Electromagnetic vacuum stresses and energy fluxes induced by a cosmic string in de Sitter spacetime

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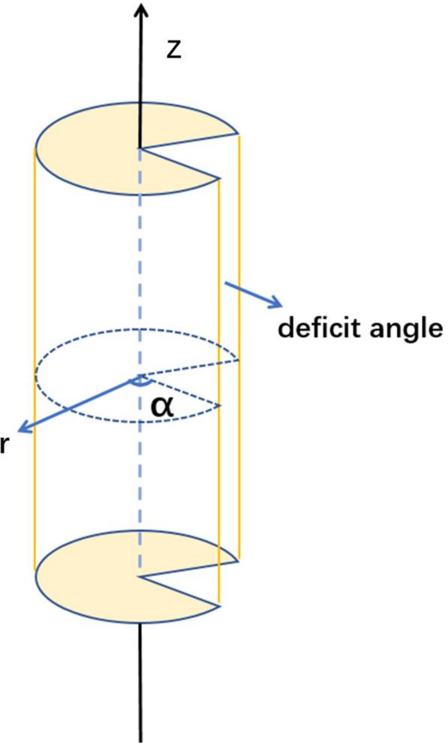
Cosmic strings



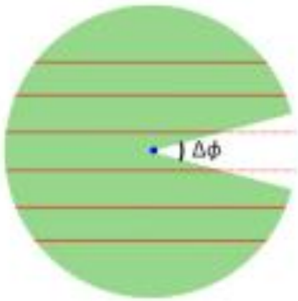
- Cosmic strings are hypothetical objects which generically arise within the framework of grand unified theories
  - could have formed in the early universe due to symmetry-breaking phase transitions (Kibble mechanism).

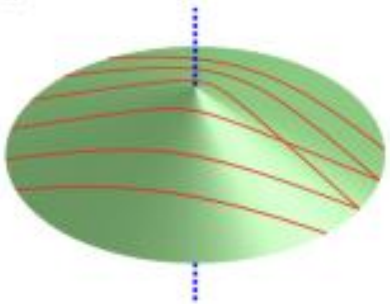
Kibble, T. W. B. (1976). "Topology of cosmic domains and strings". J. Phys. A: Math. Gen. ***9*** *(8): 1387–1398.*

- The large-scale matter clumping in the Universe was initially attributed to cosmic strings
  - but later measurements of the cosmic microwave background ruled out the significance of the cosmic strings ($\leq 10\%$)

Cosmic strings

- Nonetheless, cosmic strings are candidates for the generation of several interesting physical effects:
  - *Gravitational lensing*
  - *Generation of gravitational waves*
  - *High energy cosmic rays and gamma-ray bursts, etc.*





The de Sitter spacetime

- Approximately de Sitter (dS) spacetime is employed to solve a number of problems in standard cosmology (in most inflationary models)

- At the present epoch, the Universe is accelerating and can be well approximated by a world with a positive cosmological constant
  - the future geometry of the Universe will tend to dS spacetime

- Due to the high symmetry, numerous physical problems are exactly solvable on the dS background, and a better understanding of physical effects in this bulk could serve as a handle to deal with more complicated geometries

Our publications

A. A. Saharian, V. Kh. Kotanjyan et al., Electromagnetic vacuum stresses and energy fluxes induced by a cosmic string in de Sitter spacetime. Int. J. Mod. Phys. D 32, 2350042 (2023).

1. Bellucci, S., Oliveira dos Santos, W., Bezerra de Mello, E.R. et al. Cosmic string and brane-induced effects on the fermionic vacuum in AdS spacetime. J. High Energ. Phys. 2022, 21 (2022),
2. de Mello, ER Bezerra, and A. A. Saharian. "Topological Casimir effect in compactified cosmic string spacetime." Classical and Quantum Gravity 29.3 (2012): 035006,
3. Bezerra de Mello, E. R., et al. "Vacuum currents induced by a magnetic flux around a cosmic string with finite core." Physical Review D 91.6 (2015): 064034 etc.

Problem setup and energy-momentum tensor

- (D+1)-dimensional spacetime described by the interval

$$ds^2_{\mathrm{dS}} = dt^2 - e^{2t/\alpha}[dr^2 + r^2 d\phi^2 + (d\mathbf{z})^2],\ \mathbf{z} = (z^3, ..., z^D),$$

$$-\infty < z^i < +\infty,\ -\infty < t < +\infty,\ 0 \leqslant r < \infty,\ \text{and}\ 0 \leqslant \phi \leqslant \phi_0$$

- For $\phi_0 < 2\pi$ and at the points $r > 0$, the local geometrical characteristics coincide with those for dS spacetime
  - sourced by the cosmological constant $\Lambda = D(D-1)/2\alpha^2$
- The complete set of electromagnetic field modes is found for the vector potential: $\{A_{(\beta)i}(x), A^*_{(\beta)i}(x)\}$
- Vacuum expectation value of the energy-momentum tensor

$$\langle T_i^l \rangle = \frac{1}{4\pi} \sum_\beta \left[\frac{1}{4} \delta_i^l F_{(\beta)}^{mn} F^*_{(\beta)mn} - F_{(\beta)i}^{\cdot m} F_{(\beta)\cdot m}^{l*} \right]$$

- Contribution induced by cosmic string (topological part) $\langle T_i^l \rangle_{\mathrm{t}} = \langle T_i^l \rangle - \langle T_i^l \rangle^{(\mathrm{dS})}$

$$\langle T_i^l \rangle^{(\mathrm{dS})} = \mathrm{const} \cdot \delta_i^l$$

⇦ VEV of the energy-momentum tensor in dS spacetime in the absence of the cosmic string

Topological parts in the energy-momentum tensor

- Topological contribution in the VEVs of the nonzero components of the energy-momentum tensor (no summation over *i*):

$$\langle T_i^i\rangle_t = \frac{2\alpha^{-D-1}}{(2\pi)^{D/2+1}}\left[\sum_{j=1}^{[q/2]}{}' t^{(i)}(r/\eta, s_j) - \frac{q}{\pi}\sin(q\pi)\int_0^\infty dz \frac{t^{(i)}(r/\eta,\cosh z)}{\cosh(2qz)-\cos(q\pi)}\right],$$

$$\langle T_0^1\rangle_t = \frac{8(D-3)r}{(2\pi)^{D/2+1}\alpha^{D+1}\eta}\left[\sum_{j=1}^{[q/2]} t^{(01)}(r/\eta, s_j) - \frac{q}{\pi}\sin(q\pi)\int_0^\infty dy \frac{t^{(01)}(r/\eta,\cosh y)}{\cosh(2qy)-\cos(q\pi)}\right],$$

$\alpha r/\eta$ ⇐ Proper distance from cosmic string

Notations: $q = 2\pi/\phi_0$, $\eta = \alpha e^{-t/\alpha}$, $s_j = \sin(j\pi/q)$

$$t^{(i)}(x,y) = \int_0^\infty du\, u^{\frac{D}{2}} e^{u-2ux^2y^2}\sum_{l=1,2} K_{\frac{D}{2}-l}(u) f_l^{(i)}(x,y,u),$$

$$t^{(01)}(x,y) = y^2\int_0^\infty du\, u^{\frac{D}{2}}\left(1-uy^2x^2\right)K_{\frac{D}{2}-1}(u)e^{u-2x^2y^2u}$$

$$f_l^{(i)}(x,y,u) = \left[4b_l^{(i)}ux^2y^2 - 2(b_l^{(i)}+d_l^{(i)})ux^2 - ((D-2)a_l^{(i)}+2b_l^{(i)})\right]y^2 + e_l^{(i)},\ i\neq 1,2,$$

$$f_l^{(i)}(x,y,u) = 2(b_l^{(i)}-d_l^{(i)})ux^2y^2 + e_l^{(i)},\ i = 1,2.$$

Properties of the vacuum energy-momentum tensor

- Because of the maximal symmetry of dS spacetime, the EMT depends on the time and radial coordinates in terms of the proper radial distance from the cosmic string core, given by the combination $\alpha r/\eta = re^{t/\alpha}$
- Vacuum energy-momentum tensor has a non-zero off-diagonal component describing energy flux along the radial direction
- Energy flux vanishes for D=3 and is directed towards the cosmic string, $\langle T_0^1 \rangle_{\rm t} < 0$, for D>3
- In the limit $\alpha \to \infty$ the cosmic string induced vacuum energy-momentum tensor is obtained in the Minkowski bulk. In this case the stresses along the directions parallel to the core of the defect are equal to the energy density and all the diagonal components behave as $1/r^{D+1}$. Off-diagonal component is zero.

Vacuum energy-momentum tensor: small distances

- For points near the cosmic string, the influence of the gravitational field on the diagonal components of the topological terms is weak. The leading terms in the expansion over r/η coincide with the corresponding results in the Minkowski bulk where the radial distance is replaced by the proper distance $\alpha r/\eta$

$$\langle T_i^i\rangle_{\mathrm{t}} \propto 1/(\alpha r/\eta)^{D+1},\ \langle T_l^l\rangle_{\mathrm{t}} \approx \langle T_0^0\rangle_{\mathrm{t}},\, l=3,\ldots,D,\, \langle T_2^2\rangle_{\mathrm{t}} \approx -D\langle T_1^1\rangle_{\mathrm{t}}$$

  Energy density near the cosmic string is negative in spatial dimensions *D* = 3, 4. Near the cosmic string the radial stress is negative and the azimuthal stress is positive, $\langle T_1^1\rangle_{\mathrm{t}} < 0, \langle T_2^2\rangle_{\mathrm{t}} > 0$

- Energy flux along the radial direction is an effect induced by the gravity and the asymptote of the corresponding component of the energy-momentum tensor near the cosmic string is given by

$$\langle T_0^1\rangle_{\mathrm{t}} \approx -\frac{(D-3)^2\Gamma\left((D+1)/2\right)}{(4\pi)^{(D+1)/2}\,\alpha^{D+1}\,(r/\eta)^D} c_{D-1}(q),\ r/\eta \ll 1$$

Vacuum energy-momentum tensor: large distances

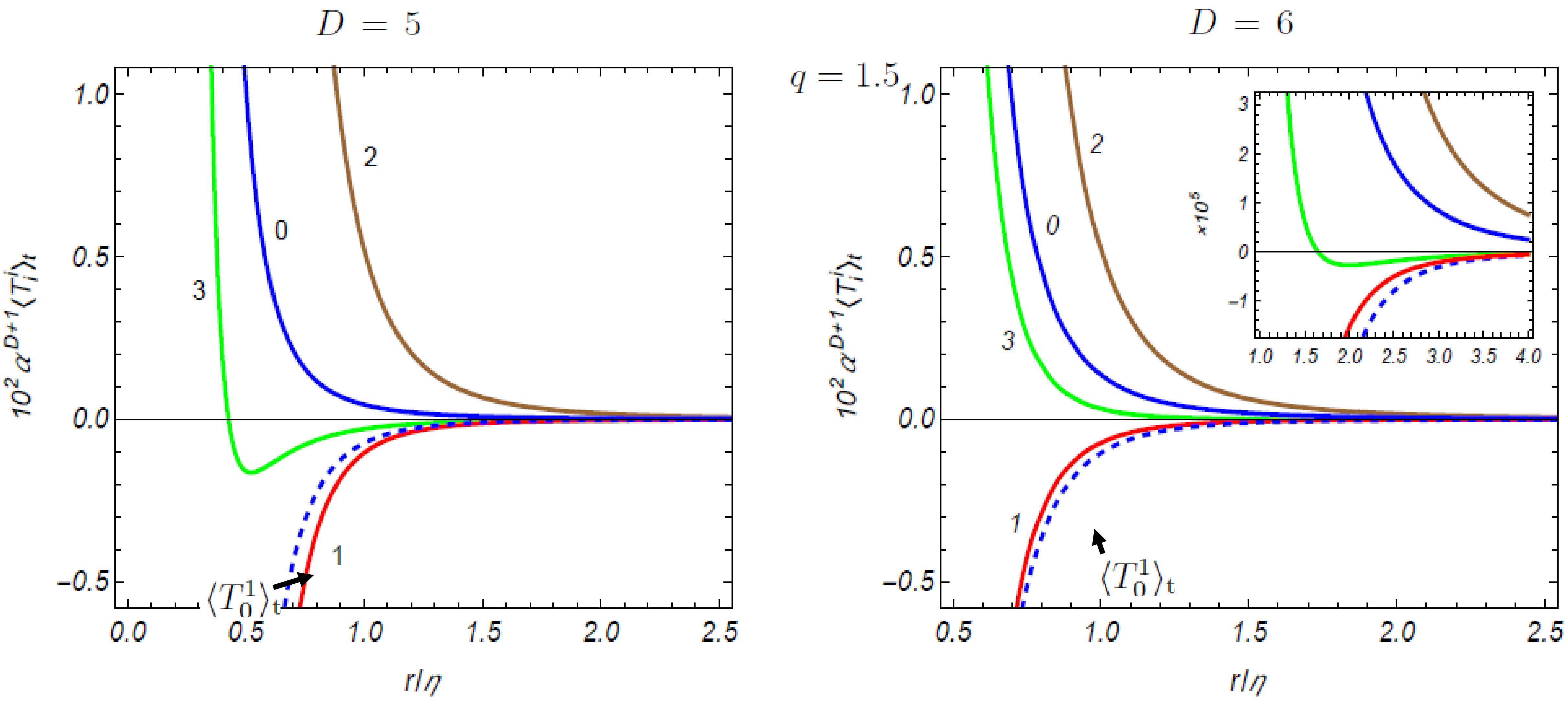
- Effects of gravity on the topological contributions in the VEV of the energy-momentum tensor are essential at proper distances from the cosmic string larger than the dS curvature radius

- At large distances one has $r/\eta \gg 1$ and the leading terms in the corresponding asymptotic expansions are given by (no summation over *i*)

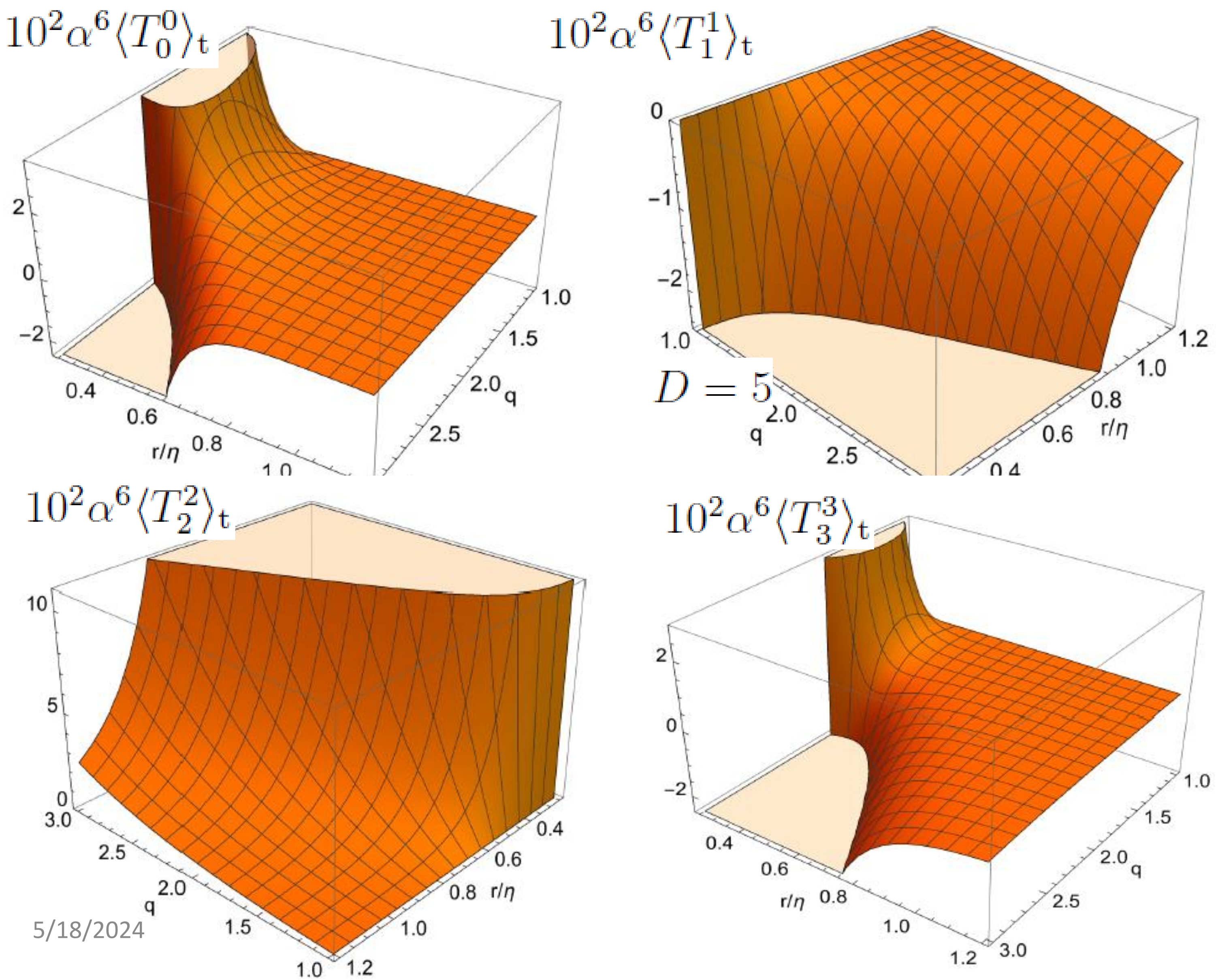
$$\langle T_i^i \rangle_t \approx \frac{\Gamma(D/2-1)t_0^{(i)}c_4(q)}{32\pi^{D/2+1}\alpha^{D+1}(r/\eta)^4},\ \langle T_0^1 \rangle_t \approx -\frac{(D-3)\Gamma(D/2-1)c_4(q)}{8\pi^{D/2+1}\alpha^{D+1}(r/\eta)^5}$$

- Topological contributions in the diagonal components decay at large distances as the inverse fourth power of the proper distance from the cosmic string in all spatial dimensions $D \geq 3$ (the exception is the energy density in 4-dimensional space). This behavior is in contrast to the geometry of a defect in the Minkowski bulk where the VEV decays like $1/r^{D+1}$

Radial dependence of the energy-momentum tensor



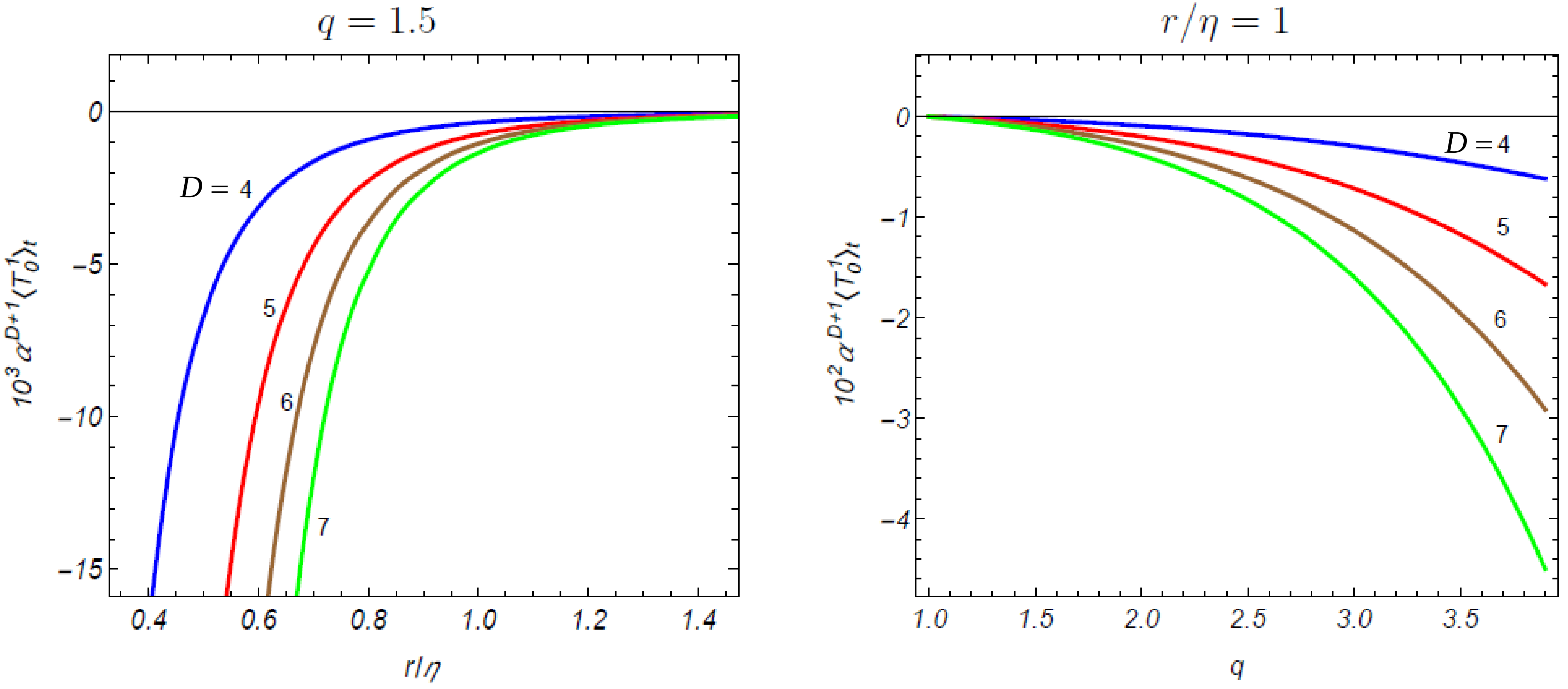
The dependence on the distance and the planar angle deficit



- Depending on the values of *q* and *r/η*, the energy density and the axial stress corresponding to the topological contributions can be either positive or negative

- Radial and azimuthal stresses are monotonic functions of both variables

The radial dependence and dependence on the angle deficit of the energy flux

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