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10/04/24

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Uncertainties on systematics and an application to the *7-8* TeV ATLAS-CMS top-quark mass combination

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Motivation

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"*Uncertain systematics*"   "*Errors-on-errors*"

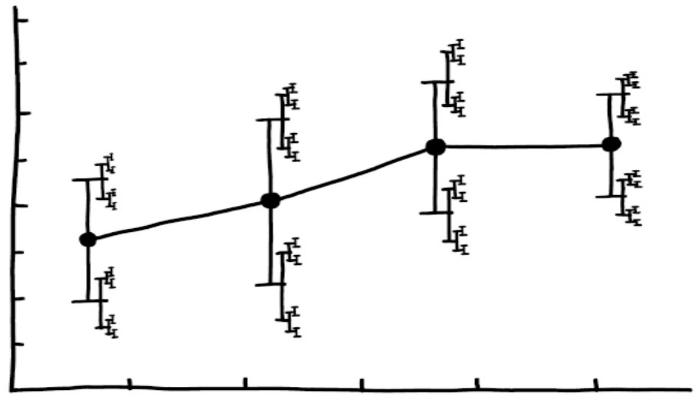
1) *Some* ***systematic uncertainties*** can be well estimated:

- **Related to stat. error of control measurements**
- **Related to size of MC event sample**

2) But they can also be ***quite uncertain***:

- **Theory systematics**
- **Two points systematics (*$\sim \frac{1}{\sqrt{2}} \cong$70% relative error*)**

https://xkcd.com/2110/



Goal: Implement errors-on-errors in a combination, non-trivial consequences!

see: G. Cowan, Eur. Phys. J. C (2019) 79:133; arXiv:1809.05778,
Canonero, E., Brazzale, A.R. & Cowan, G. *Eur. Phys. J. C* **83**, 1100 (2023)

Formulation of the problem

- Suppose measurements $\boldsymbol{y}$ have a probability density $P(\boldsymbol{y}|\boldsymbol{\mu},\boldsymbol{\theta})$
  - $\boldsymbol{\mu}$ = Parameters of interest
  - $\boldsymbol{\theta}$ = Nuisance parameters

- Auxiliary Measurements $\boldsymbol{u}$ are used to provide info on nuisance parameters and are (often) assumed to be independently Gaussian distributed

- The resulting Likelihood is:

Can be a real measurement or just our best guess based on theoretical reasons

$$L(\boldsymbol{\mu},\boldsymbol{\theta}) = P(\boldsymbol{y},\boldsymbol{u}|\boldsymbol{\mu},\boldsymbol{\theta}) = P(\boldsymbol{y}|\boldsymbol{\mu},\boldsymbol{\theta})\times\prod_i \frac{1}{\sqrt{2\pi}\sigma_{u_i}} e^{-(u_i-\theta_i)^2/2\sigma_{u_i}^2}$$

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- And the log Likelihood:

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Let systematic errors be potentially uncertain!

Gamma Variance Model (GVM)

- The original **quadratic terms** in the log likelihood replaced by **logarithmic terms:**

$$\sum_i \frac{(u_i - \theta_i)^2}{2\sigma_{u_i}^2} \longrightarrow \sum_i \left(1 + \frac{1}{2\varepsilon_i^2}\right) \log\left(1 + 2\varepsilon_i^2 \frac{(u_i - \theta_i)^2}{\sigma_{u_i}^2}\right)$$

ε = error-on-error parameter

ε = 0.3 means 30% uncertainty on σ

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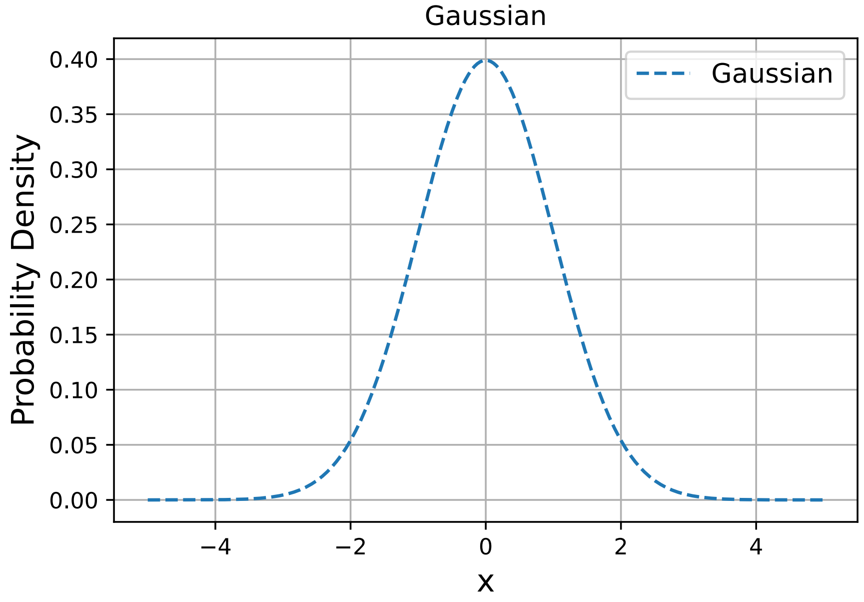
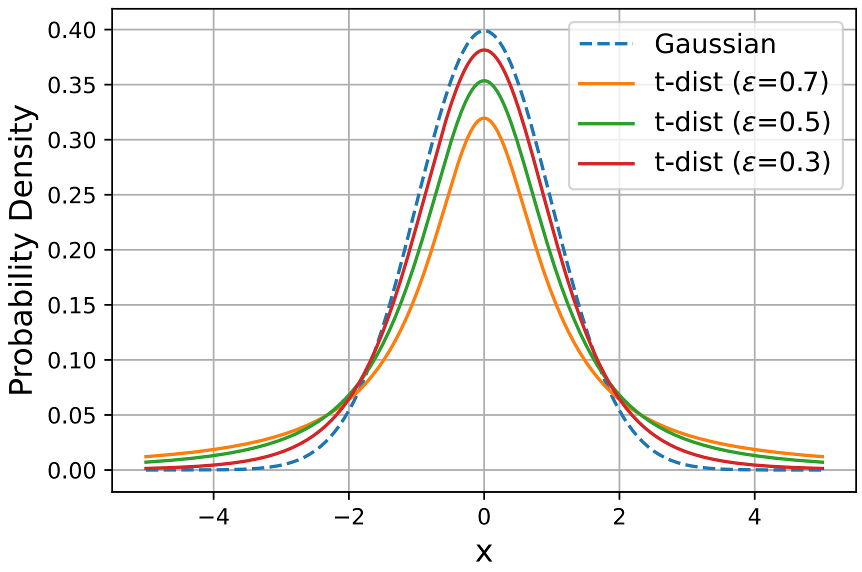
- Equivalent to switch from **Gaussian constraints** to **Student's t constraints** for systematics:





Sensitivity to outliers

- Suppose we want to average 4 measurements all with **statistical** and **syst errors** equal to **1**. Also assume they all have equal ***errors-on-errors*** $\boldsymbol{\varepsilon}$ (auxiliary measurements set to zero):

$$\log L_P(\boldsymbol{\mu}, \boldsymbol{\theta}) = -\frac{1}{2}\sum_i \frac{(y_i - \mu - \theta_i)^2}{\sigma_{y_i}^2} - \frac{1}{2}\sum_i \left(1 + \frac{1}{2\boldsymbol{\varepsilon}_i^2}\right) \log\left(1 + 2\boldsymbol{\varepsilon}_i^2 \frac{{\theta_i}^2}{\sigma_{u_i}^2}\right)$$

- Suppose the measurements are internally compatible (no outliers), errors on errors have a small impact:

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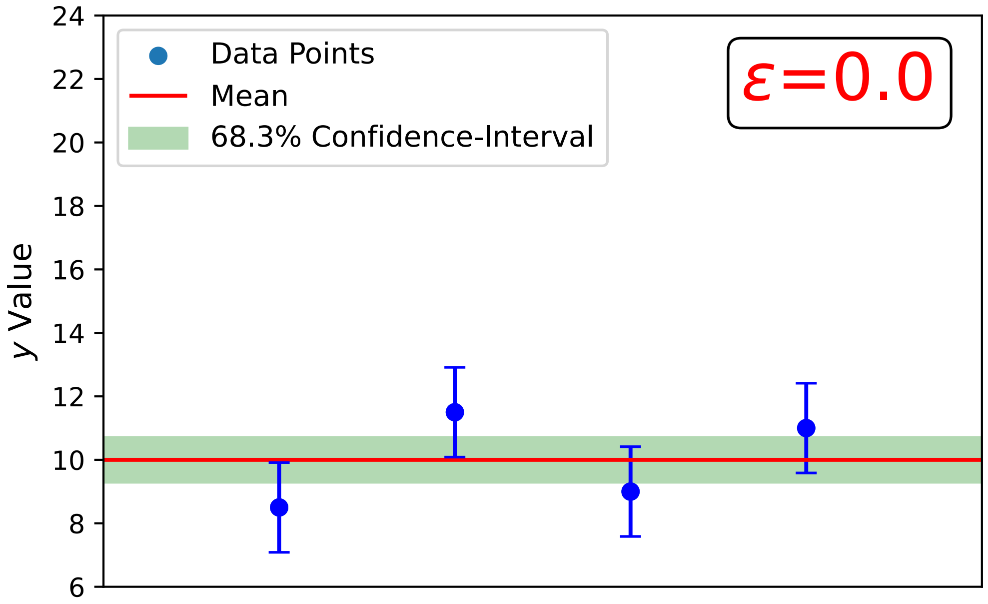

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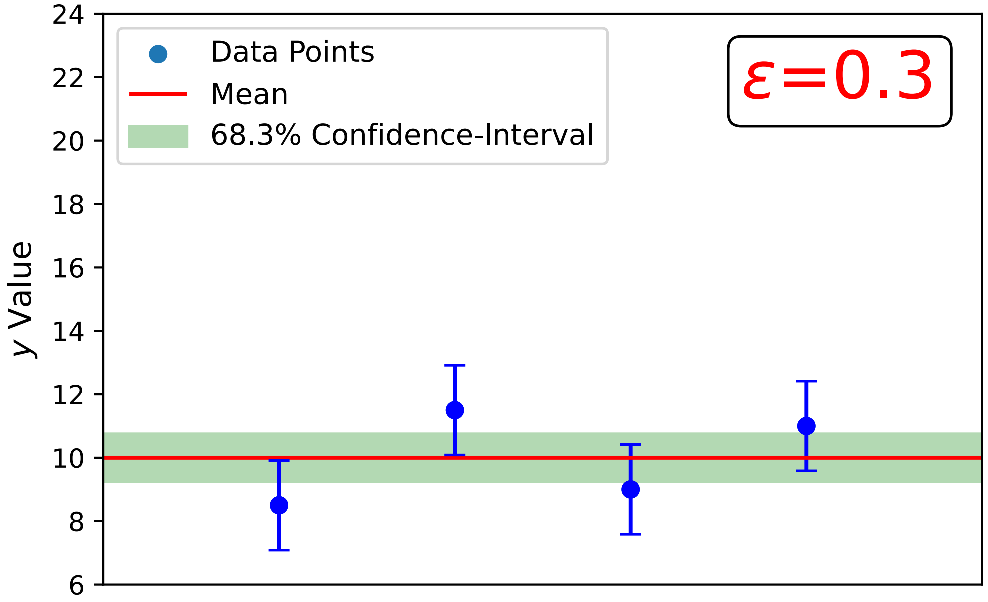

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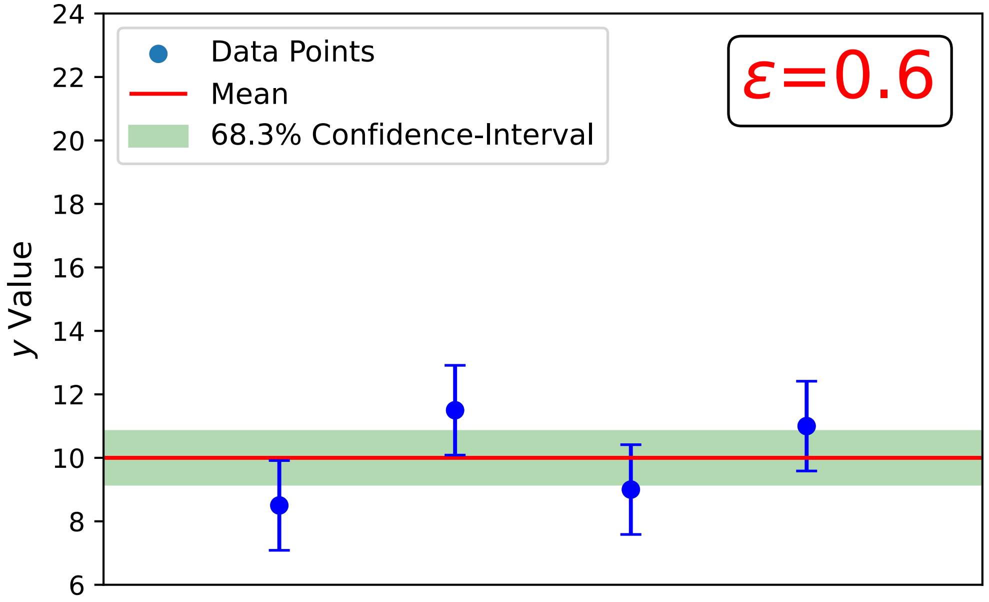

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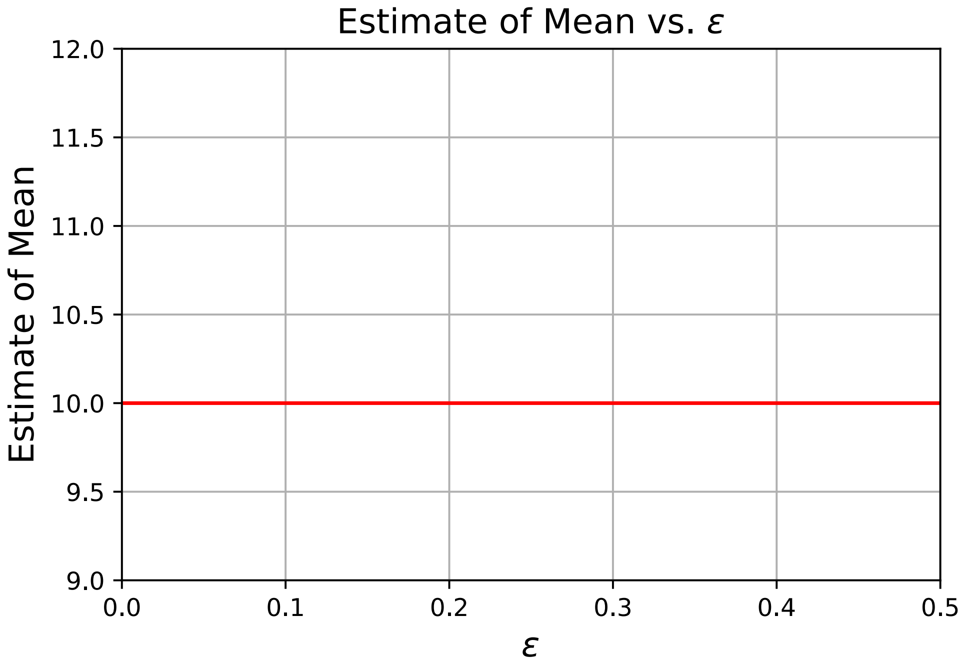
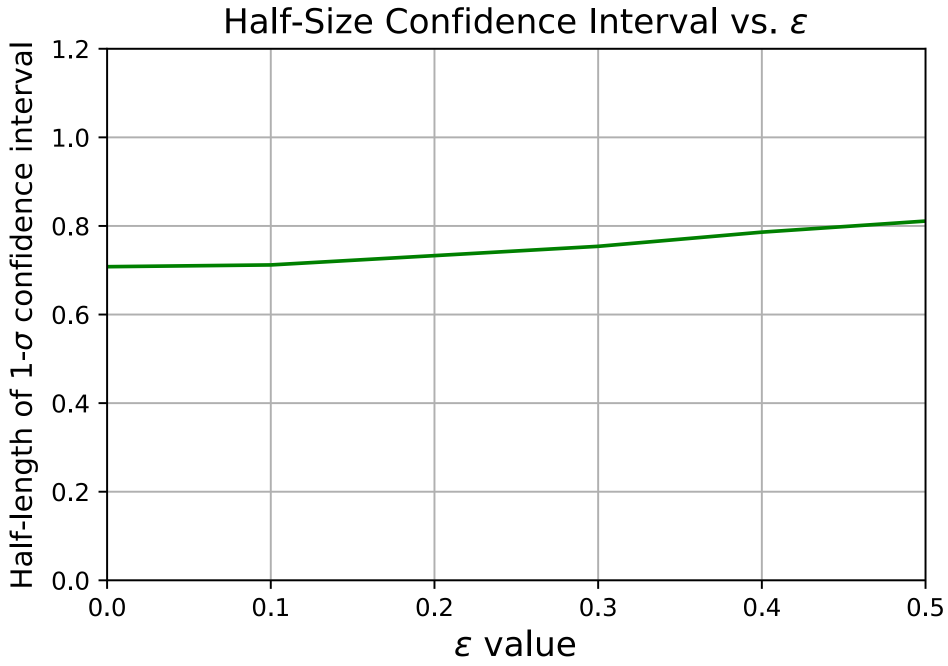
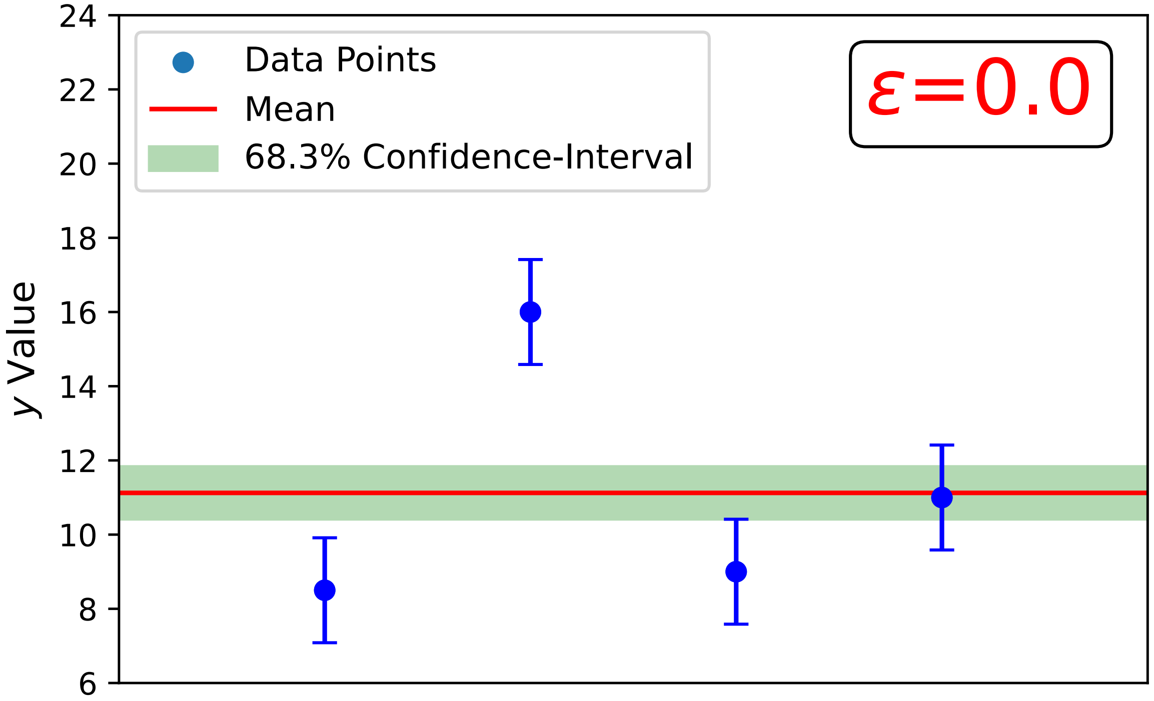




1. The estimate of the mean does not change when we increase ε
2. The size of the confidence interval for the mean only slightly increases, reflecting the extra degree of uncertainty introduced by errors-on-errors
3. If data are internally compatible results are only slightly modified

Sensitivity to outliers

- Suppose one of the measurements is an outlier
- If data are internally incompatible important changes can be observed



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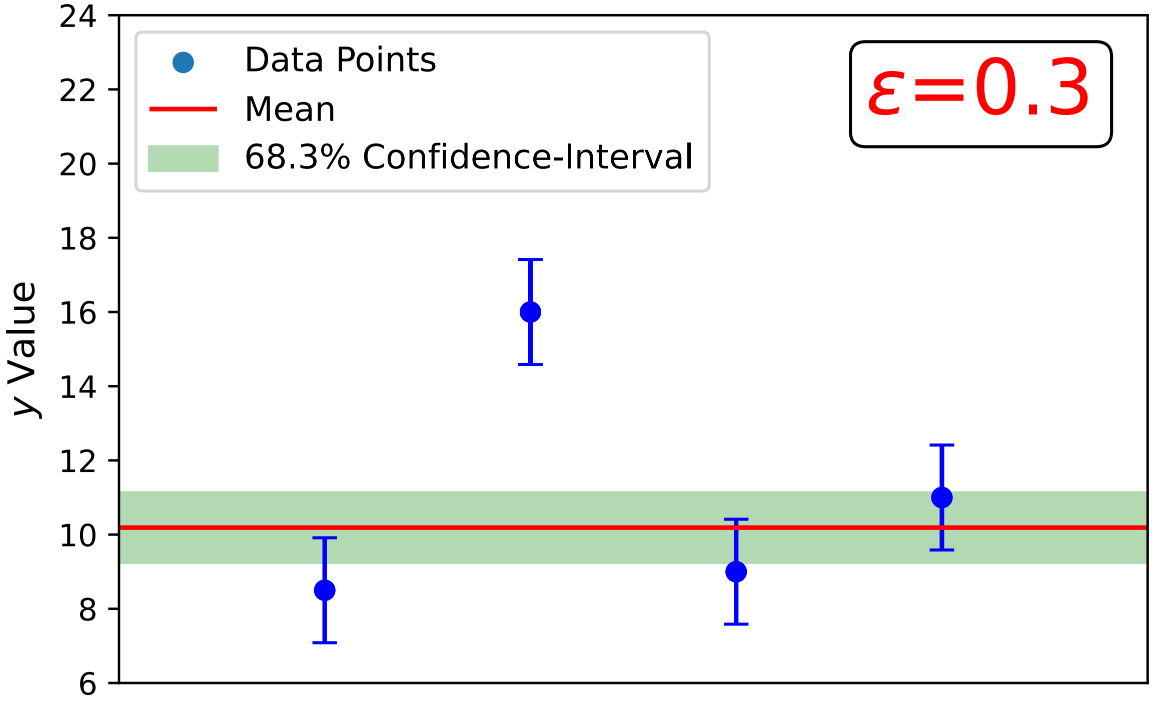

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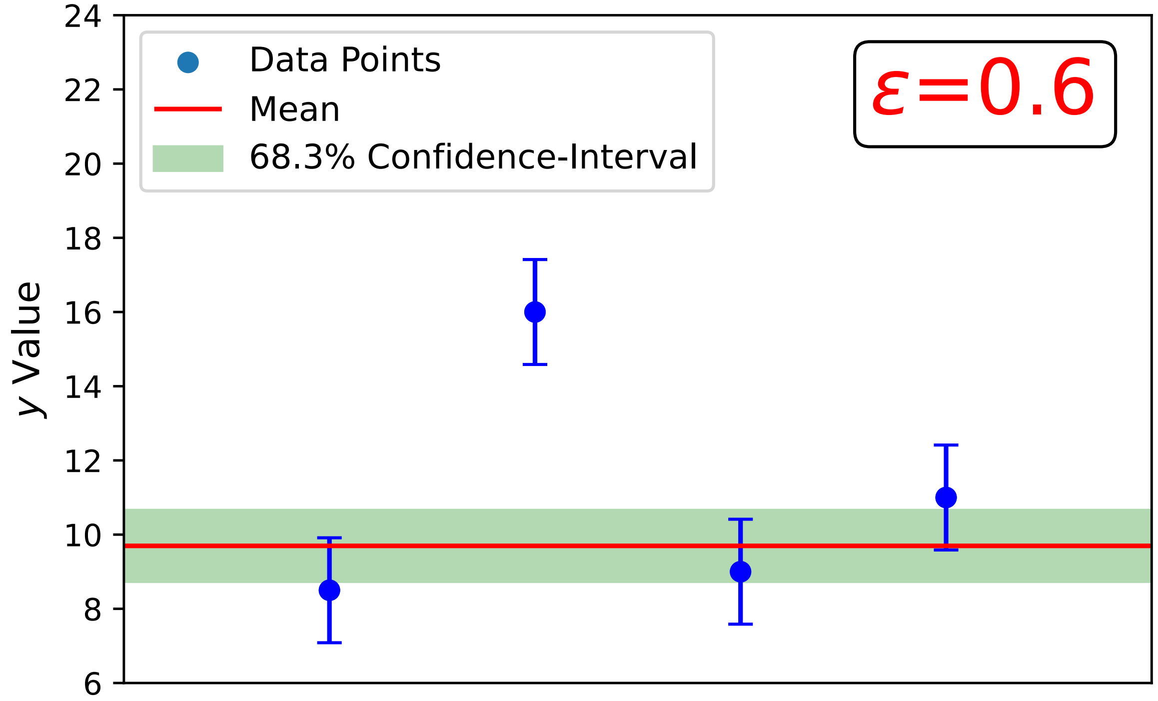

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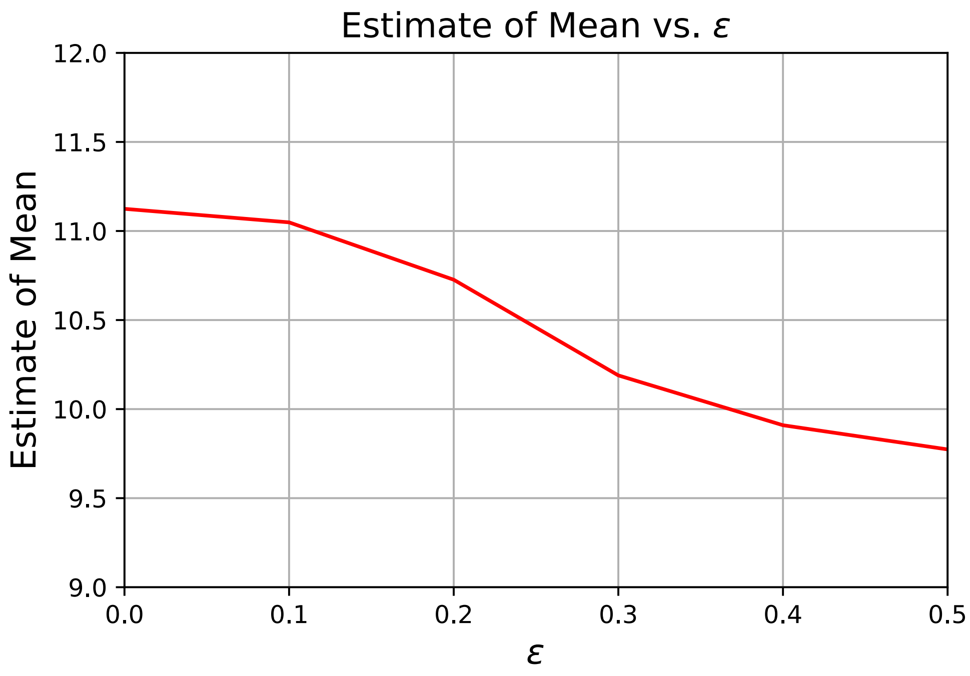
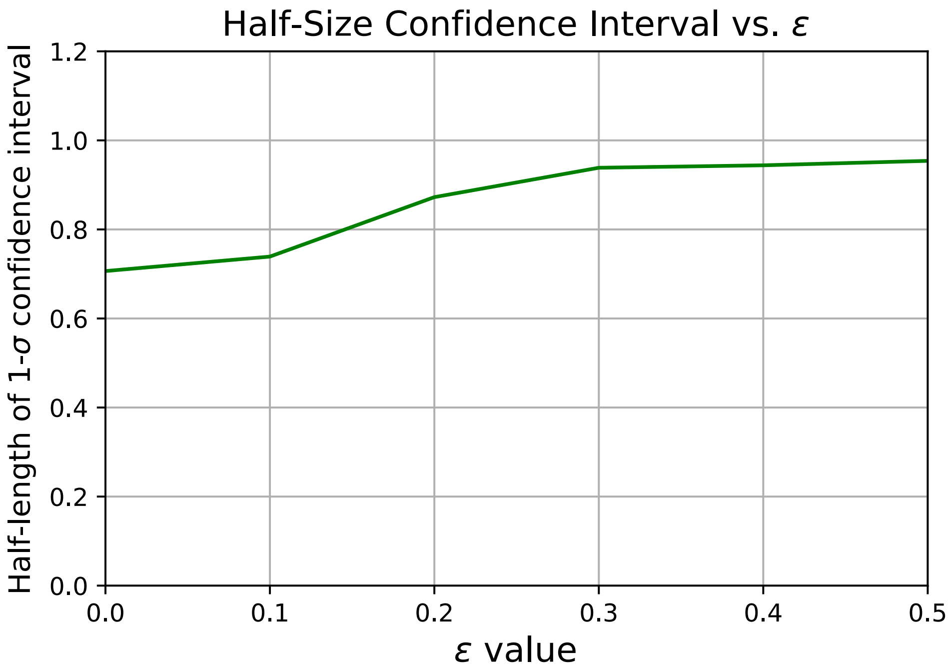




1. With increasing ε, the estimate of mean is pulled less strongly by the outlier

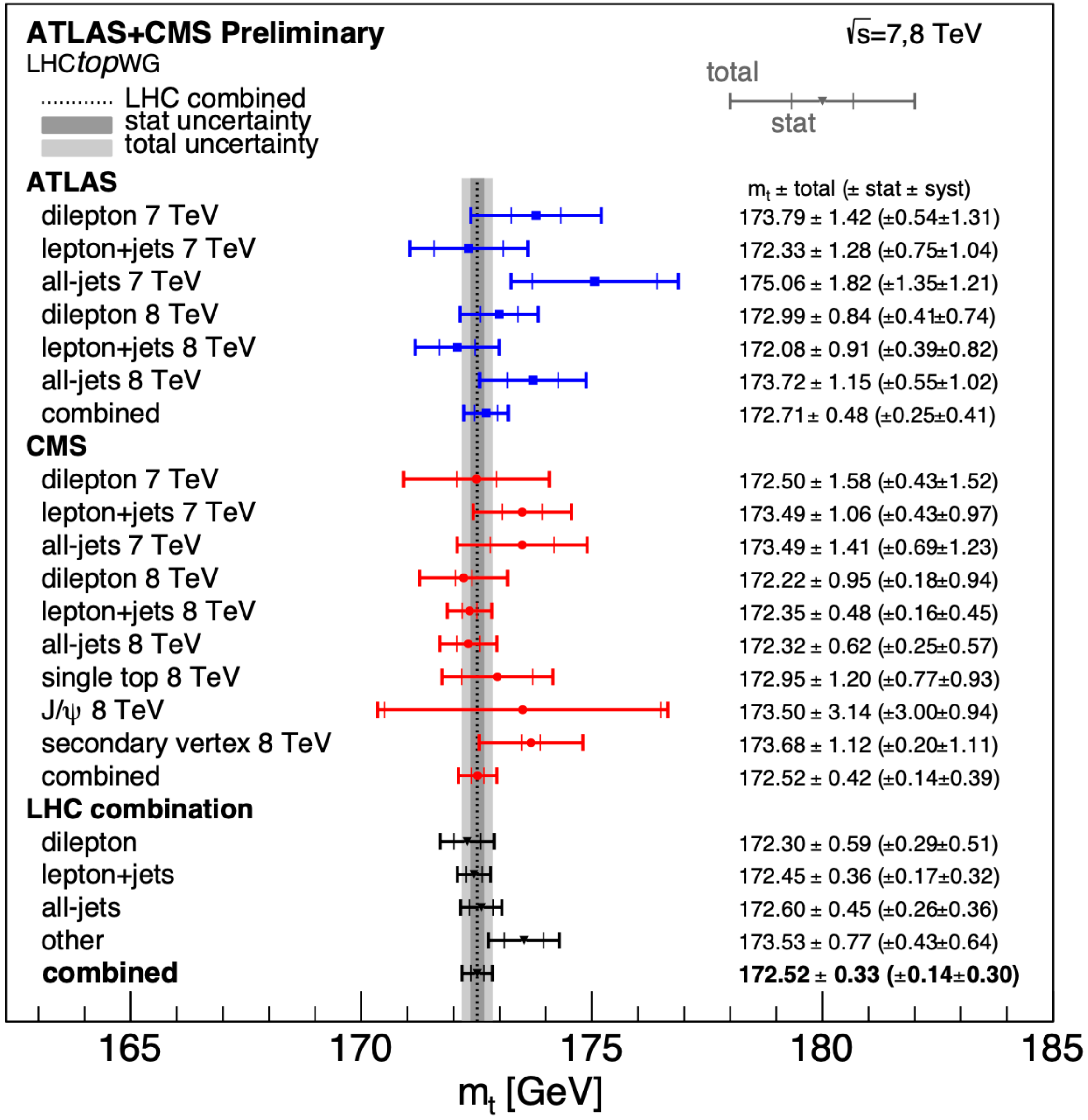
2. The error bar grows more significantly: the GVM treats internal incompatibility as an additional source of uncertainty

3. The model is sensitive to internal compatibility of the data

7-8 TeV ATLAS-CMS top-quark mass combination

Why a top combination?

1. Top-mass measurements are becoming systematics dominated
2. Potentially affected by QCD modelling systematics
3. Outliers may be present in future combination



Combination Paper: [Arxiv:2402.08713]

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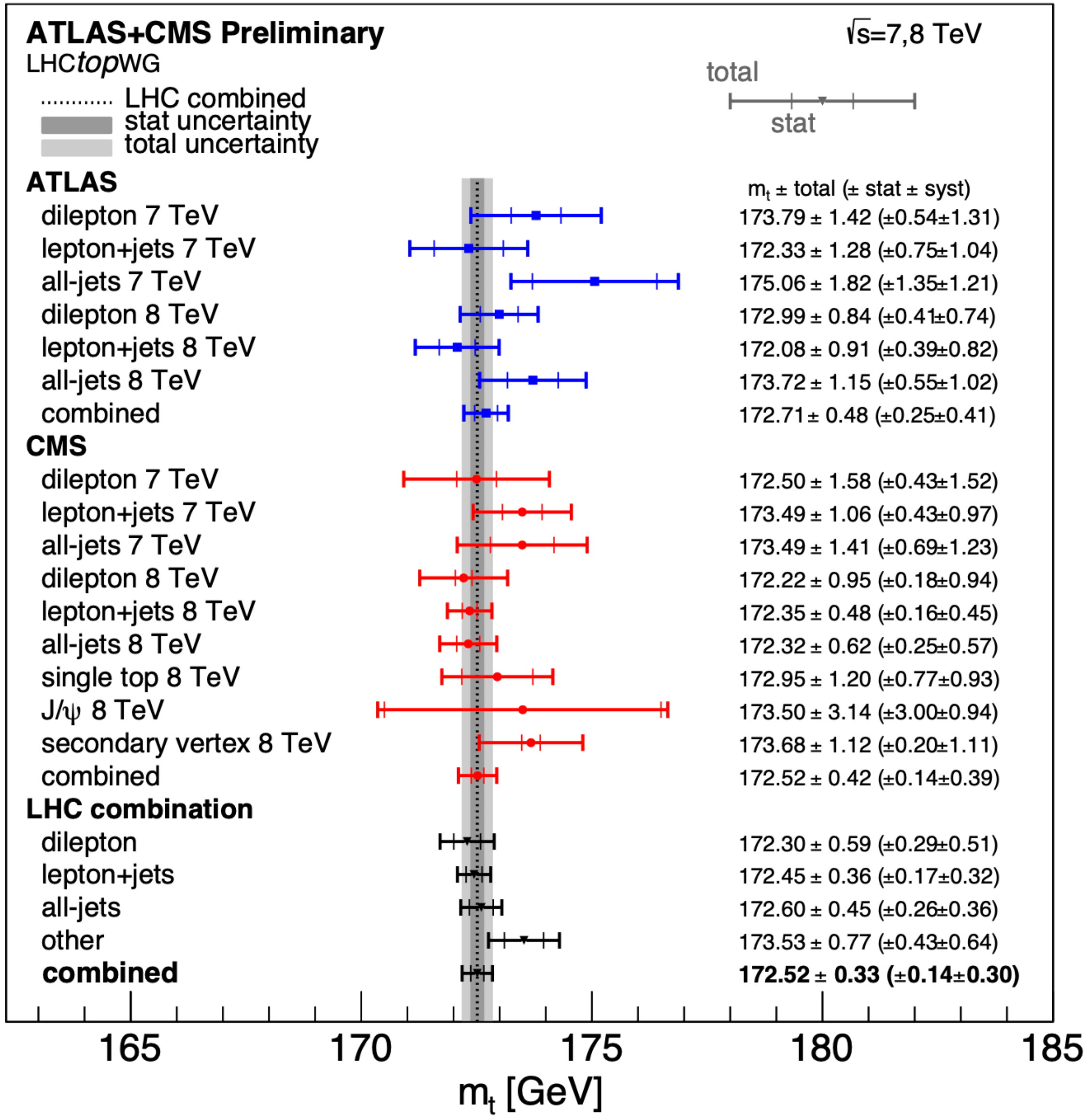
Goals:

1. Demonstrate the generalization of the BLUE method to incorporate errors-on-errors
2. Prove that the original BLUE combination result is retrieved as errors-on-errors approach zero
3. Check if the combination is robust to the presence of errors-on-errors



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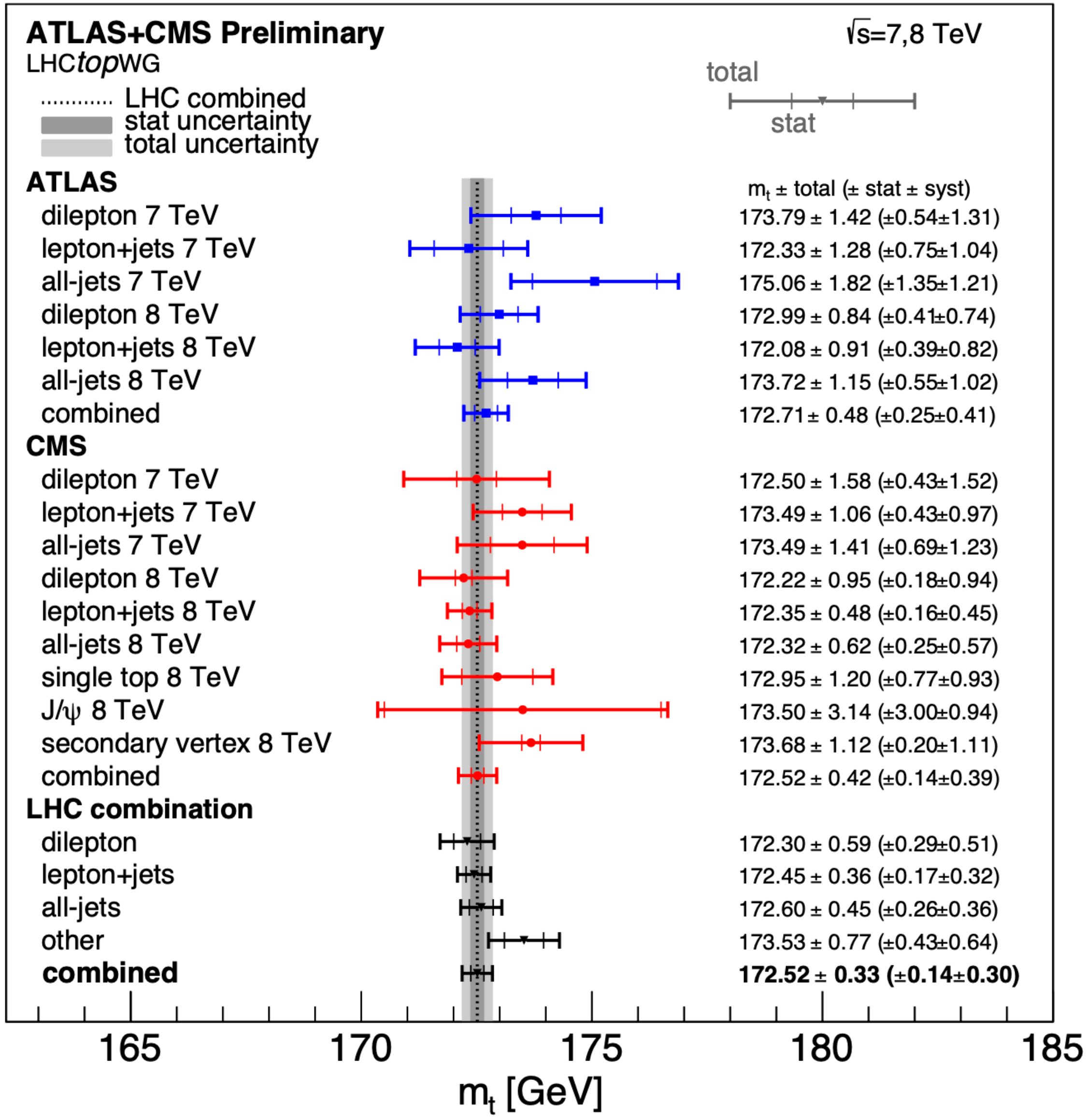
This is a parallel analysis based on public results



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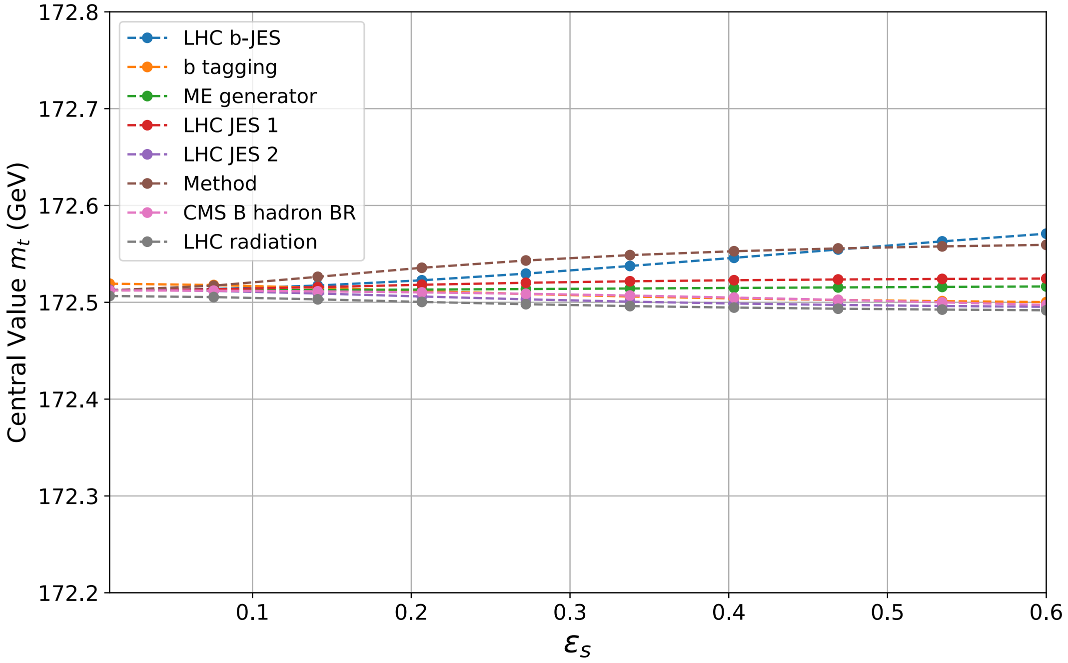
Impact of errors-on-errors on combination

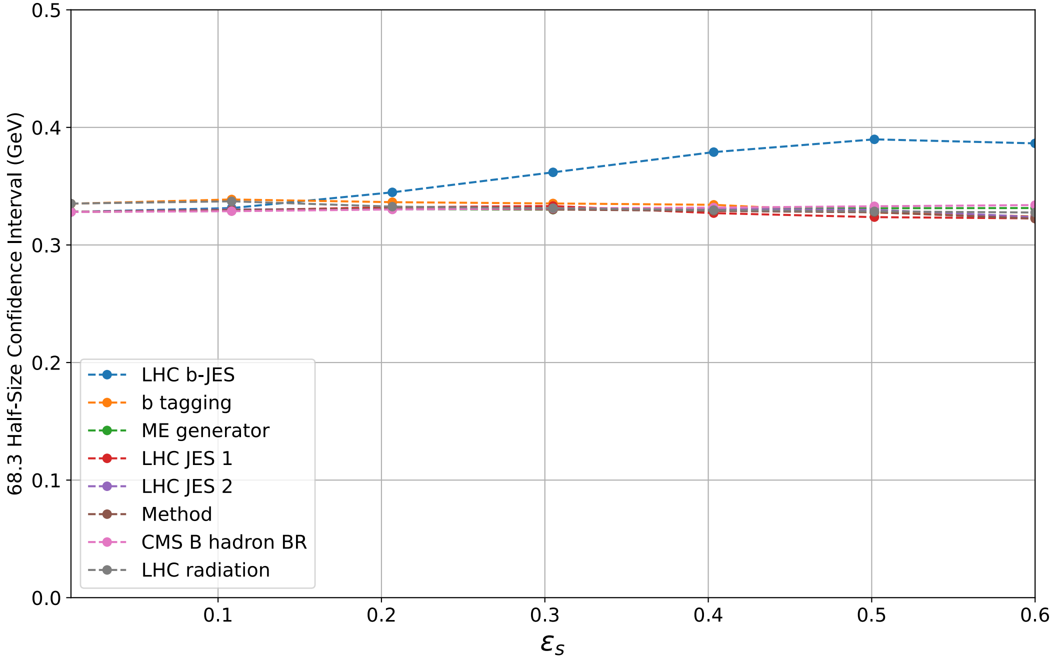
- We identify the eight largest systematic sources in the combination as potentially uncertain.
- An error-on-error parameter, denoted as ε_s, is assigned to each of these systematics.
- We study how the central value and the confidence interval when one ε_s is varied at time.
- This selection aims to show the impact of different assumptions on the combination, not to suggest which systematics should be treated as uncertain

From: Arxiv:2402.08713

Uncertainty category	Uncertainty impact [GeV] LHC	ATLAS	CMS
LHC b-JES	0.18	0.17	0.25
b tagging	0.09	0.16	0.03
ME generator	0.08	0.13	0.14
LHC JES 1	0.08	0.18	0.06
LHC JES 2	0.08	0.11	0.10
Method	0.07	0.06	0.09
CMS B hadron BR	0.07	—	0.12
LHC radiation	0.06	0.07	0.10
Leptons	0.05	0.08	0.07
JER	0.05	0.09	0.02
Top quark p_T	0.05	—	0.07
Background (data)	0.05	0.04	0.06
Color reconnection	0.04	0.08	0.03
Underlying event	0.04	0.03	0.05
LHC g-JES	0.03	0.02	0.04
Background (MC)	0.03	0.07	0.01
Other	0.03	0.06	0.01
LHC l-JES	0.03	0.01	0.05
CMS JES 1	0.03	—	0.04
Pileup	0.03	0.07	0.03
LHC JES 3	0.02	0.07	0.01
LHC hadronization	0.02	0.01	0.01
p_T^{miss}	0.02	0.04	0.01
PDF	0.02	0.06	<0.01
Trigger	0.01	0.01	0.01
Total systematics	0.30	0.41	0.39
Statistical	0.14	0.25	0.14
Total	0.33	0.48	0.42

Impact of errors-on-errors on combination





1. As $\boldsymbol{\varepsilon_s} \to \mathbf{0}$ the model reproduces BLUE results, as expected.
2. The central value is robust to the presence of uncertain systematic errors:
   - The change in the central value remains always within *0.1* GeV, well within the confidence interval of approximately *0.3* GeV.
3. The confidence intervals are also stable, though they exhibit non-negligible sensitivity to uncertainties in the *LHC b-JES* uncertainty.

Sensitivity to outliers

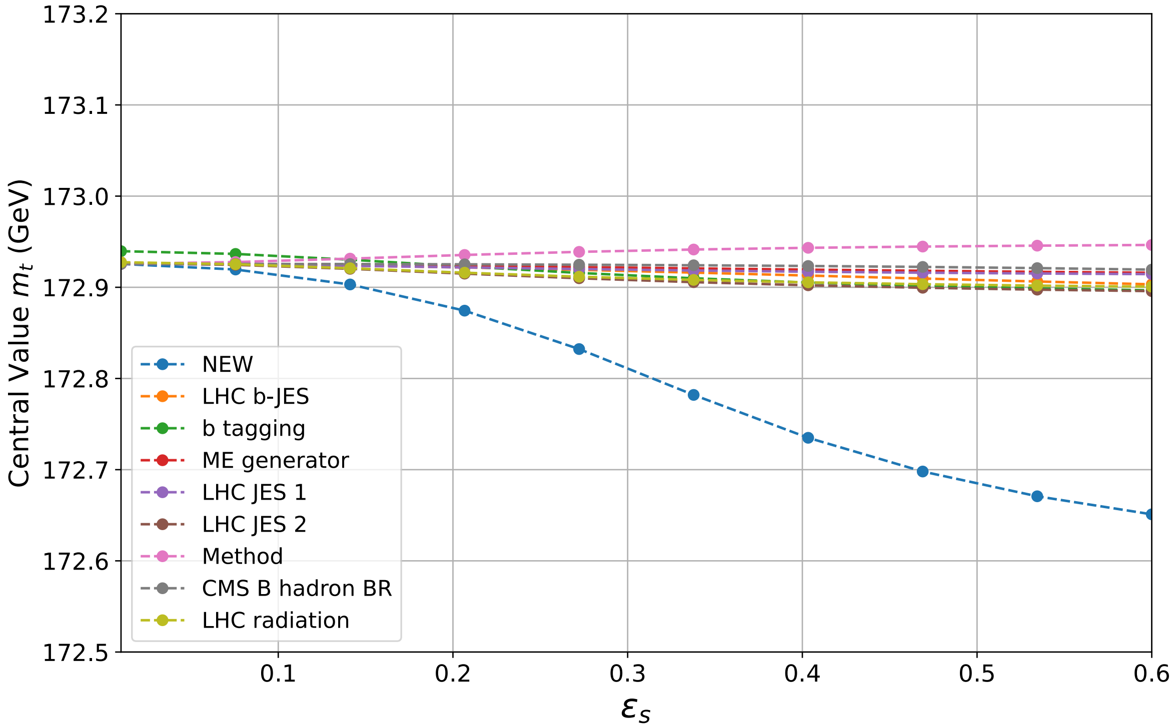
- Show how the combination would be impacted if any of the combination inputs exhibited tension with the rest of the measurements.

- Relevant for future LHC –Tevatron combinations, or for a LHC run 2 combination that includes the top mass measurement exploiting a leptonic invariant mass (arXiv:2209.00583).

- We introduce a fictitious measurement to explore GVM properties in these scenarios.

- We add a measurement with $m_t^{NEW} = 174.5$ GeV, statistical uncertainty of *0.4* GeV, and global systematic uncertainty of *0.5* GeV.

Sensitivity to outliers – central value

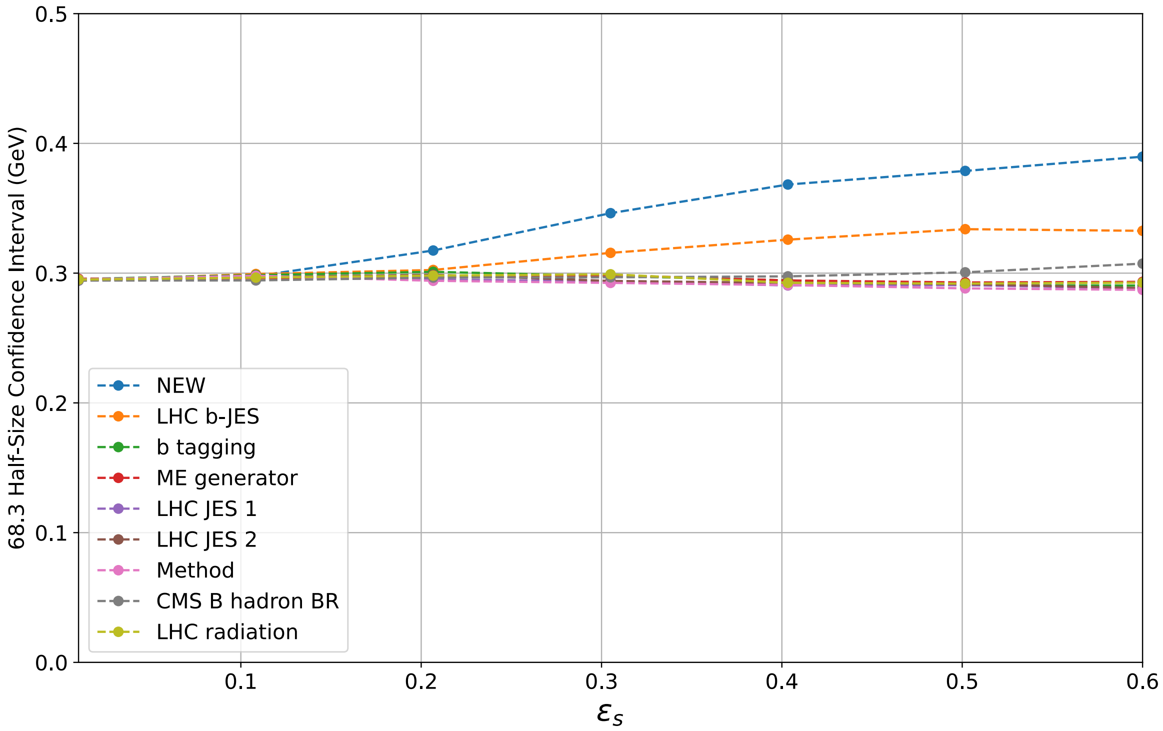


- When all the errors-on-errors are zero, the central value of the combination is biased:

172.52 GeV → 172.91 GeV

- If the new measurement is affected by large uncertain systematic, it shifts back to the original value

Sensitivity to outliers – confidence interval



- When all the errors-on-errors are zero, adding the new measurements shrinks the CI:

0.33 GeV → 0.29 GeV

- If the new measurement is affected by large uncertain systematic, the CI inflates
- This is because the GVM treats the tension in the dataset as an additional source of uncertainty resulting in an inflated confidence interval

Conclusions

- The Gamma Variance Model (GVM) is a statistical framework designed to account for uncertainties in error parameters.

- GVM proves to be particularly relevant for combining measurements (e.g., top mass, W mass, Hubble constant, ...), especially when systematic uncertainties are dominant.

- GVM is sensitive to internal compatibility of input data
  - If the data is internally compatible, the results are only marginally affected
  - If the data is incompatible, errors-on-errors significantly alter both the central values and confidence intervals in a non-linear manner.

- GVM serves as a valuable tool for robustness studies, enabling researchers to determine whether an analysis is sensitive to uncertainties in a systematic source

Thank you for your attention

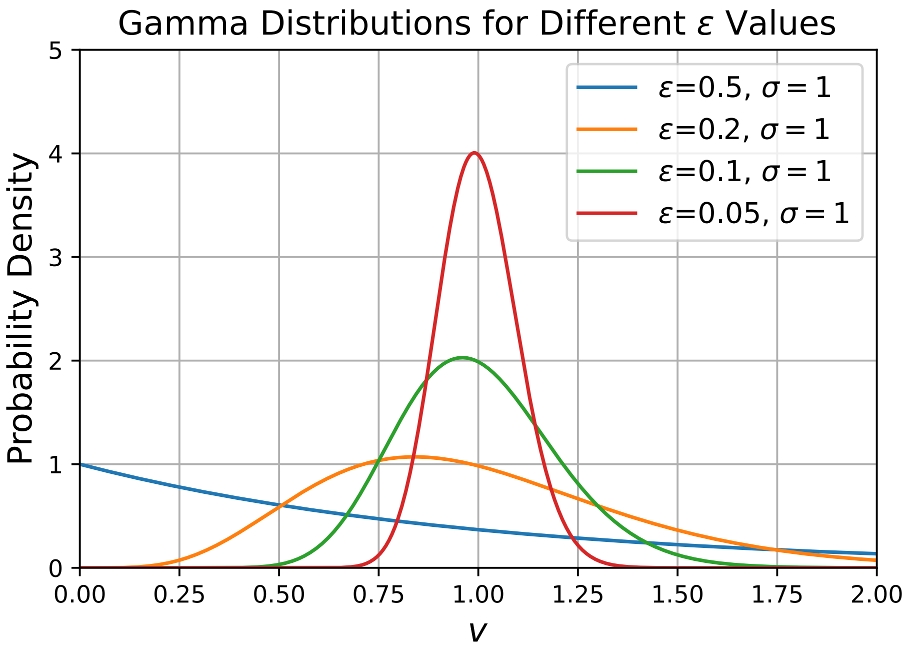
Back-up slides

Gamma Distributions

- Treat the systematic variances $\sigma_{u_i}^2$ are *adjustable parameters* (*nuisance parameter*).
- Suppose their best estimates v_i are gamma distributed:

$$v \sim \frac{\beta^{\alpha}}{\Gamma(\alpha)} v^{\alpha-1} e^{-\beta v}$$

$$\alpha = \frac{1}{4\varepsilon_i^2} \qquad \beta = \frac{1}{4\varepsilon_i^2 \sigma_{u_i}^2}$$



- σ_{u_i} Systematic Error
- $\varepsilon_i = \frac{1}{2}\frac{\sigma_{v_i}}{\sigma_{u_i}^2} \cong \frac{\sqrt{v_i}}{\sigma_{u_i}}$ relative error on σ_{u_i}: "***Error on error***"

Gamma Variance Model (GVM)

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- The likelihood is modified as follows:

$$L(\boldsymbol{\mu}, \boldsymbol{\theta}, \boldsymbol{\sigma}_{\boldsymbol{u}_i}^2) = P(\boldsymbol{y}|\boldsymbol{\mu}, \boldsymbol{\theta}) \times \prod_i \frac{1}{\sqrt{2\pi}\sigma_{u_i}} e^{-(u_i-\theta_i)^2/2\sigma_{u_i}^2} \times \frac{\boldsymbol{\beta}_i^{\boldsymbol{\alpha}_i}}{\boldsymbol{\Gamma}(\boldsymbol{\alpha}_i)} \boldsymbol{v}_i^{\boldsymbol{\alpha}_i - 1} \boldsymbol{e}^{-\boldsymbol{\beta}_i \boldsymbol{v}_i}$$

- One can profile over $\boldsymbol{\sigma}_{\boldsymbol{u}_i}^2$ in closed form:

$$\log L_P(\boldsymbol{\mu}, \boldsymbol{\theta}) = \log P(\boldsymbol{y}|\boldsymbol{\mu}, \boldsymbol{\theta}) - \frac{1}{2}\sum_i \left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{2}\boldsymbol{\varepsilon}_i^2}\right) \mathbf{log}\left(\mathbf{1} + \mathbf{2}\boldsymbol{\varepsilon}_i^2 \frac{(\boldsymbol{u}_i - \boldsymbol{\theta}_i)^2}{\boldsymbol{v}_i}\right)$$

- We call this model the Gamma Variance Model (GVM)

(see: G. Cowan, Eur. Phys. J. C (2019) 79:133; arXiv:1809.05778)

Motivation for the GVM

- Gamma distributions allow to parametrize distributions of positive defined variables (like estimates of variances)

- Using Gamma distributions it is possible to profile in close form over σ_i^2

Motivation for the GVM

- Gamma distributions include the case where the variance is estimate from a real dataset of control measurements:

$$v_i = \frac{1}{n_i - 1} \sum \left(u_{i,j} - \bar{u_i}\right)^2$$

- $(n-1)v_i/\sigma_{u_i}^2$ follows a χ_{n-1}^2 distribution and v_{i} a Gamma distribution with:

$$\alpha_i = \frac{n_i - 1}{2}$$

$$\beta_i = \frac{n_i - 1}{2\sigma_{u_i}^2}$$

From BLUE to the Gamma Variance Model

- BLUE approach to combinations:

$$\chi^2 = \sum_i (y_i - \mu) V_{ij}^{-1} (y_j - \mu)$$

$$V_{ij} = V_{ij}^{(stat)} + V_{ij}^{(syst)}$$

- $V_{ij}^{(stat)}$: Statistical covariance matrix.
- $V_{ij}^{(syst)}$: Covariance matrix induced by systematic source.
- We assume the presence of a single systematic source. If multiple sources exist, simply sum over *syst*.

From BLUE to the Gamma Variance Model

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- Switch to a nuisance parameters approach:

$$\chi^2 = \sum_i \frac{(y_i - \mu - \theta_i)^2}{\sigma_i^2} + \sum_{ij} \theta_i C_{ij}^{-1} \theta_j$$

$$C_{ij} = V_{ij}^{(syst)}$$

- Substitute quadratic term with log-constraint:

$$\sum_{ij} \boldsymbol{\theta_i C_{ij}^{-1} \theta_j} \longrightarrow \sum_i \left(\boldsymbol{N + \frac{1}{2\varepsilon_i^2}} \right) \mathbf{log}\left(\boldsymbol{1 + 2\varepsilon_i^2 \theta_i C_{ij}^{-1} \theta_j} \right)$$

Motivation for the GVM

- $n_i = 1 + 1/2\epsilon_i^2$

- So, since for a two-points systematic $n_i = 2$:

$$\epsilon_i = 1/\sqrt{2}$$

Uncertainty category	ρ	Scan range	$\Delta m_t/2$ [MeV]	$\Delta\sigma_{m_t}/2$ [MeV]
LHC JES 1	0	—	—	—
LHC JES 2	0	$[-0.25, +0.25]$	8	7
LHC JES 3	0.5	$[+0.25, +0.75]$	1	<1
LHC b-JES	0.85	$[+0.5, +1]$	26	5
LHC g-JES	0.85	$[+0.5, +1]$	2	<1
LHC l-JES	0	$[-0.25, +0.25]$	1	<1
CMS JES 1	—	—	—	—
JER	0	$[-0.25, +0.25]$	5	1
Leptons	0	$[-0.25, +0.25]$	2	2
b tagging	0.5	$[+0.25, +0.75]$	1	1
p_T^{miss}	0	$[-0.25, +0.25]$	<1	<1
Pileup	0.85	$[+0.5, +1]$	2	<1
Trigger	0	$[-0.25, +0.25]$	<1	<1
ME generator	0.5	$[+0.25, +0.75]$	<1	4
LHC radiation	0.5	$[+0.25, +0.75]$	7	1
LHC hadronization	0.5	$[+0.25, +0.75]$	1	<1
CMS B hadron BR	—	—	—	—
Color reconnection	0.5	$[+0.25, +0.75]$	3	1
Underlying event	0.5	$[+0.25, +0.75]$	1	<1
PDF	0.85	$[+0.5, +1]$	1	<1
Top quark p_T	—	—	—	—
Background (data)	0	$[-0.25, +0.25]$	8	2
Background (MC)	0.85	$[+0.5, +1]$	2	<1
Method	0	—	—	—
Other	0	—	—	—

	ATLAS						
	2011 (7 TeV)			2012 (8 TeV)			comb.
	dil	lj	aj	dil	lj	aj	
m_t	173.79	172.33	175.06	172.99	172.08	173.72	172.71
LHC JES 1	0.54	0.33	0.38	0.35	0.28	0.40	0.18
LHC JES 2	0.30	0.30	0.20	0.41	0.39	0.42	0.11
LHC JES 3	0.43	0.07	0.24	0.08	0.05	0.12	0.07
LHC b-JES	0.68	0.06	0.62	0.30	0.03	0.34	0.17
LHC g-JES	0.03	0.28	0.10	0.02	0.21	0.05	0.02
LHC l-JES	0.02	0.24	0.02	0.01	0.10	0.06	0.01
JER	0.19	0.22	0.01	0.09	0.20	0.10	0.09
Leptons	0.13	0.04	—	0.14	0.16	0.01	0.08
b tagging	0.07	0.50	0.16	0.04	0.38	0.10	0.16
p_T^{miss}	0.04	0.15	0.02	0.01	0.05	0.01	0.04
Pileup	0.01	0.02	0.02	0.05	0.15	0.01	0.07
Trigger	0.01	—	0.01	—	0.01	0.08	0.01
ME generator	0.26	0.22	0.30	0.09	0.16	0.18	0.13
LHC radiation	0.47	0.32	0.22	0.23	0.08	0.10	0.07
LHC hadronization	0.53	0.18	0.50	0.22	0.15	0.64	0.01
Color reconnection	0.14	0.11	0.22	0.03	0.19	0.12	0.08
Underlying event	0.05	0.15	0.08	0.10	0.08	0.12	0.03
PDF	0.10	0.25	0.09	0.05	0.09	0.09	0.06
Background (data)	0.04	0.11	0.35	0.07	0.05	0.17	0.04
Background (MC)	0.01	0.29	—	0.03	0.13	—	0.07
Method	0.09	0.11	0.42	0.05	0.13	0.11	0.06
Other	0.07	0.12	0.24	0.02	0.10	0.03	0.06
Total systematics	1.31	1.04	1.21	0.74	0.82	1.02	0.41
Statistical	0.54	0.75	1.35	0.41	0.39	0.55	0.25
Total	1.42	1.28	1.82	0.84	0.91	1.15	0.48

	CMS									
	2011 (7 TeV)			2012 (8 TeV)						comb.
	dil	lj	aj	dil	lj	aj	t	J/ψ	vtx	
m_t	172.50	173.49	173.49	172.22	172.35	172.32	172.95	173.50	173.68	172.52
LHC JES 1	0.77	0.24	0.69	0.31	0.10	0.16	0.40	<0.01	0.11	0.06
LHC JES 2	0.54	0.02	0.35	0.17	0.12	0.19	0.21	<0.01	0.13	0.10
LHC JES 3	0.06	0.01	0.08	0.03	0.01	0.02	0.05	<0.01	0.01	0.01
LHC b-JES	0.70	0.61	0.49	0.37	0.32	0.29	0.38	—	—	0.25
LHC g-JES	—	—	—	0.07	0.08	0.02	—	—	—	0.04
LHC l-JES	—	—	—	0.04	0.06	0.01	0.07	—	—	0.05
CMS JES 1	0.58	0.11	0.58	—	—	—	—	—	—	0.04
JER	0.14	0.23	0.15	—	0.03	0.02	0.05	<0.01	0.05	0.02
Leptons	0.14	0.02	—	0.25	0.01	—	0.05	0.10	0.24	0.07
b tagging	0.09	0.12	0.06	0.01	0.06	0.02	0.10	—	0.02	0.03
p_T^{miss}	0.12	0.06	—	0.01	0.04	—	0.15	—	—	0.01
Pileup	0.11	0.07	0.06	0.05	0.06	0.06	0.14	0.07	0.05	0.03
Trigger	—	—	0.24	—	—	0.01	—	0.02	—	0.01
ME generator	0.04	0.02	0.19	0.07	0.12	0.16	—	0.37	0.42	0.14
LHC radiation	0.58	0.30	0.33	0.24	0.09	0.18	0.35	0.74	0.20	0.10
LHC hadronization	—	—	—	0.38	0.01	0.04	—	0.30	0.54	0.01
CMS B hadron BR	—	—	—	0.12	0.16	0.13	0.15	—	0.16	0.12
Color reconnection	0.13	0.54	0.15	0.13	0.01	0.16	0.05	0.12	0.08	0.03
Underlying event	0.05	0.15	0.20	0.11	0.08	0.14	0.20	0.13	—	0.05
PDF	0.09	0.07	0.06	0.17	0.04	0.03	0.11	0.11	0.04	<0.01
Top quark p_T	—	—	—	0.51	0.02	0.06	—	—	—	0.07
Background (data)	—	—	0.13	—	—	0.20	—	—	0.44	0.06
Background (MC)	0.05	0.13	—	—	0.03	—	0.17	0.01	—	0.01
Method	0.40	0.06	0.13	—	0.04	0.06	0.39	0.22	0.62	0.09
Other	—	—	—	0.03	—	—	0.25	0.09	0.09	0.01
Total systematics	1.52	0.97	1.23	0.94	0.45	0.57	0.93	0.94	1.11	0.39
Statistical	0.43	0.43	0.69	0.18	0.16	0.25	0.77	3.00	0.20	0.14
Total	1.58	1.06	1.41	0.95	0.48	0.62	1.20	3.14	1.12	0.42