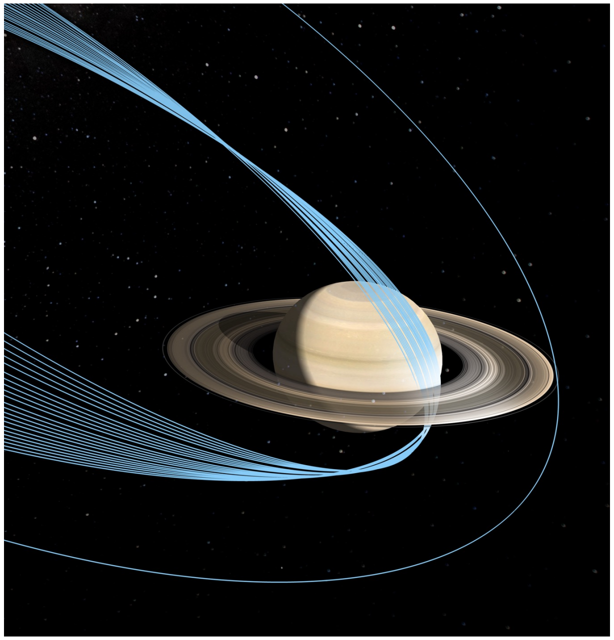
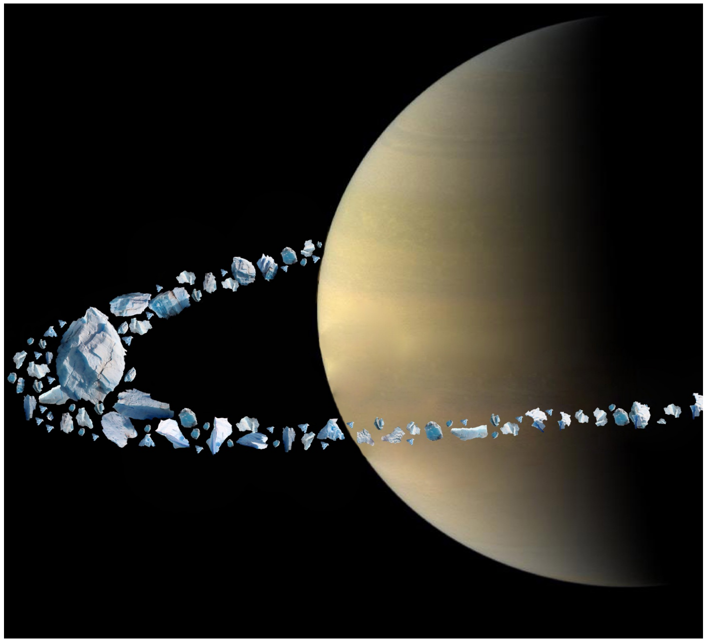
NASA mission Cassini to Saturn. How did that planet become the Lord of the Rings?





B. Militzer, W. B. Hubbard, J. Wisdom, R. Dbouk,
UC Berkeley U Arizona MIT MIT
F. Nimmo, B. Downey, R. French
UCSC UCSC Wellesley

Outline

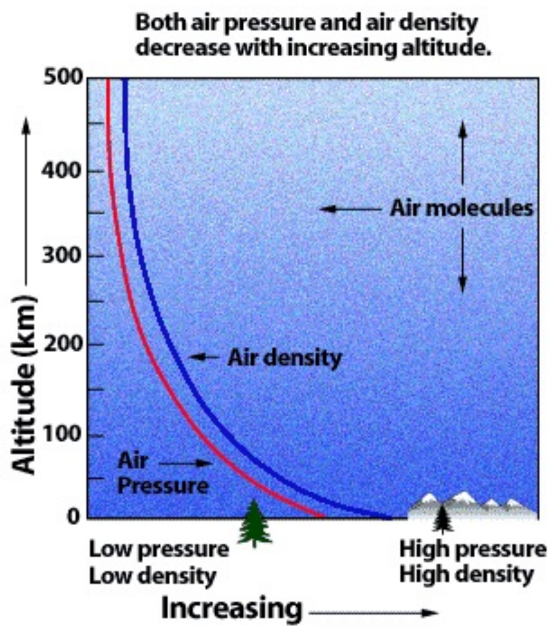
1. Quadratic Monte Carlo
2. **Cassini** mission to Saturn
3. **Resonances** in planetary science?
4. How did Saturn become the
   Lord of the Rings?

1. New General-Purpose Monte Carlo Method

Monte Carlo Simulations in Physics – Canonical Ensemble

Move particles so that the resulting distributions

$$P(h) \sim \exp(-E(h) \,/\, k_b T)$$



Monte Carlo Simulations in Physics – Canonical Ensemble

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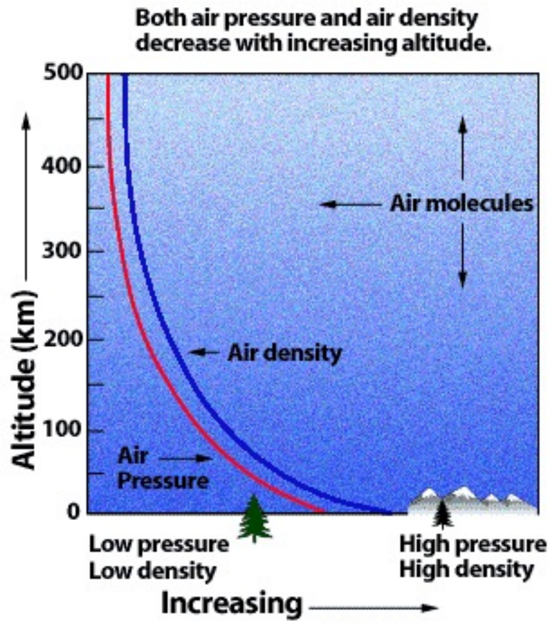
$$P(h) \sim \exp(-E(h) \;/\; k_b T)$$

Here is how the algorithm works:

- **The altitude of our one particle is h. Its energy is E(h).**
- **Propose a new altitude**

  h_{new} = h + stepSize * [RandomNumber() – 0.5]

  E_{new} = $E(h_{new})$



Monte Carlo Simulations in Physics – Canonical Ensemble

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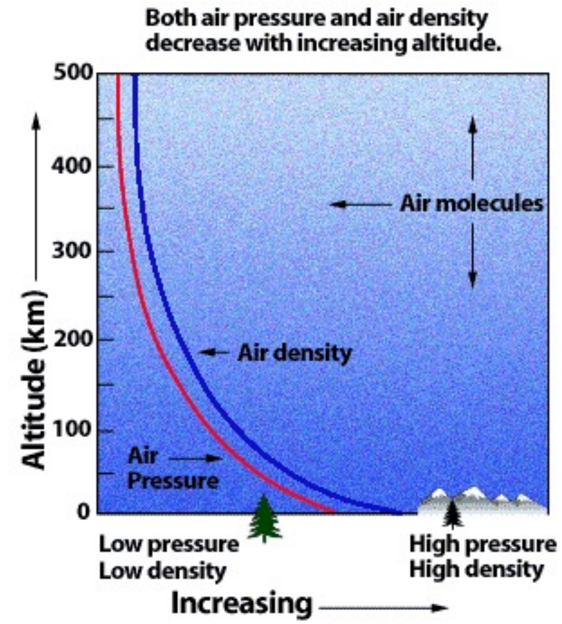
Here is how the algorithm works:

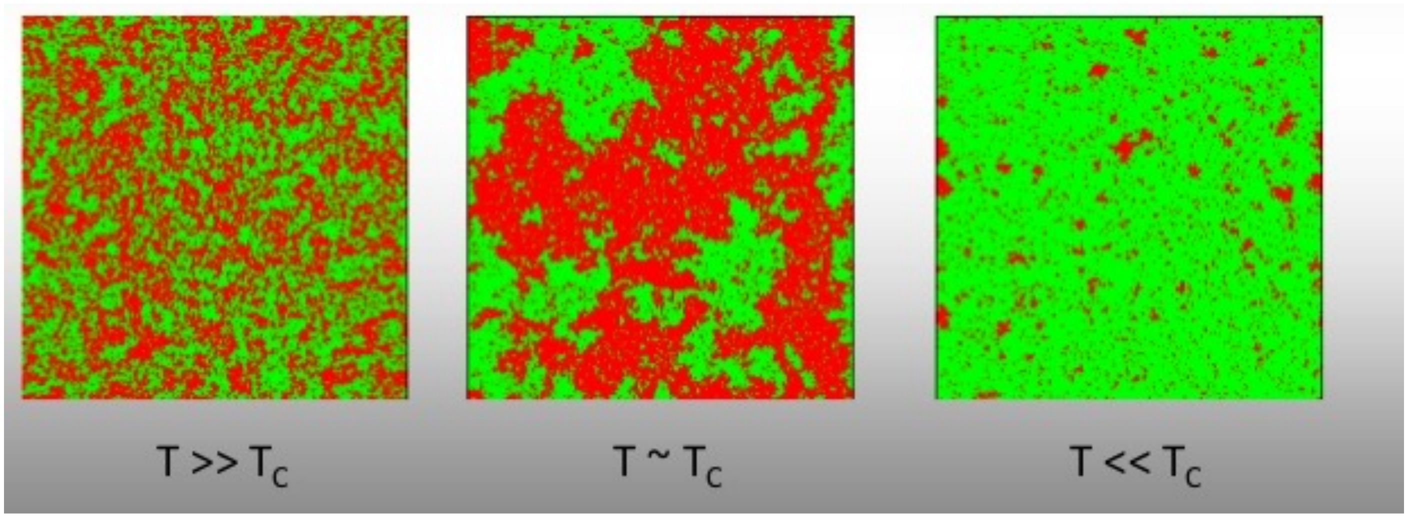
- **The altitude of our one particle is h. Its energy is E(h).**
- **Propose a new altitude**
  - **h_{new} = h + stepSize * [RandomNumber() – 0.5]**
  - **E_{new} = E(h_{new})**

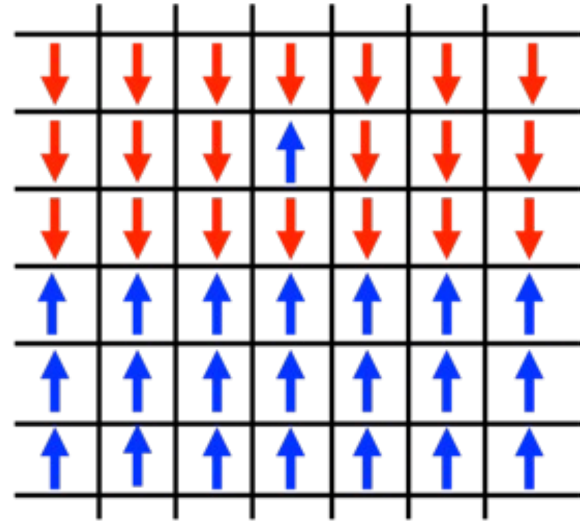
- **If (E_{new} < E) → Always accept this move.** (Always go downhill in energy.)
- **If (E_{new} > E) → Accept this move with probability**

$$P_{accept} = \frac{P(E_{new})}{P(E_{old})} = \frac{exp(-E_{new}/k_b\,T)}{exp(-E_{old}/k_b\,T)} = exp[-(E_{new} - E_{old})/k_b\,T]$$



Monte Carlo Simulations in Physics – Ising Spin System





Flip spins to generate a canonical ensemble

$$P(s) \sim \exp(-E(s)\ /\ k_b T)$$

Apply our QMC method to Dilute Core Models

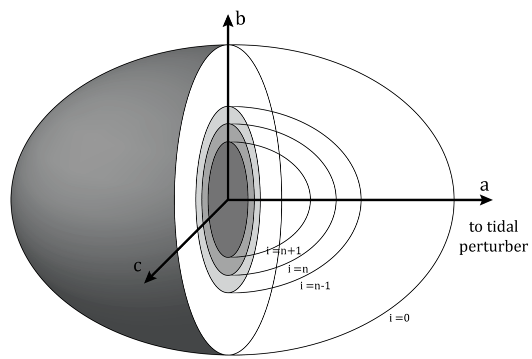
Parameters of planet model:

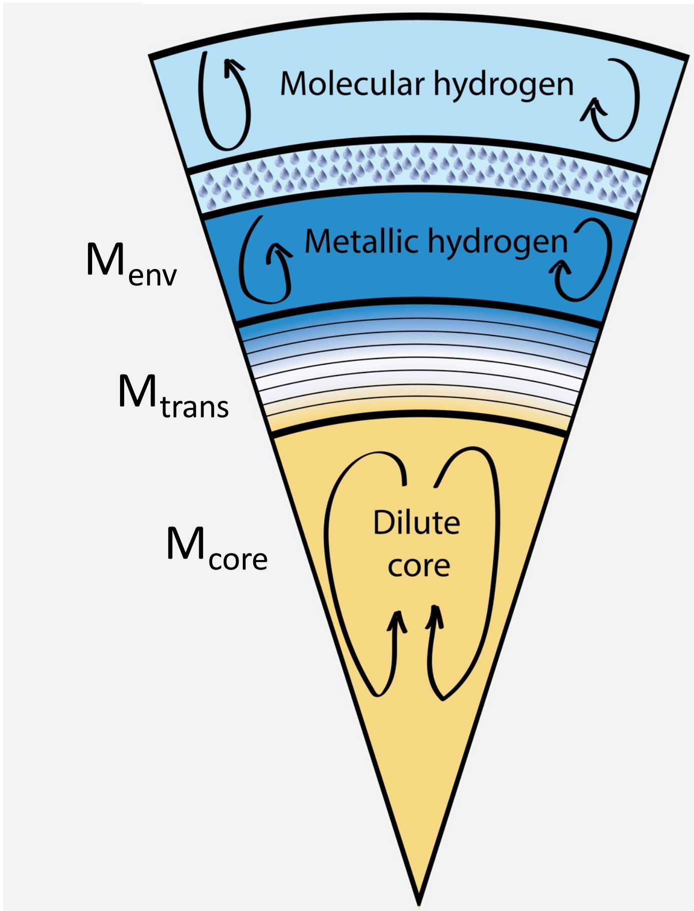
- How much helium is in various layers?
- Mass fraction of heavy elements, Z
- Pressures at layer boundaries
- Size of the core
- Entropy in deep interior

Then the gravity coefficients, J_n, with CMS

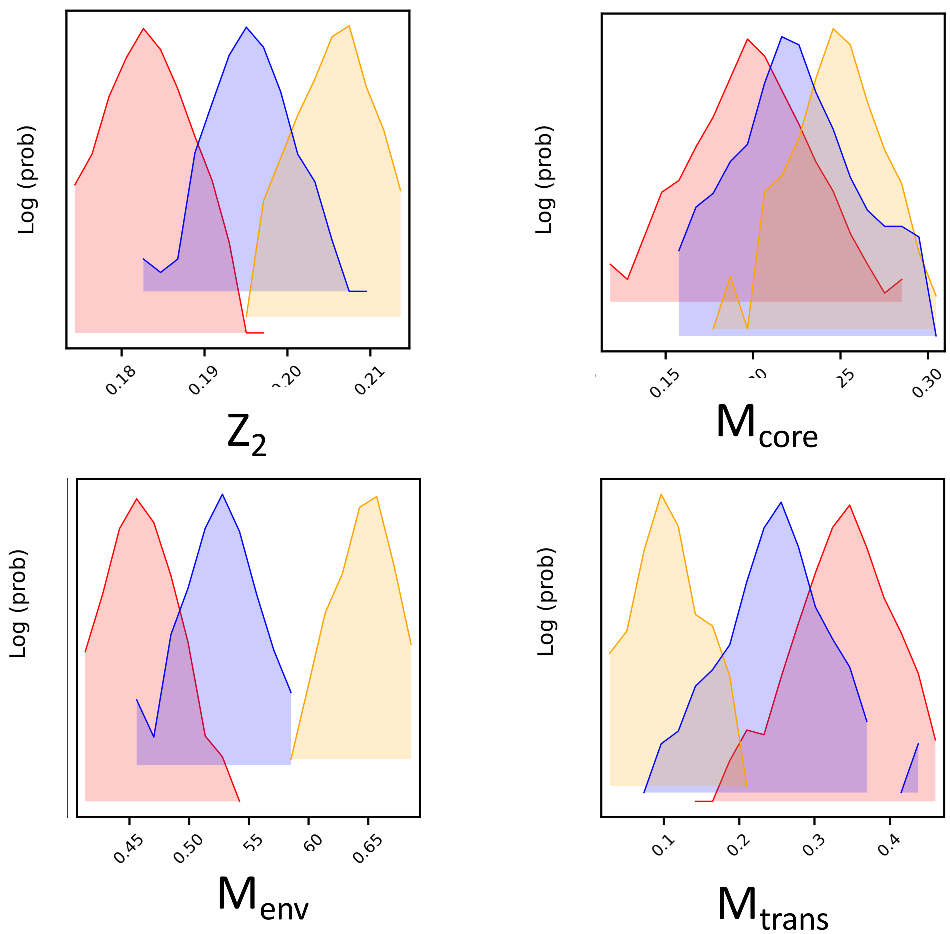
$$\chi^2 = \sum_n \left(\frac{J_n^{Juno} - J_n^{model}}{\delta J_n^{Juno}} \right)^2$$

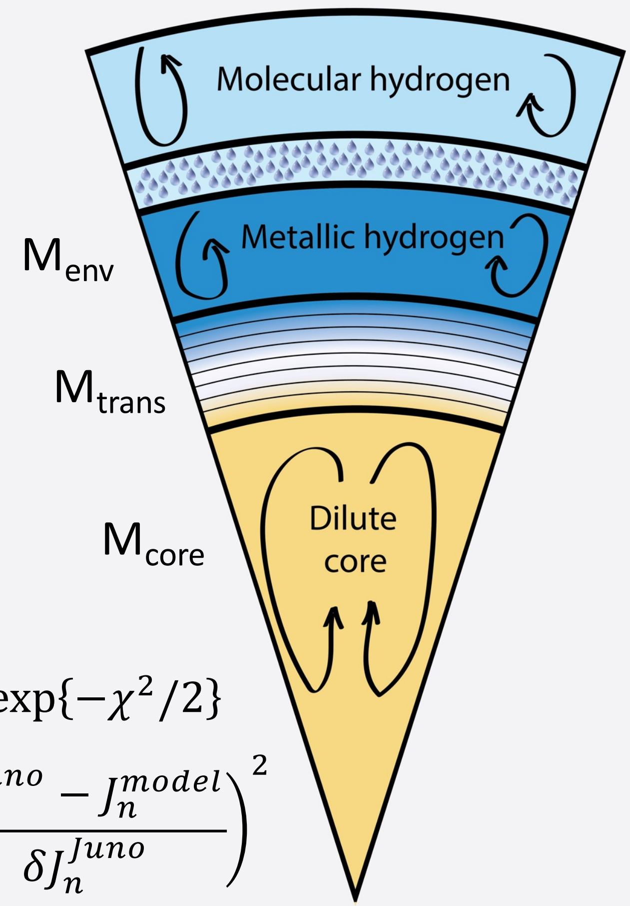
Probability $\sim \exp\{-\chi^2/2\}$





Apply our QMC method Dilute Core Models

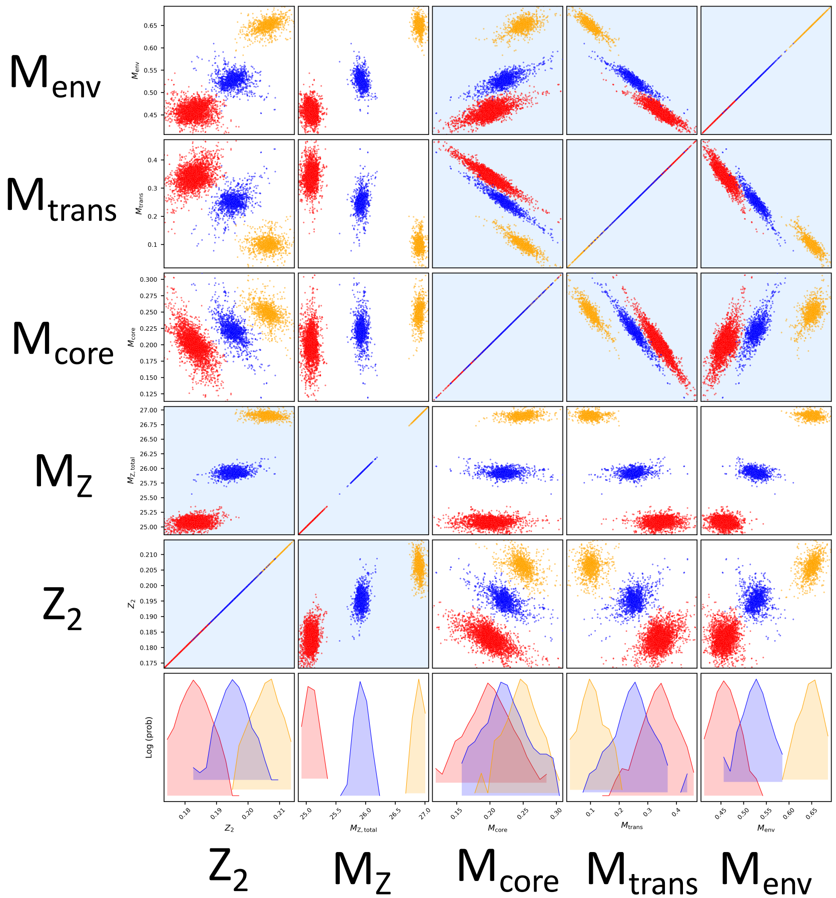


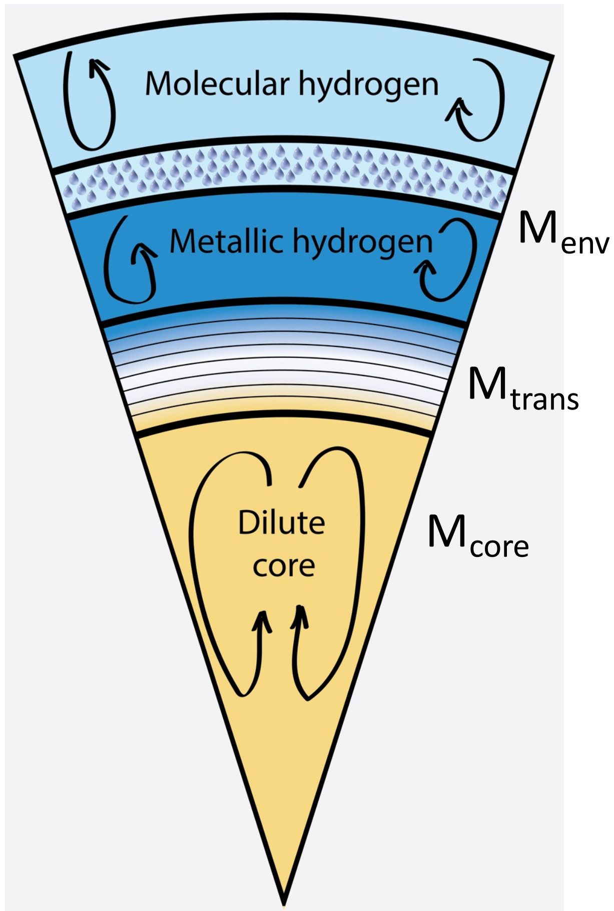


Probability $\sim \exp\{-\chi^2/2\}$

$$\chi^2 = \sum_n \left(\frac{J_n^{Juno} - J_n^{model}}{\delta J_n^{Juno}} \right)^2$$

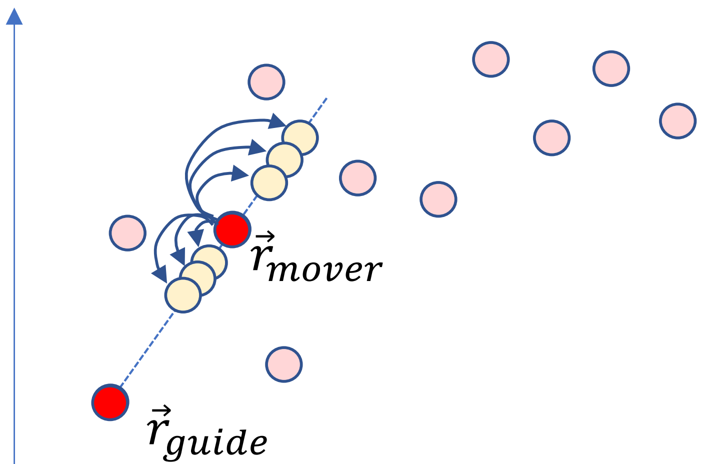
Apply our QMC method Dilute Core Models





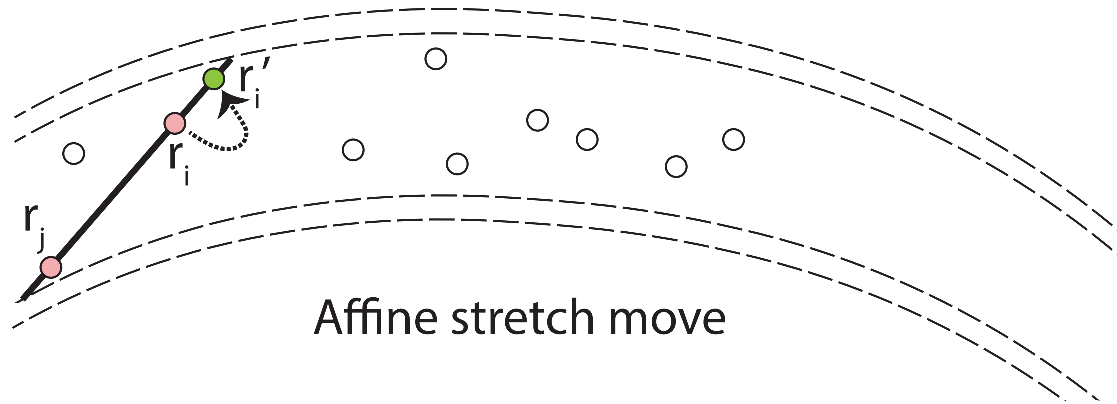
Affine Invariance MCMC by Goodman & Weare

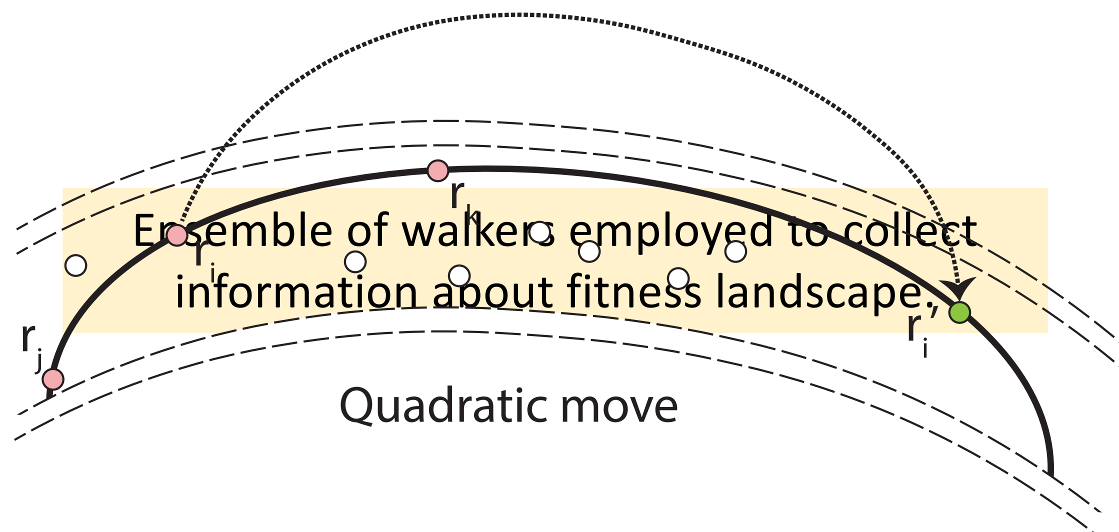
JONATHAN GOODMAN AND JONATHAN WEARE



Affine Stretch move

$$g(z) \propto \begin{cases} \dfrac{1}{\sqrt{z}} & \text{if } z \in \left[\dfrac{1}{a}, a\right], \\ 0 & \text{otherwise.} \end{cases}$$





Our Quadratic Monte Carlo Method Explained at http://militzer.berkeley.edu/QMC

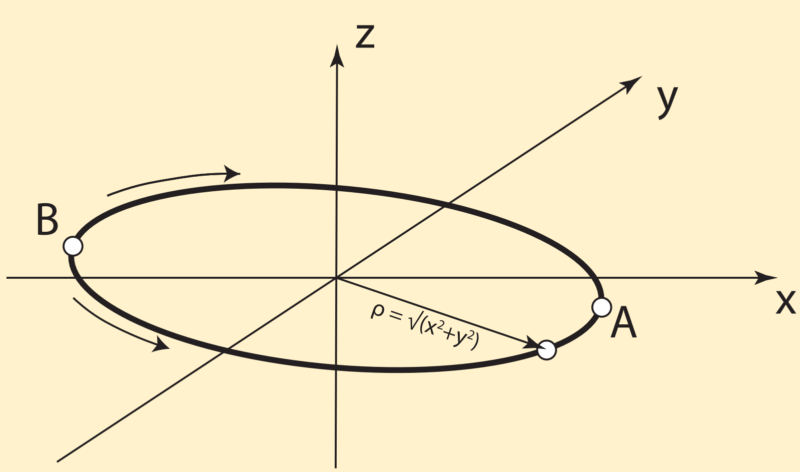
```
Array1 <S> s = SetUpNStates(nWalkers);           // Set up nWalkers different states of type S
for(int iBlock=0; i<nBlocks; iBlock++) {         // loop over blocks
   for(int iStep=0; i<nStepsPerBlock; iStep++) { // loop over steps
      for(int i=0; i<nWalkers; i++) {            // try moving every walker once per step
         int j,k;
         SelectTwoOtherWalkersAtRandom(i,j,k);

         const double tj = -1.0;
         const double tk = +1.0;
         const double ti   = SampleT(a); // sample t space
         const double tNew = SampleT(a); // sample t space one more time

         const double wi = LagrangeInterpolation(tNew,ti,tj,tk);
         const double wj = LagrangeInterpolation(tNew,tj,tk,ti);
         const double wk = LagrangeInterpolation(tNew,tk,ti,tj);

         S sNew = s[0]; // Create new state by coping over an existing one.
         for(int d=0; d<nDim; d++) {
            sNew[d] = wi*s[i][d] + wj*s[j][d]+ wk*s[k][d]; // set nDim parameters of new state
         }

         if (sNew.Valid()) {                 // check if state sNew is valid before calling Evaluate()
            sNew.Evaluate();                 // Sets the energy sNew.y which defines the state's probability = exp(-y/temp)
            double dy    = sNew.y - s[i].y;  // difference in energy between new and old state
            double prob = pow(fabs(wi),nDim) * exp(-dy/temp);    // Note |wi|^nDim and Boltzmann factors
            bool accept = (prob>Random());   // Random() returns a single random number between 0 and 1.

            if (accept) {
               for(int d=0; d<nDim; d++) {
                  s[i][d] = sNew[d]; // copy over state sNew
               }
               s[i].y = sNew.y;      // copy also its energy
            }
         }

      } // look over all walkers
      ComputeDifferentEnsembleAverages(s);
   }    // end of loop over steps
   PrintEndOfBlockStatement();
}       // end of loop over blocks
PrintEndOfRunStatement();
```

New lines

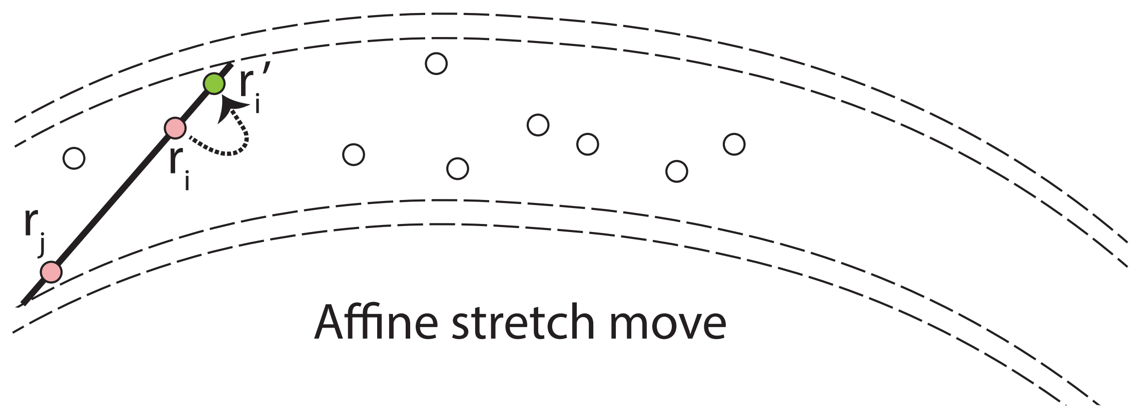
Prefactor requires discussion

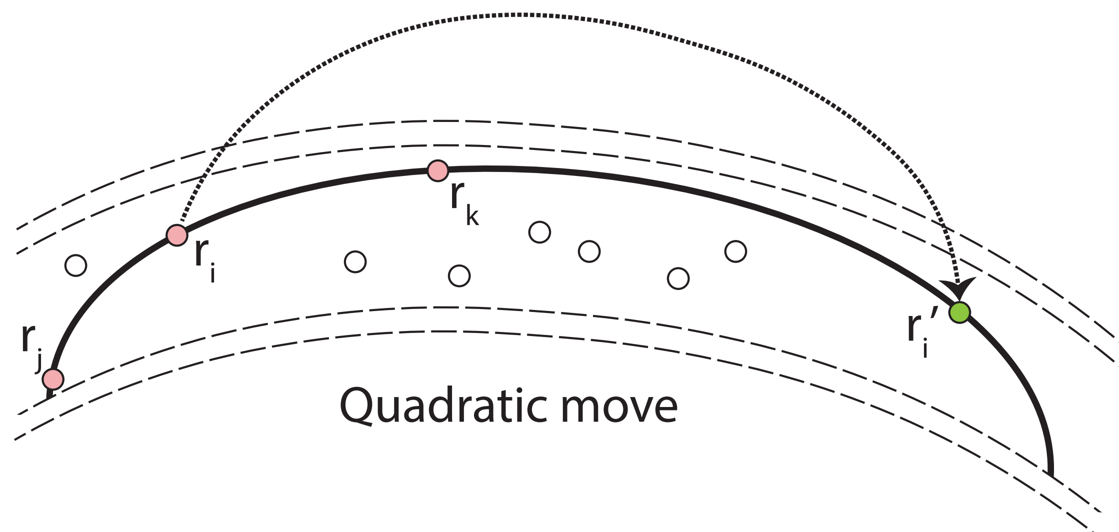
Our Test Case: A Ring Potential

Test case: Ring potential

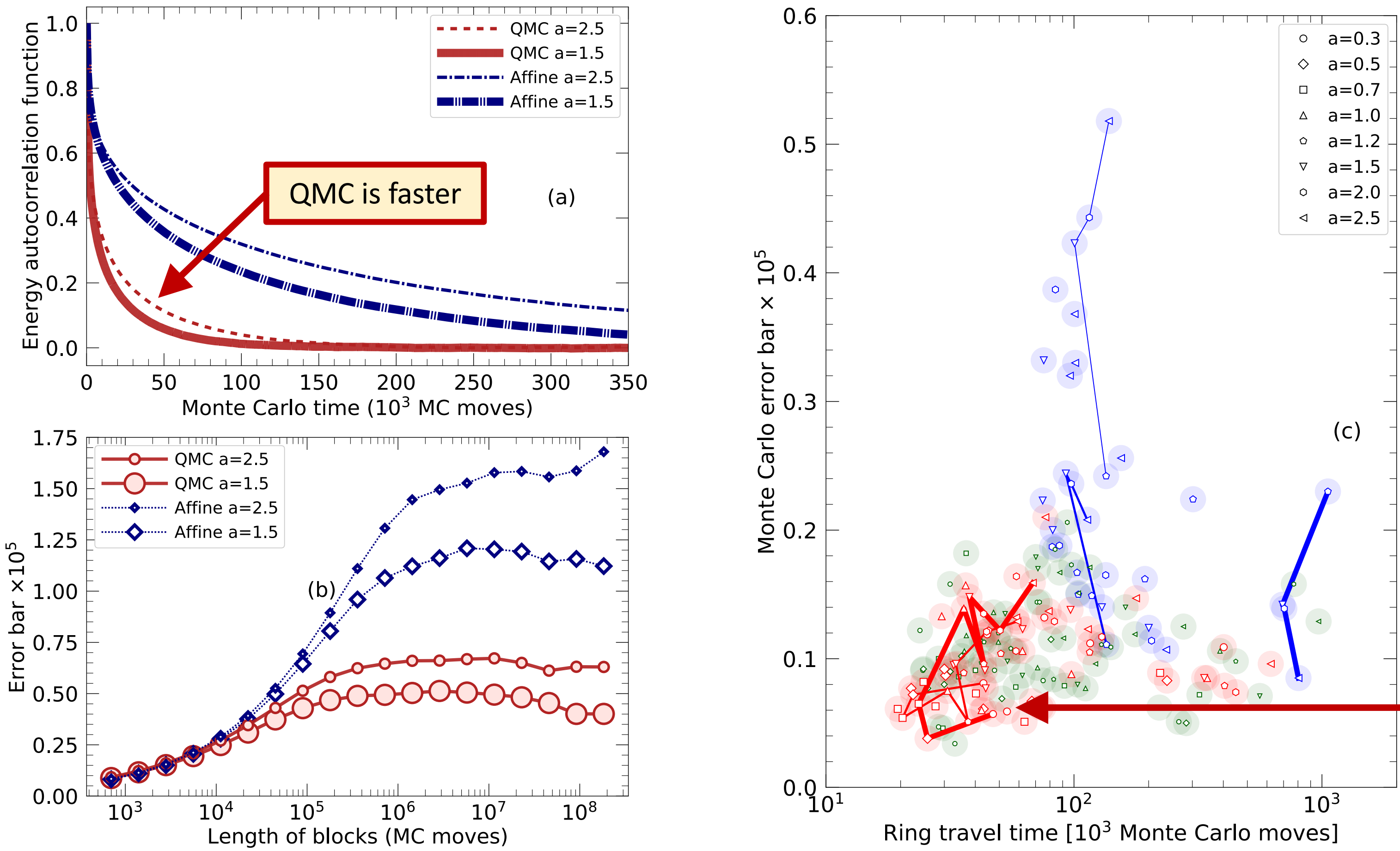


$$V(\vec{r}) = (2m)^{2m}\left[(\rho - R)^{2m} + \sum_{i=3}^{N} r_i^{2m}\right] - Cr_1$$





Two Performance Criteria: Autocorrelation & Travel Time



- Quadratic moves yield smaller error bars and shorter travel times.
- But don't use too many walkers.
- We suggest N_W=2D…3D

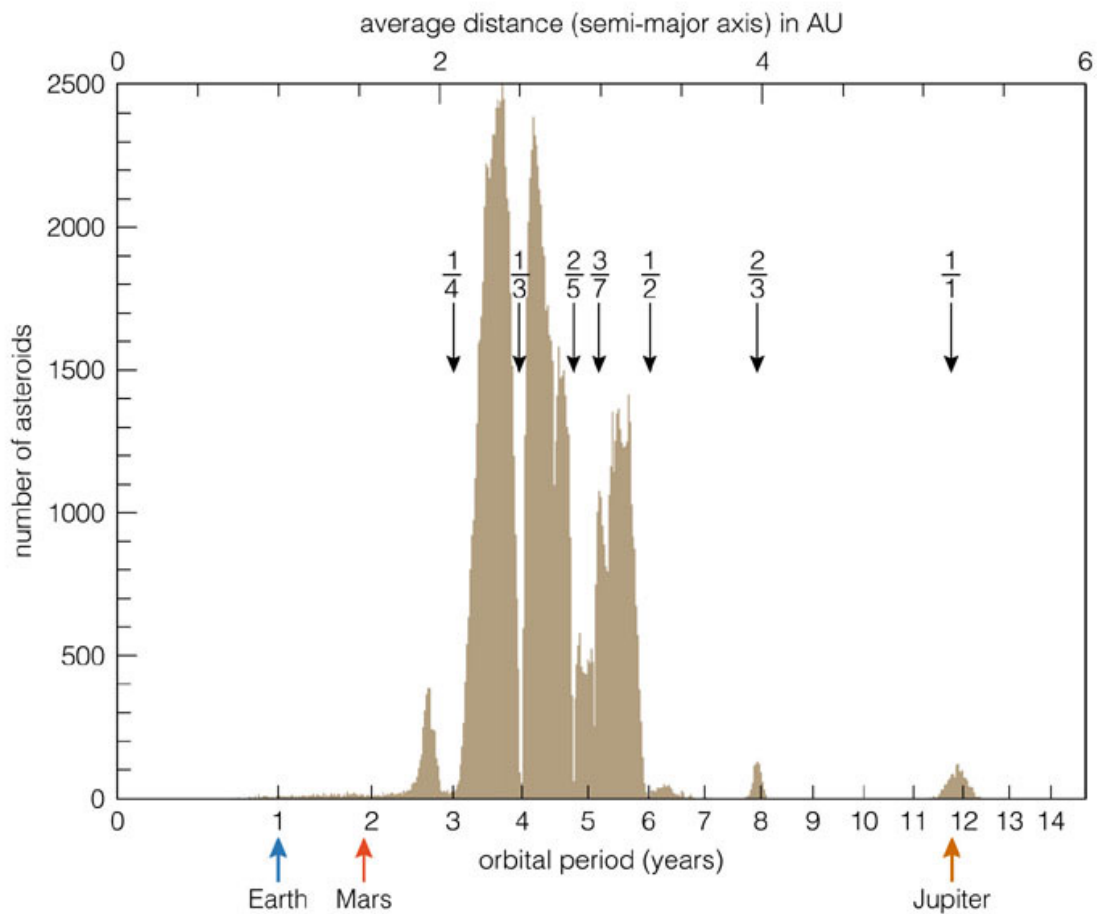
Planets in our Solar System (not to scale)

2. Resonances

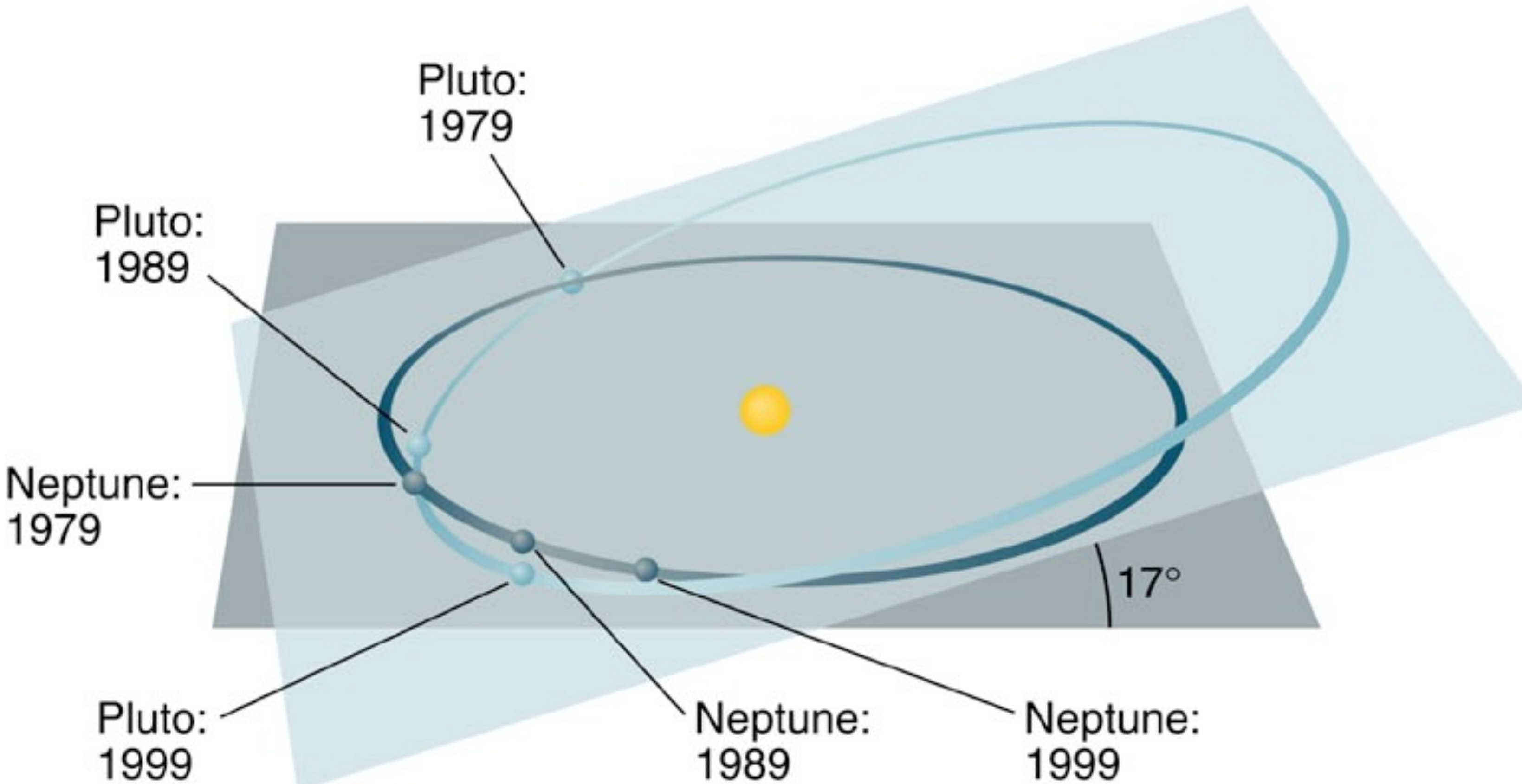
(occur in many different scenarios)

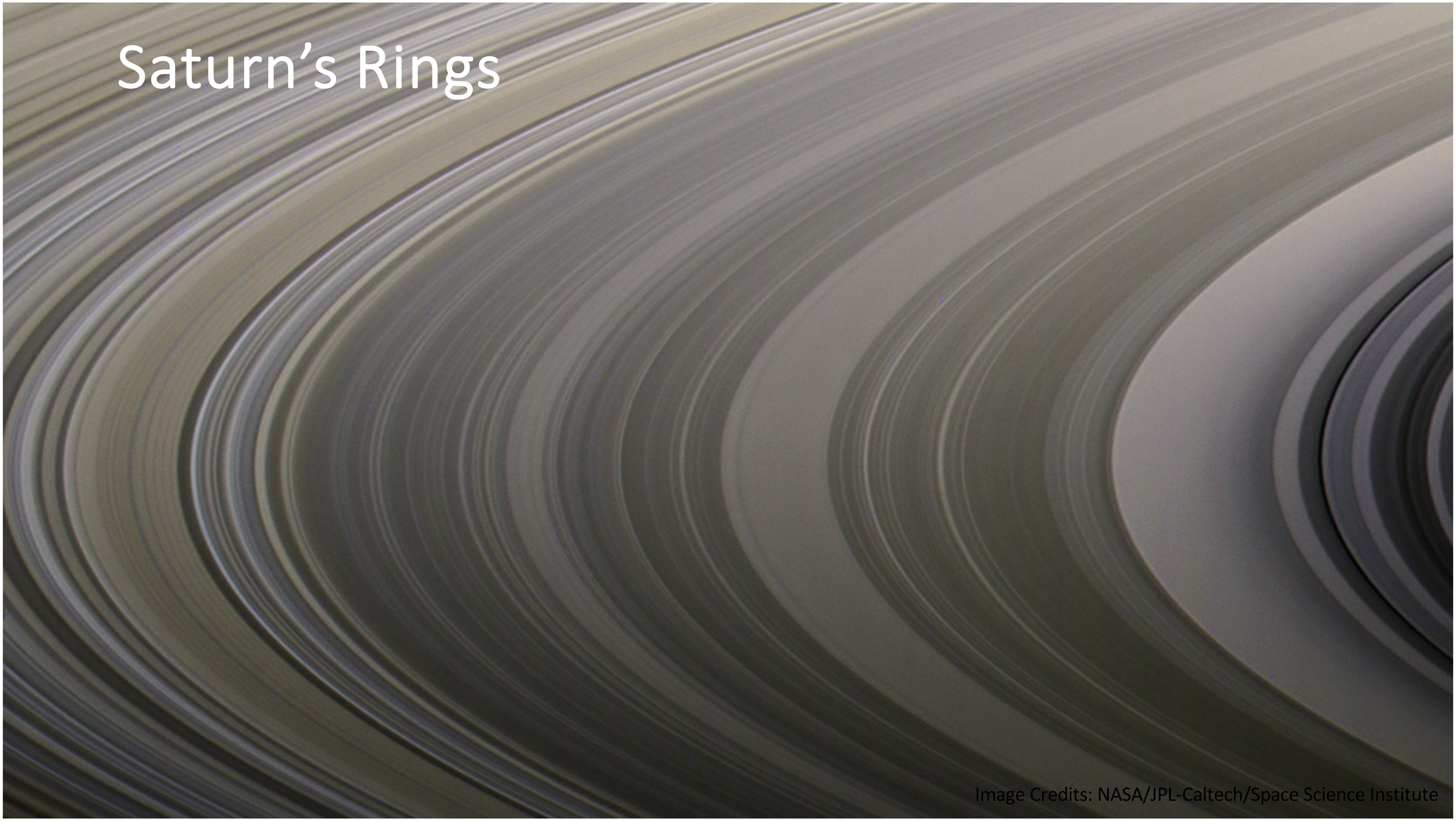
The animation represents a map of the increased count of all known asteroids in the solar system between Jan. 1, 1999 and Jan. 31, 2018. (Blue represents near-Earth asteroids. Orange represents main-belt asteroids between the orbits of Mars and Jupiter.)

Gravitational interactions (orbital resonances) with Jupiter have ejected asteroids on certain orbits.

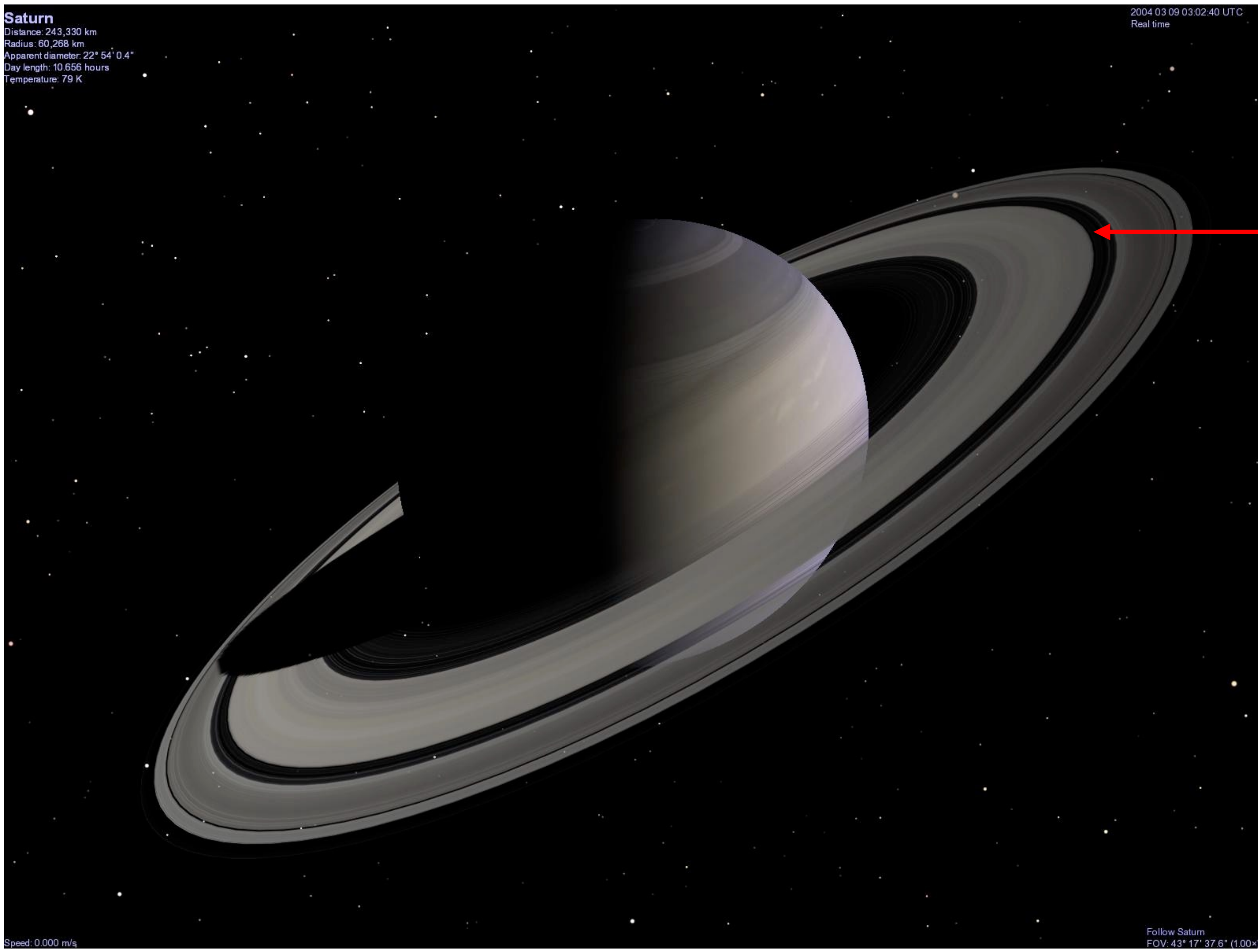


Pluto's & Neptune's orbit intersect but Pluto will never collide with Neptune because of a 3:2 orbital resonance.

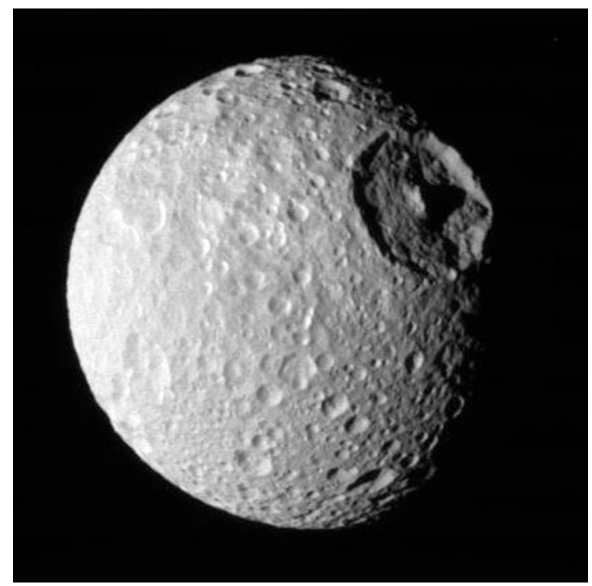



Saturn’s Rings
Image Credits: NASA/JPL-Caltech/Space Science Institute

Orbital resonance with moon Mimas made Cassini division



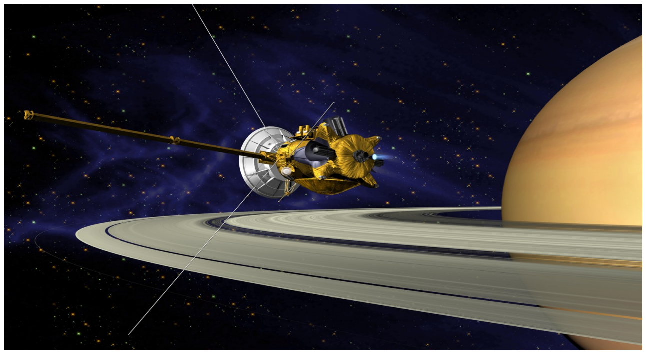
Cassini division in a 2:1 resonance with Mimas



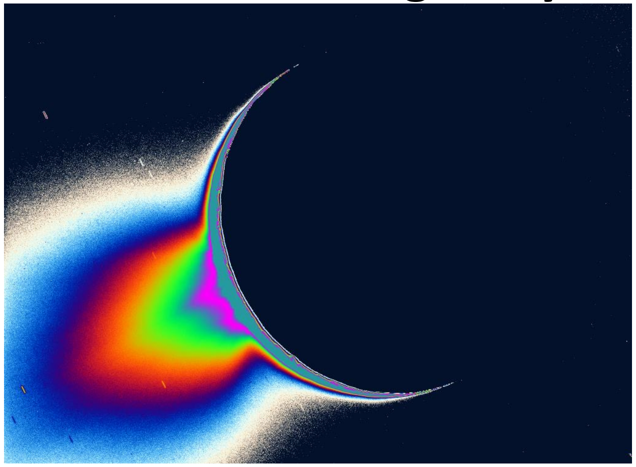
3. How did Saturn become the Lord of the Rings?

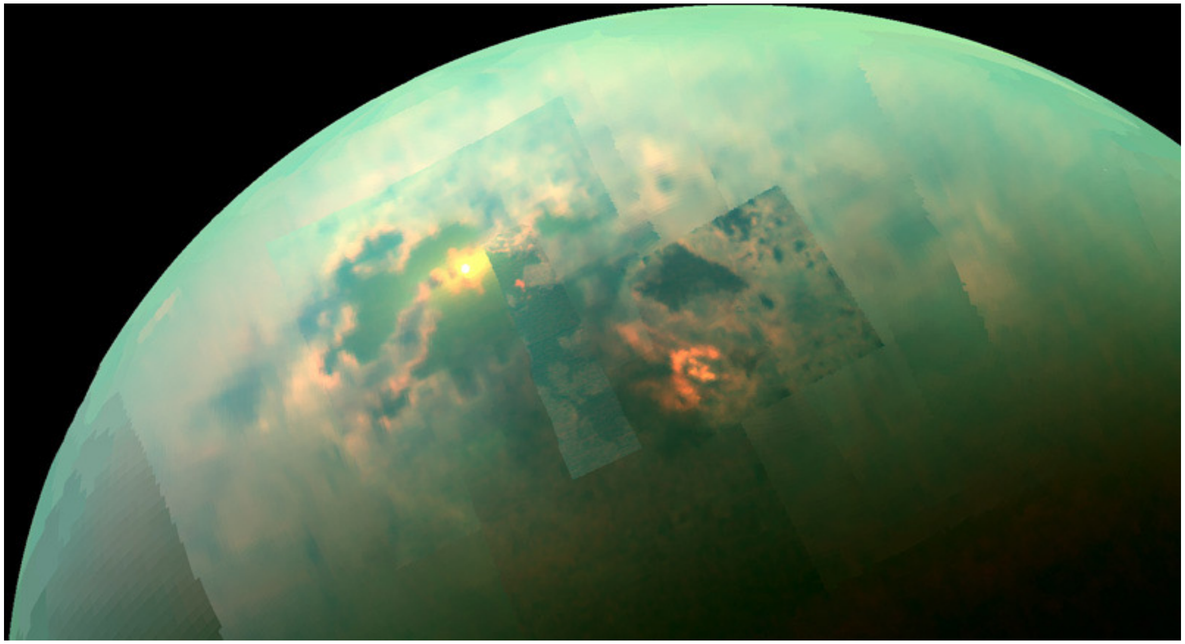
What is so Unusual About Planet Saturn?

Cassini Mission to Saturn



My contribution: Modeling Saturn's unusual gravity field

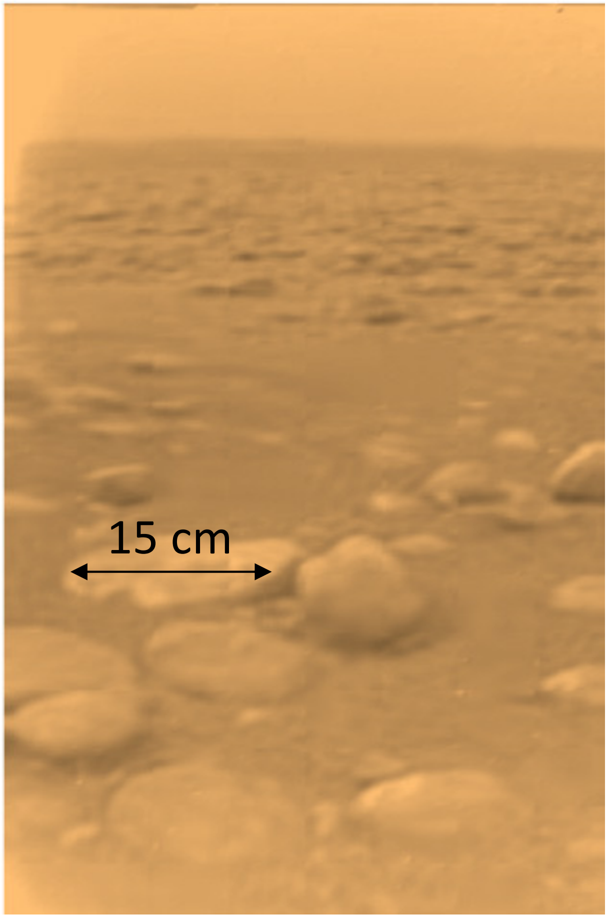


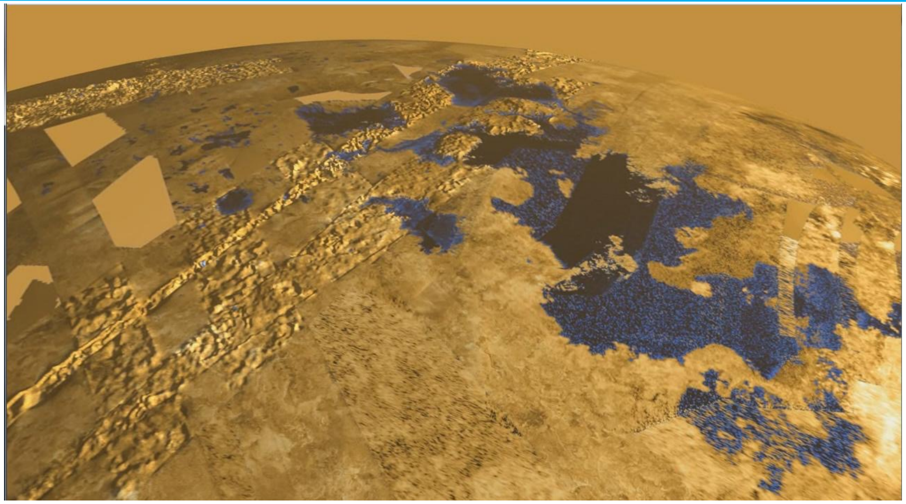


Mission Timeline:

- **Launch, October 1997**
- **Inserted into orbit around Saturn in July 2004**
- **Burned up in Saturn's atmosphere in 2017**

Cassini Mission to Saturn

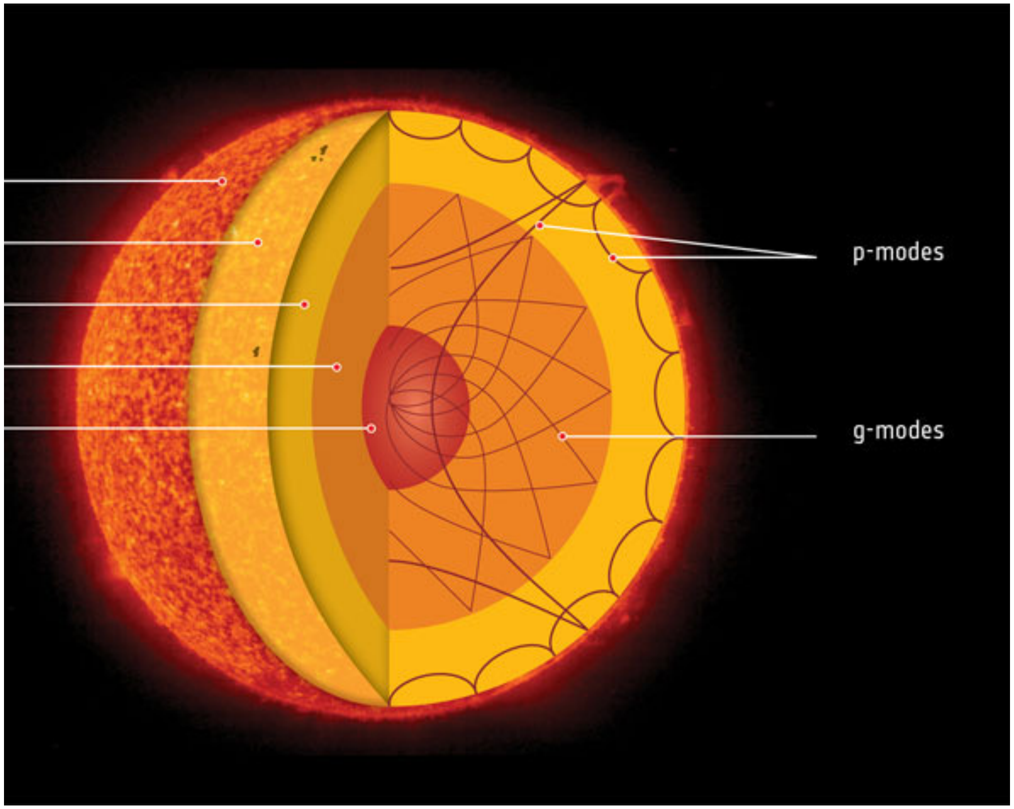




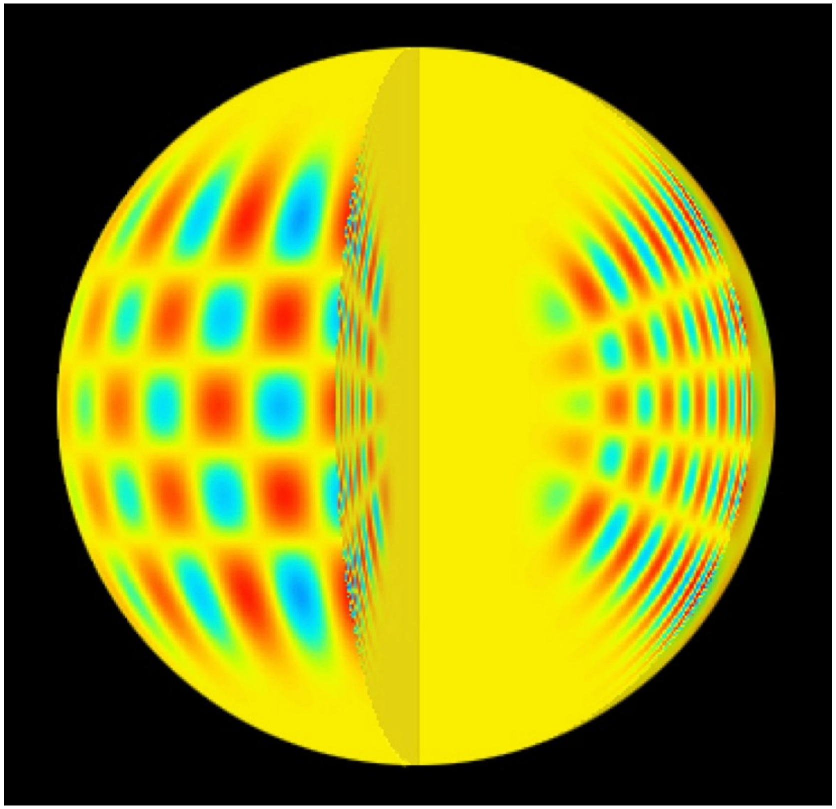
Mission Timeline:

- **Launch, October 1997**
- **Inserted into orbit around Saturn in July, 2004**
- **Huygens probe lands on Titan in January, 2005**
- **Dove inside Saturn's rings in August, 2017**
- **After 13 years in orbit it burned up in atmosphere in September, 2017**

Helioseismology

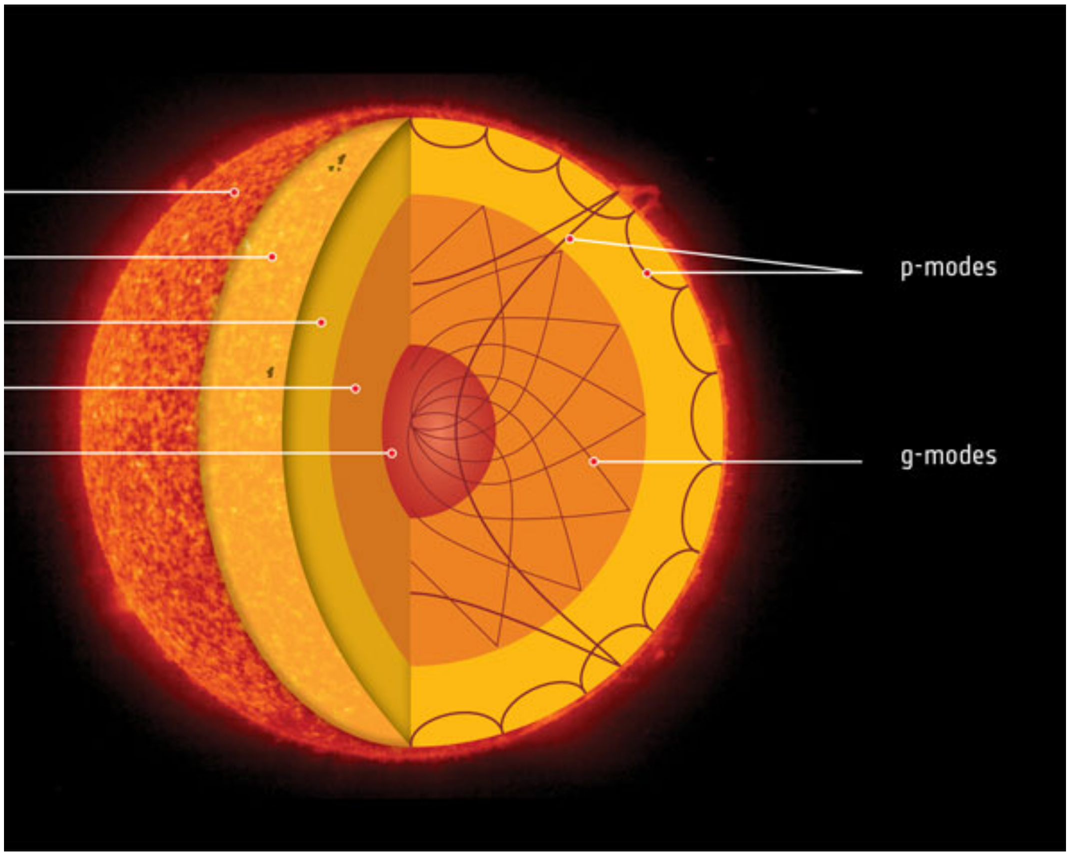


NASA SOHO

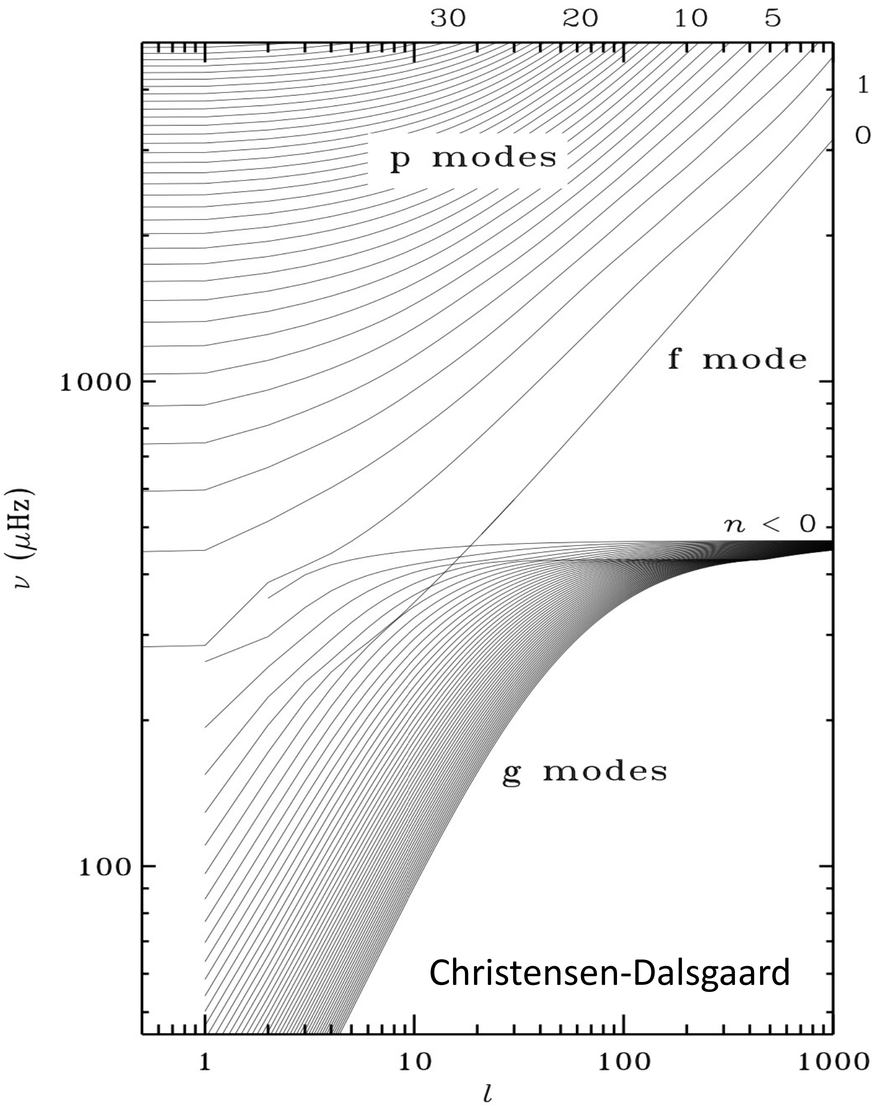


P modes

Helioseismology



NASA SOHO



Christensen-Dalsgaard

Helioseismology

P modes: "Pressure" waves

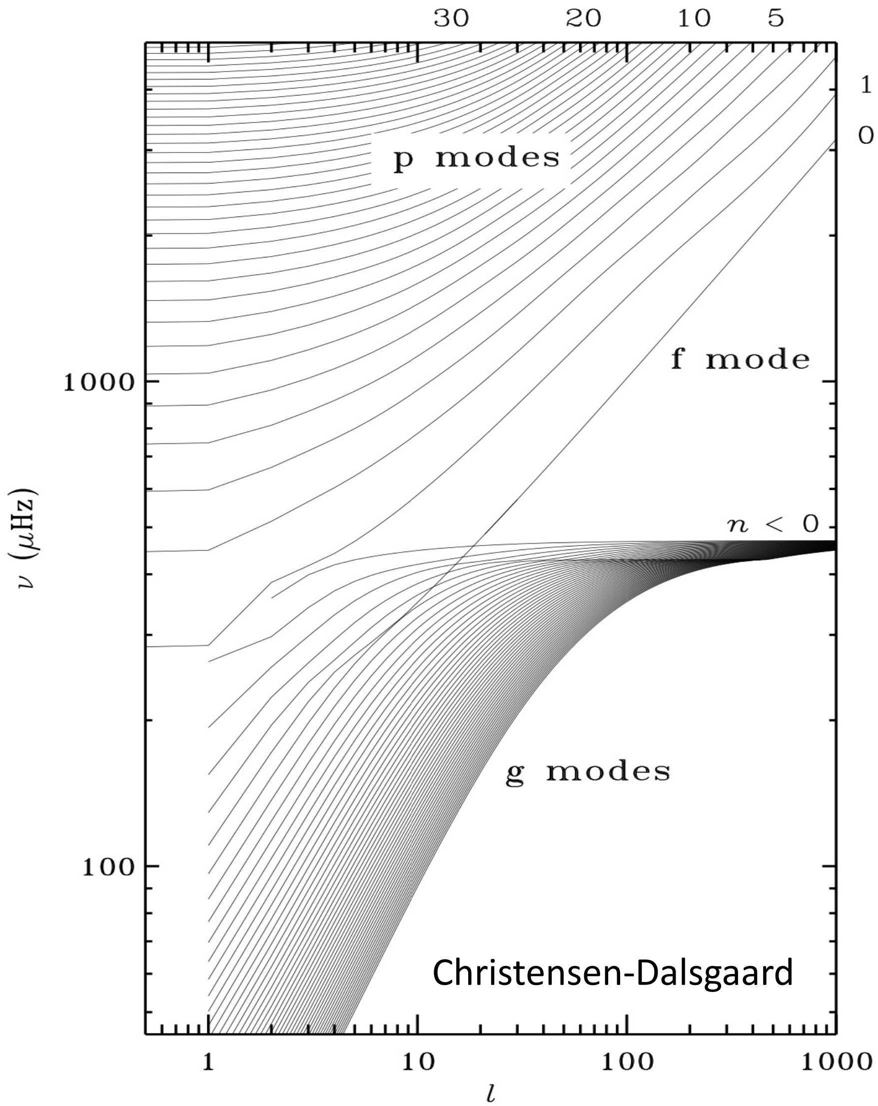
- Primary restoring force is pressure
- High frequency limit: acoustic waves

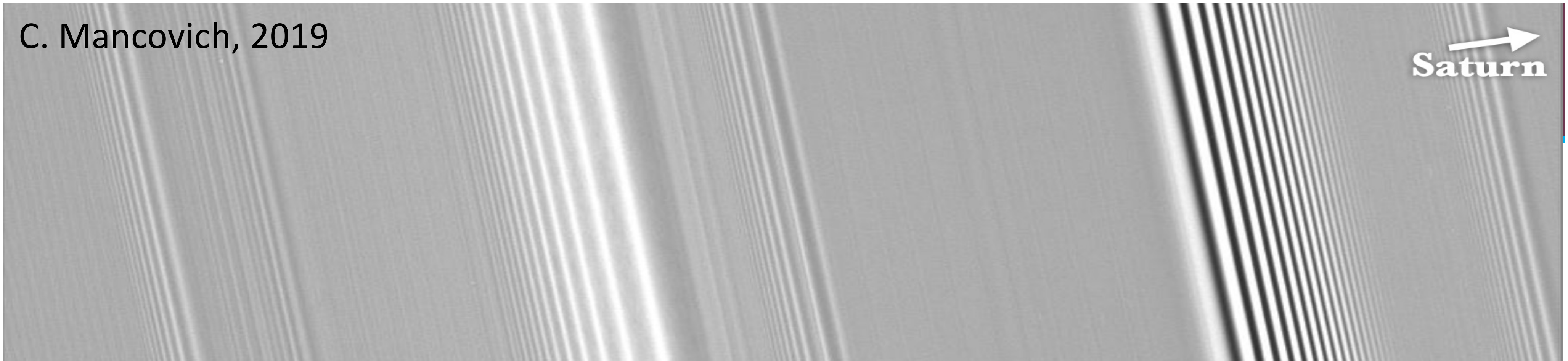
F modes: "Fundamental" modes

- Are the limit of p modes as radial order n goes to zero (long wavelength limit)
- Also known as surface gravity wave, no nodes in interior. Deforms like a soccer ball
- No compression involved.

G Modes: "Gravity" waves

- Low frequency waves
- Primary restoring force is buoyancy
- Requires stable stratification, no convection





Waves at satellite resonances are well known.

Lindblad resonance:
eccentricity; spiral density waves

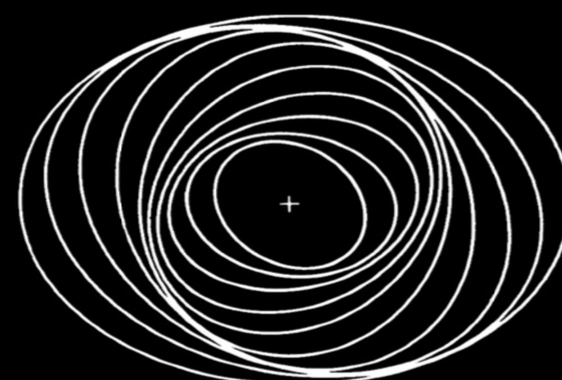
$$|m|(n - \Omega_{\rm pat}) = \kappa$$

azimuthal wavenumber

mean motion

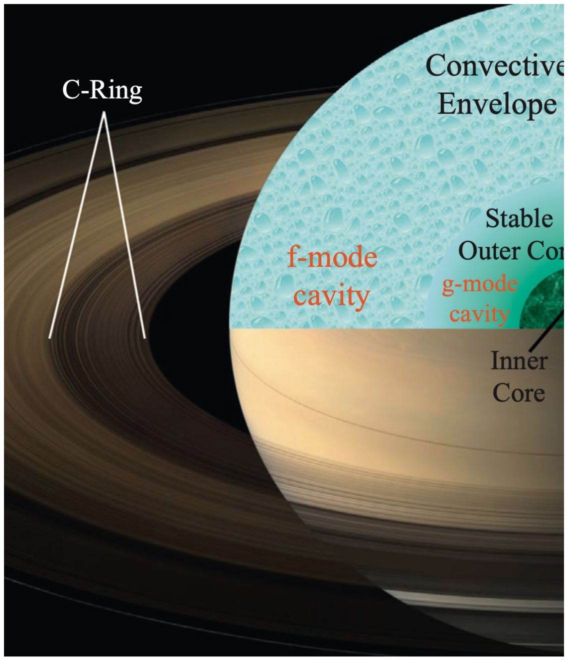
frequency of perturbing pattern

epicyclic frequency

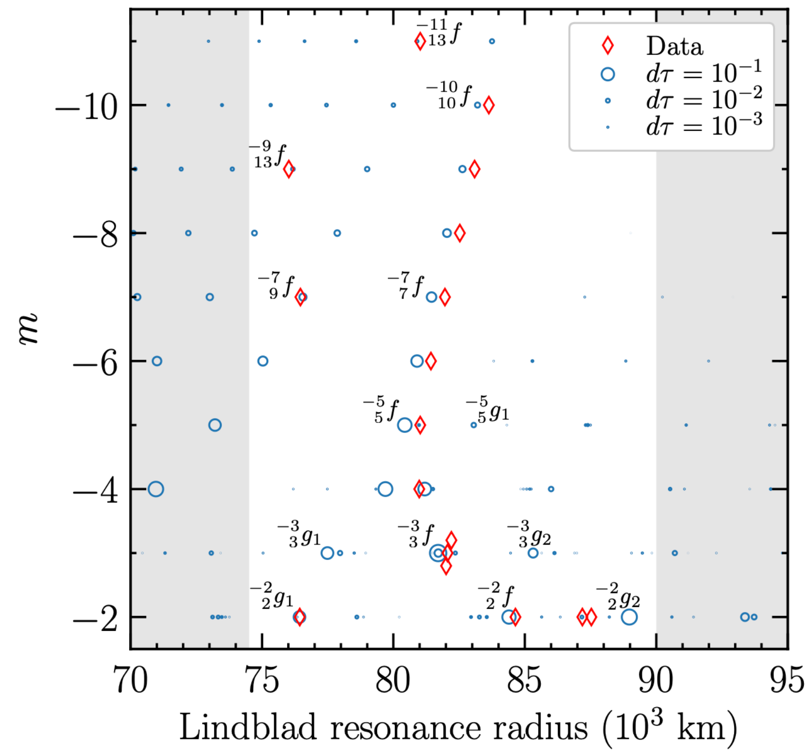


e.g.,
Goldreich & Tremaine 1979
Cuzzi, Lissauer, & Shu 1981
Shu 1984

Splitting of f-Modes implies Stably Stratified Layer

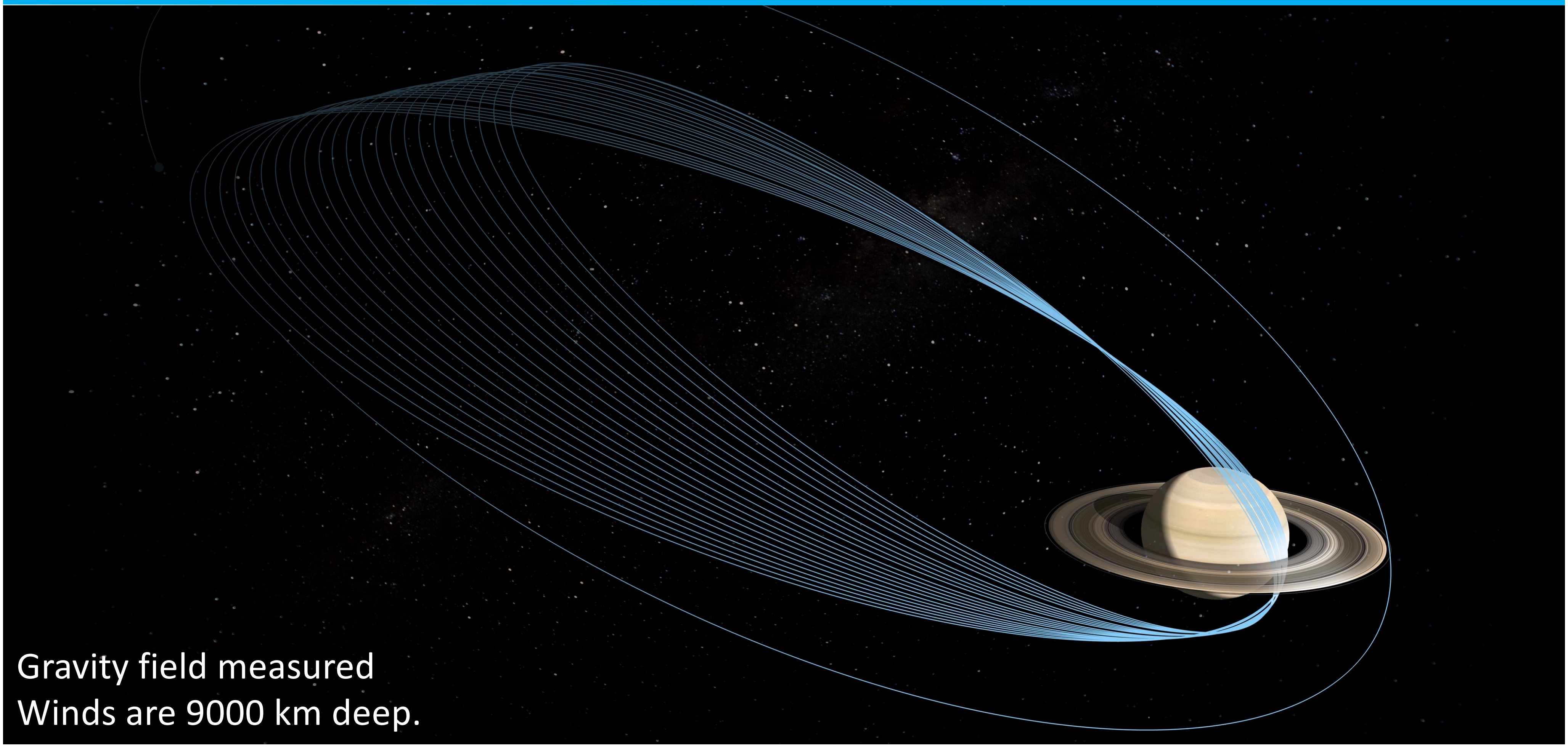


Fuller (2014)



Fuller & Mankovich (2022): Coupling f and g modes can explain observed splitting of f modes. g modes are caused by stably stratified layer with a nonzero Brunt-Väisälä-Frequency, N. Stratification may be caused by helium rain or core erosion. Unclear which one dominates.

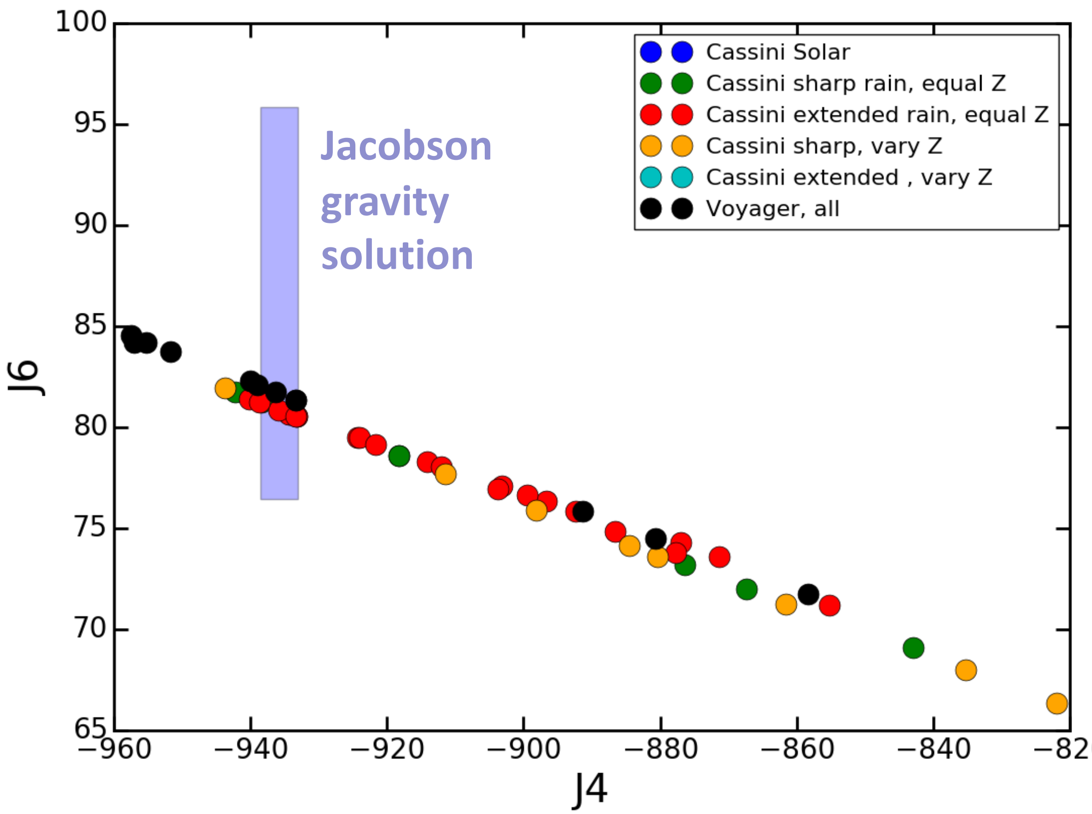
What is so unusual about Planet Saturn?



Preliminary Models Constructed Without Cassini Data

Models match mass, radius, and J_2 from Jacobson solution. All J_{2n} multiplied by 10^6.

	Range of Model Predictions assuming Uniform Rotation	
J_4	-938.619	-933.187
J_6	80.532	81.737
J_8	-8.950	-8.680
J_{10}	1.076	1.129
J_{12}	-0.157	-0.147
J_{14}	0.0215	0.0234



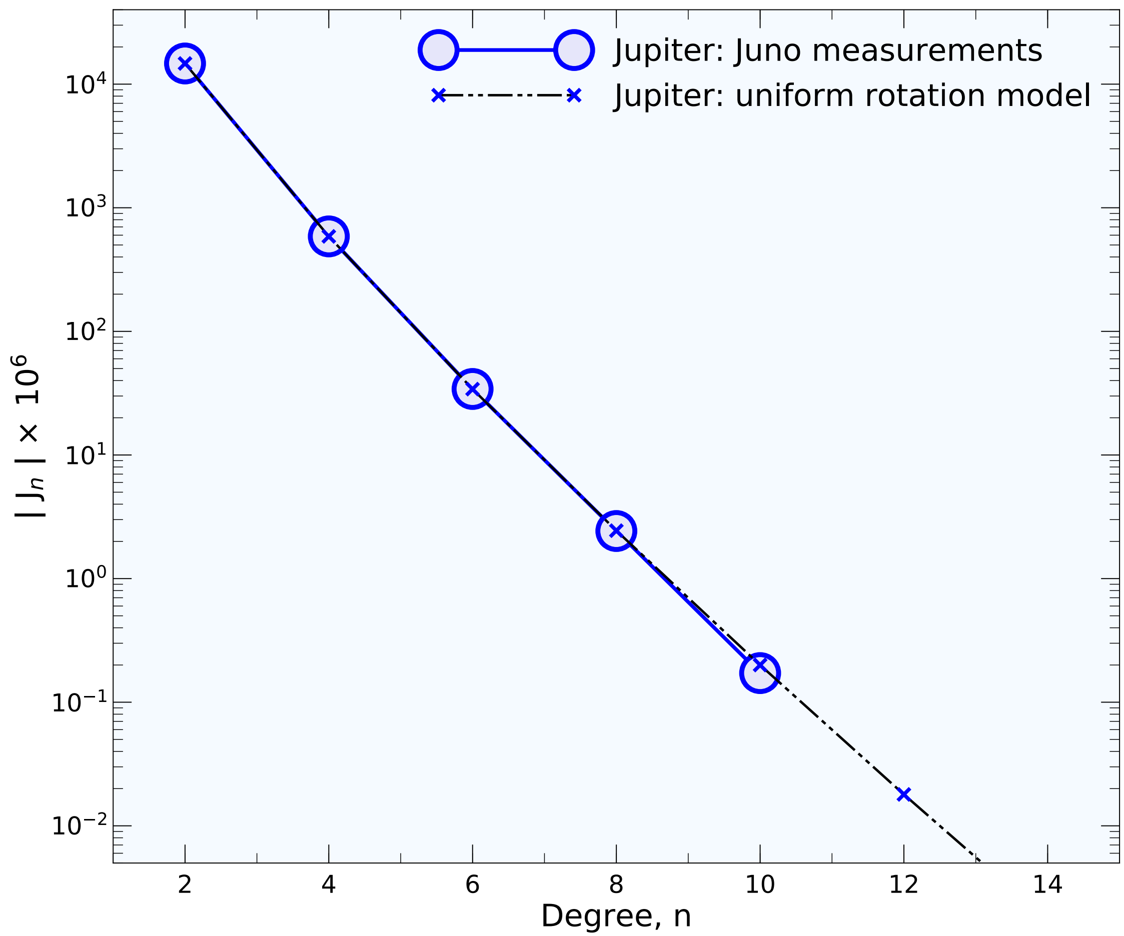
Puzzling Gravity Data with unusually large J_6, J_8 and J_{10}

Models match mass, radius, and J_2 from Jacobson solution. All J_{2n} multiplied by 10^6.

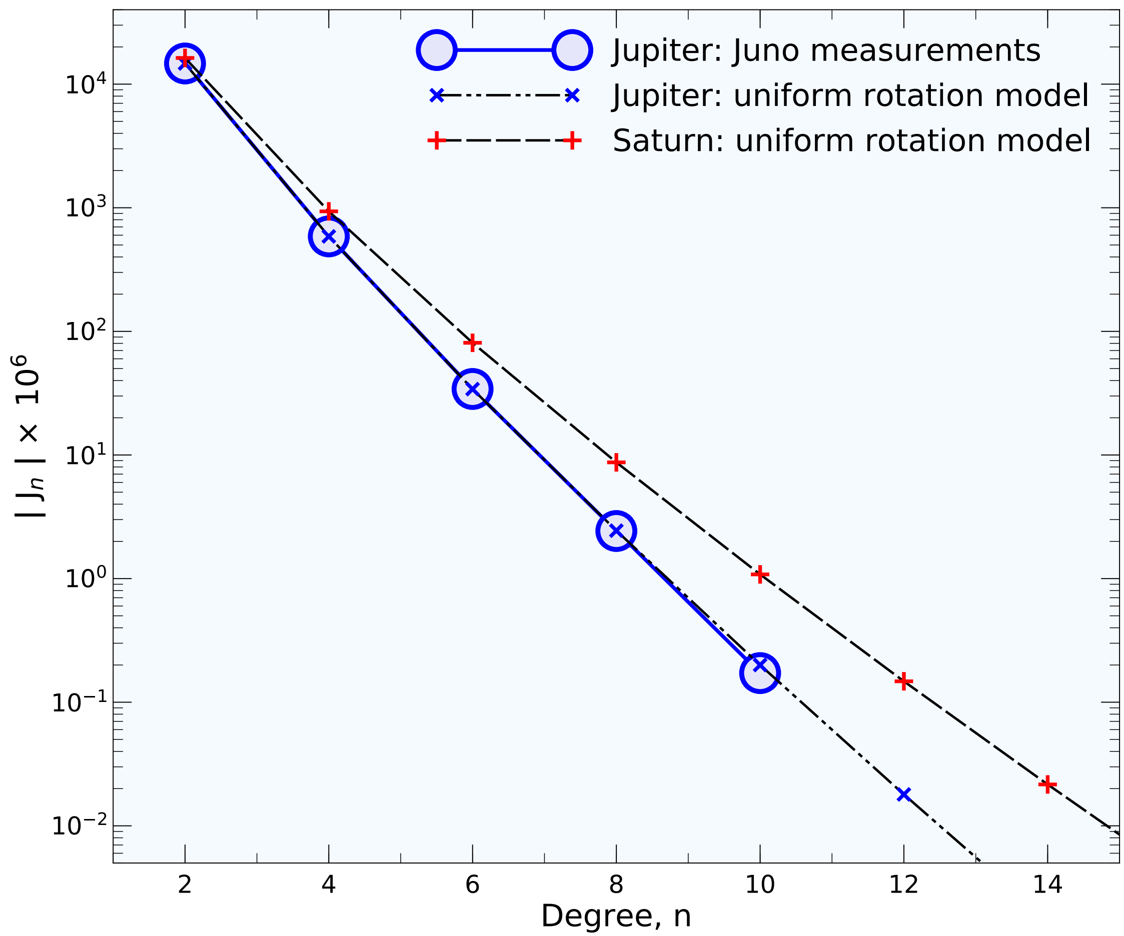
	Range of Model Predictions assuming Uniform Rotation		Cassini Data Rev 273+274 solution
J_4	-938.619	-933.187	-934.5792
J_6	80.532	81.737	**86.5215**
J_8	-8.950	-8.680	**-14.3704**
J_{10}	1.076	1.129	**4.9910**
J_{12}	-0.157	-0.147	-0.6670
J_{14}	0.0215	0.0234	0.5332

No model could match the observations.

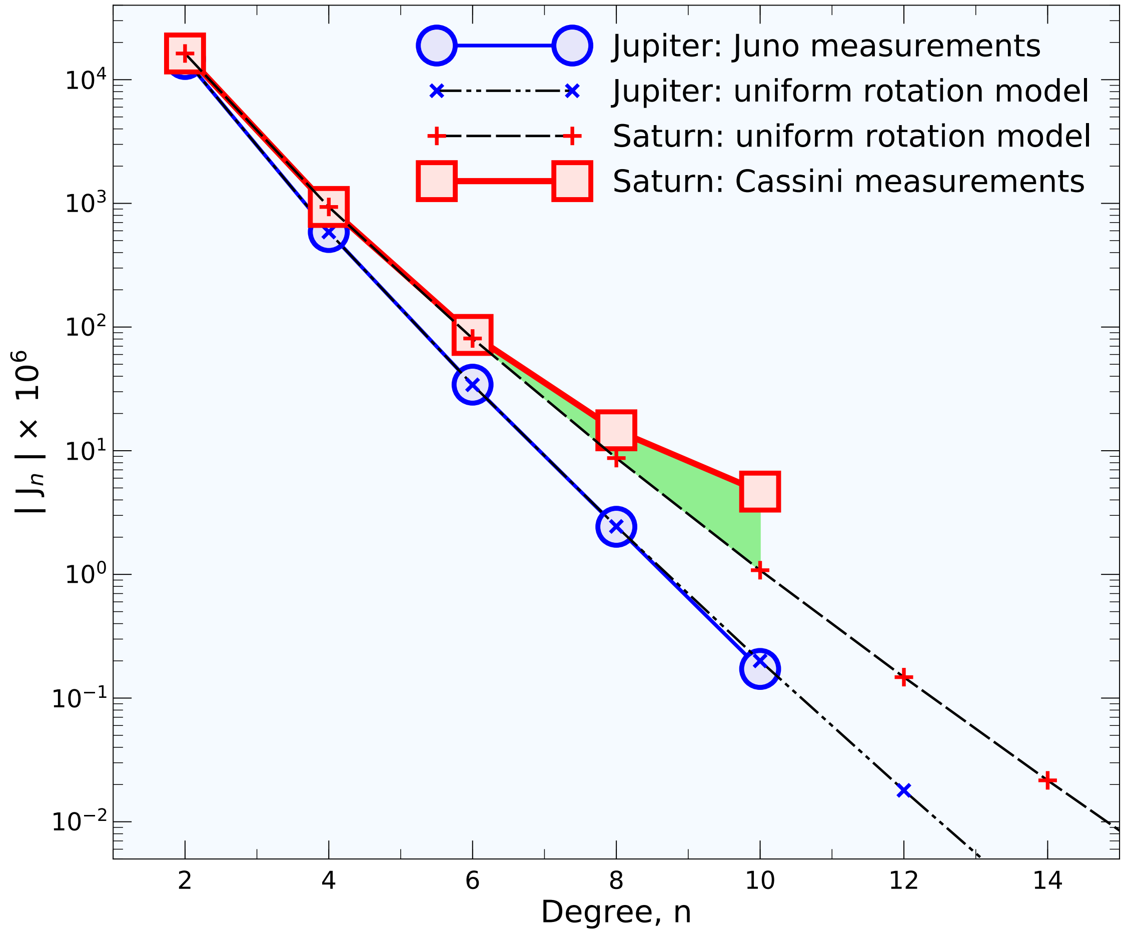
The Even Coefficients of Jupiter's Gravity Field ,J_n, can be matched with Uniform Rotation Models



The Even Coefficients of Jupiter's Gravity Field ,J_n, can be matched with Uniform Rotation Models

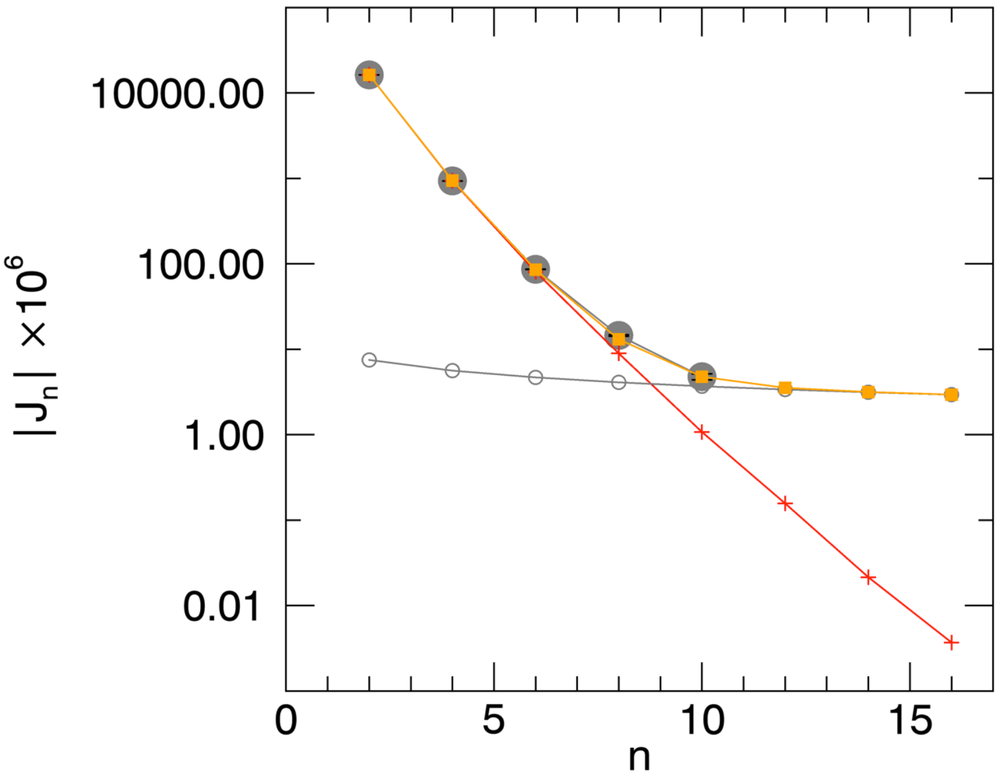


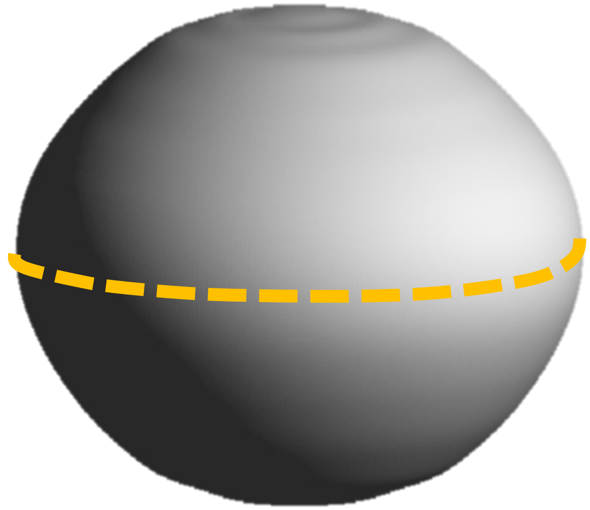
Saturn's Gravity is highly unusual, cannot be matched with Uniform Rotation Models



L. Iess,
B. Militzer,
et al.
Science
17 Jan 2019

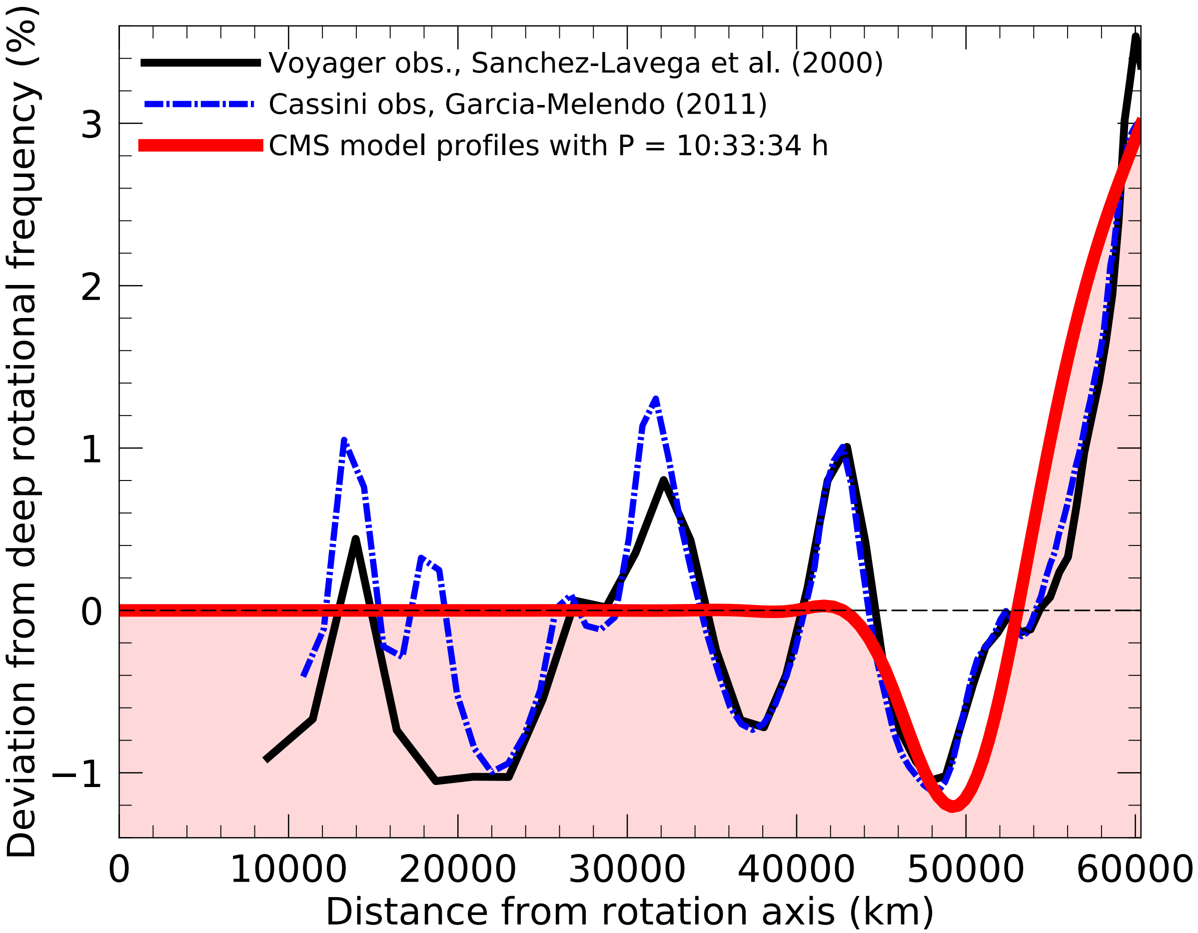
Bill Hubbard's Thin-Cord Model

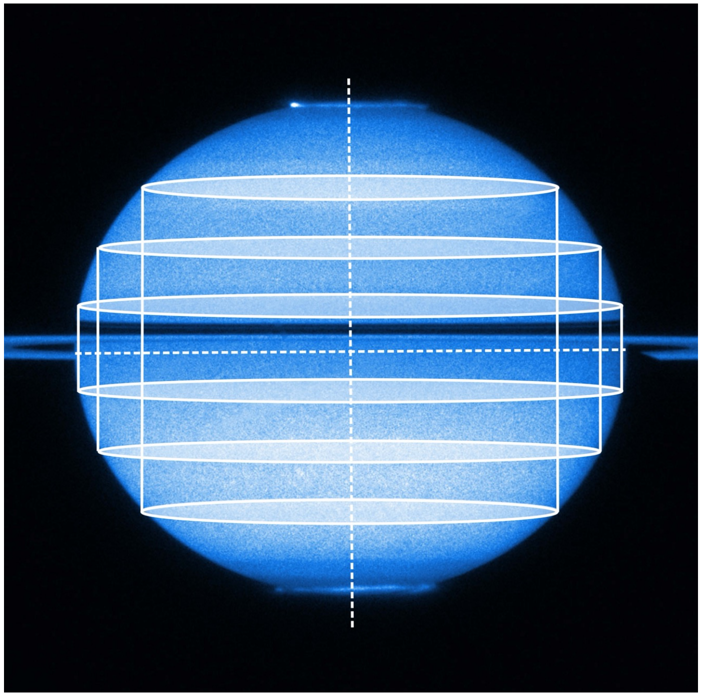




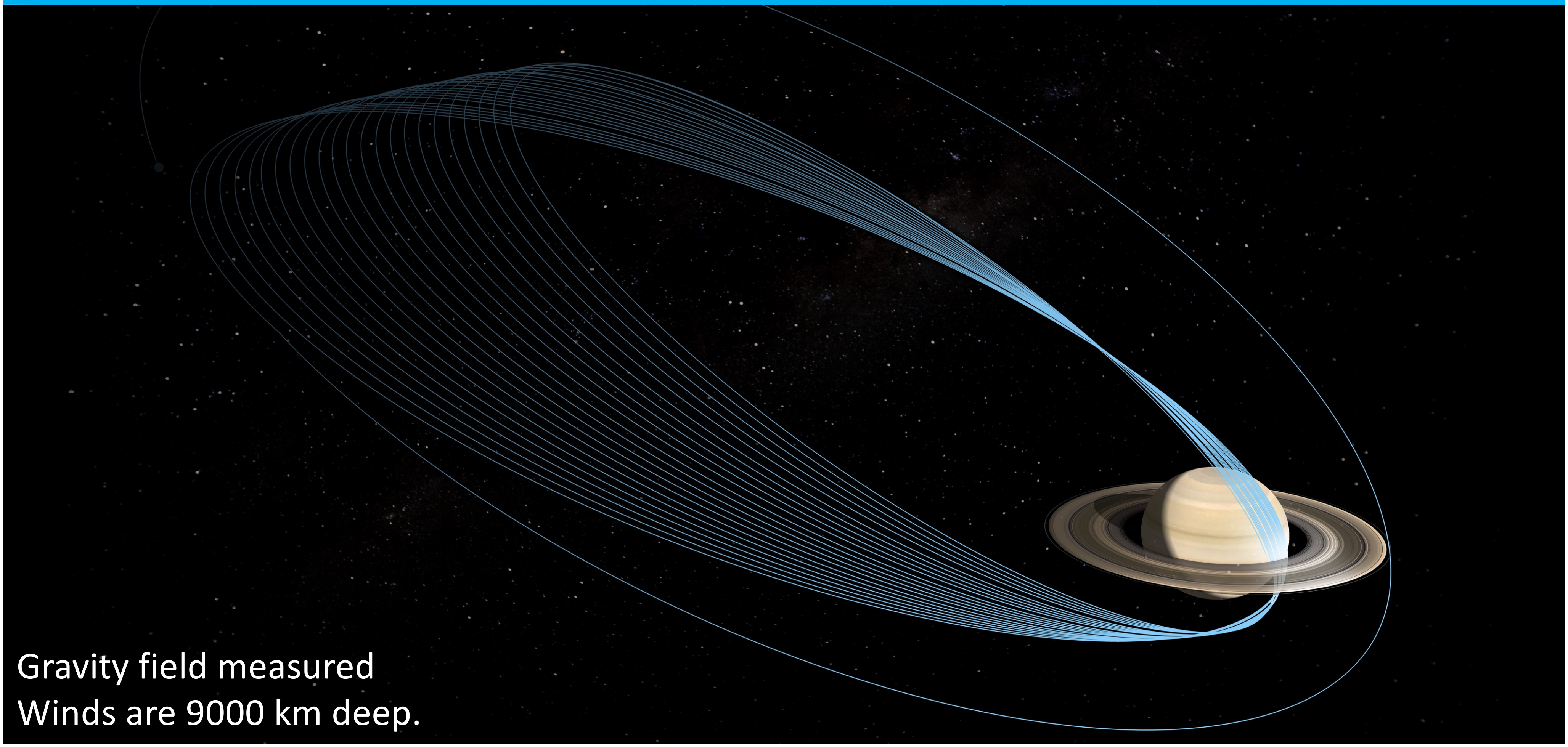
Thin cord of mass 1.5 x 10^{-5} M_0 added to uniform-rotation model (orange).

Gravity Data can be matched by assuming the observed winds are 10000km deep and rotation period P=10:33:34





What is so unusual about Planet Saturn?



First Determination of Saturn's Ring Mass from Gravity

Directly from the gravity signal, we determined a total mass of the main rings A, B and C =

0.41 $\pm$ 0.13 Mimas masses.

(2000 Mimas masses = 1 lunar mass)
(16000 Mimas masses = 1 Earth mass)

Iess, BM, et al.
Science (2019)

First Determination of Saturn's Ring Mass from Gravity

Directly from the gravity signal, we determined a total mass of the main rings A, B and C =

0.41 ± 0.13 Mimas masses.

(2000 Mimas masses = 1 lunar mass)
(16000 Mimas masses = 1 Earth mass)

Iess, BM, et al.
Science (2019)

An indication that the rings are young,
were formed only 10-100 million years ago.

Problem: We were unsure by what mechanism the rings formed recently!!

How did Saturn become the Lord of the Rings?

The Origin of Saturn's Obliquity and Young Rings

Saturn was tilted by a resonance with Neptune that was disrupted when a primordial satellite scattered and formed the rings.

Jack Wisdom,[1*] Rola Dbouk,[1] Burkhard Militzer,[2] William Hubbard,[3] Francis Nimmo,[4] Brynna Downey[4], Richard French[5]

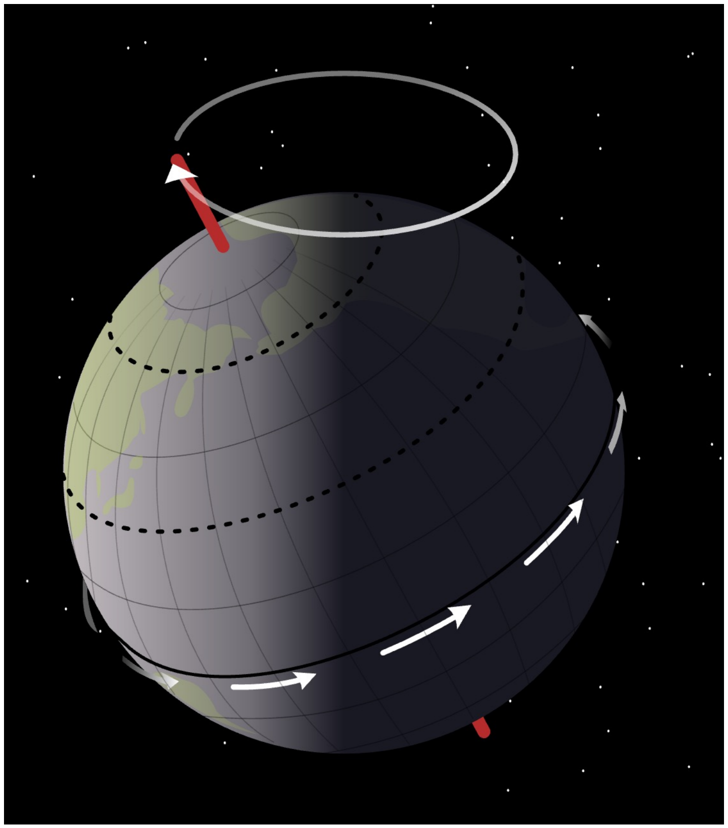
Science **377**, 1285–1289 (2022) 16 September 2022

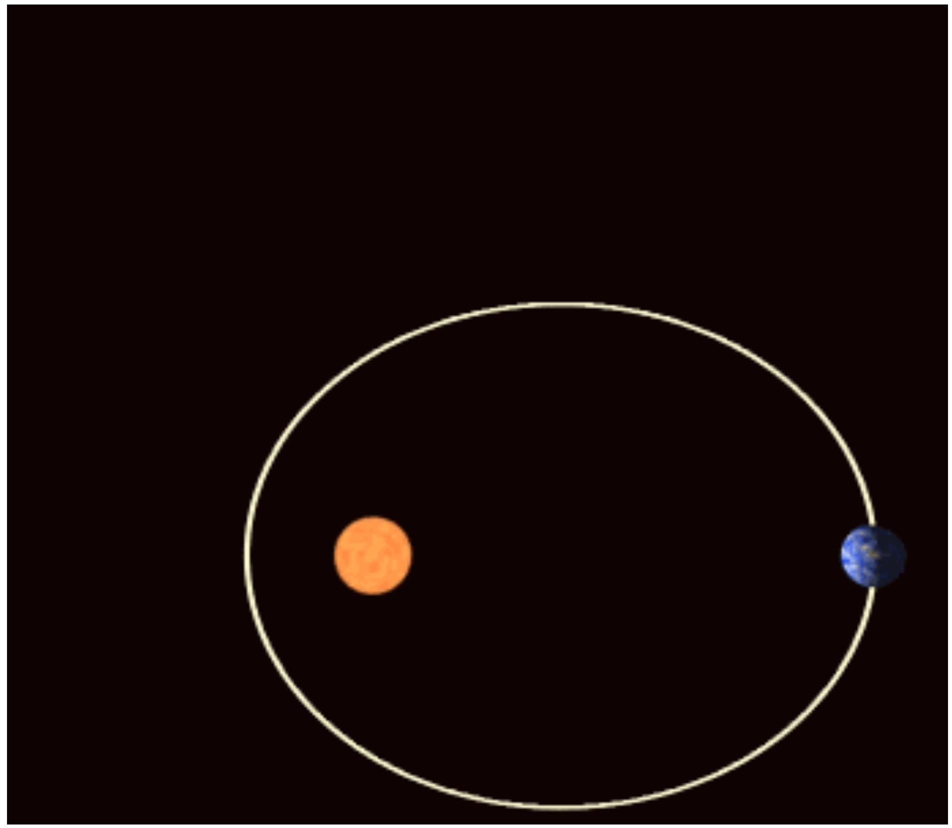
What is a Cassini State?

What is a Cassini State?

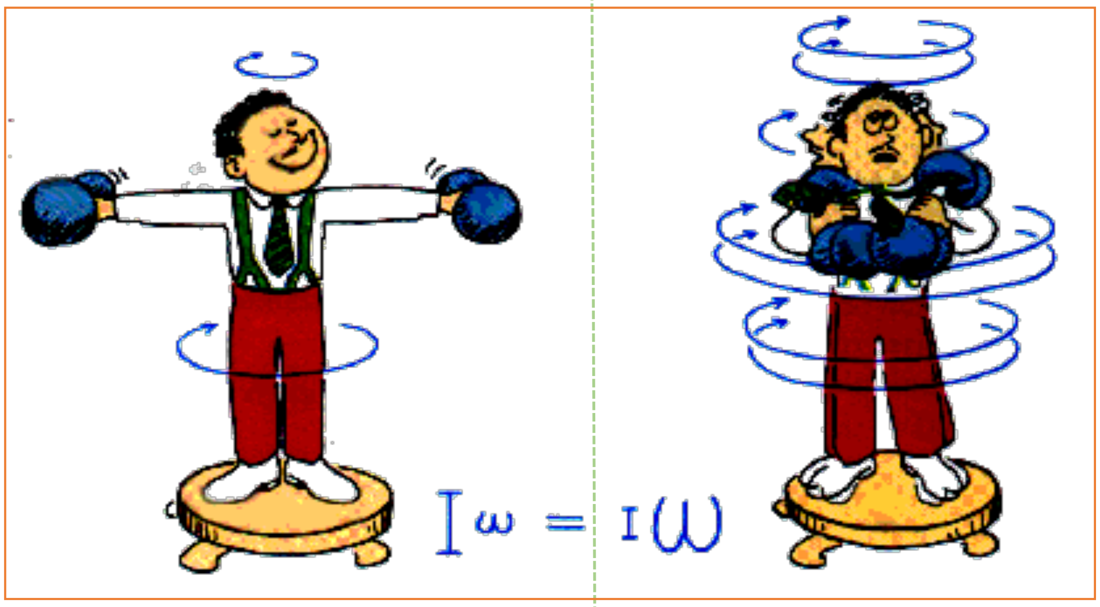
Answer: Spin-Orbit Coupling between Planets or Moons

What is a Cassini State?





Rotating Chair Demo



$$\vec{L} = m\,\vec{v} \times \vec{r}$$

$$L = I \times \omega$$

$$I = \iiint d^3r\, \rho(x, y, z)(x^2 + y^2)$$

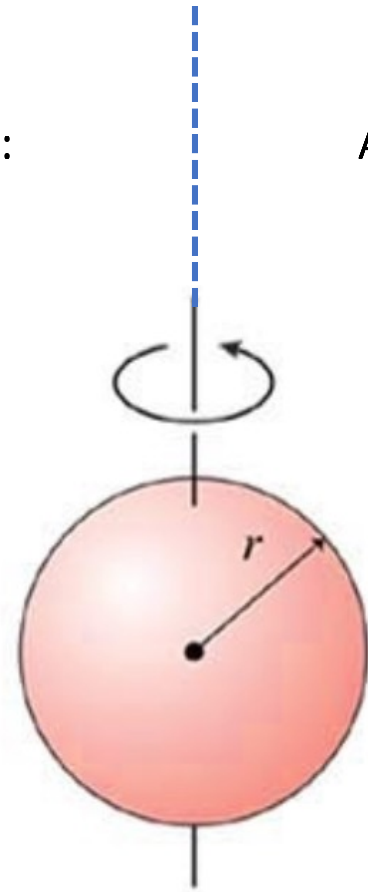
Before: Radius large
Angular velocity small

After: Radius small
Angular velocity large

Rotating Solid Sphere of Uniform Density

Moment of inertia
of a sphere of uniform density:

$$\mathrm{MoI} \equiv \frac{C}{MR_e^2} = \frac{2}{5}$$



Angular momentum
Assuming a uniform angular velocity

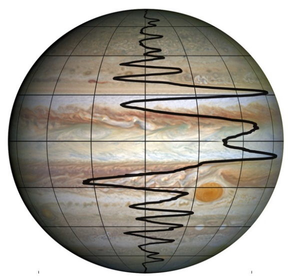
$$\mathrm{J} = \mathrm{I} * \omega \equiv \mathrm{C} * \omega$$

Moment of Inertia vs Angular Momentum

Moment of inertia definition:

$$\mathrm{MoI} \equiv \frac{C}{MR_e^2} = \frac{2\pi}{MR_e^2} \int_{-1}^{+1} d\mu \int_0^{R(\mu)} dr\, r^2\, l^2\, \rho(r,\mu)$$

(Cannot be measured!)



Angular momentum definition:

$$\mathcal{J}_{\mathrm{norm}}^{\mathrm{DR}} = \frac{2\pi\sqrt{q_{\mathrm{rot}}}}{MR_e^2} \int_{-1}^{+1} d\mu \int_0^{R(\mu)} dr\, r^2\, l^2\, \rho(r,\mu) \frac{v(r,\mu)}{\bar{v}(l)}$$

Effective MoI for differentially rotating planets:

$$\bar{C}^{\mathrm{DR}}/MR_e^2 = \mathcal{J}_{\mathrm{norm}}^{\mathrm{DR}}/\sqrt{q_{\mathrm{rot}}} \qquad q_{\mathrm{rot}} = \frac{\omega^2 R_e^3}{GM}$$

We call the difference the between the two the **direct effect** of differential rotation on moment of inertia. The direct is very small:

- For Jupiter we calculated +0.0015% (not measurable by Juno)
- For Saturn, we calculated −0.13%. (potentially measurable by Juno-type mission)

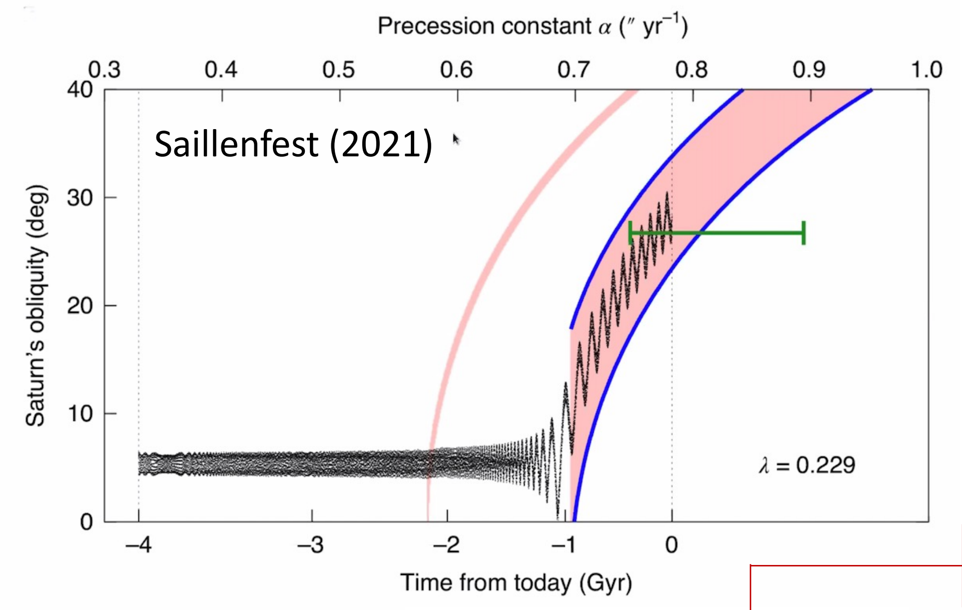
But there is a much larger indirect effect. See B. Militzer, W. B. Hubbard, *Planet. Sci. J.* **4** (2023) 95

Demo

Precession Demo 1+2

Spin-Orbit Coupling

This Saturn in Resonance with Neptune?
This depends on Saturn's Angular Momentum



Torques acting on Saturn

Precession constant: $\alpha = \frac{3}{2} n \frac{n}{\omega} \frac{J_2 + q}{\lambda + l}$

Torques acting on moons' orbits including Titan

Saturn's moment of inertia: C/MR^2

Moons' contribution to angular momentum

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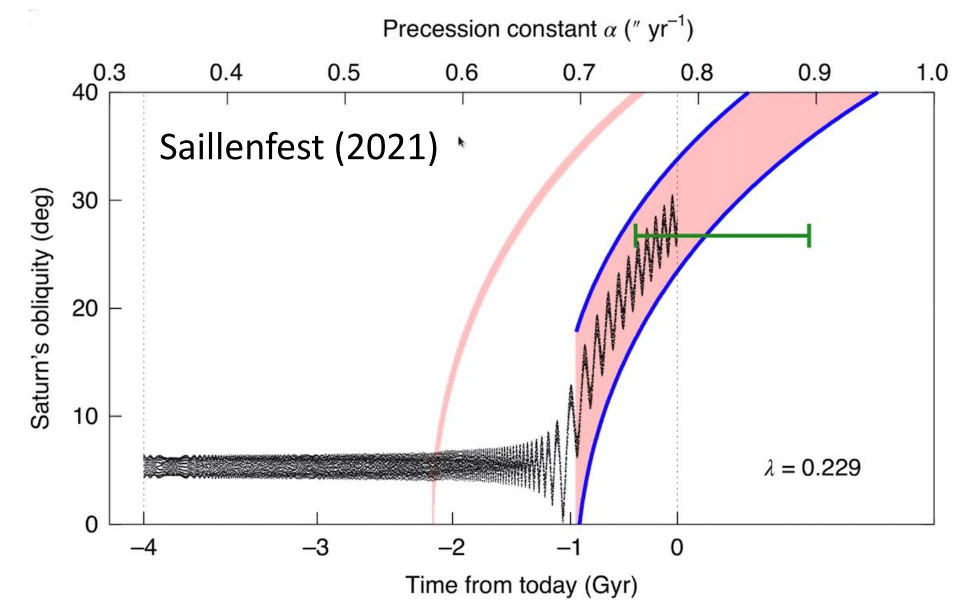
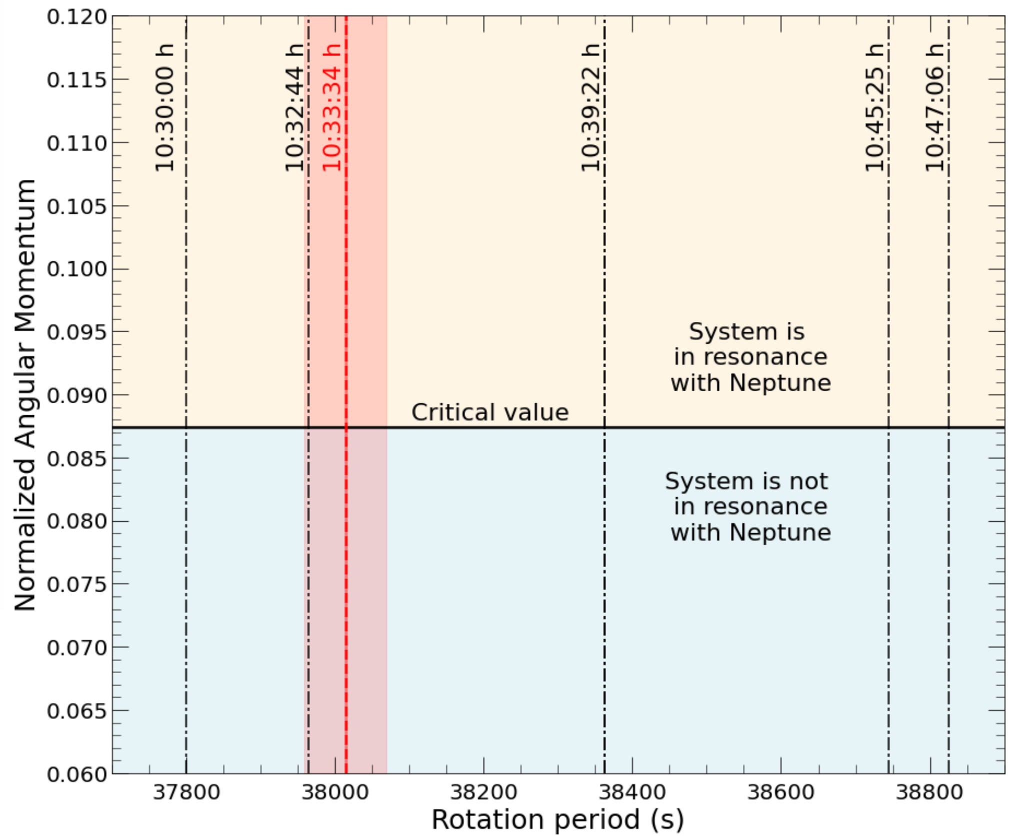
This depends on Saturn's Angular Momentum





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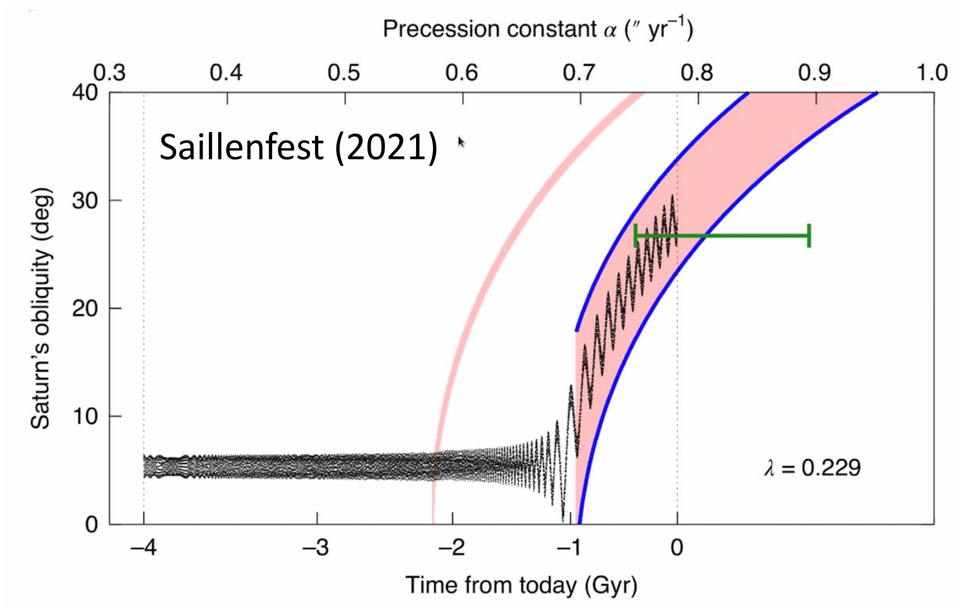

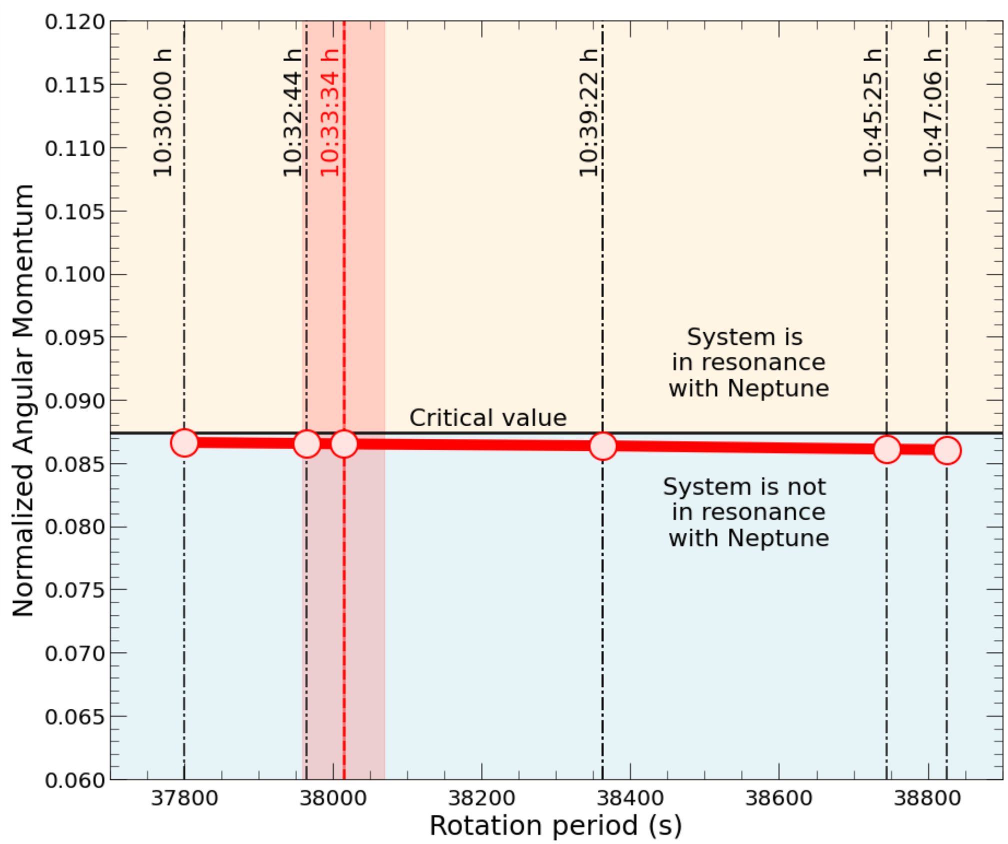

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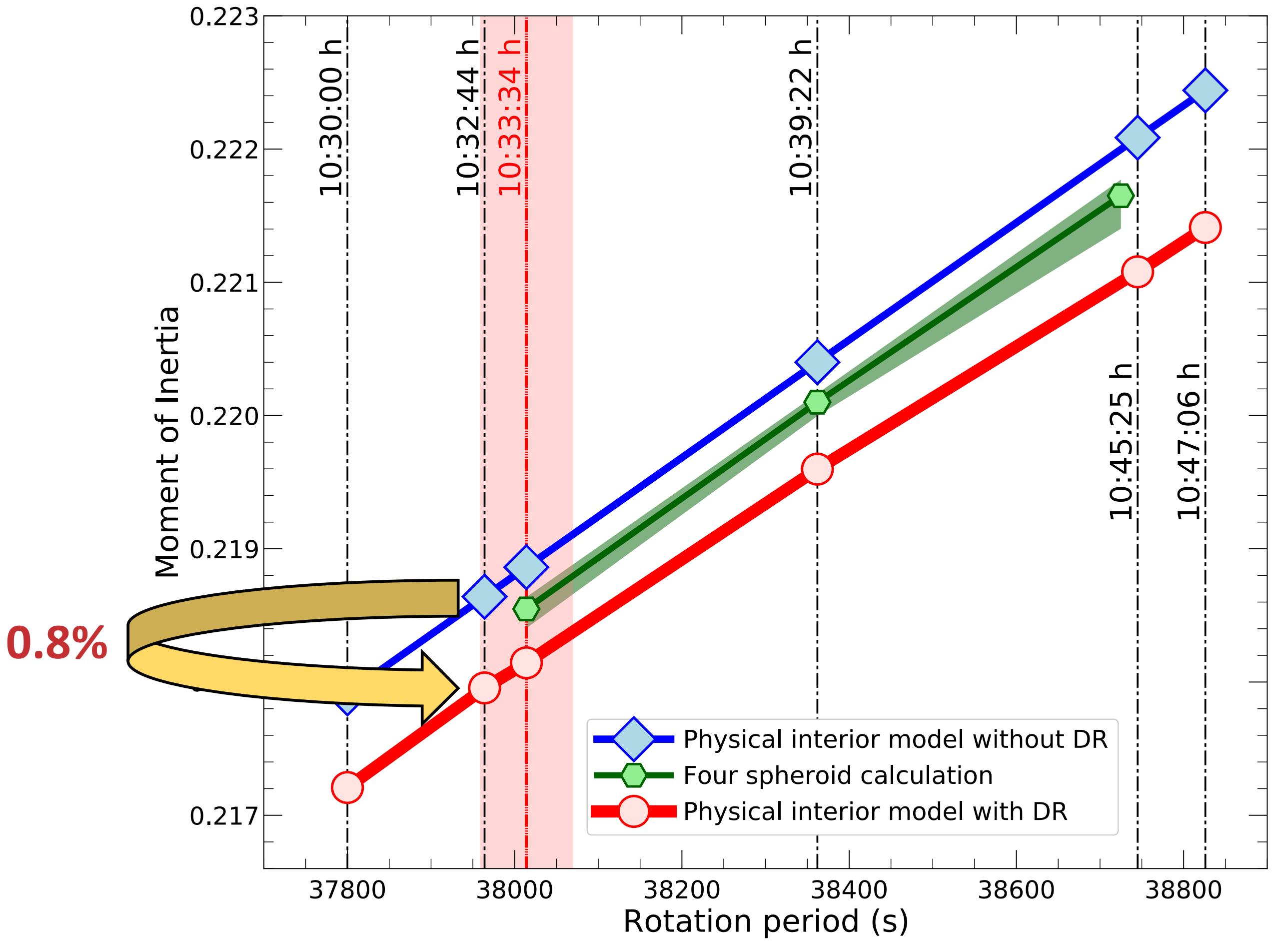
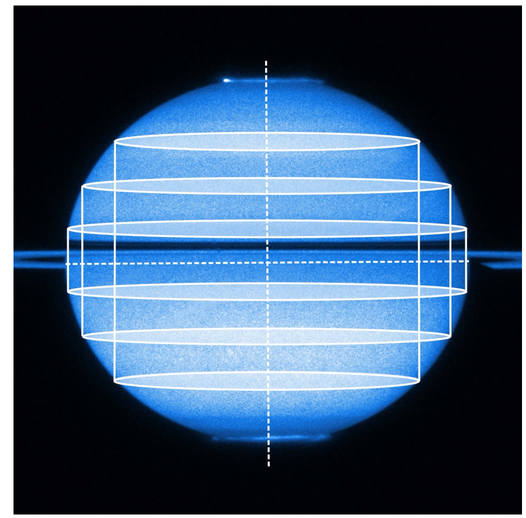
Saturn's moment of inertia: C/MR^2

Moons' contribution to angular momentum

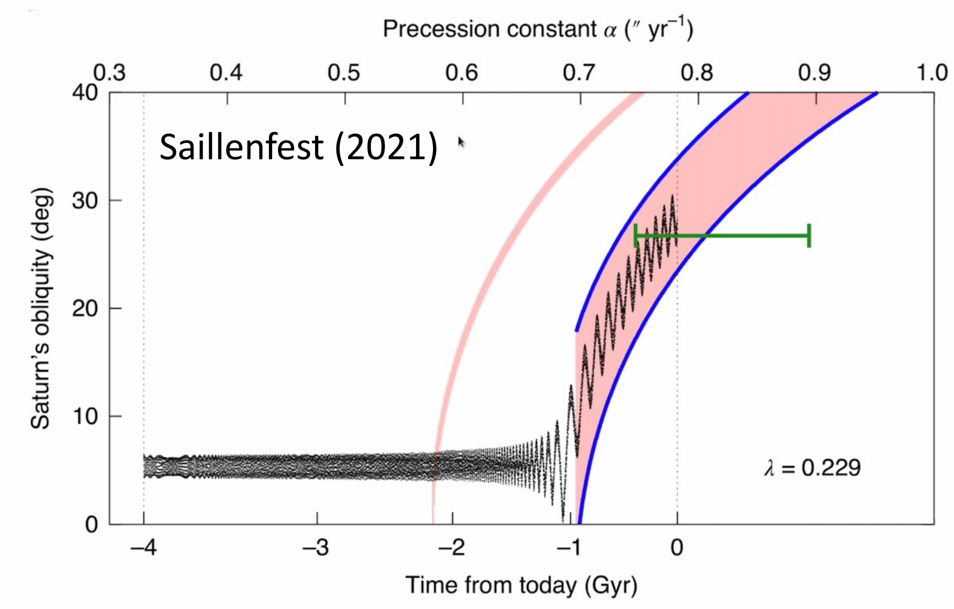
Saturn Models with DR that Match J_2, J_4, and J_6 predict a Smaller MOI than Models Without DR

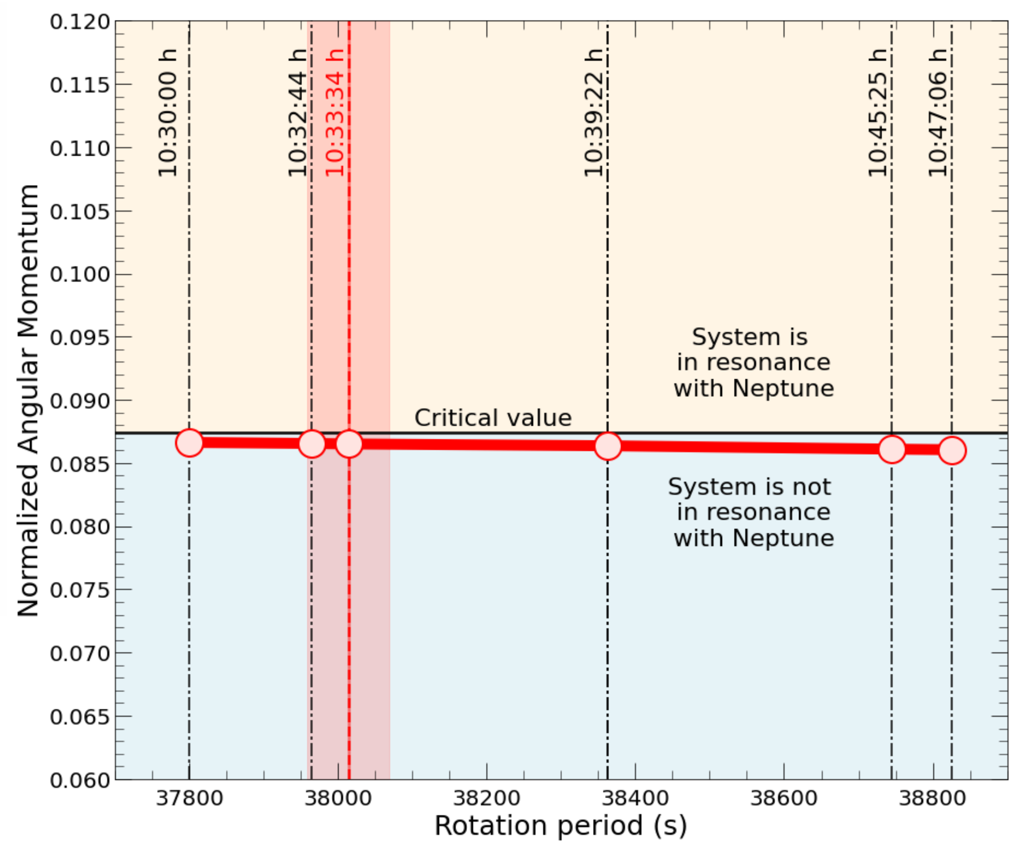




B. Militzer, W. B. Hubbard, "Relation of Gravity, Winds, and the Moment of Inertia of Jupiter and Saturn", *Planet. Sci. J.* **4** (2023) 95

This Saturn in Resonance with Neptune?
This depends on Saturn's Angular Momentum





Precession constant: $\alpha = \frac{3}{2} n \frac{n}{\omega} \frac{J_2 + q}{\lambda + l}$

Saturn's moment of inertia: C/MR^2

Moons' contribution to angular momentum

Our scenario for the formation of Saturn's rings

1. The Saturnian system formed with an **additional moon, Chrysalis**. Saturn's spin axis was perpendicular to its orbital plane.
2. Chrysalis **gave Neptune an extra "handle" to tilt Saturn's spin axis** (via a spin-orbit resonance) to the large value that we see today, 27°.
3. Saturn's moon **Titan started to migrate out**. About 160 million years ago, it entered into a **resonance with the moon Chrysalis** destabilizing its orbit.
4. As a result, Chrysalis came so close to Saturn that it was sheared apart by Saturn's intense gravity (**tidal disruption**). Most of the material fell into Saturn but out of 1%, the rings formed.
5. With Chrysalis gone, Neptune could no longer change Saturn's spin axis. So the planet was left spinning at an angle of 27°.

Our scenario for the formation of Saturn's rings is supported by the following lines of evidence:

1. It predicts a young age for **Saturn's rings of only 100 million years** approximately. This is in agreement with the ring color and Cassini's measurements of the ring mass.
2. It explains **why Saturn's spin axis is tilted** rather than being vertical, which it was when the planet formed.
3. It also explains **why Saturn's moment of inertia is so close to the critical value** to be in a spin-orbit resonance with Neptune but just outside of the critical region.
4. It is consistent with **Titan's observed migration** and offers an explanation why its orbit is slightly elliptical.

Take-Away Points

1. **Saturn's rings are young**, only 100 millions years old. Enjoy while the last (about another 100 Ma)
2. They formed from an **early moon** that encountered a grazing impact on Saturn.
3. **Neptune** tilted Saturn's spin axis.

For manuscripts on planets
and ab initio simulations see
http://militzer.berkeley.edu