# LFV, LNV processes and new physics at the same-sign muon colliders

Jin-Lei. Yang, Chao-Hsi. Chang and Tai-Fu. Feng, Chin. Phys. C 48 (2024) 4, 043101

Hebei University Jin-Lei Yang





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# 1 Introduction

The motivation of studying the LFV and LNV processes at the same-sign muon colliders

LFV: 
$$\mu^{\pm}\mu^{\pm} \rightarrow \tau^{\pm}\tau^{\pm}, \mu^{\pm}\mu^{\pm} \rightarrow e^{\pm}e^{\pm}$$
  
LNV:  $\mu^{\pm}\mu^{\pm} \rightarrow W^{\pm}W^{\pm}$ 

1, the LFV processes do not depend on the large flavor mixing parameters, for example





# 1 Introduction

2, strict constraints from the nuclear  $0v2\beta$  decays





3, the muon collider can reach higher collider energy comparted with the electron collider

$$rac{m_{\mu}}{m_{e}} \approx 207$$

#### 1 Introduction

#### Possible contributions in NP



#### ► The meaning of observing LFV and LNV at the same-sign muon collider

1, providing definite evidences of NP beyond the SM

2, identifying the nature of neutrinos

3, searching new particle: doubly charged Higgs

## ► Types of NP

	Particle contents contribute to LFV and LNV in NP
Type I new physics (TI-NP)	Majorana neutrinos; $W_L$
Type II new physics (TII-NP)	Majorana neutrinos; $W_L$ ; $W_R$
Type III new physics (TIII-NP)	Majorana neutrinos; $W_L$ ; $W_R$ ; doubly charged Higgs

**TI-NP:** B-LSSM, NMSSM, ...

representative NP models

**TII-NP:** LRSM without triplet

**TIII-NP:** LRSM, LRSSM,...

The needed Lagrangian for **TI-NP** can be extracted from B-LSSM, and for **TII-NP**, **TIII-NP** can be extracted from LRSM.

#### Majorana neutrinos in the B-LSSM

New  $U(1)_{B-L}$ , three right-handed neutrinos and two scalar singlets are introduced in the B-LSSM, then the tiny neutrino masses can be obtained by the Type-I seesaw mechanism

$$\begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix}$$

The mass matrix above can be diagonalized by a unitary matrix  $U_{\nu}$ 

$$U_{\nu}^{T} \begin{pmatrix} 0 & M_{D}^{T} \\ M_{D} & M_{R} \end{pmatrix} U_{\nu} = \begin{pmatrix} \widehat{m}_{\nu} & 0 \\ 0 & \widehat{M}_{N} \end{pmatrix}$$

where  $\widehat{m}_{\nu} = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}), \ \widehat{m}_N = \text{diag}(m_{N_1}, m_{N_2}, m_{N_3})$ . The unitary matrix  $U_{\nu}$  reads

$$U_{\nu} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

The Lagrangian involving leptons and W boson in the B-LSSM

$$\mathcal{L}_{W}^{\mathrm{BL}} = \frac{ig_{2}}{\sqrt{2}} \sum_{j=1}^{3} \left[ U_{ij} \bar{e}_{i} \gamma^{\mu} P_{L} \nu_{j} W_{L,\mu} + S_{ij} \bar{e}_{i} \gamma^{\mu} P_{L} N_{j} W_{L,\mu} + h.c. \right]$$

The relevant couplings of leptons and Goldstones are

$$\mathcal{L}_{G}^{\mathrm{BL}} = \frac{ig_{2}}{\sqrt{2M_{W_{L}}}} \sum_{j=1}^{3} \left\{ \bar{e}_{i} \left[ \left( M_{D}^{\dagger} \cdot T^{*} \right)_{ij} P_{R} - (\widehat{m}_{l} \cdot U)_{ij} P_{L} \right] \nu_{j} G_{L} + \bar{e}_{i} \left[ \left( M_{D}^{\dagger} \cdot V^{*} \right)_{ij} P_{R} - (\widehat{m}_{l} \cdot S)_{ij} P_{L} \right] N_{j} G_{L} + h.c. \right\}$$

We can define  $S_i^2 = \sum_{j=1}^3 |S_{ij}^2| (i = e, \mu, \tau)$  to describe the strength of light-heavy neutrino mixings.

#### Majorana neutrinos in the LRSM

New  $SU(2)_R$ , three right-handed neutrinos and two scalar trilets are introduced in the LRSM, then the tiny neutrino masses can be obtained by the Type-I+II seesaw mechanism

$$\begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix}$$

The mass matrix can also be diagonalized by the unitary matrix  $U_{\nu}$ . The W boson mass matrix in the LRSM can be written as

$$\frac{g_2^2}{4}(W_L, W_R) \begin{pmatrix} v_1^2 + v_2^2 + 2v_L^2 & 2v_1v_2 \\ 2v_1v_2 & v_1^2 + v_2^2 + 2v_R^2 \end{pmatrix} \begin{pmatrix} W_L \\ W_R \end{pmatrix}$$

The physical masses of the two W bosons are

$$M_{W_1} \approx \frac{g_2}{2} \left( v_1^2 + v_2^2 \right)^{\frac{1}{2}}, \qquad M_{W_2} \approx \frac{g_2}{\sqrt{2}} v_R$$

The mass eigenstates  $W_1$ ,  $W_2$  are related to the gauge eigenstates  $W_L$ ,  $W_R$  by

$$\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = \begin{pmatrix} \cos \zeta & \sin \zeta \\ -\sin \zeta & \cos \zeta \end{pmatrix} \begin{pmatrix} W_L \\ W_R \end{pmatrix}$$

Where  $\tan \zeta = \frac{2v_1v_2}{(v_R^2 - v_L^2)}$ . The Lagrangian involving leptons and W boson in the LRSM  $\mathcal{L}_W^{LR} = \frac{ig_2}{\sqrt{2}} \sum_{j=1}^3 \left[ \bar{e}_i \Big( \cos \zeta \, U_{ij} \gamma^\mu P_L + \sin \zeta \, T_{ij}^* \gamma^\mu P_R \Big) v_j W_{1,\mu} + \bar{e}_i \Big( \cos \zeta \, T_{ij}^* \gamma^\mu P_R - \sin \zeta \, U_{ij} \gamma^\mu P_L \Big) v_j W_{2,\mu} \right. \\ \left. + \bar{e}_i \Big( \cos \zeta \, S_{ij} \gamma^\mu P_L + \sin \zeta \, V_{ij}^* \gamma^\mu P_R \Big) N_j W_{1,\mu} + \bar{e}_i \Big( \cos \zeta \, V_{ij}^* \gamma^\mu P_R - \sin \zeta \, S_{ij} \gamma^\mu P_L \Big) N_j W_{2,\mu} + h.c. \right]$ 

The relevant couplings of leptons and Goldstones in the LRSM are

$$\mathcal{L}_{G}^{LR} = \frac{ig_{2}}{\sqrt{2}M_{W_{L}}} \sum_{j=1}^{3} \left\{ \bar{e}_{i} \left[ \lambda_{1,ij}P_{L} + \lambda_{2,ij}P_{R} \right] \nu_{j}G_{L} + \bar{e}_{i} \left[ \lambda_{3,ij}P_{L} + \lambda_{4,ij}P_{R} \right] N_{j}G_{L} + \bar{e}_{i} \left( \lambda_{5,ij}P_{L} \right) \nu_{j}G_{R} + \bar{e}_{i} \left( \lambda_{6,ij}P_{L} \right) N_{j}G_{R} + h.c. \right\}$$

where 
$$\lambda_1 = -m_l^{\dagger} \cdot U$$
,  $\lambda_2 = \widehat{M}_D^{\dagger} \cdot T^*$ ,  $\lambda_3 = -m_l^{\dagger} \cdot S$ ,  $\lambda_4 = \widehat{M}_D^{\dagger} \cdot V^*$ ,  $\lambda_5 = \frac{M_{W_1}}{M_{W_2}} M_R^{\dagger} \cdot T$ ,  $\lambda_6 = \frac{M_{W_1}}{M_{W_2}} M_R^{\dagger} \cdot V^*$ 

The Lagrangian involving leptons and doubly charged Higgs in the LRSM  $\mathcal{L}_{\Delta ll}^{\mathrm{LR}} = i2Y_{L,ij}\bar{e}P_L e_j^C \Delta_L^{--} + i2Y_{R,ij}\bar{e}P_R e_j^C \Delta_R^{--} + h.c.$ 

The Lagrangian involving W bosons and doubly charged Higgs in the LRSM

$$\mathcal{L}_{\Delta WW}^{\mathrm{LR}} = i\sqrt{2}g_2^2 v_L \Delta_L^{--} W_1^{\mu+} W_{1\mu}^+ + i\sqrt{2}g_2^2 v_L \sin\zeta \Delta_L^{--} W_1^{\mu+} W_{2\mu}^+ + i\sqrt{2}g_2^2 v_R \sin\zeta \Delta_R^{--} W_1^{\mu+} W_{2\mu}^+ + i\sqrt{2}g_2^2 v_R \Delta_R^{--} W_2^{\mu+} W_{2\mu}^+ + h.c.$$

#### • The relevant Lagrangian for TI-NP, TII-NP and TIII-NP

**TII-NP:** 
$$\mathcal{L}_{W}^{\text{BL}}$$
,  $\mathcal{L}_{G}^{\text{BL}}$   
**TII-NP:**  $\mathcal{L}_{W}^{\text{LR}}$ ,  $\mathcal{L}_{G}^{\text{LR}}$   
**TIII-NP:**  $\mathcal{L}_{W}^{\text{LR}}$ ,  $\mathcal{L}_{G}^{\text{LR}}$ ,  $\mathcal{L}_{AU}^{\text{LR}}$ ,  $\mathcal{L}_{AWM}^{\text{LR}}$ 

The leading order contributions from Majorana neutrinos and doubly charged Higgs to the LFV processes



Fig. 1. The leading order contributions from Majorana neutrinos (left) and doubly charged Higgs (right) to the LFV processes in the Feynman gauge.

The amplitude can be simplified by neglecting charged lepton masses

$$\mathcal{M}\left(\mu^{\pm}\mu^{\pm} \rightarrow l^{\pm}l^{\pm}\right) = \frac{i}{16\pi^2} \sum_{i=1}^{\prime} \sum_{X,Y=L,R} C_i^{XY} \mathcal{O}_i^{XY}$$

where

$$\begin{aligned} \mathcal{O}_{1}^{XY} &= \bar{u}(k_{1})\gamma_{\mu}P_{X}u^{c}(k_{2})\overline{u^{c}}(p_{2})\gamma^{\mu}P_{Y}u(p_{1}) \\ \mathcal{O}_{2}^{XY} &= \bar{u}(k_{1})p_{1}^{\alpha}\gamma_{\alpha}P_{X}u^{c}(k_{2})\overline{u^{c}}(p_{2})k_{1}^{\beta}\gamma_{\beta}P_{Y}u(p_{1}) \\ \mathcal{O}_{3}^{XY} &= \bar{u}(k_{1})P_{X}u^{c}(k_{2})\overline{u^{c}}(p_{2})P_{Y}u(p_{1}) \\ \mathcal{O}_{4}^{XY} &= \bar{u}(k_{1})\gamma_{\mu}P_{X}u^{c}(k_{2})\overline{u^{c}}(p_{2})\gamma^{\mu}k_{1}^{\beta}\gamma_{\beta}P_{Y}u(p_{1}) \\ \mathcal{O}_{5}^{XY} &= \bar{u}(k_{1})\gamma_{\mu}p_{1}^{\alpha}\gamma_{\alpha}P_{X}u^{c}(k_{2})\overline{u^{c}}(p_{2})\gamma^{\mu}P_{Y}u(p_{1}) \\ \mathcal{O}_{6}^{XY} &= \bar{u}(k_{1})P_{X}u^{c}(k_{2})\overline{u^{c}}(p_{2})k_{1}^{\beta}\gamma_{\beta}P_{Y}u(p_{1}) \\ \mathcal{O}_{7}^{XY} &= \bar{u}(k_{1})p_{1}^{\alpha}\gamma_{\alpha}P_{X}u^{c}(k_{2})\overline{u^{c}}(p_{2})P_{Y}u(p_{1}) \end{aligned}$$

The coefficients contributed by doubly charged Higgs can be read directly

$$C_{3}^{LR}(\Delta_{L}^{\pm\pm}) = \frac{4Y_{L,22}Y_{L,jj}}{(p_{1}+p_{2})^{2} - M_{\Delta_{L}^{\pm\pm}}^{2} + iM_{\Delta_{L}^{\pm\pm}}\Gamma_{\Delta_{L}^{\pm\pm}}}, \quad C_{3}^{RL}(\Delta_{R}^{\pm\pm}) = \frac{4Y_{R,22}Y_{R,jj}}{(p_{1}+p_{2})^{2} - M_{\Delta_{R}^{\pm\pm}}^{2} + iM_{\Delta_{R}^{\pm\pm}}\Gamma_{\Delta_{R}^{\pm\pm}}}$$

The dominant decay channels of doubly charged Higgs:

$$\Delta_{L}^{\pm\pm} \to l^{\pm}l^{\pm}, W_{1}^{\pm}W_{1}^{\pm}$$
$$\Delta_{R}^{\pm\pm} \to l^{\pm}l^{\pm}, W_{2}^{\pm}W_{2}^{\pm(*)}, W_{2}^{\pm}W_{2}^{\pm}$$

The leading order contributions from Majorana neutrinos and doubly charged Higgs to the LNV processes



Fig. 2. The leading order contributions from Majorana neutrinos (1, 2) and doubly charged Higgs (3) to the LNV processes.

Summing up the fermions' spin and gauge bosons' polarizations, the squared amplitude for the LNV processes can be obtained

$$\begin{split} \left| \mathcal{M} \left( \mu^{\pm} \mu^{\pm} \to W_{1}^{\pm} W_{1}^{\pm} \right) \right|^{2} &\approx \frac{g_{2}^{4}}{M_{W_{1}}^{4}} \Big\{ \left( \left| C_{t}^{11} \right|^{2} + \left| C_{u}^{11} \right|^{2} \right) M_{W_{1}}^{2} (2p_{1} \cdot k_{1}p_{1} \cdot k_{2} + \\ &\qquad M_{W_{1}}^{2}p_{1} \cdot p_{2}/2) + 2 \left| C_{t}^{11} \right|^{2}k_{1} \cdot k_{2}(p_{1} \cdot k_{1})^{2} + 2 \left| C_{u}^{11} \right|^{2}k_{1} \cdot k_{2}(p_{1} \cdot k_{2})^{2} + \\ &\qquad \mathcal{R} (C_{t}^{11} C_{u}^{11*}) [2p_{1} \cdot p_{2}(k_{1} \cdot k_{2})^{2} + M_{W_{1}}^{2} \left( 3M_{W_{1}}^{2}p_{1} \cdot p_{2} - 4p_{1} \cdot k_{1}p_{1} \cdot k_{2} \right) - \\ &\qquad 2k_{1} \cdot k_{2}((p_{1} \cdot k_{1})^{2} + (p_{1} \cdot k_{2})^{2}) \Big\} \\ \left| \mathcal{M} \left( \mu^{\pm} \mu^{\pm} \to W_{1}^{\pm} W_{2}^{\pm} \right) \right|^{2} &\approx \frac{g_{2}^{4}}{2M_{W_{1}}^{2}} \left\{ \left| C_{t}^{12} \right|^{2} [4M_{W_{1}}^{2}M_{W_{2}}^{2}p_{1} \cdot k_{1}(p_{2} \cdot k_{1} - \\ &\qquad p_{1} \cdot p_{2}) + 8M_{W_{1}}^{2}p_{1} \cdot k_{1}p_{2} \cdot k_{2}(k_{1} \cdot k_{2} - p_{1} \cdot k_{2}) - M_{W_{1}}^{4}(M_{W_{2}}^{2}p_{1} \cdot p_{2} + \\ &\qquad 2p_{2} \cdot k_{2}p_{1} \cdot k_{2}) + 4(p_{1} \cdot k_{1})^{2}(M_{W_{2}}^{2}p_{1} \cdot p_{2} + 2p_{2} \cdot k_{2}p_{1} \cdot k_{2}) \right] + \\ &\quad \left| C_{u}^{12} \right|^{2} [4M_{W_{1}}^{2}M_{W_{2}}^{2}p_{1} \cdot k_{2}(p_{2} \cdot k_{2} - p_{1} \cdot p_{2}) + 8M_{W_{2}}^{2}p_{1} \cdot k_{2}p_{2} \cdot k_{1}(k_{1} \cdot k_{2} - p_{1} \cdot k_{1}) - M_{W_{2}}^{4}(M_{W_{1}}^{2}p_{1} \cdot p_{2} + 2p_{2} \cdot k_{1}p_{1} \cdot k_{1}) + \\ &\qquad 4(p_{1} \cdot k_{2})^{2}(M_{W_{1}}^{2}p_{1} \cdot p_{2} + 2p_{2} \cdot k_{1}p_{1} \cdot k_{1}) \right] \Big\} \end{split}$$

$$\begin{split} \left| \mathcal{M} \left( \mu^{\pm} \mu^{\pm} \to W_{2}^{\pm} W_{2}^{\pm} \right) \right|^{2} &\approx \frac{g_{2}^{4}}{M_{W_{1}}^{4}} \Big\{ \left( \left| C_{t}^{22} \right|^{2} + \left| C_{u}^{22} \right|^{2} \right) M_{W_{2}}^{2} (2p_{1} \cdot k_{1}p_{1} \cdot k_{2} + \\ & \qquad M_{W_{2}}^{2} p_{1} \cdot p_{2}/2) + 2 \left| C_{t}^{22} \right|^{2} k_{1} \cdot k_{2} (p_{1} \cdot k_{1})^{2} + 2 \left| C_{u}^{11} \right|^{2} k_{1} \cdot k_{2} (p_{1} \cdot k_{2})^{2} + \\ & \qquad \mathcal{R} \left( C_{t}^{22} C_{u}^{22*} \right) [2p_{1} \cdot p_{2} (k_{1} \cdot k_{2})^{2} + M_{W_{2}}^{2} \left( 3M_{W_{2}}^{2} p_{1} \cdot p_{2} - 4p_{1} \cdot k_{1}p_{1} \cdot k_{2} \right) - \\ & \qquad 2k_{1} \cdot k_{2} ((p_{1} \cdot k_{1})^{2} + (p_{1} \cdot k_{2})^{2}) \Big\} \end{split}$$

where

$$\begin{aligned} C_t^{11} &= \cos^2 \zeta \left( S_{2j} \right)^2 \frac{M_{N_j}}{t - M_{N_j}^2} + \frac{2\sqrt{2}Y_{L,22}v_L}{s - M_{\Delta_L}^2 + i\Gamma_{\Delta_L}M_{\Delta_L}} \\ C_u^{11} &= \cos^2 \zeta \left( S_{2j} \right)^2 \frac{M_{N_j}}{u - M_{N_j}^2} + \frac{2\sqrt{2}Y_{L,22}v_L}{s - M_{\Delta_L}^2 + i\Gamma_{\Delta_L}M_{\Delta_j}} \\ C_t^{12} &= \cos^2 \zeta \left( \frac{T_{2j}^*U_{2j}}{t - m_{\nu_j}^2} + \frac{V_{2j}^*S_{2j}}{t - M_{N_j}^2} \right) \\ C_u^{12} &= \cos^2 \zeta \left( \frac{T_{2j}^*U_{2j}}{u - m_{\nu_j}^2} + \frac{V_{2j}^*S_{2j}}{u - M_{N_j}^2} \right) \end{aligned}$$

$$C_{t}^{22} = \cos^{2} \zeta \left( V_{2j}^{*} \right)^{2} \frac{M_{N_{j}}}{t - M_{N_{j}}^{2}} + \frac{2\sqrt{2}Y_{R,22}v_{R}}{s - M_{\Delta_{R}}^{2} + i\Gamma_{\Delta_{R}}M_{\Delta_{R}}}$$
$$C_{u}^{22} = \cos^{2} \zeta \left( V_{2j}^{*} \right)^{2} \frac{M_{N_{j}}}{u - M_{N_{j}}^{2}} + \frac{2\sqrt{2}Y_{R,22}v_{R}}{s - M_{\Delta_{R}}^{2} + i\Gamma_{\Delta_{R}}M_{\Delta_{R}}}$$

 $|\mathcal{M}(\mu^{\pm}\mu^{\pm} \to W_1^{\pm}W_1^{\pm})|^2$  in **TI-NP** and **TII-NP** can be obtained by setting  $Y_{L,22} = 0$ ;  $|\mathcal{M}(\mu^{\pm}\mu^{\pm} \to W_2^{\pm}W_2^{\pm})|^2$ in **TI-NP** can be obtained by setting  $Y_{R,22} = 0$ . The results of  $\sigma(\mu^{\pm}\mu^{\pm} \to W_1^{\pm}W_1^{\pm})$  in **TI-NP** and **TII-NP** are similar; The results of  $\sigma(\mu^{\pm}\mu^{\pm} \to W_1^{\pm}W_2^{\pm})$  in **TII-NP** and **TIII-NP** are similar.

#### **Experimental constraints**

For simplicity, the mass matrix of heavy neutrinos is assumed to be diagonal, the Yukawa coupling of charged leptons and doubly charged Higgs is assumed to be diagonal  $Y_L = \text{diag}(Y_{ee}, Y_{\mu\mu}, Y_{\tau\tau})$ . Considering the sensitivity of future HL-LHC, we take

 $S_{\mu}^2 \leq 0.01, \ S_{\tau}^2 \leq 0.01$ 

 $Y_{ee}$  and  $S_e^2$  suffer strict constraints from the nuclear  $0v2\beta$  decays experimentally,

 $Y_{ee} < 0.04, \ S_e^2 \le 10^{-5}$ 

The experimental constraints on the right-handed W boson, doubly charged Higgs and neutrino masses

$$M_{W_2} > 4.8 \text{ TeV}, \ M_{\Delta_L^{\pm\pm}} > 0.8 \text{ TeV}, \ M_{\Delta_R^{\pm\pm}} > 0.65 \text{ TeV}, \ \sum_{i=1}^{3} m_{\nu_i} < 0.12 \text{ eV}$$

#### **Numerical results for the LFV processes**



Fig. 3. Taking  $M_{N_1} = 1.0 \text{ TeV}, M_{N_3} = 3.0 \text{ TeV}, S_e^2 = 10^{-5}, S_\tau^2 = 0.01 \text{ and } \sqrt{s} = 5 \text{ TeV}, \sigma(\mu^{\pm}\mu^{\pm} \rightarrow l^{\pm}l^{\pm})$  versus  $S_{\mu}^2$  are plotted, where (a) for  $l = \tau$ , (b) for l = e. The solid, dashed and dotted curves denote the results for  $M_{N_2} = 1.0, 2.0, 3.0 \text{ TeV}$ . The black curves denotes the results in **TII-NP**, the red curves denotes the results in **TII-NP** with  $M_{W_2} = 5 \text{ TeV}$ , the blue curves denotes the results in **TII-NP** with  $M_{W_2} = 5 \text{ TeV}$ , the blue curves denotes the results in **TIII-NP** with  $M_{W_2} = 5.0 \text{ TeV}, M_{\Delta^{\pm\pm}} = 3.0 \text{ TeV}, Y_{ee} = 0.04, Y_{\mu\mu} = 1.0, Y_{\tau\tau} = 1.0$ .



Fig. 4. Taking  $M_{N_1} = 1.0 \text{ TeV}, M_{N_2} = 2.0 \text{ TeV}, M_{N_3} = 3.0 \text{ TeV}, S_e^2 = 10^{-5}, S_\mu^2 = 10^{-4} \text{ and } S_\tau^2 = 0.01, \sigma(\mu^{\pm}\mu^{\pm} \rightarrow l^{\pm}l^{\pm}) \text{ versus } \sqrt{s}$  are plotted, where (a) for  $l = \tau$ , (b) for l = e. The black curves denotes the results in **TI-NP**. The red solid, dashed, dotted curves denote the results in **TII-NP** with  $M_{W_2} = 5$  TeV for  $M_{W_2} = 5, 7, 9$  TeV respectively. The blue curves denote the results in **TIII-NP** with  $M_{W_2} = 5.0 \text{ TeV}, Y_{ee} = 0.04, Y_{\mu\mu} = 1.0, Y_{\tau\tau} = 1.0$ , where the solid curves denote the results for  $M_{\Delta_L^{\pm\pm}} = 10.0 \text{ TeV}, M_{\Delta_R^{\pm\pm}} = 11.0 \text{ TeV}$ , the dashed curves denote the results for  $M_{\Delta_L^{\pm\pm}} = 5.0 \text{ TeV}, M_{\Delta_R^{\pm\pm}} = 15.0 \text{ TeV}$ , the dotted curves denote the results for  $M_{\Delta_L^{\pm\pm}} = 4.0 \text{ TeV}$ .



Fig. 5.  $M_{N_1} = 1.0 \text{ TeV}$ ,  $M_{N_2} = 2.0 \text{ TeV}$ ,  $M_{N_3} = 3.0 \text{ TeV}$ ,  $S_e^2 = 10^{-5}$ ,  $S_\tau^2 = 0.01$  and  $\sqrt{s} = 5.0 \text{ TeV}$ , the angle distributions of the processes  $\mu^{\pm}\mu^{\pm} \rightarrow l^{\pm}l^{\pm}$  are plotted, where (a) for  $l = \tau$ , (b) for l = e. The solid, dashed and dotted curves denote the results for  $S_{\mu}^2 = 10^{-6}$ ,  $10^{-4}$ ,  $10^{-2}$  respectively. The black curves denotes the results in **TI-NP**, the red curves denotes the results in **TII-NP** with  $M_{W_2} = 5.0 \text{ TeV}$ , the blue curves denotes the results in **TII-NP** with  $M_{W_2} = 5.0 \text{ TeV}$ , the blue curves denotes the results in **TIII-NP** with  $M_{W_2} = 5.0 \text{ TeV}$ ,  $M_{\Delta_L^{\pm\pm}} = M_{\Delta_R^{\pm\pm}} = 3.0 \text{ TeV}$ ,  $Y_{ee} = 0.04$ ,  $Y_{\mu\mu} = 1.0$  and  $Y_{\tau\tau} = 1.0$ .

#### **Numerical results for the LNV processes**



Fig. 6. Taking  $M_{N_2} = 2.0 \text{TeV}, \sqrt{s} = 15.0 \text{ TeV}, \sigma \left(\mu^{\pm} \mu^{\pm} \rightarrow W_i^{\pm} W_j^{\pm}\right)$  versus  $S_{\mu}^2$  are plotted, where the black solid curve denotes the results in **TI-NP** for  $W_i W_j = W_1 W_1$ , the red dashed and red dotted curves denote the results in **TII-NP** with  $M_{W_2} = 5.0 \text{ TeV}$  for  $W_i W_j = W_1 W_2$  and  $W_i W_j = W_2 W_2$  respectively, the blue solid and blue dotted curves denote the results in **TIII-NP** with  $M_{W_2} = 5.0 \text{ TeV}$  for  $M_{\Delta_L^{\pm\pm}} = 10.0 \text{ TeV}, M_{\Delta_R^{\pm\pm}} = 11.0 \text{ TeV}, Y_{\mu\mu} = 1.0 \text{ for } W_i W_j = W_1 W_1$  and  $W_i W_j = W_2 W_2$  respectively.



Fig. 7. Taking  $M_{N_2} = 2.0 \text{ TeV}$ ,  $S_{\mu}^2 = 10^{-4}$ ,  $\sigma \left( \mu^{\pm} \mu^{\pm} \rightarrow W_i^{\pm} W_j^{\pm} \right)$  versus  $\sqrt{s}$  are plotted, where the solid, dashed, dotted curves denote  $W_i W_j = W_1 W_1$ ,  $W_1 W_j = W_1 W_2$ ,  $W_1 W_j = W_2 W_2$  respectively. The black curves denotes the results in **TII-NP**. The red curves denotes the results in **TII-NP** with  $M_{W_2} = 5.0 \text{ TeV}$ . The blue curves denotes the results in **TIII-NP** with  $M_{W_2} = 5.0 \text{ TeV}$ . The blue curves denotes the results in **TIII-NP** with  $M_{W_2} = 5.0 \text{ TeV}$ . The blue curves denotes the results in **TIII-NP** with  $M_{W_2} = 5.0 \text{ TeV}$ . The blue curves denotes the results in **TIII-NP** with  $M_{W_2} = 5.0 \text{ TeV}$ ,  $Y_{\mu\mu} = 1.0$ , where (a) for  $M_{\Delta_L^{\pm\pm}} = 10.0 \text{ TeV}$ ,  $M_{\Delta_R^{\pm\pm}} = 11.0 \text{ TeV}$  and (b) for  $M_{\Delta_L^{\pm\pm}} = 5.0 \text{ TeV}$ ,  $M_{\Delta_R^{\pm\pm}} = 15.0 \text{ TeV}$ .



Fig. 8. Taking  $M_{N_2} = 2.0$  TeV,  $S_{\mu}^2 = 10^{-4}$ ,  $M_{W_2} = 5.0$  TeV, the angle distributions of the LNV processes are plotted. (a): the angle distributions of  $\mu^{\pm}\mu^{\pm} \rightarrow W_1^{\pm}W_1^{\pm}$  with  $\sqrt{s} = 5.0$  TeV, the black curve denotes the results in **TI-NP**, the blue solid, blue dashed, blue dotted curves denote the results in **TIII-NP** with  $Y_{\mu\mu} = 1.0$  for  $M_{\Delta_L^{\pm\pm}} = M_{\Delta_R^{\pm\pm}} = 3$ , 5, 7 TeV respectively. (b): the angle distributions of  $\mu^{\pm}\mu^{\pm} \rightarrow W_1^{\pm}W_2^{\pm}$  with  $\sqrt{s} = 7.0$  TeV, where the red curve denotes the results in **TII-NP**. (c): the angle distributions of  $\mu^{\pm}\mu^{\pm} \rightarrow W_2^{\pm}W_2^{\pm}$  with  $\sqrt{s} = 12.0$  TeV, the red curve denotes the results in **TII-NP**, the blue solid, blue dashed, blue dotted curves denote the results in **TIII-NP** with  $Y_{\mu\mu} = 1.0$  for  $M_{\Delta_L^{\pm\pm}} = M_{\Delta_R^{\pm\pm}} = 7, 12, 15$  TeV respectively.

1. The contributions in the three types of NP to the LFV and LNV processes can be identified by observing the cross sections of these processes at the same-sign muon colliders.

2. The angle distributions of LFV processes are flat in all three types of NP, and observing the angle distributions of LNV processes at the same-sign muon colliders can help to identify the contributions to these LNV processes come from Majorana neutrinos or doubly charged Higgs.

3. The high-energy same-sign muon colliders are effective to observe the doubly charged Higgs through the LFV and LNV processes.



Jin-Lei Yang 2024.04.20