Critical Behavior in Strongly Interacting Matter

Helmut Satz

Universität Bielefeld, Germany

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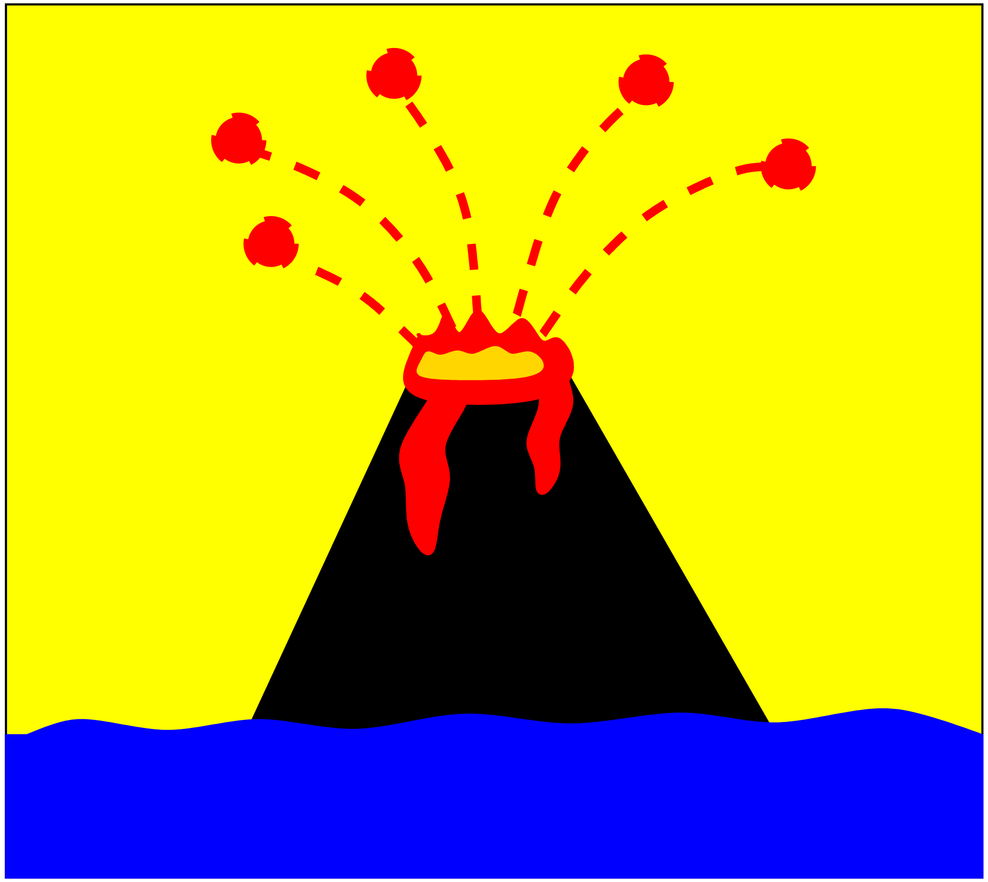
1. Introduction

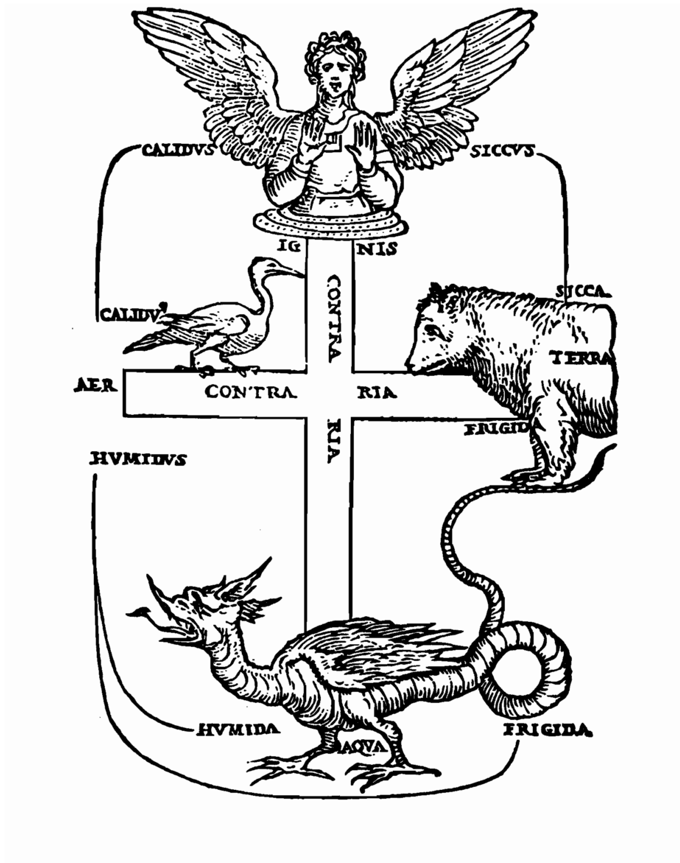
2. Hadronic Matter

3. Deconfined Quarks

The States of Matter 500 B. C. - Experiment



The States of Matter 500 B. C. - Theory



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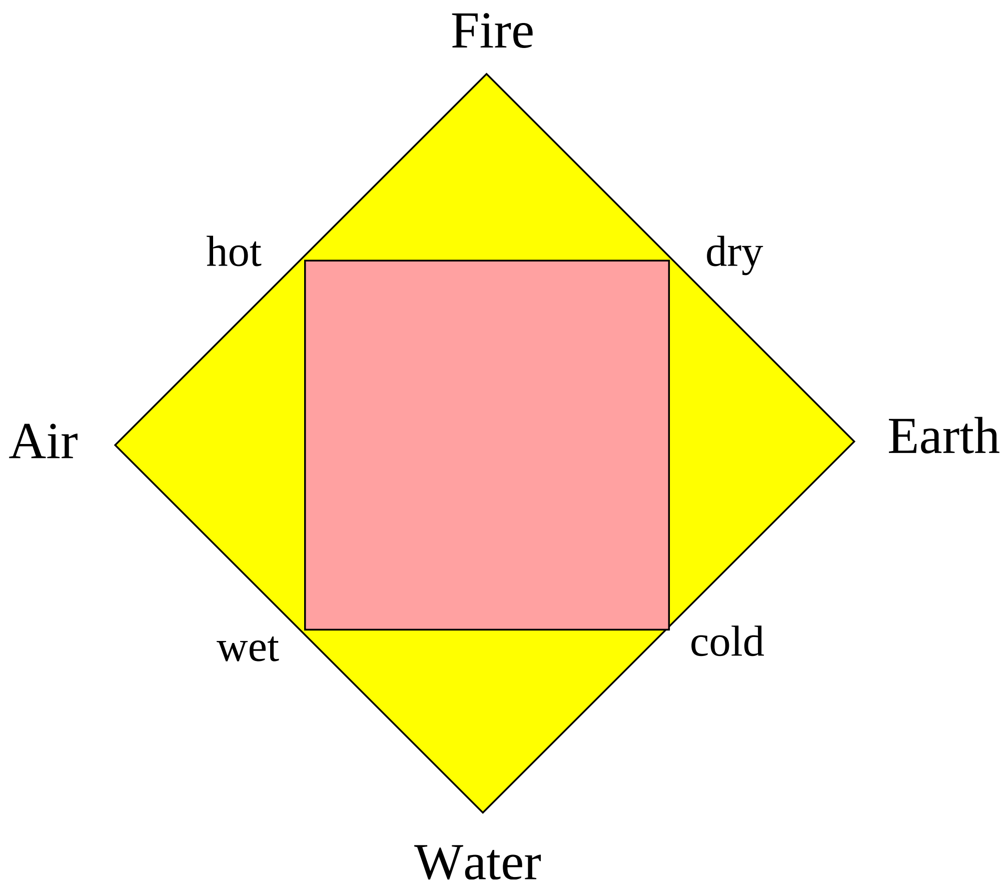


Advent of strong interaction:
what happens to strongly interacting matter
as function of temperature and density?

- **I. Ya. Pomeranchuk, Doklady Akad. Nauk SSSR 1951:**

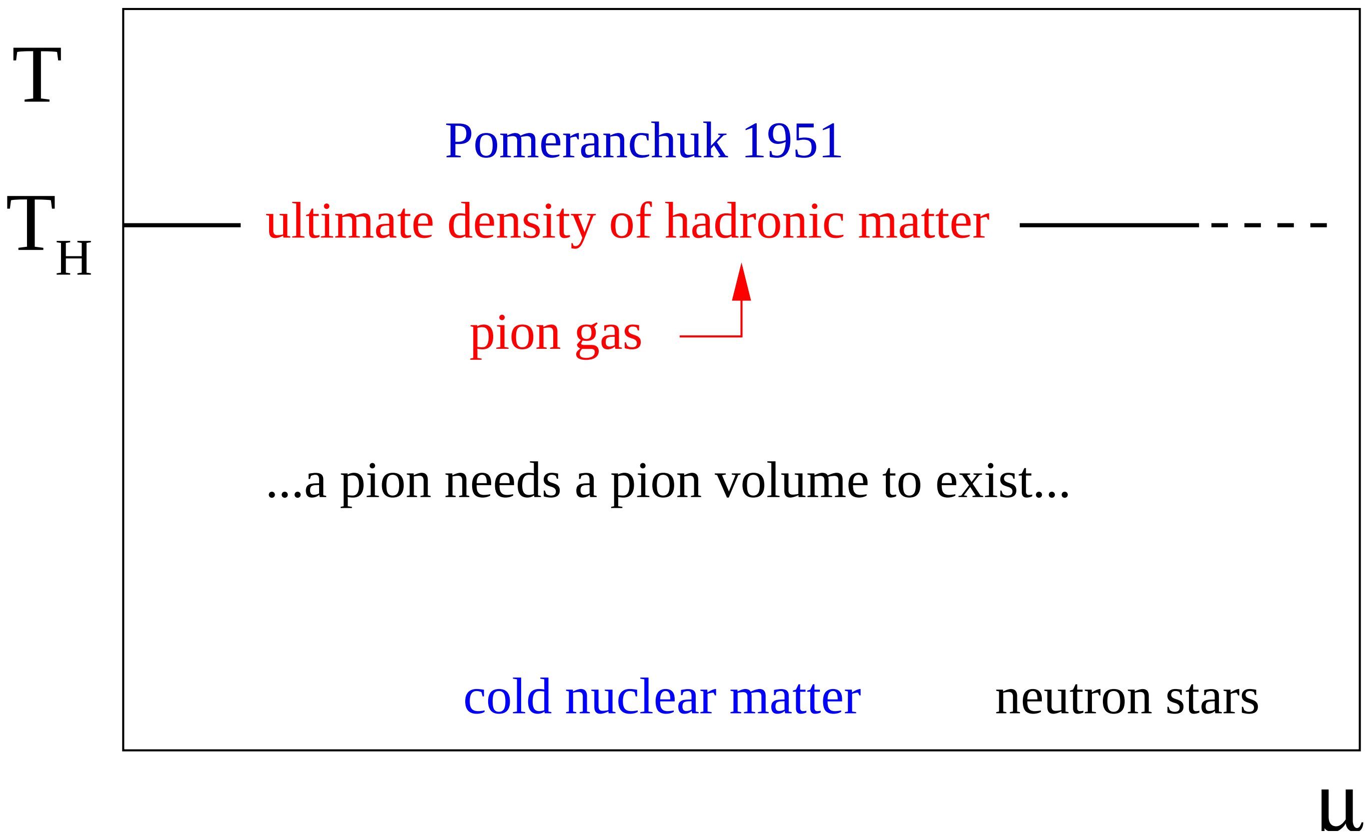
  ...the finite size of hadrons implies a density limit to hadronic matter.

- **Ya. B. Zel'dovich, JETP Letters 1959:**

  ...use the equation of state to establish how many different baryons are really elementary.

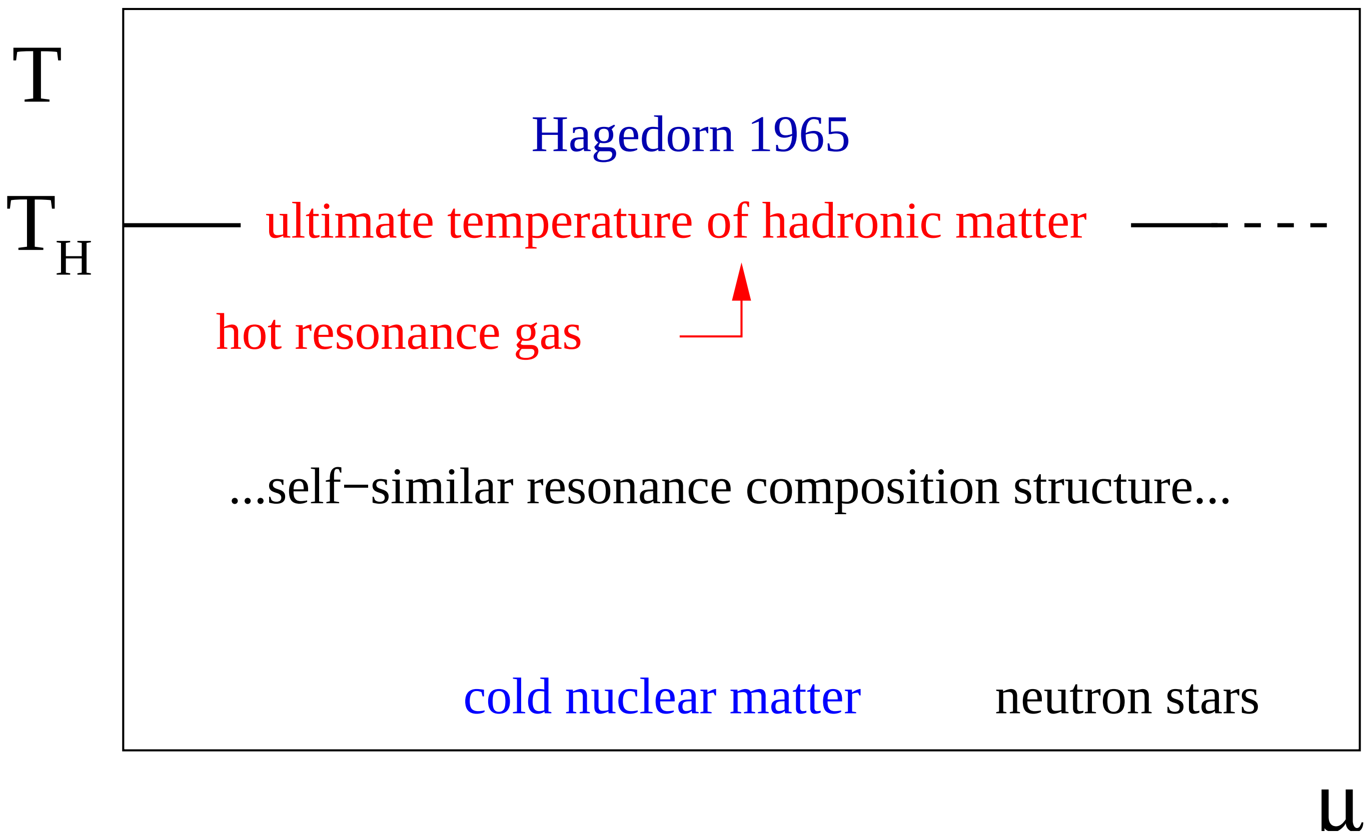
The States of Strongly Interacting Matter

1951



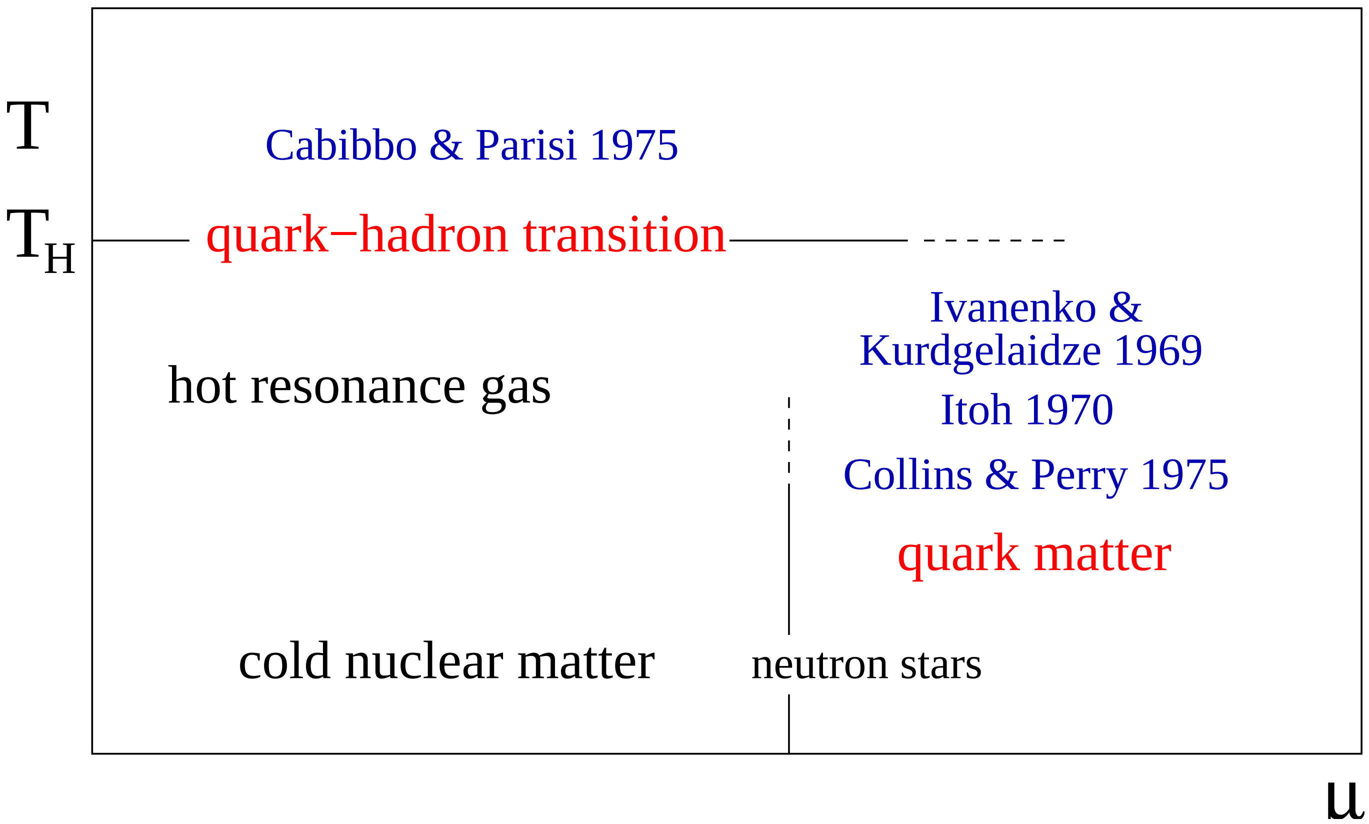
The States of Strongly Interacting Matter

1965



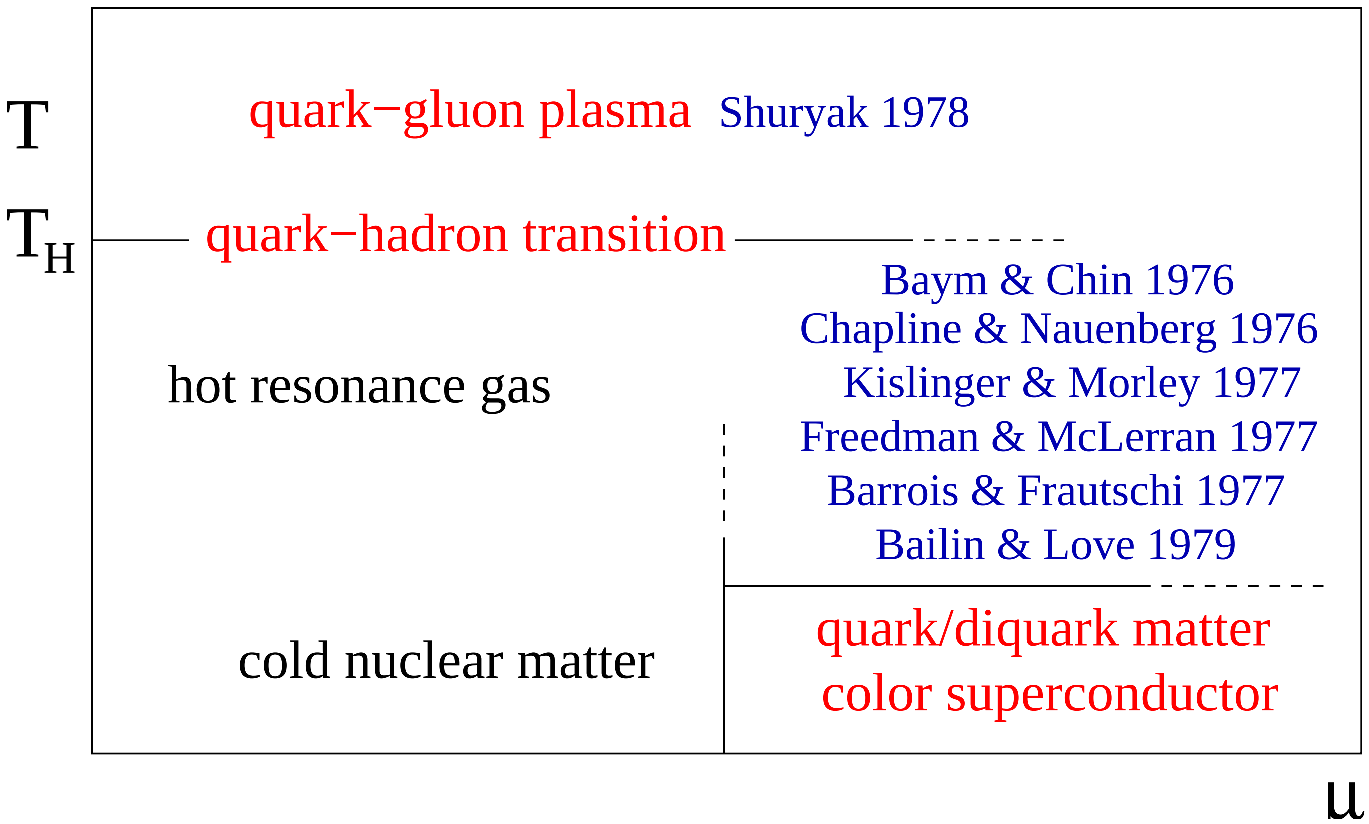
The States of Strongly Interacting Matter

1975



The States of Strongly Interacting Matter

1980



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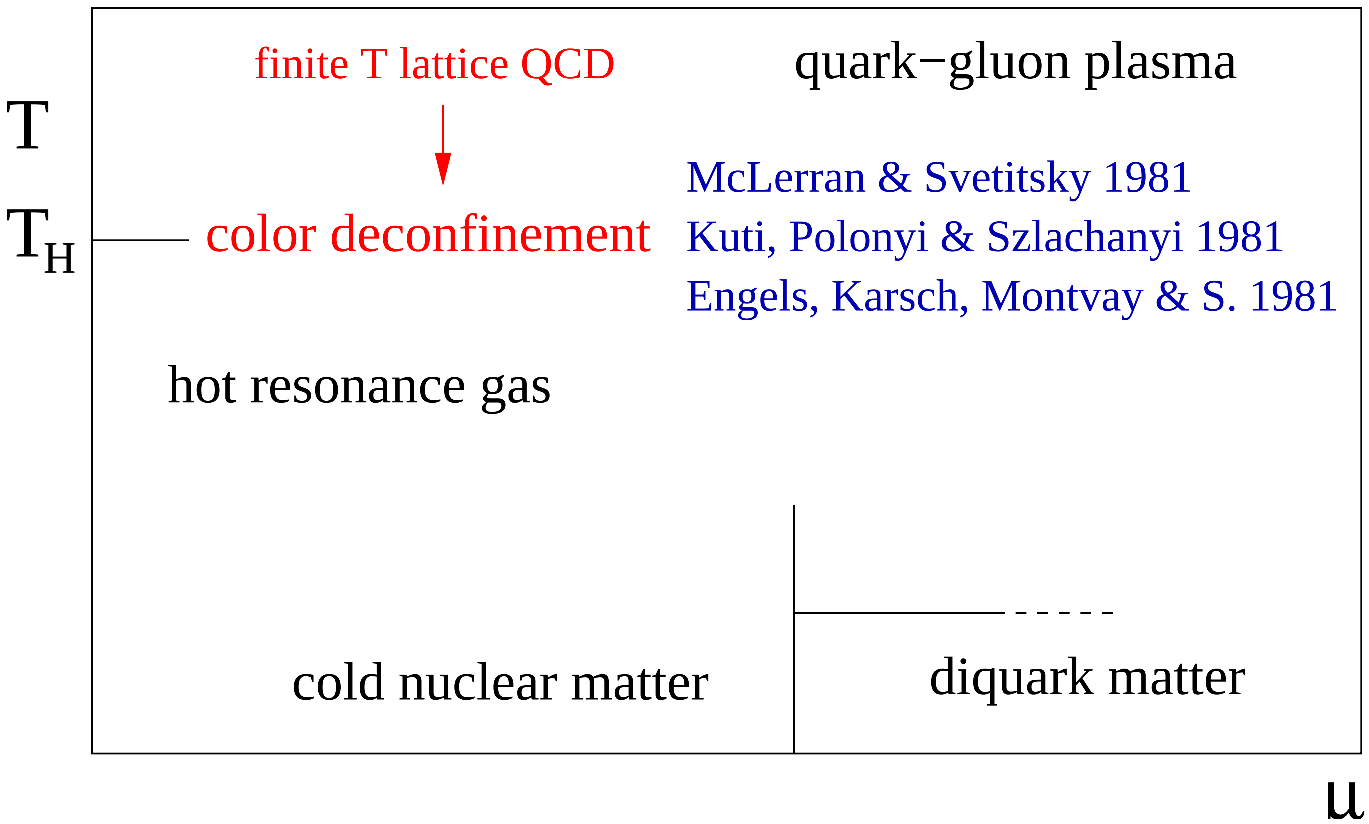
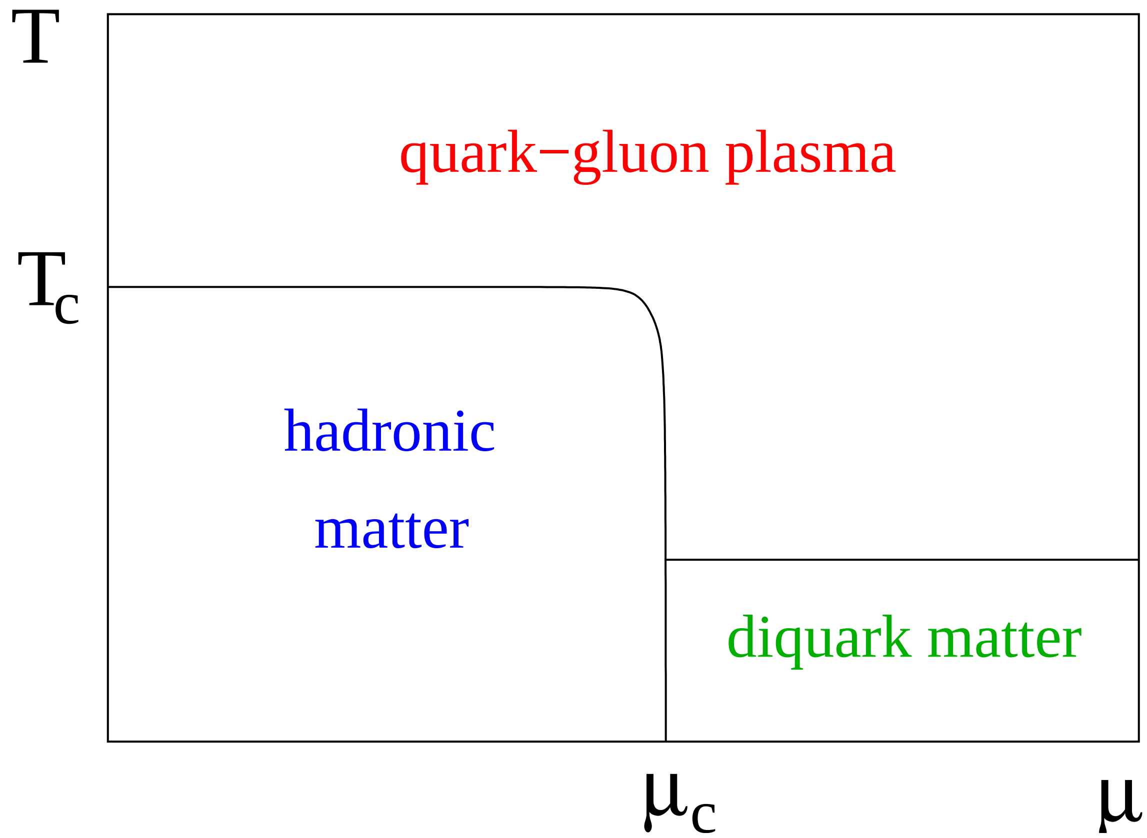
1981



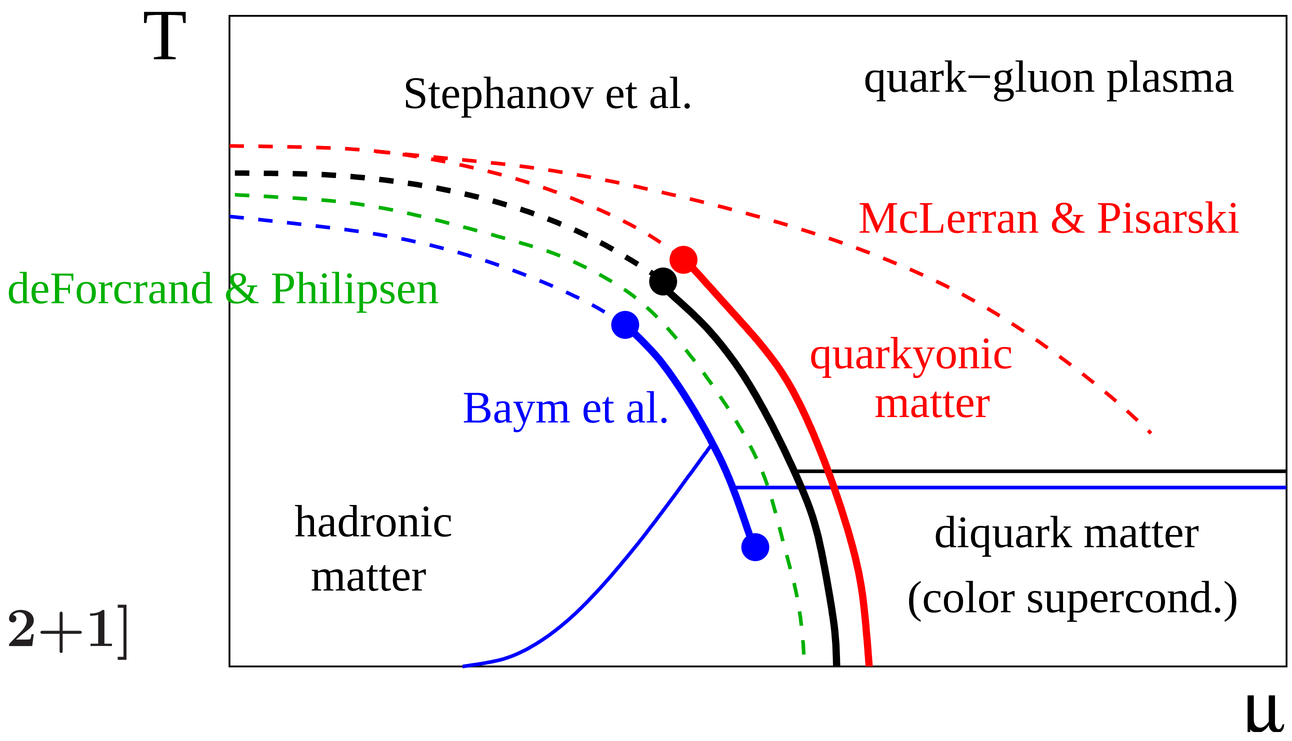
The States of Strongly Interacting Matter

1990



The States of Strongly Interacting Matter

2009



[for 2+1]

Back to basics: How does the underlying physics depend on where we are in the phase diagram?

Conventional Basis of Critical Behavior

- confinement/deconfinement $\sim$ spontaneous Z_2/Z_N symmetry breaking McLerran & Svetitsky 1981, Svetitsky & Yaffe 1982
- dynamical mass generation $\sim$ spontaneous chiral symmetry breaking Pisarski & Wilczek 1984

consider phase structure for $\mu = 0$:

genuine thermal phase transitions

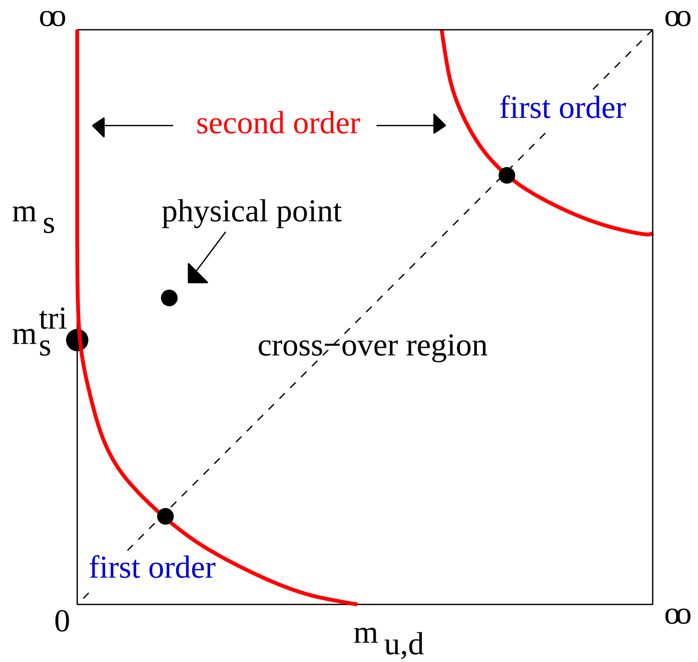
(singularities in partition function)

only for special values of $m_{u,d}, m_s$

but always $\exists$ "transition region"

with sharp variation of thermal

observables: "rapid cross-over"

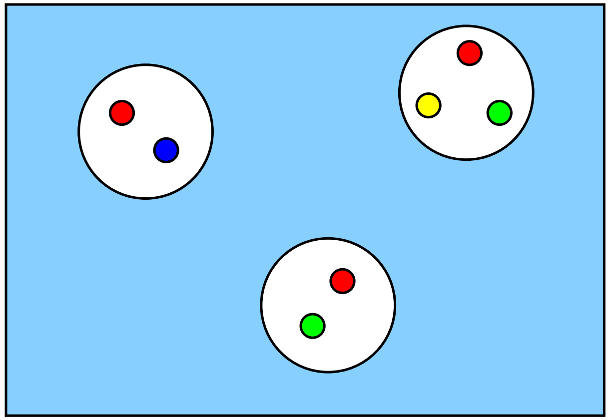


How to understand this? What about density?

What is deconfinement?

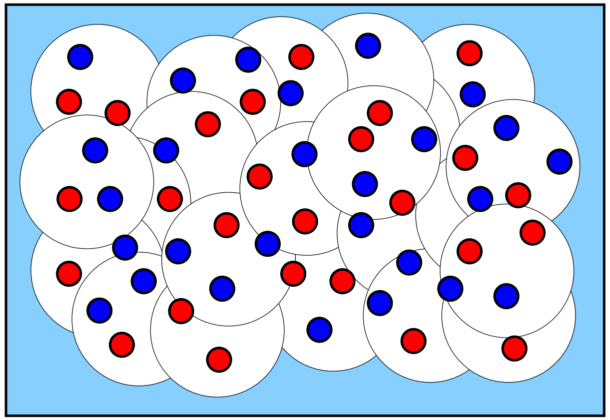
confinement:

a quark has within a range of about 1 fm one antiquark or two quarks to form a color singlet
→ low density phenomenon



deconfinement:

a quark has within a range of about 1 fm so many quarks and antiquarks that pairing becomes meaningless
→ high density phenomenon



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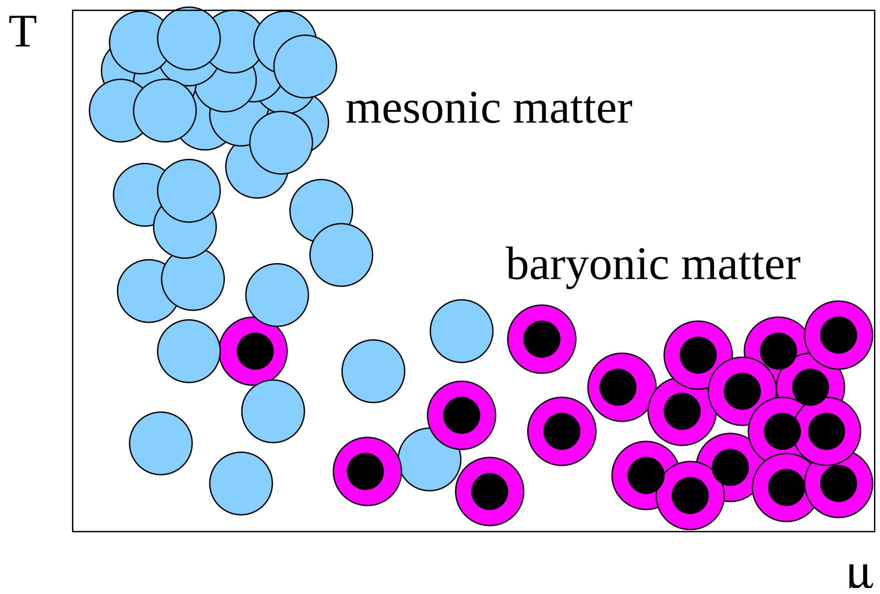
Constituent Structure of Hadronic Matter



- low μ: with increasing T, mesonic medium of increasing density
  mesons attract → resonance formation
  mesons are permeable (overlap) → resonances ∼ same size

- low T: with increasing μ, baryonic medium of increasing density
  nucleons attract → formation of nuclei
  nucleons repel (hard core) → nuclei grow linearly with A

In both cases, $\exists$ clustering

$\exists$ relation between clustering and critical behavior? Frenkel 1939
Essam & Fisher1963

consider spin systems, e.g., Ising model

- for $H = 0$,
  spontaneous Z_2 symmetry breaking $\longrightarrow$ magnetization transition

- but this can be translated into cluster formation and fusion
  critical behavior via cluster fusion: percolation $\equiv$
  critical behavior via spontaneous symmetry breaking

  Fisher 1967, Fortuin & Kasteleyn 1972, Coniglio & Klein 1980

- for $H \neq 0$, Isakov 1984
  partition function is analytic, no thermal critical behavior
  but clustering & percolation persists Kertész 1989

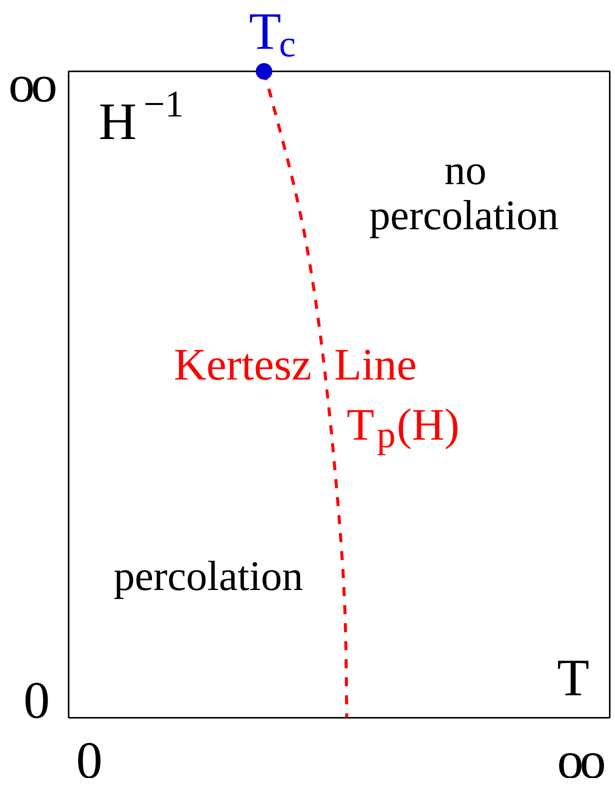
$\exists$ geometic critical behavior

In spin systems,

$\exists$ geometric critical behavior
for all values of H;

for $H = 0$, this can become identical
to thermal critical behavior, with
non-analytic partition function
& Z_2 exponents

for $H \neq 0$, $\exists$ Kertész line
geometric transition with
singular cluster behavior
& percolation exponents



For spin systems,

thermal critical behavior $\subset$ geometric critical behavior

Also in QCD? Hadrons have intrinsic size, with increasing density
they form clusters & eventually percolate

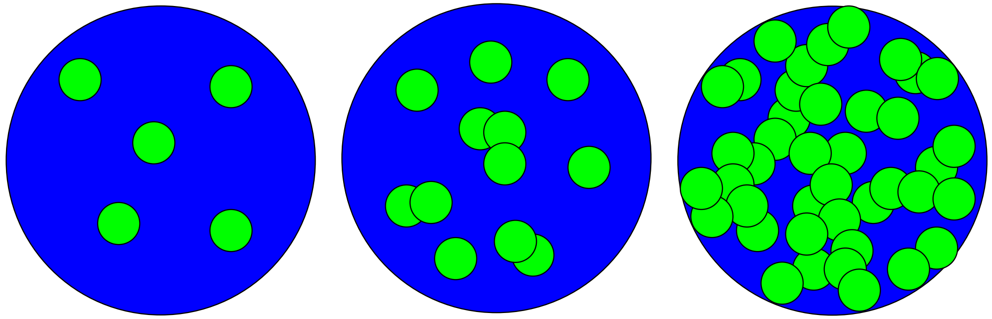
Hadron Percolation ∼ Color Deconfinement

Pomeranchuk 1951

Baym 1979, Çelik, Karsch & S. 1980

Recall percolation

- 2-d, with overlap:
  lilies on a pond



isolated disks   clusters   percolation

- 3-d: N spheres of volume V_h in box of volume V, with overlap
  increase density $n = N/V$ until largest cluster spans volume:
  percolation

  critical percolation density $n_p \simeq 0.34/V_h$

  at $n = n_P$, 30 % of space filled by overlapping spheres,
  70 % still empty

how dense is the percolating cluster? Digal, Fortunato & S. 2004

critical cluster density $n_m \simeq 1.2/V_h$

$$R_h \simeq 0.8 \text{ fm} \Rightarrow n_m \simeq \frac{0.6}{\text{fm}^3} \quad \text{as deconfinement density}$$

so far, cluster constituents were allowed arbitrary overlap

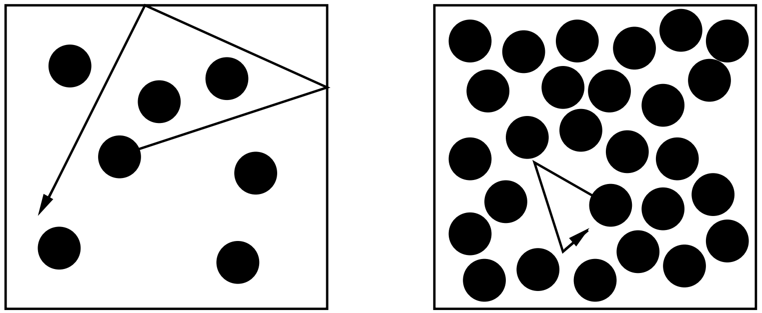
what if they have a hard core?

then ∃ jamming

at high density, constituents
have restricted spatial mobility

∃ <u>jamming transition</u>

with mobility ∼ order parameter



Karsch & S. 1980

percolation for spheres of radius R_0
with a hard core of radius $R_{hc} = R_0/2$ Kratky 1988

hard cores tend to prevent dense clusters;

higher density needed to achieve percolating jammed clusters

$$n_b \simeq \frac{2.0}{V_0} = \frac{0.25}{V_{hc}} \simeq \frac{1.0}{\mathrm{fm}^3} \simeq 6\ \mathrm{n}_0$$

for the deconfinement density of baryonic matter

NB: additional uniform attractive potential
$\longrightarrow$ first order thermal transition

$\exists$ two percolation thresholds in strongly interacting matter:

- mesonic matter, full overlap: $n_m \simeq 0.6/\mathrm{fm}^3$
- baryonic matter, hard core: $n_b \simeq 1.0/\mathrm{fm}^3$

now apply to determine critical behavior

If interactions are resonance dominated,

interacting medium $\equiv$ ideal resonance gas

Beth & Uhlenbeck 1937; Dashen, Ma & Bernstein 1969

consider ideal resonance gas of all PDG states for $M \leq 2.5$ GeV

partition function

$$\ln\ Z(T,\mu,\mu_S,V) = \ln\ Z_M(T,\mu_S,V)\ +\ \ln\ Z_B(T,\mu,\mu_S,V)$$

with

$$\ln\ Z_M(T,V,\mu_S) = \sum_{\text{mesons } i} \ln\ Z_M^i(T,V,\mu_S)$$

$$\ln\ Z_B(T,\mu,\mu_S,V) = \sum_{\text{baryons } i} \ln\ Z_B^i(T,\mu,\mu_S,V)$$

for mesonic and baryonic contributions; enforce $S = 0$

• low baryon-density limit: percolation of overlapping hadrons

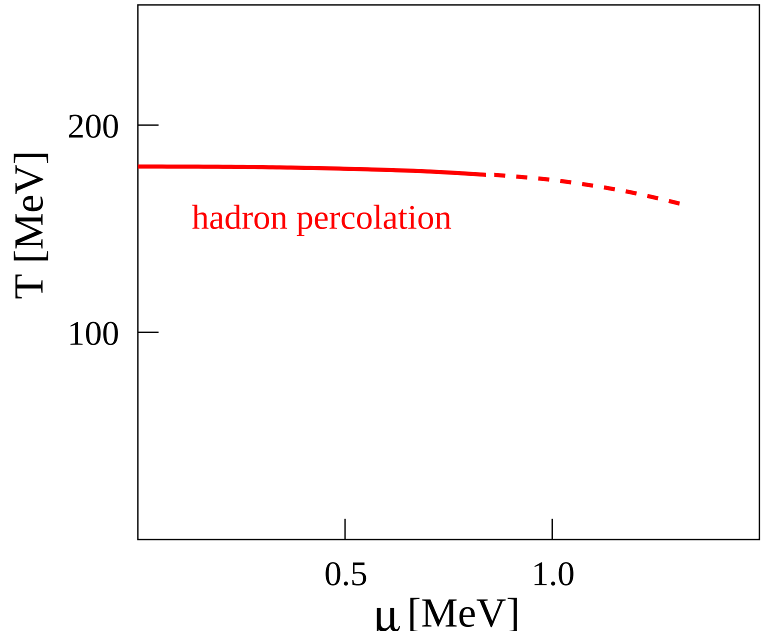
$$n_h(T_h, \mu) = \frac{\ln Z(T, \mu, V)}{V} = 0.6/\text{fm}^3$$

Obtain at $\mu = 0$

$$T_h \simeq 180 \text{ MeV}$$

deconfinement temperature

based on hadron percolation



baryons included, but hard core effects ignored

slow decrease of transition temperature with μ,
due to associated production

• high baryon-density limit:

percolation/jamming of hard-core baryons

density of pointlike baryons

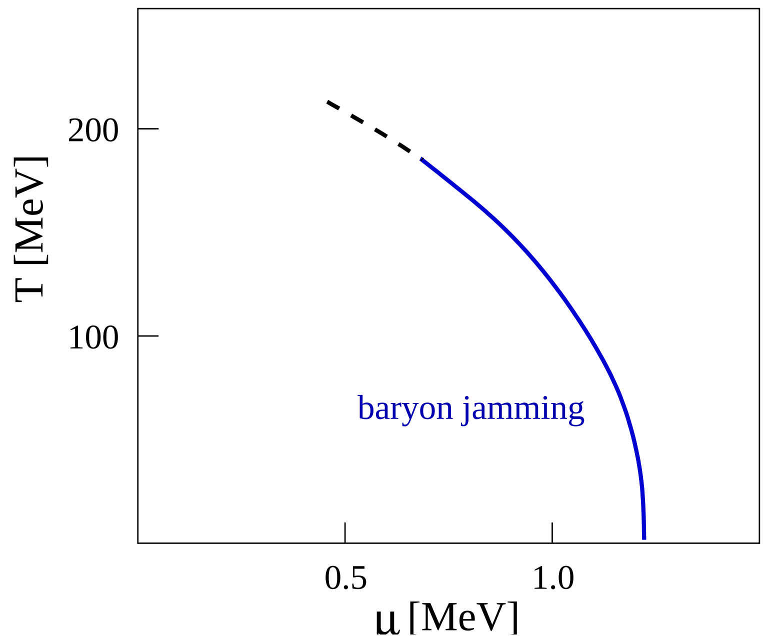
$$n_b^0 = \frac{1}{V}\left(\frac{\partial\ T \ln Z_B(T,\mu,V)}{\partial\mu}\right)$$

hard core ⇒ excluded volume
(Van der Waals)

$$n_b = \frac{n_b^0}{1 + V_{hc} n_b^0}$$

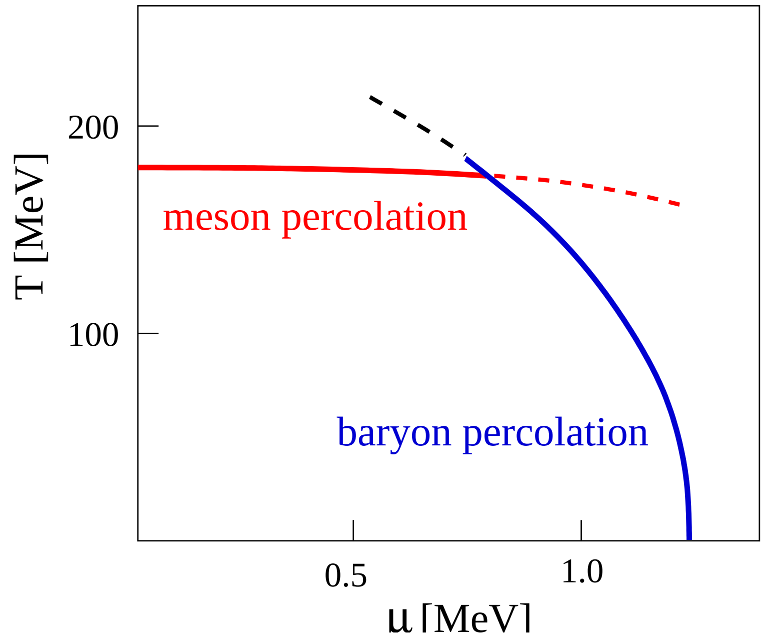
jamming threshold
→ transition line

$$n_b^c(T,\mu) = \frac{2.0}{V_0} = \frac{1.0}{\text{fm}^3}$$



combine the two mechanisms:

phase diagram of hadronic matter



• low baryon density:

percolation of overlapping hadrons

clustering $\sim$ attraction

• high baryon density:

percolation of hard-core baryons

nuclear attraction plus hard-core repulsion $\longrightarrow$ 1^{st} order transition

clustering and percolation can provide

a conceptual basis for the limits of hadronic matter

in the QCD phase diagram

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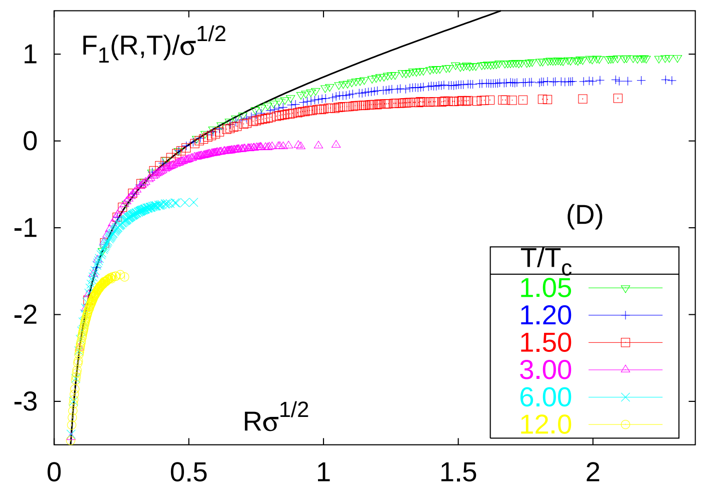
What happens beyond the limits?

There are two roads to deconfinement:

- Increase quark density so that several quarks/antiquarks within confinement radius $\longrightarrow$ pairing ambiguous or meaningless.
- Increase temperature so much that gluon screening forbids communication between quarks/antiquarks distance r apart.

Illustration of the second case:
heavy quark correlations



Quarks separated by about 1 fm
no longer "see" each other for $T \geq T_c$

mesonic matter:
when quark density is high enough,
gluon screening radius is short enough, so both coincide

baryonic matter?

in hadrons & in hadronic matter $\exists$ chiral symmetry breaking

$\Rightarrow$ confined quarks acquire effective mass $M_q \simeq 300$ MeV

effective size $R_q \simeq R_h/3 \simeq 0.3$ fm

through surrounding gluon cloud

what happens at deconfinement? Possible scenarios:

- **plasma of massless quarks and gluons,**
  ground state shift re physical vacuum $\longrightarrow$ bag pressure B
- **plasma of massive "constituent" quarks, all gluon effect in M_q**

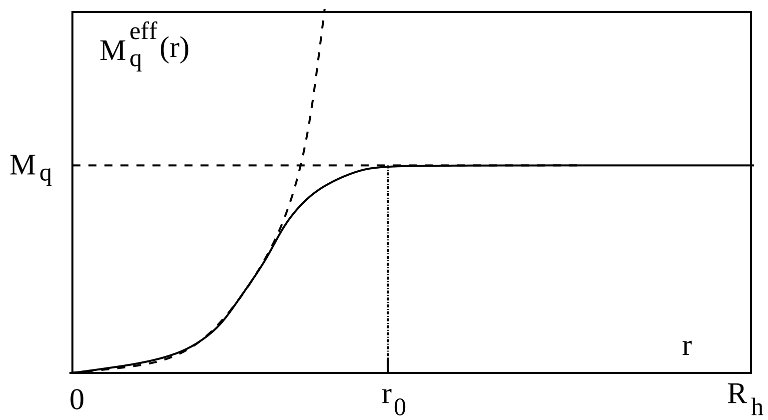
"effective" quark? $\sim$ depends on how you look: **Shuryak 1988**

- **short distance, hard probe: bare current quark**
  (deep inelastic scattering)
- **larger distance, softer probe: massive constituent quark**
  (additive quark model)

Origin of constituent quark mass?

quark polarizes gluon medium → gluon cloud around quark

$$M_q \sim m_q + \epsilon_g r^3$$



where ϵ_g is the change in energy density of the gluon field due to the presence of the quark

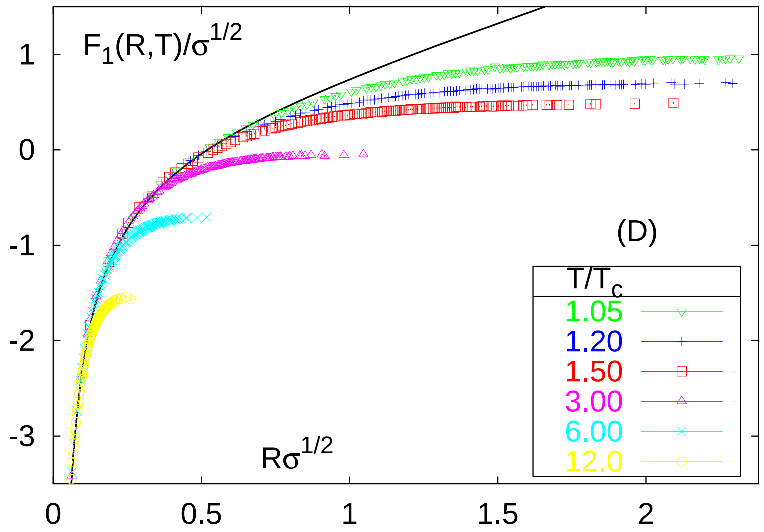
QCD:

non-abelian gluon screening limits "visibility" range to r_g

→ energy density of gluon cloud and screening radius determine "asymptotic" constituent quark mass ∼ gluon cloud

how does this change in a hot deconfined medium?

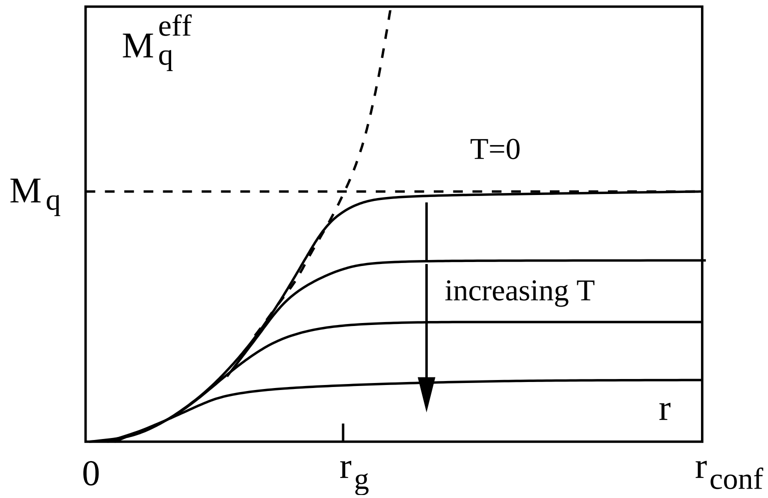
heavy quark correlation studies: Bielefeld Lattice Group 2002



screening radius and “mass” of polarization cloud decrease with increasing temperature (quenched - i.e. gluon effect)

expect corresponding T dependence of constituent quark mass

with increasing temperature,
constituent quark becomes
smaller and lighter



high temperature, short distance limit $\longrightarrow$ current quark

now consider different $T-\mu$ regions:

- $\mu \simeq 0,\ T \simeq T_c$:

  interquark distance $\sim$ 1 fm and hot gluon medium

  $\Rightarrow\ M_q^{\text{eff}} \simeq 0$

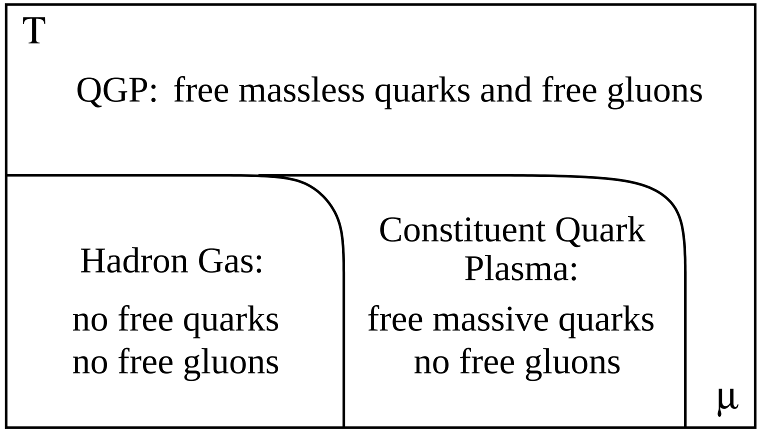
- $T \simeq 0,\ \mu \simeq \mu_c$:

  interquark distance ~ 1 fm and cold medium, no gluon screening

  $\Rightarrow\ M_q^{\rm eff} \simeq M_q$

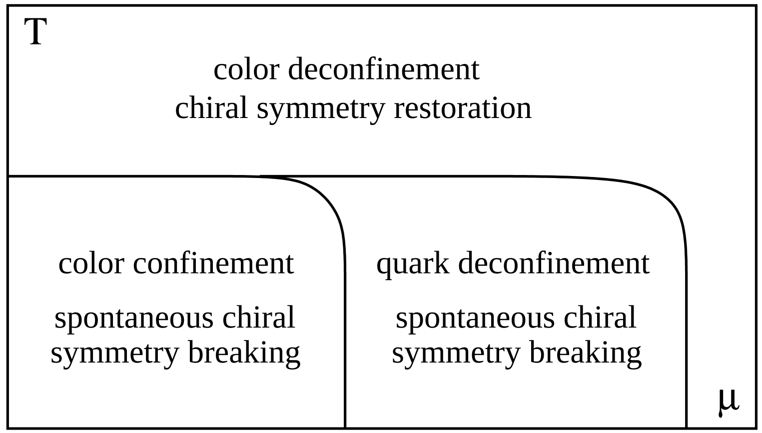
for cold dense matter,
$M_q^{\rm eff} \longrightarrow 0$ requires
very short interquark distance

intermediate constituent quark
plasma possible for $0.3 < r < 1$ fm



Speculative Scenario:

- at small μ and $T \geq T_c$, hot gluon medium rules out a constituent gas phase, direct transition to QGP
  (alternative view: hard gluons $k_g \sim 3\ T_c$ resolve gluon cloud)
- at large $\mu \geq \mu_c$ and small T, constituent quarks can survive and form a massive quark plasma between hadronic matter and QGP



in terms of deconfinement
& chiral symmetry:

in terms of large N_c,
effective degrees of freedom

- hadron gas: $d_{\text{eff}} = 1$
- constituent quark plasma: $d_{\text{eff}} = N_c$
- quark-gluon plasma: $d_{\text{eff}} = N_c^2$

crucial aspect:
∃ an intermediate phase with only quark degrees of freedom
("quarkyonic"?), gluons make constituent quark mass;

for $r < r_g$, transition to quark-gluon plasma, "gluon liberation" consider constituent quark plasma:

– massive quarks and (at higher T) some massive antiquarks

– no gluons

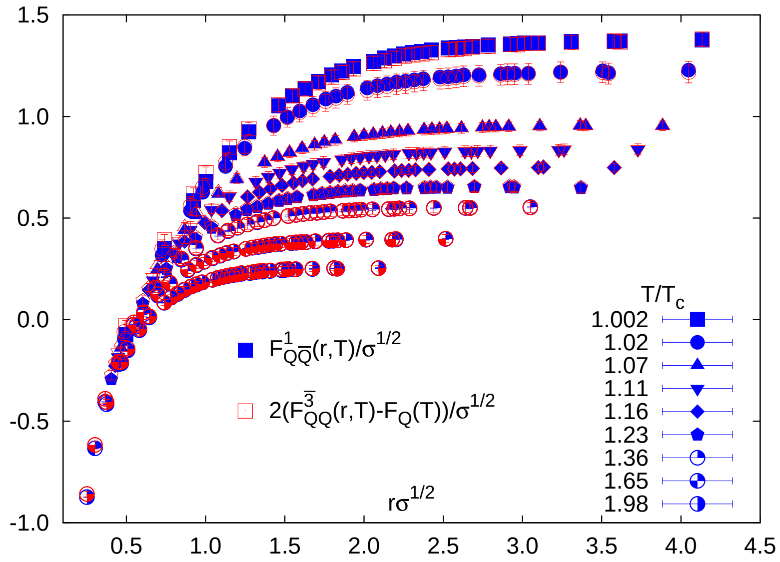
no color confinement, but conventional bound states possible

attractive interaction for

$qq \longrightarrow$ color anti-triplet,

$q\bar{q} \longrightarrow$ color singlet,

with same functional form of potential in r, T



Bielefeld Lattice Group 2002

NB: anti-triplet qq bound state = diquark
(genuine two-body state, not Cooper pair)

constituent quark plasma can be structurally similar to hadron gas:

- (antitriplet) diquark and (singlet) $q\bar{q}$ states
- higher excitations (colored resonance gas)
- also possible: glueballs
- all states have intrinsic finite size (and mass), hence ∃ percolation limit

Essential prerequisite for "third, intermediate" state:

quark degrees of freedom, gluons only modify quark properties

other alternative: string gas... Miyazawa 1979; Goloviznin & S. 1996

Conclusion

- **Three State Phase Diagram (apart from color superconductor)**
- **Hadronic mattter:**
  quarks and gluons confined to hadrons, broken chiral symmetry
- **Constituent quark plasma:**
  massive deconfined quarks, broken chiral symmetry
- **Quark-gluon plasma:**
  deconfined massless quarks and gluons, restored chiral symmetry