Spectral properties of non-Abelian gauge theories both in and out-of-thermal equilibrium

Ravi Shanker

The Institute of Mathematical Sciences

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Harshit Pandey, R.S.,Sayantan Sharma, work in preparation

Introduction

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- We want to study how well this conjecture is realized in an interacting quantum field theory.
- We use first-principles lattice gauge theory techniques to study
  - Properties implementing a particular chaotic classical realization of SU(3) gauge theory
  - A quantum phase of SU(3) and show that it belongs to a particular RMT universality class

Our motivation comes from the general picture of heavy-ion collisions

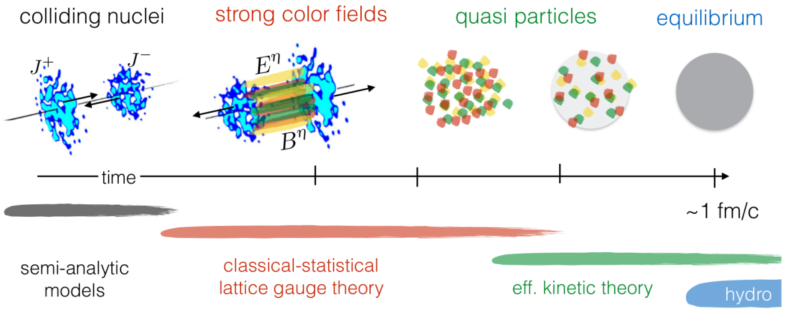

Fig. courtesy S. Schlichting, Quark Matter 2015

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- We thus study the spectrum of this quantum version of the SU(3) gauge theory in thermal equilibrium both with and without fermions.

The classical system: SU(3) theory away from equilibrium

- A classical realization of $SU(3)$ consists of with over-occupied infra-red gluon modes motivated from colour glass condensate effective theory of QCD

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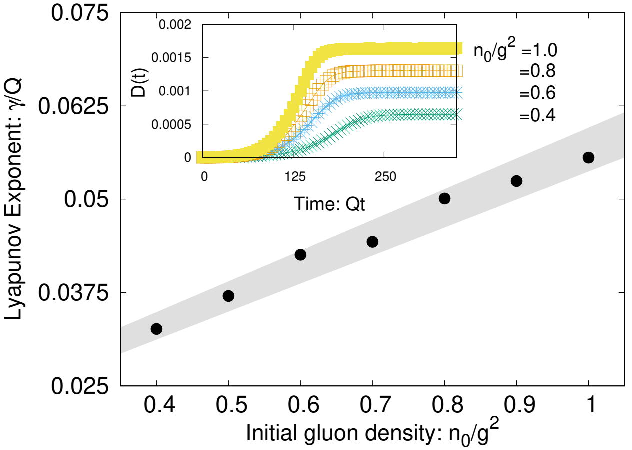
- The separation between two gauge trajectories starting with n_0 and $n_0 + \Delta n_0$, $\Delta n_0 = 0.001$ as a function of time, characterized by a gauge-invariant distance measure defined as,

  [B. Müller and A. Traynov, Phys. Rev. Lett. 68, 23 (1992)]

$$D(U_l, U_l^{'}, t) = \frac{1}{2N_p} \sum_p |\mathrm{tr}\ U_P - \mathrm{tr}\ U_P^{'}| \ .$$



A positive Lyapunov exponent is a signal of chaos.

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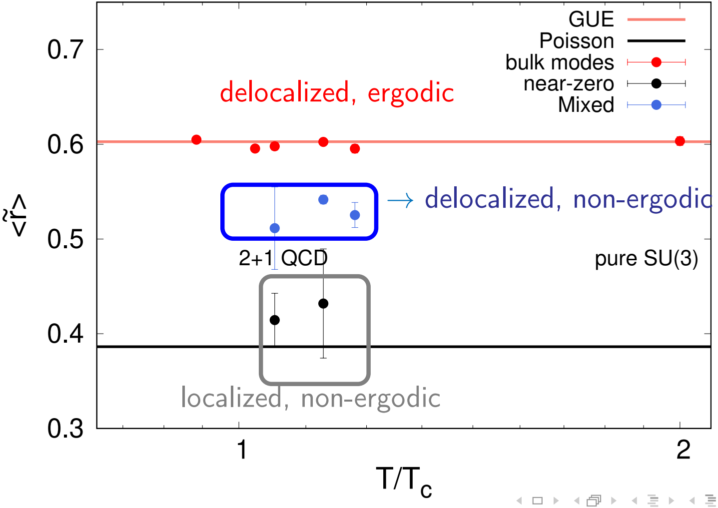
- We study spectral properties of the Dirac operator representing a massless quark which acts as a probe of the thermalized non-Abelian medium.

- We measure the ratios of spacing between adjacent eigenvalues λ of the Dirac operator defined as

$$r_n = \frac{\lambda_{n+2} - \lambda_{n+1}}{\lambda_{n+1} - \lambda_n}$$

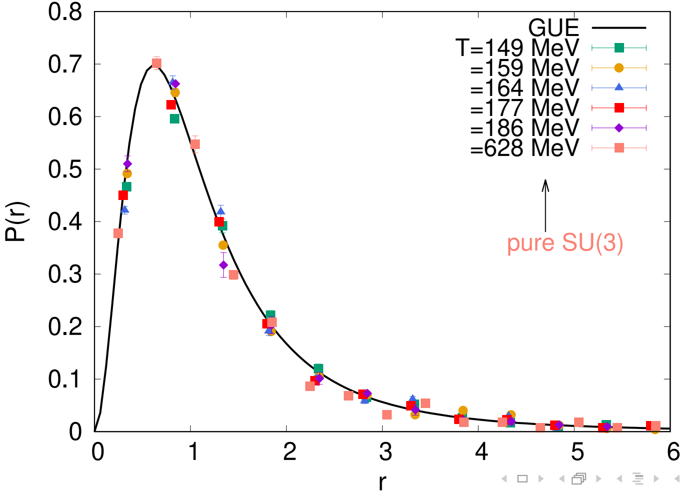
Is the spectrum fully described by RMT?

- We calculate $\langle \tilde{r} \rangle$, where $\tilde{r}_n = \min(r_n, \frac{1}{r_n})$.
- Three distinct regimes!

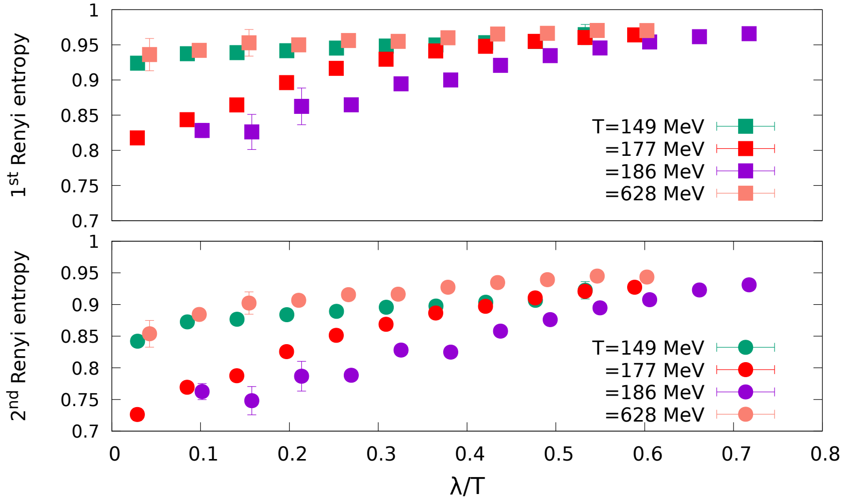


The probability distribution of ratio of level spacings

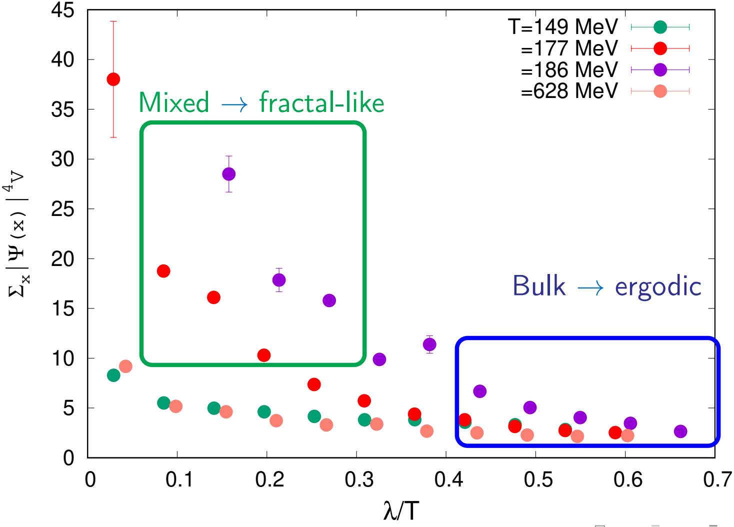
- A robust observable independent of the systematics! Ergodic bulk modes show clear agreement with predictions from RMT belonging to Gaussian Unitary Ensemble.



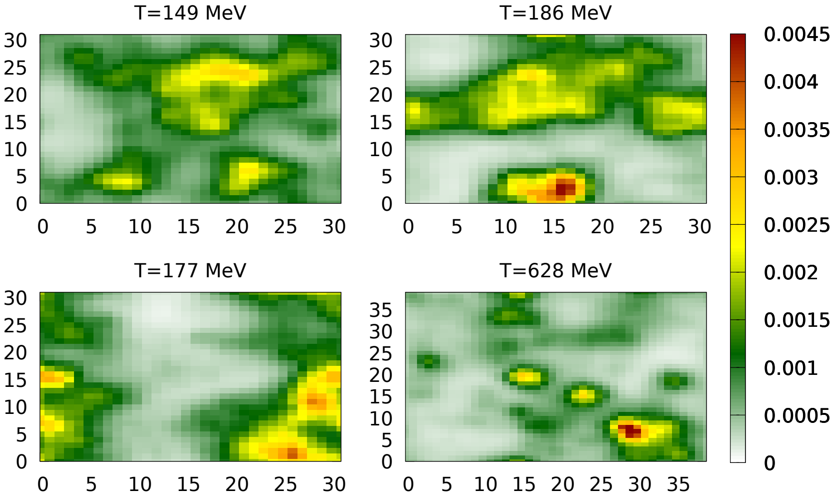
Renyi Entropy quantifies the amount of randomness



The randomness is also in the eigenvectors: Inverse participation ratio



Can the mixed phase be understood in terms of fractals?



Summary and Outlook

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- We thus observe a realization of BGS conjecture in an interacting quantum field theory.
- We want to further understand the implications of this study in the context of thermalization of gauge theories.

Thanks