m_ϕ and the axion decay constant f_ϕ:

$$m_\phi = \frac{\sqrt{m_u m_d}}{m_u + m_d} \frac{m_\pi f_\pi}{f_\phi} \simeq 5.7 \times \left(\frac{10^{12}\,\text{GeV}}{f_\phi}\right) \mu\text{eV}.$$

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Axion couplings to the Standard Model

The couplings between the axions and the Standard Model particles can be worked using chiral perturbation theory. Most notable ones are

- **Couplings to fermions:** axion-electron, axion-neutron, ...

$$\mathcal{L} \supset \frac{\partial_\mu \phi}{2 f_\phi} \bar{\psi} c_\psi \gamma^\mu \gamma_5 \psi$$

- **Coupling to photon:** modifies Electrodynamics, most promising discovery channel

$$\mathcal{L} \supset \frac{1}{4} \underbrace{C_{\phi\gamma} \frac{\alpha_{\mathrm{EM}}}{2\pi f_\phi}}_{\equiv g_{\phi\gamma}} \underbrace{F \widetilde{F}}_{\mathbf{E}\cdot\mathbf{B}} .$$

All couplings are suppressed by f_ϕ^{-1}!

QCD Axion and Axion-Like-Particles (ALPs)

An "axion-like-particle (ALP)" is defined as a scalar field ϕ with the following effective Lagrangian at low energies:

$$\mathcal{L}_{\text{ALP}} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \Lambda_b^4(T)\left[1-\cos\left(\frac{\phi}{f_\phi}\right)\right] - \frac{g_{\phi\gamma}}{4}\phi F_{\mu\nu}\widetilde{F}^{\mu\nu} + \ldots$$

The mass (barrier-height) is in general temperature-dependent:

$$\Lambda_b^4(T) \approx m_\phi^2 \times \begin{cases} \left(\frac{T_c}{T}\right)^\gamma & , T \geq T_c \\ 1 & , T < T_c \end{cases}$$

QCD axion	Generic ALP
$m_\phi^2 f^2 \approx (76\,\text{MeV})^4$, $\gamma \approx 8$, $T_c \approx 150\,\text{MeV}$ Couplings to photons, nucleons, electrons, etc...	m_ϕ, f, γ, T_c are free parameters. Might not have any coupling to SM.

This talk: A generic ALP with a constant mass, i.e. $\gamma = 0$.

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ALP dark matter: The standard paradigm

The cosmology of an ALP field ϕ is determined by the evolution equation:

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2}{a^2}\phi + V'(\phi) = 0, \quad V(\phi, T) = m_\phi^2(T) f_\phi^2 \left[1 - \cos\left(\frac{\phi}{f_\phi}\right)\right].$$

One also needs to specify the initial conditions that depends on the time of the symmetry breaking that has generated the ALP as the pNGB.

- **Post-inflationary:** Different initial conditions in each Hubble patch. Inhomogeneous.
- **Pre-inflationary:** Random initial angle $\theta \equiv \phi/f_\phi \in [-\pi, \pi)$ in observable universe. Homogeneous.

Assuming pre-inflationary scenario and negligible initial kinetic energy

$$\rho_\phi \propto \begin{cases} \text{constant}, & m(T) \ll H(T) \\ a^{-3}, & m(T) \gg H(T) \end{cases}.$$

The relic density for ALP dark matter is determined by $0 \leq |\theta_i| < \pi$.

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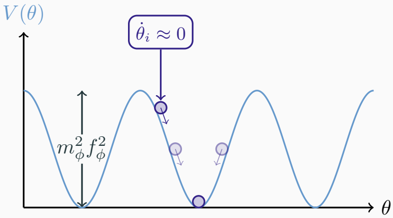
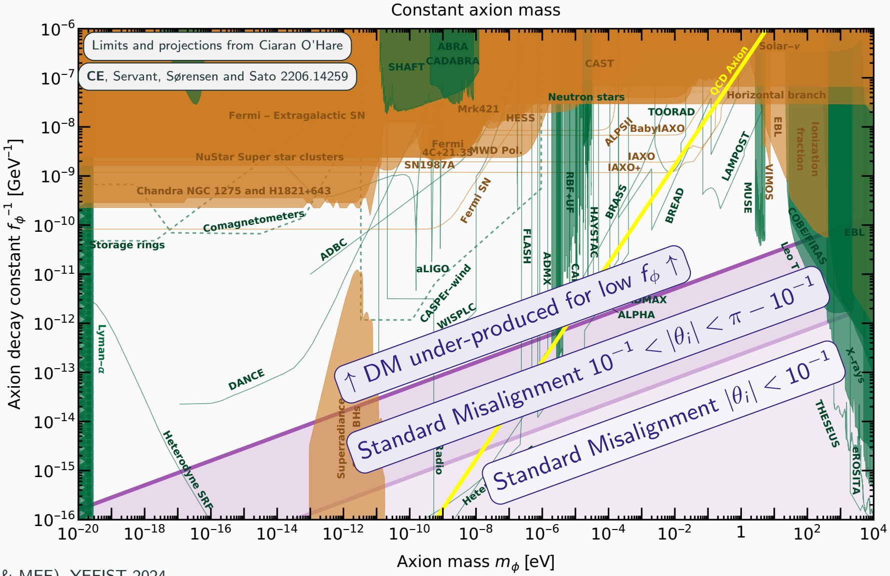


ALP dark matter parameter space in the standard paradigm (with $g_{\theta\gamma} = (\alpha_{em}/2\pi)(1.92/f_\phi)$)



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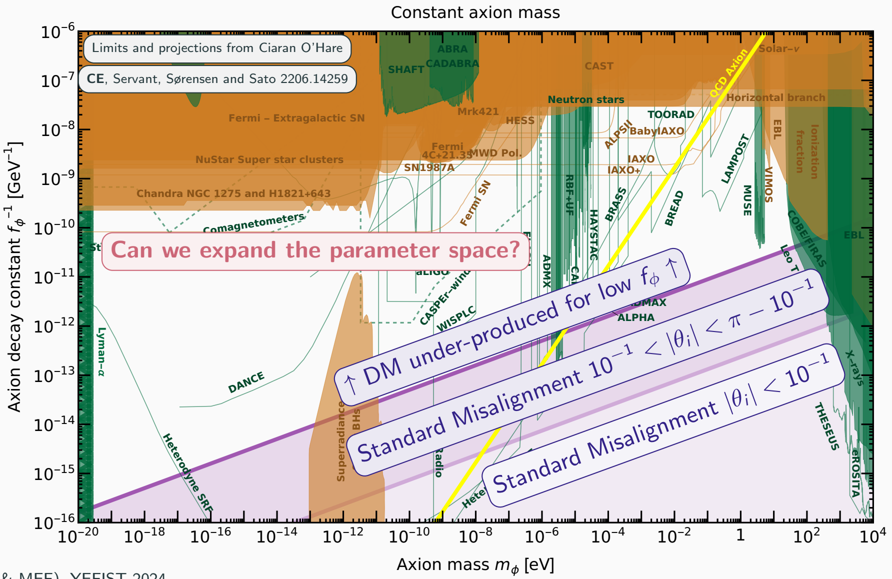

Extending the parameter space to lower f_ϕ values

- Modify the initial conditions
  - **Large misalignment:** Choose the initial angle very close to the top, i.e. $|\pi - \theta_i| \ll 1$.
    Zhang,Chiueh 1705.01439; Arvanitaki et al. 1909.11665
  - **Kinetic misalignment:** Start with a large initial kinetic energy.
    Co et al. 1910.14152; Chang et al. 1911.11885
- Modify the potential to a non-periodic one:
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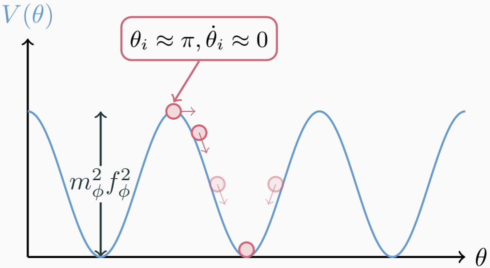

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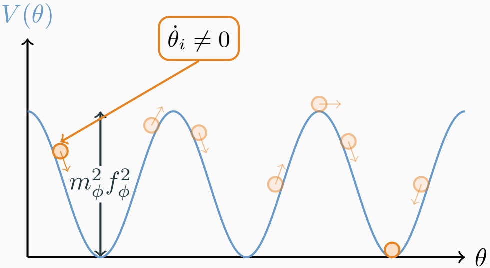

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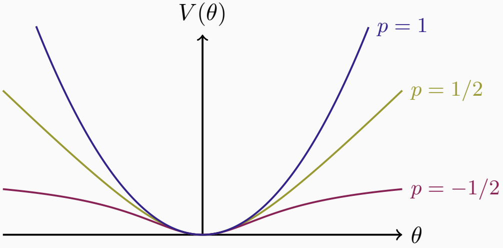
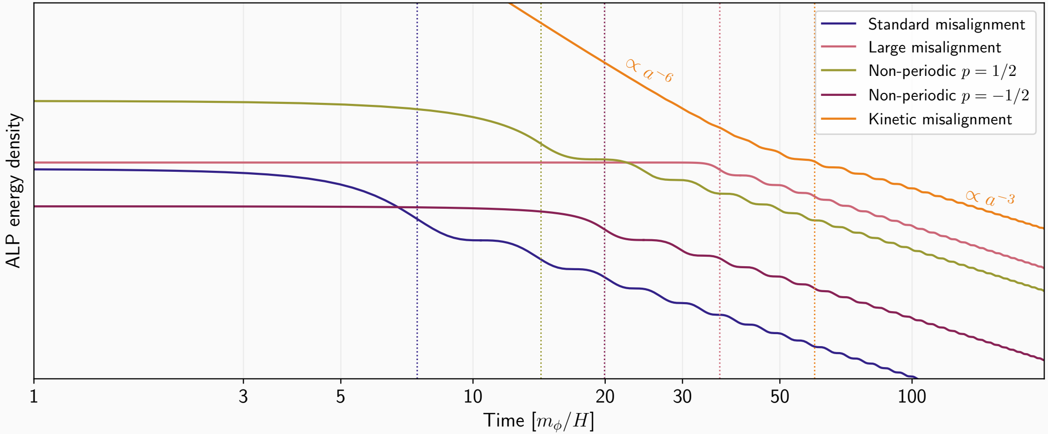
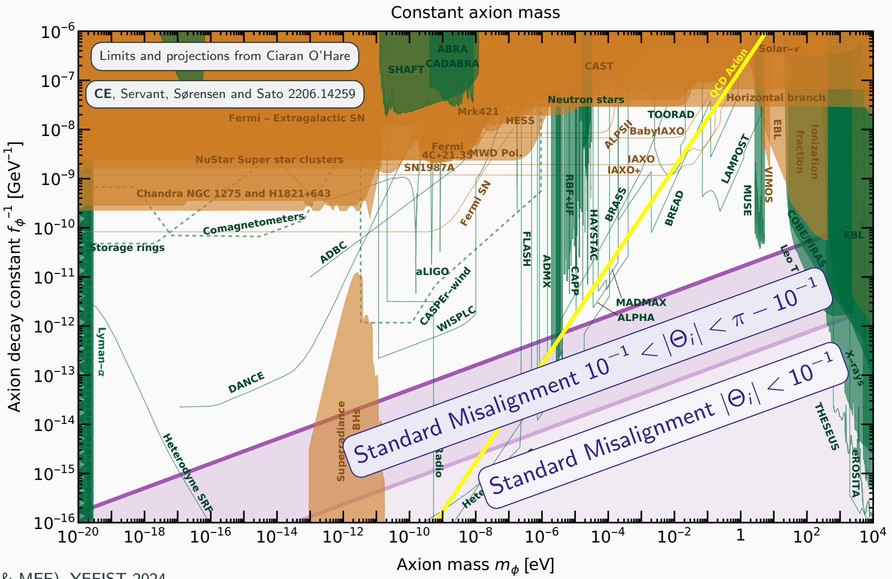


Expanding the parameter space to lower f_ϕ values



Common property of all these is that the onset of oscillations got delayed which boosts the dark matter abundance, and extends the ALP dark matter parameter space to lower decay constants.

ALP parameter space (with KSVZ-like photon coupling $g_{\theta\gamma} = (\alpha_{em}/2\pi)(1.92/f_\phi)$)



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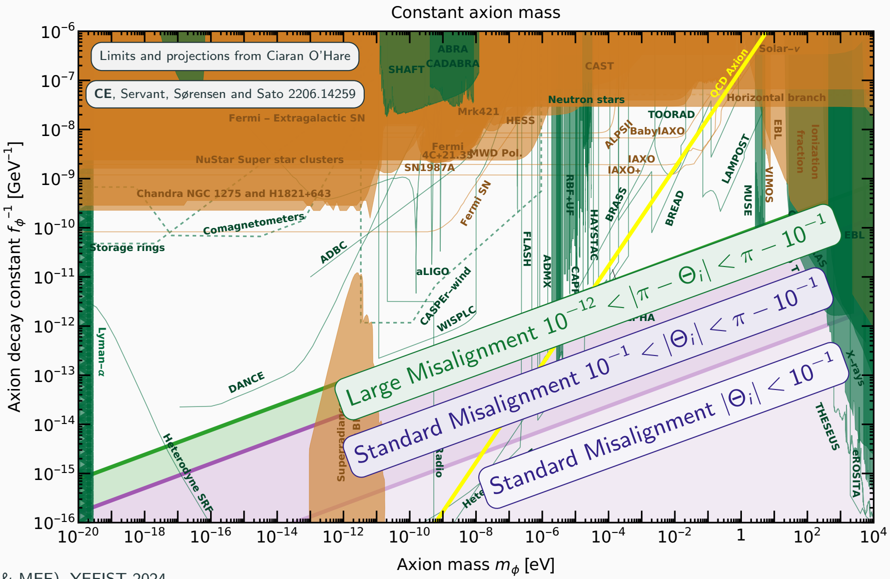

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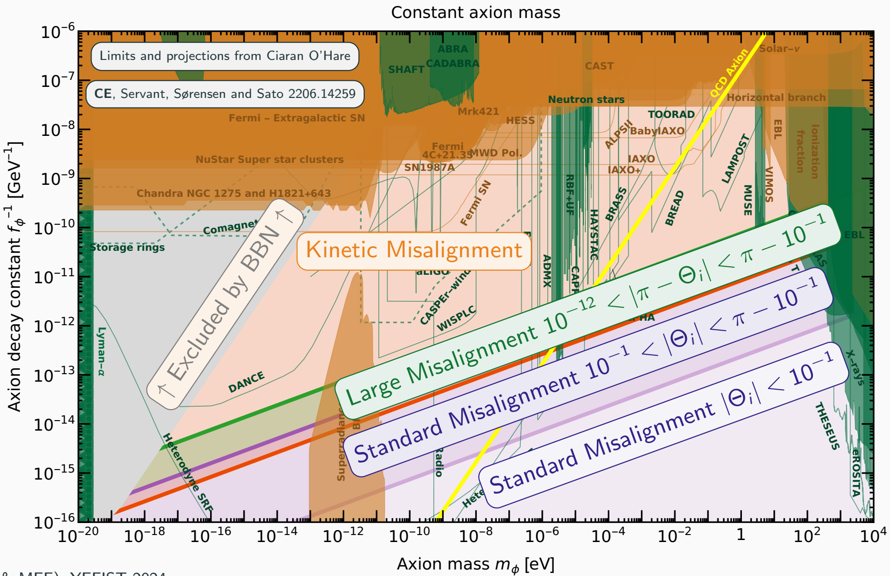

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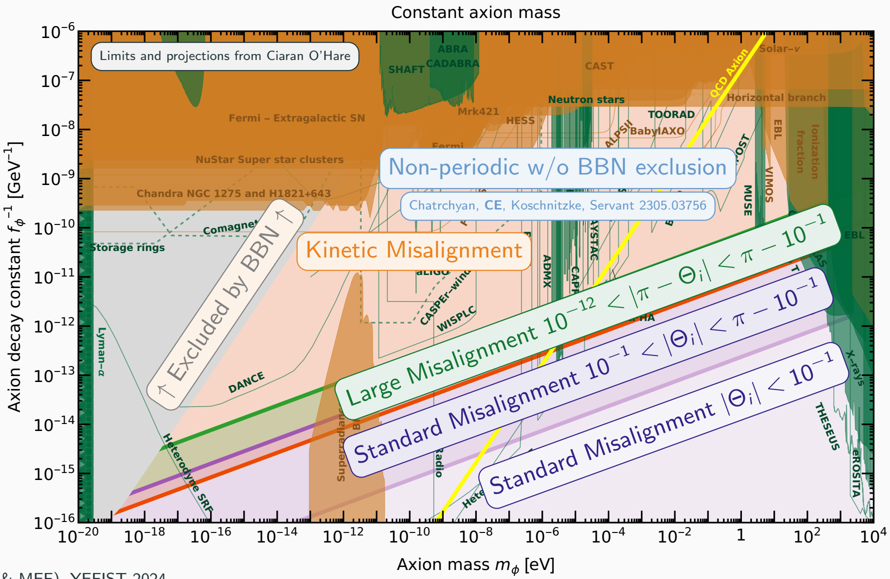

ALP fluctuations and the mode functions

- Even in the pre-inflationary scenario ALP field has some fluctuations on top of the homogeneous background which can be described by the mode functions in the Fourier space.

$$\theta(t, \mathbf{x}) = \Theta(t) + \int \frac{d^3k}{(2\pi)^3} \theta_k e^{i\vec{k}\cdot\vec{x}} + \text{h.c.}$$

- These fluctuations are seeded by adiabatic and/or isocurvature perturbations:

Adiabatic perturbations (This work)

- Due to the energy density perturbations of the dominating component, **unavoidable**.
- Initial conditions in the super-horizon limit:

$$\delta_i/(1 + w_i) = \delta_j/(1 + w_j)$$

Isocurvature perturbations

- If ALPs exist during inflation and are light $m \ll H_{\text{inf}}$, they pick up quantum fluctuations:

$$\delta\theta \sim H_{\text{inf}}/(2\pi f_{\text{inf}})$$

- Can be avoided/suppressed if ALP has a large mass during inflation, or $f_{\text{inf}} \gg f_{\text{today}}$.

Exponential growth of the mode functions

The equation of motion for the mode functions can be derived from the FRLW metric including the curvature perturbations:

$$\mathrm{d}s^2 = -[1-2\Phi(t,\mathbf{x})] + a^2(t)[1+2\Phi(t,\mathbf{x})]\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j\,, \quad \Phi_k(t,\mathbf{x}) = 3\Phi_k(0)\left[\frac{\sin t_k - t_k\cos t_k}{t_k^3}\right],\ t_k = \frac{k/a}{\sqrt{3}H}$$

For small fluctuations $\delta\theta \ll \Theta$ the equation of motion for the mode functions become

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \underbrace{\left[\frac{k^2}{a^2} + V''(\phi)\Big|_{\bar{\phi}}\right]}_{\text{eff. frequency}}\phi_k = \underbrace{2\,\Phi_k\ V'(\phi)\Big|_{\bar{\phi}} - 4\dot{\Phi}_k\dot{\bar{\phi}}}_{\text{source term}}$$

The EoM is unstable when the effective frequency

- becomes negative $\Rightarrow$ tachyonic instability
- is oscillating $\Rightarrow$ parametric resonance

Kofman et al. hep-ph/9704452; Felder, Kofman hep-ph/0606256
Greene et al. hep-ph/9808477; Jaeckel et al. 1605.01367
Cedeno et al. 1703.10180; Berges et al. 1903.03116
Fonseca et al. 1911.08472; Morgante et al. 2109.13823

Instability exists except for a free theory where $V''(\phi) = m^2$.

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The size of fluctuations is determined by the density contrast:

$$\delta_\rho(\vec{\mathbf{x}}, t) \equiv \frac{\rho(\vec{\mathbf{x}}, t) - \overline{\rho}(t)}{\overline{\rho}(t)}$$

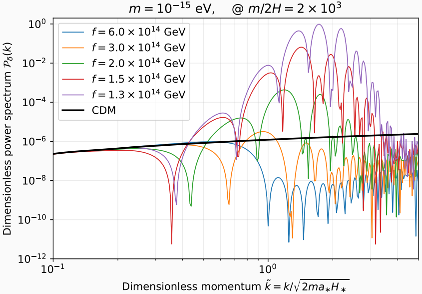
The power spectrum (two-point function) determines the distribution of structures today:

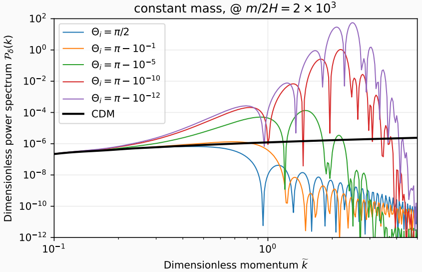
$$\mathcal{P}_\delta(k) = \frac{k^3}{2\pi^2} \left\langle \left| \tilde{\delta}_\rho(\vec{\mathbf{k}}, t) \right|^2 \right\rangle$$

After the parametric resonance the power spectrum can reach to $\mathcal{O}(1)$ values:

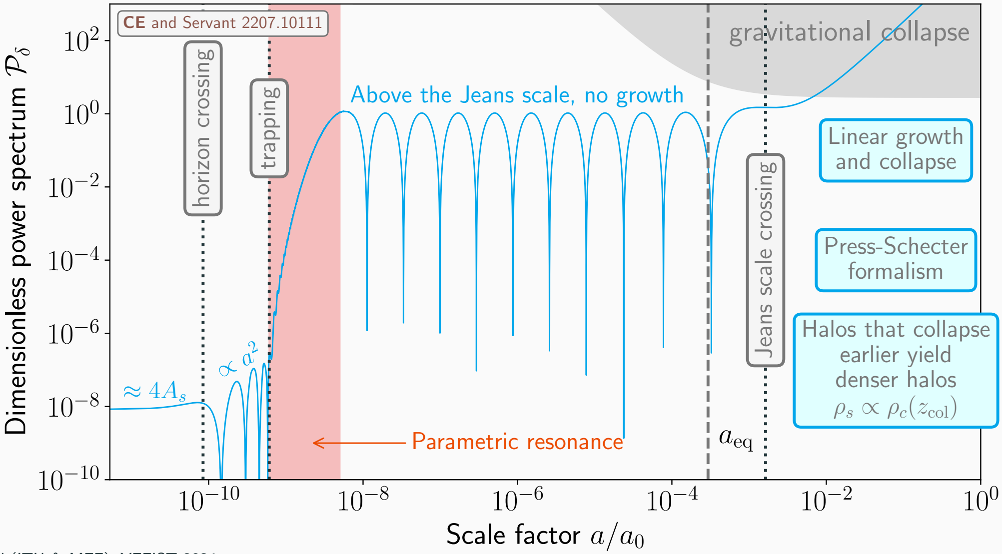
Dense and compact ALP mini-clusters can also be formed in the pre-inflationary scenario!

Growth rate of the perturbations depend exponentially on $m_\phi / H\big|_{\text{osc}}$.





Lifetime of a fluctuation mode



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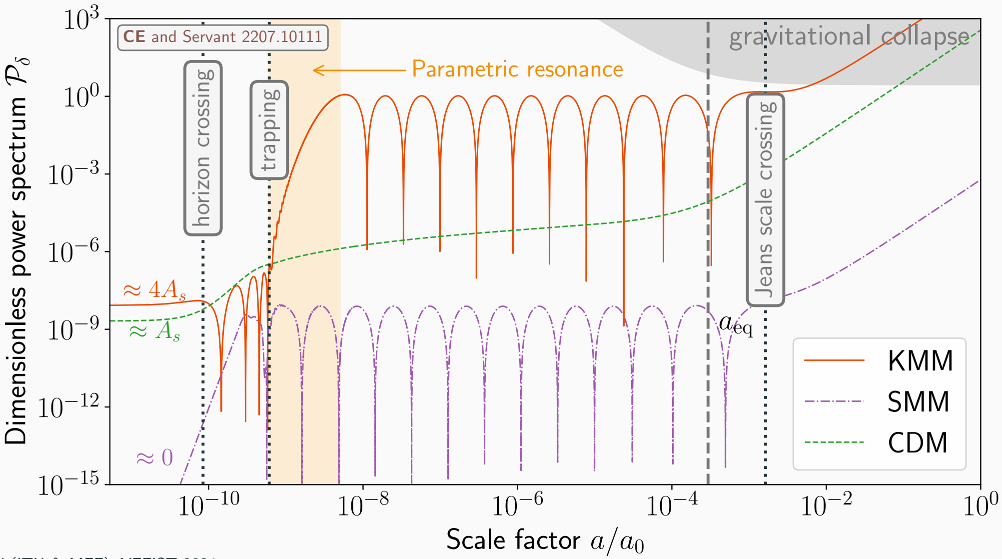
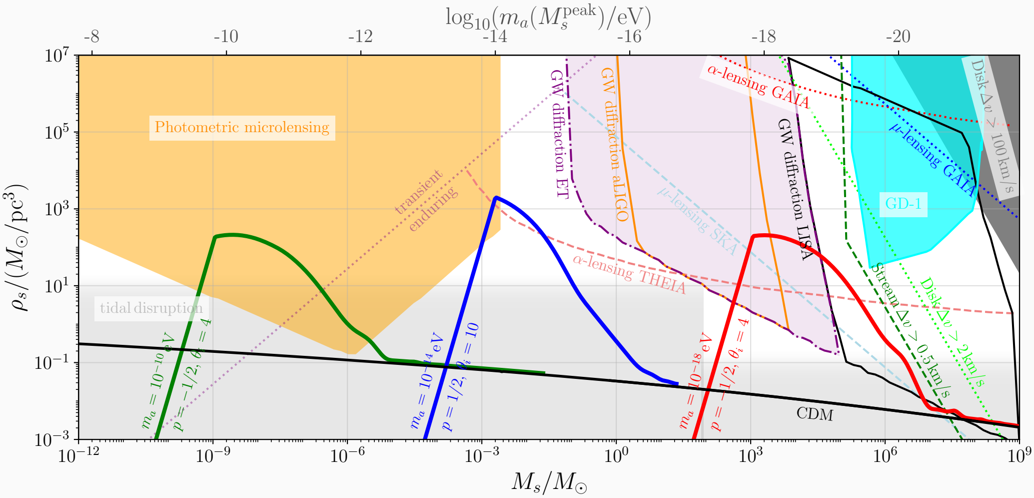


Observational prospects



Experimental prospects from Tilburg et al. 1804.01991; Arvanitaki et al. 1909.11665; Ramani et al. 2005.03030

Conclusions and Outlook

- The Standard Misalignment Mechanism is not sufficient to account for the correct dark matter abundance in the ALP parameter space where the experiments are most sensitive.
- This parameter space can be opened by considering models where the initial energy budget is increased, and the onset of oscillations is delayed from the conventional value $m_{\rm osc}/H_{\rm osc} \sim 3$.
- In these models which go beyond the standard paradigm, the fluctuations can grow exponentially, and dense ALP mini-clusters can be formed even in the pre-inflationary scenario.

Thank you for listening!

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Cosmological evolution of the ALP field

Start with the action for the ALP field (neglect Standard Model interactions):

$$S = \int \mathrm{d}^4x \sqrt{-g}[-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)].$$

Take the background geometry to be the FRWL geometry with the metric

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a^2(t)\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j$$

Then, the ALP field obeys the following equation of motion:

$$\ddot{\theta} + 3H\dot{\theta} - \frac{\nabla^2}{a^2}\theta + \frac{1}{f_\phi^2}\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0, \quad H = \frac{\dot{a}}{a} := \text{Hubble parameter}.$$

This is a second order ODE which can be solved after specifying the initial conditions.

How to get a large initial kinetic energy?

A large initial kinetic energy for the ALP field can be motivated in various UV completions:

- Explicit breaking of the PQ symmetry at very large energies. Co et al. 1910.14152; 2004.00629; 2006.05687
- Trapped misalignment Luzio et al. 2102.00012; 2102.01082

Today's ALP energy density is Co et al. 1910.14152

CE, Servant, Sørensen, Sato 2206.14259

$$h^2\Omega_{\phi,0} \approx 0.12\left(\frac{m_\phi}{5\times 10^{-3}\,\mathrm{eV}}\right)\left(\frac{Y}{40}\right), \quad Y = \frac{f_\phi \dot{\phi}(T)}{s(T)}$$

The yield parameter Y is conserved after the kick, and determines the ALP relic density today.

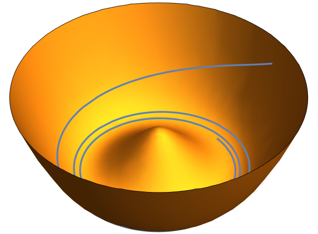
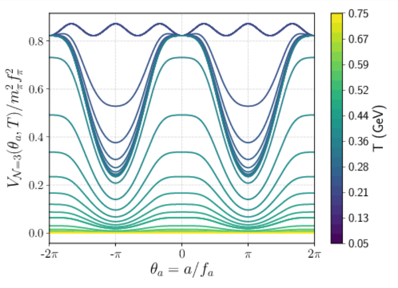


Figure credit: Philip Sørensen



Luzio et al. 2102.01082

Evolution of the density contrast at late times

Calculating the evolution of the density contrast until today by numerically solving the mode function equation of motion is very time- and resource-consuming. Luckily, we can use an effective description using the WKB approximation: Park et al., 1207.3124

$$\Theta(t) = a^{-3/2}[\Theta_+ \cos(mt) + \Theta_- \sin(mt)],$$
$$\theta_k(t) = \theta_+(k,t)\cos(mt) + \theta_-(k,t)\sin(mt).$$

The evolution of the density contrast for sub-horizon modes, $k/a \gg H$ obeys the differential equation (valid both in radiation and matter eras):

$$\ddot{\delta}_k + 2H\dot{\delta}_k + \left(c_{s,\mathrm{eff}}^2 \frac{k^2}{a^2} - 4\pi G\bar{\rho}\right)\delta_k = 0, \quad \overbrace{c_{s,\mathrm{eff}}^2 \approx \frac{1}{4}\frac{k^2}{a^2 m^2}\left(1 + \frac{1}{4}\frac{k^2}{a^2 m^2}\right)^{-1} \approx \frac{1}{4}\frac{k^2}{a^2 m^2}}^{\text{effective sound speed of the ALP}}.$$

The evolution for Cold Dark Matter (CDM) is recovered in the limit $c_{s,\mathrm{eff}}^2 \to 0$ or $m \to \infty$.

Axion Jeans Scale

For sub-horizon $k/a \gg H$ and non-relativistic $k/a \ll m$ modes, the density contrast evolution is

$$\ddot{\delta}_k + 2H\dot{\delta}_k + \left[\underbrace{\frac{1}{4}\frac{(k/a)^4}{m^2}}_{\text{"pressure" term}} - \underbrace{4\pi G\bar{\rho}}_{\text{gravitational instability}} \right] \delta_k = 0.$$

The scale at which the "pressure" term and gravitational instability becomes equal is called the Axion Jeans scale:

$$k_J(a) = (16\pi G a \bar{\rho})^{1/4}\sqrt{m} = 66.5 \times a^{1/4} \left(\frac{h^2\Omega_\Theta}{h^2\Omega_{\rm DM}} \right) \sqrt{\frac{m}{10^{-22}\,\text{eV}}}\,\text{Mpc}^{-1}.$$

The behavior of the density contrast depends whether it is above or below the Jeans scale:

- Modes above the Jeans scale oscillate with a frequency given by the effective sound speed both in matter- and radiation-domination.
- Modes below the Jeans scale behaves like CDM. They grow logarithmically during the radiation era, and linearly during the matter era.

Gravitational collapse

Once the matter fluctuations become sufficiently dense, they decouple from the ambient Hubble flow, and form gravitationally bound structures known as halos. This process is called gravitational collapse. Studying this process precisely is quite difficult, and requires N-body simulations. However, qualitative results can be derived by exploiting the approximate spherical symmetry.

Consider a spherical overdensity δ with physical radius r:

$$M = \frac{4\pi}{3}\overline{\rho}(1+\delta)r^3$$

Assume that the mass M in the overdense region is constant during the collapse. The evolution of the physical radius r obeys the differential equation:

$$\ddot{r} = -\underbrace{\frac{MG}{r^2}}_{\text{matter}} - \underbrace{\frac{8\pi G}{3}\rho_r r}_{\text{radiation}}$$

Critical density at collapse

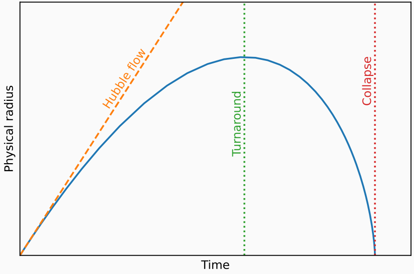
During the evolution, the physical radius r first decouples from the Hubble flow, then turns around, and finally collapses. The redshifts at which these events occur depends on the size of the initial overdensity δ_i:

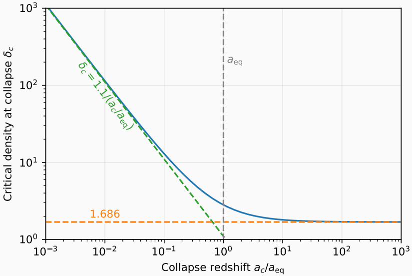
$$\left.\frac{a}{a_{\rm eq}}\right|_{\rm turnaround} \approx \frac{0.7}{\delta_i} \quad , \quad \left.\frac{a}{a_{\rm eq}}\right|_{\rm collapse} \approx \frac{1.1}{\delta_i}$$

Prediction of the linear theory at the time of collapse gives the critical density at collapse:

Ellis et al. 2006.08637; **CE** and Servant, 2207.10111

$$\delta_c(a_c) \approx \left(\frac{1.1}{a_c/a_{\rm eq}}\right)\left(1 + \frac{3}{2}\frac{a_c}{a_{\rm eq}}\right).$$





Press-Schechter Formalism

The formation of the dark matter halos can be studied analytically via the Press-Schechter (PS) formalism. Let us define the following quantities: Press and Schechter, '74

- $\mathscr{F}(>M;a)$: The fraction of matter which is inside collapsed structures of comoving size larger than R at any given scale factor a.
- $\mathscr{P}(\delta_R(a)>\delta_c(a))$: The probability of finding an overdensity $\delta_R(\mathbf{x},a)>\delta_c(a)$ where $\delta_R(\mathbf{x},a)$ is the overdensity *smoothed* at the scale R, and $\delta_c(a)$ is the critical density for collapse at scale factor a.

The PS postulate states that these quantities are equal:

$$\mathscr{F}(>M;a)=\mathscr{P}(\delta_R(a)>\delta_c(a))$$

The smoothe density contrast is obtained via a window function:

$$\delta_R(\mathbf{x},a)=\int \mathrm{d}^3\mathbf{x}'\, W_R(|\mathbf{x}-\mathbf{x}'|)\delta(\mathbf{x}',a), \quad \int \mathrm{d}^3\mathbf{x}\, W_R(\mathbf{x})=1,$$

The variance of the smoothed density contrast is

$$\sigma_R^2(a)=\left\langle|\delta_R(\mathbf{x},a)|^2\right\rangle=\int_0^\infty \frac{\mathrm{d}k}{k}\left|\widetilde{W}_R(k)\right|^2\mathcal{P}_\delta(k;a),$$

Halo mass function (HMF)

An important observable of a dark matter model is the halo mass function (HMF) which gives the number density of halos per logarithmic mass bin:

$$\frac{\mathrm{d}n(M;a)}{\mathrm{d}\ln M} = \frac{1}{2}\frac{\bar{\rho}_{m,0}}{M} f_{\mathrm{PS}}\left(\frac{\delta_c(a)}{\sigma_M(a)}\right)\left|\frac{\mathrm{d}\ln\sigma_M^2(a)}{\mathrm{d}\ln M}\right|, \quad f_{\mathrm{PS}}(\nu) = \sqrt{\frac{2}{\pi}}\,\nu\exp\left(-\frac{\nu^2}{2}\right)$$

The relation between the mass of the halo M and the comoving radius R depends on the window function. For a spherical top-hat window function

$$W_{\mathrm{STH}}(\mathbf{x}) = \left(\frac{4\pi}{3}R^3\right)^{-1} \times \begin{cases} 1, & \mathbf{x} \leq R \\ 0, & \mathbf{x} > R \end{cases}.$$

The relation is

$$M(R) = \frac{4\pi}{3}\bar{\rho}a^3R^3 \approx \frac{4\pi}{3}\bar{\rho}_{m,0}a_0^3R^3,$$

This HMF is directly related to the luminosity function which quantifies the number of galaxies per luminosity interval.

Density profiles of dark matter halos

- Useful parameters to describe the dark matter halos:

$$\underbrace{\left.\frac{\partial \ln \rho(r)}{\partial \ln r}\right|_{r=r_s} = -2}_{\text{scale radius}}, \quad \underbrace{\rho_s = \rho(r=r_s)}_{\text{scale density}}, \quad \underbrace{M_s = \int_0^{r_s} \mathrm{d}^3\vec{\mathbf{r}}\, \rho(r) = 16\pi \rho_s r_s^3 \left(\ln 2 - \frac{1}{2}\right)}_{\text{scale mass}}.$$

- In order to determine these parameters, we need to know the density profile.

CDM

On all scales, the profile is well-approximated by the Navarro-Frenk-White (NFW) profile

Navarro et al. astro-ph/9611107

$$\rho_{\mathrm{NFW}}(r) = \frac{4\rho_s}{(r/r_s)(1+r/r_s)^2}.$$

ALP

The profile is scale dependent:

- **Large scales:** NFW
- **Small scales:** Soliton profile Schive et al. 1406.6586

$$\rho_{\mathrm{sol}}(r) \approx \frac{2.9\rho_s}{\left(1+\left(r/\sqrt{7}r_s\right)^2\right)^8} \;\Rightarrow\; \rho_s \propto m^6 M_s^4$$

- The scale density is closely related to the energy density at collapse:

$$\rho_s \propto \rho_c(z_{\mathrm{col}}) \;\Rightarrow\; \text{Fluctuations that collapse earlier create denser halos.}$$