## Path Integral Monte Carlo Simulations of Warm Dense Matter



## Plan for L5, L9, L12 and T1 \& T3

L5: Path integral Monte Carlo (PIMC) simulations and First Principles Equation of state (FPEOS) database

L9: NASA mission Juno to Jupiter, dilute core
T1: "Build that Planet" with SPH method L12: NASA mission Cassini to Saturn. How did that planet become the Lord of the Rings?
T3: FPEOS tutorial

## Software needed for T1 \& T3

T1: "Built that Planet" with SPH method
Python + Jupyter notebooks (installation on laptop required, for example with Anaconda)
Alternative: use Google Colab (no installation, no animation)
T3: FPEOS tutorial
Requires a C++ compiler for all calculations, uses Python for all graphics.

## Outline of lecture 1

- Path integral Monte Carlo (PIMC) method
- Comparison with different experiments
- First Principles Equation of state (FPEOS) database


## C(MEC CENTER FOR MATTER UNDER EXTREME CONDITIONS <br> enter for atter under xtreme onditions (CMEC)




## Study planetary interiors in the laboratory: shock wave experiments



Two-stage gas gun (Livermore) 0.2 Mbar


Nova laser (Livermore) 3.4 Mbar


Z capacitor bank (Sandia) 2 Mbar


National Ignition Facility $\mathbf{7 0 0}$ Mbar

## Shock wave measurements determine the Equation of State on the Hugoniot curve



Conservation of mass, momentum and energy yields:

$$
\begin{aligned}
& \rho=\rho_{0} \frac{u_{s}}{u_{s}-u_{p}} \\
& P=P_{0}+\rho_{0} u_{s} u_{p} \\
& E=E_{0}+\frac{1}{2}\left(V_{0}-V\right)\left(P+P_{0}\right)
\end{aligned}
$$



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X
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$$



## Comparison of Simulation Results and Shock Wave experiments of Deuterium



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## Comparison of Simulation Results and Shock Wave experiments of Deuterium



$$
\begin{gathered}
\text { I. } \\
\text { Path Integral } \\
\text { Monte Carlo }
\end{gathered}
$$

## Density functional molecular dynamics at lower T



Born-Oppenheimer approx. MD with classical nuclei:

$$
F=m a
$$

Forces derived DFT with electrons in the instantaneous ground state.

## Path integral Monte Carlo at high $\mathrm{T}>10^{4} . . .10^{6} \mathrm{~K}$




## Starting from Restricted PIMC Simulations of Hydrogen



Equation of State of the Hydrogen Plasma by Path Integral Monte Carlo Simulation

$$
\text { C. Pierleoni, }{ }^{1,2, *} \text { D. M. Ceperley, }{ }^{3} \text { B. Bernu, }{ }^{1} \text { and W. R. Magro }{ }^{3}
$$

Volume 76, Number $8 \quad$ PHYSICAL REVIEW LETTERS $\quad 19$ February 1996

Molecular Dissociation in Hot, Dense Hydrogen
W.R. Magro, ${ }^{1}$ D.M. Ceperley, ${ }^{2}$ C. Pierleoni, ${ }^{3}$ and B. Bernu ${ }^{4}$

## Canonical Ensembles: Classical

## Boltzmann factor

$$
e^{-E / k_{B} T}
$$

Thermodynamic averages:

$$
Z_{C l}=\sum_{S} e^{-\beta E_{S}}
$$



## Canonical Ensembles: Classical Quantum

## Boltzmann factor

$$
e^{-E / k_{B} T}
$$

## Density matrix

$$
\hat{\rho}=e^{-\beta \hat{\mathrm{H}}}
$$

$$
\begin{aligned}
& \rho\left(R, R^{\prime}, \beta\right)=\langle R| e^{-\beta \hat{\mathrm{H}}}\left|R^{\prime}\right\rangle \\
& \rho\left(R, R^{\prime}, \beta\right)=\sum_{S} e^{-\beta E_{S}} \Psi_{S}^{*}(R) \Psi_{S}\left(R^{\prime}\right)
\end{aligned}
$$

Thermodynamic averages:

$$
Z_{C l}=\sum_{S} e^{-\beta E_{S}}
$$

$$
\begin{aligned}
& Z_{Q}=\operatorname{Tr}[\hat{\rho}]=\int d R\langle R| e^{-\beta \hat{\mathrm{H}}}|R\rangle \\
& \langle\hat{O}\rangle=\frac{\operatorname{Tr}[\hat{O} \hat{\rho}]}{\operatorname{Tr}[\hat{\rho}]}
\end{aligned}
$$

Step 1 towards the path integral Matrix squaring property of the density matrix

Matrix squaring in operator notation:

$$
\hat{\rho}=e^{-\beta \hat{\mathrm{H}}}=\left(e^{-(\beta / 2) \hat{\mathrm{H}}}\right)\left(e^{-(\beta / 2) \hat{\mathrm{H}}}\right), \quad \beta=\frac{1}{k_{B} T}
$$

Matrix squaring in real-space notation:

$$
\langle R| \hat{\rho}\left|R^{\prime}\right\rangle=\int d R_{1}\langle R| e^{-(\beta / 2) \hat{H}}\left|R_{1}\right\rangle\left\langle R_{1}\right| e^{-(\beta / 2) \hat{H}}\left|R^{\prime}\right\rangle
$$

Matrix squaring in matrix notation:

$$
\left[\begin{array}{ccc}
\cdots & R^{\prime} & \ldots \\
R & \ddots & \vdots \\
\cdots & \ldots & \ldots .
\end{array}\right]=\left[\begin{array}{ccc}
\cdots & R_{1} & \ldots \\
R & \ddots & \vdots \\
\cdots & \ldots & \ldots . .
\end{array}\right] *\left[\begin{array}{ccc}
\cdots & R^{\prime} & \ldots \\
R_{1} & \ddots & \vdots \\
\cdots & \ldots & \ldots
\end{array}\right]
$$

## Repeat the matrix squaring step

Matrix squaring in operator notation:

$$
\hat{\rho}=e^{-\beta \hat{\mathrm{H}}}=\left(e^{-(\beta / 4) \hat{\mathrm{H}}}\right)^{4}, \beta=\frac{1}{k_{B} T}
$$

Matrix squaring in real-space notation:
$\langle R| \hat{\rho}\left|R^{\prime}\right\rangle=\int d R_{1} \int d R_{2} \int d R_{3}\langle R| e^{-(\beta / 4) \hat{\mathrm{H}}}\left|R_{1}\right\rangle\left\langle R_{1}\right| e^{-(\beta / 4) \hat{\mathrm{H}}}\left|R_{2}\right\rangle\left\langle R_{2}\right| e^{-(\beta / 4) \hat{\mathrm{H}}}\left|R_{3}\right\rangle\left\langle R_{3}\right| e^{-(\beta / 4) \hat{\mathrm{H}}}\left|R^{\prime}\right\rangle$

## Path Integrals in Imaginary Time

## Every particle is represented by a path, a ring polymer.

$$
\text { Density matrix: } \hat{\rho}=e^{-\beta \hat{\mathrm{H}}}=\left(e^{-\tau \hat{\mathrm{H}}}\right)^{M}, \beta=\frac{1}{k_{B} T}, \tau=\frac{\beta}{M} \quad\langle\hat{O}\rangle=\frac{\operatorname{Tr}[\hat{O} \hat{\rho}]}{\operatorname{Tr}[\hat{\rho}]}
$$

## Trotter break-up:

$$
\langle R| \hat{\rho}\left|R^{\prime}\right\rangle=\langle R|\left(e^{-x \hat{t} \hat{t}}\right)^{M}\left|R^{\prime}\right\rangle=\int d R_{1} \ldots \int d R_{M-1}\langle R| e^{-t \hat{f} \hat{t}}\left|R_{1}\right\rangle\left\langle R_{1}\right| e^{-x \hat{f}}\left|R_{2}\right\rangle \ldots\left\langle R_{M-1}\right| e^{-\hat{t h}}\left|R^{\prime}\right\rangle
$$

## Path Integrals in Imaginary Time

 Simplest form for the paths' action: primitive approx.Density matrix: $\hat{\rho}=e^{-\beta \hat{H}}=\left(e^{-\tau \hat{H}}\right)^{M}, \beta=\frac{1}{k_{B} T}, \tau=\frac{\beta}{M}$
$\langle\hat{O}\rangle=\frac{\operatorname{Tr}[\hat{O} \hat{\rho}]}{\operatorname{Tr}[\hat{\rho}]}$
Trotter break-up:

Trotter formula:

$$
e^{-\beta(\hat{T}+\hat{V})}=\lim _{M \rightarrow \infty}\left[e^{-\tau \hat{T}} e^{-\tau \hat{V}}\right]^{M}
$$

Path integral and primitive action S:

$\langle R| \hat{\rho}\left|R^{\prime}\right\rangle=\oint_{R \rightarrow R^{\prime}} d R_{e} e^{-S\left[R_{i}\right]}$
$S\left[R_{t}\right]=\sum_{i=1}^{M} \frac{\left(R_{i+1}-R_{i}\right)^{2}}{4 \lambda \tau}+\frac{\tau}{2}\left[V\left(R_{i}\right)+V\left(R_{i+1}\right)\right]$



## Path Integrals in Imaginary Time include ZeroPoint Motion and some Tunnelling Effects

$$
\begin{aligned}
& \langle R| \hat{\rho}\left|R^{\prime}\right\rangle=\oint_{R \rightarrow R^{\prime}} d R_{t} e^{-S\left[R_{t}\right]} \\
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\end{aligned}
$$



zero-point energy

tunneling

## Path Integrals in Imaginary Time include ZeroPoint Motion and some Tunnelling Effects

| $\begin{aligned} & \langle R\| \hat{\rho}\left\|R^{\prime}\right\rangle=\oint_{R-1} d R_{e} e^{-S R} R_{i, l} \\ & S\left[R_{t}\right]=\sum_{i=1}^{m}\left(\frac{\left(R_{i+1}-R_{i}\right)^{2}}{4 \lambda \tau}+\frac{\tau}{2}\left[V\left(R_{i}\right)+V\left(R_{i+1}\right)\right]\right. \end{aligned}$ |  |
| :---: | :---: |



## Bosonic and Fermionic Density Matrices

Bosonic density matrix:
Sum over all symmetric eigenstates.

$$
\rho_{B}\left(R, R^{\prime}, \beta\right)=\sum_{i} e^{-\beta E_{i}} \Psi_{S}^{[i]^{*}}(R) \Psi_{S}^{[i]}\left(R^{\prime}\right)
$$

Fermionic density matrix:
Sum over all antisymmetric eigenstates.

$$
\rho_{F}\left(R, R^{\prime}, \beta\right)=\sum_{i} e^{-\beta E_{i}} \Psi_{A S}^{[i]^{*}}(R) \Psi_{A S}^{[i]}\left(R^{\prime}\right)
$$

## Bosonic and Fermionic Path Integrals

Bosonic density matrix:
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$$

Project out the symmetric states:

$$
\rho_{B}\left(R, R^{\prime}, \beta\right)=\sum_{P}(+1)^{P} \rho_{D}\left(R, P R^{\prime}, \beta\right)
$$

Fermionic density matrix:
Sum over all antisymmetric eigenstates.

$$
\rho_{F}\left(R, R^{\prime}, \beta\right)=\sum_{i} e^{-\beta E_{i}} \Psi_{A S}^{[i]^{*}}(R) \Psi_{A S}^{[i]}\left(R^{\prime}\right)
$$

Project out the antisymmetric states:

$$
\rho_{F}\left(R, R^{\prime}, \beta\right)=\sum_{P}(-1)^{P} \rho_{D}\left(R, P R^{\prime}, \beta\right)
$$

$$
\langle R| \hat{\rho}_{F / B}\left|R^{\prime}\right\rangle=\sum_{P}( \pm 1)^{P} \int d R_{1} \ldots \int d R_{M-1}\langle R| e^{-\tau \hat{\mathrm{H}}}\left|R_{1}\right\rangle \ldots\left\langle R_{M-1}\right| e^{-\tau \hat{\mathrm{H}}}\left|P R^{\prime}\right\rangle
$$

## Bosonic and Fermionic Path Integrals

Bosonic density matrix:
Sum over all symmetric eigenstates.
$\rho_{B}\left(R, R^{\prime}, \beta\right)=\sum_{i} e^{-\beta E_{i}} \Psi_{S}^{[i]^{*}}(R) \Psi_{S}^{[i]}\left(R^{\prime}\right)$
Project out the symmetric states:

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Sum over all antisymmetric eigenstates.

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Project out the antisymmetric states:

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\rho_{F}\left(R, R^{\prime}, \beta\right)=\sum_{P}(-1)^{P} \rho_{D}\left(R, P R^{\prime}, \beta\right)
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$\langle R| \hat{\rho}_{F / B}\left|R^{\prime}\right\rangle=\sum_{P}( \pm 1)^{P} \int d R_{1} \ldots \int d R_{M-1}\langle R| e^{-\tau \hat{\mathrm{H}}}\left|R_{1}\right\rangle \ldots\left\langle R_{M-1}\right| e^{-\tau \hat{\mathrm{H}}}\left|P R^{\prime}\right\rangle$


## Bosonic and Fermionic ntegrals

Bosonic density matrix: Sum over all symmetric eigensta $\square$

## n

 $\rho_{B}\left(R, R^{\prime}, \beta\right)=\sum e^{-\beta E_{j} \cdot} \Psi^{\text {b }}$
## Project

## 

D:


## Restricted PIMC for fermions: How is the restriction applied?



Construct a fermionic trial density matrix in form of a Slater determinant of single-particle density matrices:
$\rho_{T}\left(R, R^{\prime}, \beta\right)=\left|\begin{array}{ccc}\rho\left(r_{1}, r_{1}^{\prime}, \beta\right) & \cdots & \rho\left(r_{1}, r_{N}^{\prime}, \beta\right) \\ \vdots & \ddots & \vdots \\ \rho\left(r_{N}, r_{1}^{\prime}, \beta\right) & \cdots & \rho\left(r_{N}, r_{N}^{\prime}, \beta\right)\end{array}\right|$

Enforce the following nodal condition for all time slices along the paths:

$$
\rho_{T}[R(t), R(0), t]>0
$$

This 3N-dimensional conditions eliminates all negative and some positive contribution to the path $\rightarrow$ Solves the fermion sign problem approx.

Free-particle nodes:

$$
\rho_{0}^{[1]}\left(r, r^{\prime} ; \beta\right)=\sum_{k} e^{-\beta E_{k}} \Psi_{k}(r) \Psi_{k}^{*}\left(r^{\prime}\right)
$$

## Fermionic Path Integrals Example: Closed paths of 2 free particles

Distinguishable particles:
Consider path types: NA + NX

## Bosons:

Consider path types: NA + NX + PNX
Direct fermions:
Consider path types: NA + NX - PNX

Restricted fermions:
Consider only path type: NA



## Molecular Hydrogen

Snapshot from a PIMC simulation with 32 protons and electrons


## 100\% molecules, weakly interacting

2 protons (pink spheres) and spin-up and one spin-down electron form one $\mathrm{H}_{2}$ molecule.

## Molecular Hydrogen

Snapshot from a PIMC simulation with 32 protons and electrons


2 protons (pink spheres) and spin-up and one spin-down electron form one $\mathrm{H}_{2}$ molecule.


- strongly interacting molecules, close to pressure dissociation
- Electrons are degenerate, partially delocalized
- Electron paths are permuting


## Metallic Hydrogen

Snapshot from a PIMC simulation with 32 protons and electrons


Free protons (pink spheres) and delocalized electrons.


- Pressure dissociation, free protons
- Degenerate electron gas
- High number of permutations


# Silicates: $\mathrm{MgSiO}_{3}$ 

## $\mathrm{MgSiO}_{3}$ : Principal Hugoniot Curve



Gonzalez, Soubiran, Peterson, Militzer, Phys. Rev. B 101 (2020) 024107

## $\mathrm{MgSiO}_{3}$ : Principal Hugoniot Curve



## $\mathrm{MgSiO}_{3}$ : Principal Hugoniot Curve



Gonzalez,
Soubiran,
Peterson,
Militzer,
Phys. Rev. B
101 (2020)
024107

CH plastics

# Inertial confinement fusion experiments with plastic coated spheres of liquid $\mathbf{H}_{2}$ 


(Graphics: Bachmann et al. LLNL)

## PIMC and DFT-MD simulations performed for $\mathrm{C}_{2} \mathrm{H}, \mathrm{CH}_{3} \mathrm{C}_{2} \mathrm{H}_{3}, \mathrm{CH}_{3}$ and $\mathrm{CH}_{4}$.



## All calculations performed on <br> $\underset{\text { sustained petascale computing }}{\boldsymbol{\sim}}$



## CH Shock Hugoniot Curves: Comparison of Theory and Experiments



## CH Shock Hugoniot Curves: Comparison of Theory and Experiments



## CH Shock Hugoniot Curves: Comparison of Theory and Experiments



## CH Shock Hugoniot Curves: Comparison of Theory and Experiments



# Hugoniot Curves of BN and $\mathrm{B}_{4} \mathrm{C}$ Fully interacting EOS and Linear Mixing agree quite well. 




# Hugoniot Curves of BN and $\mathrm{B}_{4} \mathrm{C}$ Fully interacting EOS and Linear Mixing agree quite well. 




## Linear Mixing at Constant $P$ and $T$ (Also called additive volume rule)



$$
\begin{aligned}
V_{\operatorname{mix}} & =N_{1} V_{1}+N_{2} V_{2} \\
m_{\operatorname{mix}} & =N_{1} m_{1}+N_{2} m_{2} \\
E_{\operatorname{mix}} & =N_{1} E_{1}+N_{2} E_{2} \\
\rho_{\operatorname{mix}} & =m_{\operatorname{mix}} / V_{\operatorname{mix}}
\end{aligned}
$$

## Hugoniot Curves of MgO and $\mathrm{MgSiO}_{3}$ Results from fully interacting EOS and experiment.



## Hugoniot Curves of MgO and $\mathrm{MgSiO}_{3}$ Fully interacting EOS and Linear Mixing agree quite well.



Linear mixing works well for $T \gtrsim 2 \times 10^{5} \mathrm{~K}$ and $\varrho / \varrho_{0} \gtrsim 3.2$

## The Journal of Chemical Physics

## Nonideal mixing effects in warm dense matter studied with first-principles computer simulations

Cite as: J. Chem. Phys. 153, 184101 (2020); doi: 10.1063/5.0023232
Submitted: 28 July 2020 • Accepted: 25 October 2020 •
Published Online: 9 November 2020
Burkhard Militzer, ${ }^{1,2, a)}$ (D) Felipe González-Cataldo, ${ }^{1}$ (D) Shuai Zhang, ${ }^{3}$ (D) Heather D. Whitley, ${ }^{4}$ (D) Damian C. Swift, ${ }^{4}$
and Marius Millot ${ }^{4}$

# Nonlinear Mixing Effects in $\mathrm{MgSiO}_{3}$ Fully interacting EOS and Linear Mixing agree quite well. 



Linear mixing works well for $T \gtrsim 2 \times 10^{5} \mathrm{~K}$ and $\varrho / \varrho_{0} \gtrsim 3.2$

## Hugoniot Curves of CO and $\mathrm{CO}_{2}$ Experimental $\mathrm{CO}_{2}$ Hugoniot agree with Linear Mixing result



# Hugoniot Curves of $\mathrm{H}_{2} \mathrm{O}_{,} \mathrm{H}_{2} \mathrm{O}_{2}$, and $\mathrm{Al}_{2} \mathrm{O}_{3}$ Experimental $\mathrm{H}_{2} \mathrm{O}$ Hugoniot agree with Linear Mixing result 



| Material | Number of isochores | Minimum density [ $\mathrm{g} \mathrm{cm}^{-3}$ ] | Maximum density [ $\mathrm{g} \mathrm{cm}^{-3}$ ] | Minimum temperature <br> [K] | Maximum temperature <br> [K] | Number of EOS points | References |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Hydrogen | 33 | 0.001 | 798.913 | 15625 | $6.400 \times 10^{7}$ | 401 | [69-74] |
| Helium | 9 | 0.387 | 10.457 | 500 | $2.048 \times 10^{9}$ | 228 | [75, 76] |
| Boron | 16 | 0.247 | 49.303 | 2000 | $5.174 \times 10^{8}$ | 314 | [77] |
| Carbon | 9 | 0.100 | 25.832 | 5000 | $1.035 \times 10^{9}$ | 162 | [78, 79] |
| Nitrogen | 17 | 1.500 | 13.946 | 1000 | $1.035 \times 10^{9}$ | 234 | [80] |
| Oxygen | 6 | 2.486 | 100.019 | 10000 | $1.035 \times 10^{9}$ | 76 | [81] |
| Neon | 4 | 0.895 | 15.026 | 1000 | $1.035 \times 10^{9}$ | 67 | [82] |
| Sodium | 9 | 1.933 | 11.600 | 1000 | $1.293 \times 10^{8}$ | 193 | [83, 84] |
| Magnesium | 23 | 0.431 | 86.110 | 20000 | $5.174 \times 10^{8}$ | 371 | [85] |
| Aluminum | 15 | 0.270 | 32.383 | 10000 | $2.156 \times 10^{8}$ | 240 | [86] |
| Silicon | 7 | 2.329 | 18.632 | 50000 | $1.293 \times 10^{8}$ | 85 | [87, 88] |
| LiF | 8 | 2.082 | 15.701 | 10000 | $1.035 \times 10^{9}$ | 91 | [89] |
| $\mathrm{B}_{4} \mathrm{C}$ | 16 | 0.251 | 50.174 | 2000 | $5.174 \times 10^{8}$ | 291 | [90] |
| BN | 16 | 0.226 | 45.161 | 2000 | $5.174 \times 10^{8}$ | 311 | [91] |
| $\mathrm{CH}_{4}$ | 16 | 0.072 | 14.376 | 6736 | $1.293 \times 10^{8}$ | 247 | [92, 93] |
| $\mathrm{CH}_{2}$ | 16 | 0.088 | 17.598 | 6736 | $1.293 \times 10^{8}$ | 248 | [92, 93] |
| $\mathrm{C}_{2} \mathrm{H}_{3}$ | 16 | 0.097 | 19.389 | 6736 | $1.293 \times 10^{8}$ | 247 | [92, 93] |
| CH | 16 | 0.105 | 21.000 | 6736 | $1.293 \times 10^{8}$ | 248 | [92, 93] |
| $\mathrm{C}_{2} \mathrm{H}$ | 16 | 0.112 | 22.430 | 6736 | $1.293 \times 10^{8}$ | 245 | [ 92,93$]$ |
| MgO | 19 | 0.357 | 71.397 | 20000 | $5.174 \times 10^{8}$ | 286 | [94] |
| $\underline{\mathrm{MgSiO}_{3}}$ | 16 | 0.321 | 64.158 | 6736 | $5.174 \times 10^{8}$ | 284 | [95, 96] |

# First-Principles Equation of State Database online http://militzer.berkeley.edu/FPEOS 

\author{

- • $\mathrm{D}^{\circ}$ く > <br> First-Principles Equation of State (FPEOS) Database for Warm Dense Matter <br> Computation <br> Authors: B. Militzer, F. Gonzalez-Cataldo, S. Zhang, K. P. Driver, F. Soubiran <br> With the goal in mind of making WDM computations more reliable and efficient, we make available our EOS tables for 11 elements and 10 compounds as well as the C++ computer codes for their interpolation. Python code is provided to generate graphs of shock Hugoniot curve, isentropes, isobars, and isotherms for compounds and user-defined mixtures. We put together this first-principles equation of state (FPEOS) database for matter at extreme conditions by combining results from path integral Monte Carlo and density functional molecular dynamics simulations of the elements $\mathrm{H}, \mathrm{He}, \mathrm{B}, \mathrm{C}, \mathrm{N}, \mathrm{O}, \mathrm{Ne}, \mathrm{Na}, \mathrm{Mg}, \mathrm{Al}$ and Si as well as the compounds $\mathrm{LiF}, \mathrm{B}_{4} \mathrm{C}, \mathrm{BN}, \mathrm{CH}_{4}, \mathrm{CH}_{2}, \mathrm{C}_{2} \mathrm{H}_{3}, \mathrm{CH}, \mathrm{C} 2 \mathrm{H}, \mathrm{MgO}$, and $\mathrm{MgSiO}_{3}$. For all these materials, we provide the pressure and internal energy over a wide density-temperature range from $\sim 0.5$ to $50 \mathrm{~g} / \mathrm{cc}$ and from $\sim 10^{4}$ to $10^{9} \mathrm{~K}$. This database encompasses the results from approximately 5000 different first-principles simulations. It allows one to compute isobars, adiabats, and shock Hugoniot curves in the regime of L and K shell ionization. Invoking the linear mixing approximation, one can study the properties of user-defined mixtures at high density and temperature. <br> Recommended citation: B. Militzer, F. Gonzalez-Cataldo, S. Zhang, K. P. Driver, F. Soubiran, "First-Principles Equation of State Database for Warm Dense Matter Computation", Physical Review E 103 (2021) 013203.
}


5000 first-principles calculations have been combined into our FPEOS database. So anyone can predict shock Hugoniot curves for a variety of compounds and mixtures. This will make warm dense matter calculations more reliable and efficient.

## NIF Gbar Experiment: Equations of State of C-O Mixtures in White Dwarf Stars

PI: D. Saumon (LANL), Blouin, Glenzer, Swift, Kritcher, Doppner, Whitley, Lazicki, Falcone, Militzer
We propose to make EOS measurements along the Hugoniot with the Gbar platform of carbon-oxygen rich materials that resemble conditions in White Dwarf stars.




Glyoxal $\mathrm{C}_{2} \mathrm{O}_{2} \mathrm{H}_{2}$


Glyoxal $\mathrm{C}_{2} \mathrm{O}_{2} \mathrm{H}_{2}$
Acetic acid $\mathrm{C}_{2} \mathrm{O}_{2} \mathrm{H}_{4}$ comp:~/fpeos> fpeos binaryMixture EOS1=6 2.0 EOS2=16 2.0 rho0 $=1.049 \mathrm{E} 0=-229.0$

## FPEOS

demo

## The End

