# LFV, LNV processes and new physics at the same-sign muon colliders 

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## 1 Introduction

- The motivation of studying the LFV and LNV processes at the same-sign muon colliders

$$
\begin{aligned}
& \mathrm{LFV}: \mu^{ \pm} \mu^{ \pm} \rightarrow \tau^{ \pm} \tau^{ \pm}, \mu^{ \pm} \mu^{ \pm} \rightarrow e^{ \pm} e^{ \pm} \\
& \mathrm{LNV}: \mu^{ \pm} \mu^{ \pm} \rightarrow W^{ \pm} W^{ \pm}
\end{aligned}
$$

1, the LFV processes do not depend on the large flavor mixing parameters, for example


## 1 Introduction

2 , strict constraints from the nuclear $0 v 2 \beta$ decays


3 , the muon collider can reach higher collider energy comparted with the electron collider

$$
\frac{m_{\mu}}{m_{e}} \approx 207
$$

## 1 Introduction

## - Possible contributions in NP



- The meaning of observing LFV and LNV at the same-sign muon collider

1, providing definite evidences of NP beyond the SM
2 , identifying the nature of neutrinos
3, searching new particle: doubly charged Higgs

## 2 Theoretical calculation

## Types of NP

|  | Particle contents contribute to LFV and LNV in NP |
| :--- | :--- |
| Type I new physics (TI-NP) | Majorana neutrinos; $W_{L}$ |
| Type II new physics (TII-NP) | Majorana neutrinos; $W_{L} ; W_{R}$ |
| Type III new physics (TIII-NP) | Majorana neutrinos; $W_{L} ; W_{R} ;$ doubly charged Higgs |
|  | TI-NP: B-LSSM, NMSSM,.. <br> representative NP models |
| TII-NP: LRSM without triplet |  |

The needed Lagrangian for TI-NP can be extracted from B-LSSM, and for TII-NP, TIII-NP can be extracted from LRSM.

## 2 Theoretical calculation

## - Majorana neutrinos in the B-LSSM

New $U(1)_{B-L}$, three right-handed neutrinos and two scalar singlets are introduced in the B-LSSM, then the tiny neutrino masses can be obtained by the Type-I seesaw mechanism

$$
\left(\begin{array}{cc}
0 & M_{D}^{T} \\
M_{D} & M_{R}
\end{array}\right)
$$

The mass matrix above can be diagonalized by a unitary matrix $U_{v}$

$$
U_{v}^{T}\left(\begin{array}{cc}
0 & M_{D}^{T} \\
M_{D} & M_{R}
\end{array}\right) U_{v}=\left(\begin{array}{cc}
\widehat{m}_{v} & 0 \\
0 & \widehat{M}_{N}
\end{array}\right)
$$

where $\widehat{m}_{v}=\operatorname{diag}\left(m_{v_{1}}, m_{v_{2}}, m_{v_{3}}\right)$, $\widehat{m}_{N}=\operatorname{diag}\left(m_{N_{1}}, m_{N_{2}}, m_{N_{3}}\right)$. The unitary matrix $U_{v}$ reads

$$
U_{v}=\left(\begin{array}{ll}
U & S \\
T & V
\end{array}\right)
$$

## 2 Theoretical calculation

The Lagrangian involving leptons and W boson in the B-LSSM

$$
\mathcal{L}_{W}^{\mathrm{BL}}=\frac{i g_{2}}{\sqrt{2}} \sum_{j=1}^{3}\left[U_{i j} \bar{e}_{i} \gamma^{\mu} P_{L} v_{j} W_{L, \mu}+S_{i j} \bar{e}_{i} \gamma^{\mu} P_{L} N_{j} W_{L, \mu}+h . c .\right]
$$

The relevant couplings of leptons and Goldstones are

$$
\begin{aligned}
\mathcal{L}_{G}^{\mathrm{BL}}= & \frac{i g_{2}}{\sqrt{2} M_{W_{L}}} \sum_{j=1}^{3}\left\{\bar{e}_{i}\left[\left(M_{D}^{\mathrm{t}} \cdot T^{*}\right)_{i j} P_{R}-\left(\widehat{m}_{l} \cdot U\right)_{i j} P_{L}\right] v_{j} G_{L}+\right. \\
& \left.\bar{e}_{i}\left[\left(M_{D}^{\mathrm{t}} \cdot V^{*}\right)_{i j} P_{R}-\left(\widehat{m}_{l} \cdot S\right)_{i j} P_{L}\right] N_{j} G_{L}+\text { h.c. }\right\}
\end{aligned}
$$

We can define $S_{i}^{2}=\sum_{j=1}^{3}\left|S_{i j}^{2}\right|(i=e, \mu, \tau)$ to describe the strength of light-heavy neutrino mixings.

## 2 Theoretical calculation

## - Majorana neutrinos in the LRSM

New $S U(2)_{R}$, three right-handed neutrinos and two scalar trilets are introduced in the LRSM, then the tiny neutrino masses can be obtained by the Type-I+II seesaw mechanism

$$
\left(\begin{array}{ll}
M_{L} & M_{D}^{T} \\
M_{D} & M_{R}
\end{array}\right)
$$

The mass matrix can also be diagonalized by the unitary matrix $U_{v}$. The W boson mass matrix in the
LRSM can be written as

$$
\frac{g_{2}^{2}}{4}\left(W_{L}, W_{R}\right)\left(\begin{array}{cc}
v_{1}^{2}+v_{2}^{2}+2 v_{L}^{2} & 2 v_{1} v_{2} \\
2 v_{1} v_{2} & v_{1}^{2}+v_{2}^{2}+2 v_{R}^{2}
\end{array}\right)\binom{W_{L}}{W_{R}}
$$

The physical masses of the two W bosons are

$$
M_{W_{1}} \approx \frac{g_{2}}{2}\left(v_{1}^{2}+v_{2}^{2}\right)^{\frac{1}{2}}, \quad M_{W_{2}} \approx \frac{g_{2}}{\sqrt{2}} v_{R}
$$

## 2 Theoretical calculation

The mass eigenstates $W_{1}, W_{2}$ are related to the gauge eigenstates $W_{L}, W_{R}$ by

$$
\binom{W_{1}}{W_{2}}=\left(\begin{array}{cc}
\cos \zeta & \sin \zeta \\
-\sin \zeta & \cos \zeta
\end{array}\right)\binom{W_{L}}{W_{R}}
$$

Where $\tan \zeta=\frac{2 v_{1} v_{2}}{\left(v_{R}^{2}-v_{L}^{2}\right)}$. The Lagrangian involving leptons and W boson in the LRSM

$$
\begin{aligned}
\mathcal{L}_{W}^{\mathrm{LR}}= & \frac{i g_{2}}{\sqrt{2}} \sum_{j=1}^{3}\left[\bar{e}_{i}\left(\cos \zeta U_{i j} \gamma^{\mu} P_{L}+\sin \zeta T_{i j}^{*} \gamma^{\mu} P_{R}\right) v_{j} W_{1, \mu}+\bar{e}_{i}\left(\cos \zeta T_{i j}^{*} \gamma^{\mu} P_{R}-\sin \zeta U_{i j} \gamma^{\mu} P_{L}\right) v_{j} W_{2, \mu}\right. \\
& \left.+\bar{e}_{i}\left(\cos \zeta S_{i j} \gamma^{\mu} P_{L}+\sin \zeta V_{i j}^{*} \gamma^{\mu} P_{R}\right) N_{j} W_{1, \mu}+\bar{e}_{i}\left(\cos \zeta V_{i j}^{*} \gamma^{\mu} P_{R}-\sin \zeta S_{i j} \gamma^{\mu} P_{L}\right) N_{j} W_{2, \mu}+\text { h.c. }\right]
\end{aligned}
$$

The relevant couplings of leptons and Goldstones in the LRSM are

$$
\begin{gathered}
\mathcal{L}_{G}^{\mathrm{LR}}=\frac{i g_{2}}{\sqrt{2 M_{W_{L}}}} \sum_{j=1}^{3}\left\{\bar{e}_{i}\left[\lambda_{1, i j} P_{L}+\lambda_{2, i j} P_{R}\right] v_{j} G_{L}+\bar{e}_{i}\left[\lambda_{3, i j} P_{L}+\lambda_{4, i j} P_{R}\right] N_{j} G_{L}\right. \\
\left.+\bar{e}_{i}\left(\lambda_{5, i j} P_{L}\right) v_{j} G_{R}+\bar{e}_{i}\left(\lambda_{6, i j} P_{L}\right) N_{j} G_{R}+\text { h.c. }\right\}
\end{gathered}
$$

where $\lambda_{1}=-m_{l}^{\dagger} \cdot U, \lambda_{2}=\widehat{M}_{D}^{\dagger} \cdot T^{*}, \lambda_{3}=-m_{l}^{\dagger} \cdot S, \lambda_{4}=\widehat{M}_{D}^{\dagger} \cdot V^{*}, \quad \lambda_{5}=\frac{M_{W_{1}}}{M_{W_{2}}} M_{R}^{\dagger} \cdot T, \quad \lambda_{6}=\frac{M_{W_{1}}}{M_{W_{2}}} M_{R}^{\dagger} \cdot V^{*}$

## 2 Theoretical calculation

The Lagrangian involving leptons and doubly charged Higgs in the LRSM

$$
\mathcal{L}_{\Delta l l}^{\mathrm{LR}}=i 2 Y_{L, i j} \bar{e} P_{L} e_{j}^{C} \Delta_{L}^{--}+i 2 Y_{R, i j} \bar{e} P_{R} e_{j}^{C} \Delta_{R}^{--}+h . c .
$$

The Lagrangian involving W bosons and doubly charged Higgs in the LRSM

$$
\begin{aligned}
\mathcal{L}_{\Delta W W}^{\mathrm{LR}}= & i \sqrt{2} g_{2}^{2} v_{L} \Delta_{L}^{--} W_{1}^{\mu+} W_{1 \mu}^{+}+i \sqrt{2} g_{2}^{2} v_{L} \sin \zeta \Delta_{L}^{--} W_{1}^{\mu+} W_{2 \mu}^{+} \\
& +i \sqrt{2} g_{2}^{2} v_{R} \sin \zeta \Delta_{R}^{--} W_{1}^{\mu+} W_{2 \mu}^{+}+i \sqrt{2} g_{2}^{2} v_{R} \Delta_{R}^{--} W_{2}^{\mu+} W_{2 \mu}^{+}+h . c .
\end{aligned}
$$

## - The relevant Lagrangian for TI-NP, TII-NP and TIII-NP

TI-NP: $\mathcal{L}_{W}^{\mathrm{BL}}, ~ \mathcal{L}_{G}^{\mathrm{BL}}$
TII-NP: $\mathcal{L}_{W}^{\mathrm{LR}}, \mathcal{L}_{G}^{\mathrm{LR}}$
TIII-NP: $\mathcal{L}_{W}^{\mathrm{LR}}, ~ \mathcal{L}_{G}^{\mathrm{LR}}, ~ \mathcal{L}_{\Delta l l}^{\mathrm{LR}}, ~ \mathcal{L}_{\Delta W W}^{\mathrm{LR}}$

## 2 Theoretical calculation

- The leading order contributions from Majorana neutrinos and doubly charged Higgs to the LFV processes


Fig. 1. The leading order contributions from Majorana neutrinos (left) and doubly charged Higgs (right) to the LFV processes in the Feynman gauge.

## 2 Theoretical calculation

The amplitude can be simplified by neglecting charged lepton masses

$$
\mathcal{M}\left(\mu^{ \pm} \mu^{ \pm} \rightarrow l^{ \pm} l^{ \pm}\right)=\frac{i}{16 \pi^{2}} \sum_{i=1}^{7} \sum_{X, Y=L, R} C_{i}^{X Y} \mathcal{O}_{i}^{X Y}
$$

where

$$
\begin{aligned}
& \mathcal{O}_{1}^{X Y}=\bar{u}\left(k_{1}\right) \gamma_{\mu} P_{X} u^{c}\left(k_{2}\right) \overline{u^{c}}\left(p_{2}\right) \gamma^{\mu} P_{Y} u\left(p_{1}\right) \\
& \mathcal{O}_{2}^{X Y}=\bar{u}\left(k_{1}\right) p_{1}^{\alpha} \gamma_{\alpha} P_{X} u^{c}\left(k_{2}\right) \overline{u^{c}}\left(p_{2}\right) k_{1}^{\beta} \gamma_{\beta} P_{Y} u\left(p_{1}\right) \\
& \mathcal{O}_{3}^{X Y}=\bar{u}\left(k_{1}\right) P_{X} u^{c}\left(k_{2}\right) \overline{u^{c}}\left(p_{2}\right) P_{Y} u\left(p_{1}\right) \\
& \mathcal{O}_{4}^{X Y}=\bar{u}\left(k_{1}\right) \gamma_{\mu} P_{X} u^{c}\left(k_{2}\right) \overline{u^{c}}\left(p_{2}\right) \gamma^{\mu} k_{1}^{\beta} \gamma_{\beta} P_{Y} u\left(p_{1}\right) \\
& \mathcal{O}_{5}^{X Y}=\bar{u}\left(k_{1}\right) \gamma_{\mu} p_{1}^{\alpha} \gamma_{\alpha} P_{X} u^{c}\left(k_{2}\right) \overline{u^{c}}\left(p_{2}\right) \gamma^{\mu} P_{Y} u\left(p_{1}\right) \\
& \mathcal{O}_{6}^{X Y}=\bar{u}\left(k_{1}\right) P_{X} u^{c}\left(k_{2}\right) \overline{u^{c}}\left(p_{2}\right) k_{1}^{\beta} \gamma_{\beta} P_{Y} u\left(p_{1}\right) \\
& \mathcal{O}_{7}^{X Y}=\bar{u}\left(k_{1}\right) p_{1}^{\alpha} \gamma_{\alpha} P_{X} u^{c}\left(k_{2}\right) \overline{u^{c}}\left(p_{2}\right) P_{Y} u\left(p_{1}\right)
\end{aligned}
$$

The coefficients contributed by doubly charged Higgs can be read directly

$$
C_{3}^{L R}\left(\Delta_{L}^{ \pm \pm}\right)=\frac{4 Y_{L, 22} Y_{L, j j}}{\left(p_{1}+p_{2}\right)^{2}-M_{\Delta_{L}^{ \pm \pm}}^{2}+i M_{\Delta_{L}^{ \pm \pm}} \Gamma_{\Delta_{L}^{ \pm \pm}}}, \quad C_{3}^{R L}\left(\Delta_{R}^{ \pm \pm}\right)=\frac{4 Y_{R, 22} Y_{R, j j}}{\left(p_{1}+p_{2}\right)^{2}-M_{\Delta_{R}^{ \pm \pm}}^{2}+i M_{\Delta_{R}^{ \pm \pm}} \Gamma_{\Delta_{R}^{ \pm \pm}}}
$$

## 2 Theoretical calculation

The dominant decay channels of doubly charged Higgs:

$$
\begin{aligned}
& \Delta_{L}^{ \pm \pm} \rightarrow l^{ \pm} l^{ \pm}, W_{1}^{ \pm} W_{1}^{ \pm} \\
& \Delta_{R}^{ \pm \pm} \rightarrow l^{ \pm} l^{ \pm}, W_{2}^{ \pm} W_{2}^{ \pm(*)}, W_{2}^{ \pm} W_{2}^{ \pm}
\end{aligned}
$$

- The leading order contributions from Majorana neutrinos and doubly charged


## Higgs to the LNV processes



Fig. 2. The leading order contributions from Majorana neutrinos (1,2) and doubly charged Higgs (3) to the LNV processes.

## 2 Theoretical calculation

Summing up the fermions‘ spin and gauge bosons' polarizations, the squared amplitude for the LNV processes can be obtained

$$
\begin{aligned}
\mid \mathcal{M}\left(\mu^{ \pm} \mu^{ \pm} \rightarrow\right. & \left.W_{1}^{ \pm} W_{1}^{ \pm}\right)\left.\right|^{2} \approx \frac{g_{2}^{4}}{M_{W_{1}}^{4}}\left\{( | C _ { t } ^ { 1 1 } | ^ { 2 } + | C _ { u } ^ { 1 1 } | ^ { 2 } ) M _ { W _ { 1 } } ^ { 2 } \left(2 p_{1} \cdot k_{1} p_{1} \cdot k_{2}+\right.\right. \\
& \left.M_{W_{1}}^{2} p_{1} \cdot p_{2} / 2\right)+2\left|C_{t}^{11}\right|^{2} k_{1} \cdot k_{2}\left(p_{1} \cdot k_{1}\right)^{2}+2\left|C_{u}^{11}\right|^{2} k_{1} \cdot k_{2}\left(p_{1} \cdot k_{2}\right)^{2}+ \\
& \mathcal{R}\left(C_{t}^{11} C_{u}^{11 *}\right)\left[2 p_{1} \cdot p_{2}\left(k_{1} \cdot k_{2}\right)^{2}+M_{W_{1}}^{2}\left(3 M_{W_{1}}^{2} p_{1} \cdot p_{2}-4 p_{1} \cdot k_{1} p_{1} \cdot k_{2}\right)-\right. \\
& \left.2 k_{1} \cdot k_{2}\left(\left(p_{1} \cdot k_{1}\right)^{2}+\left(p_{1} \cdot k_{2}\right)^{2}\right)\right\} \\
\mid \mathcal{M}\left(\mu^{ \pm} \mu^{ \pm} \rightarrow\right. & \left.W_{1}^{ \pm} W_{2}^{ \pm}\right)\left.\right|^{2} \approx \frac{g_{2}^{4}}{2 M_{W_{1}}^{2} M_{W_{2}}^{2}}\left\{| C _ { t } ^ { 1 2 } | ^ { 2 } \left[4 M _ { W _ { 1 } } ^ { 2 } M _ { W _ { 2 } } ^ { 2 } p _ { 1 } \cdot k _ { 1 } \left(p_{2} \cdot k_{1}-\right.\right.\right. \\
& \left.p_{1} \cdot p_{2}\right)+8 M_{W_{1}}^{2} p_{1} \cdot k_{1} p_{2} \cdot k_{2}\left(k_{1} \cdot k_{2}-p_{1} \cdot k_{2}\right)-M_{W_{1}}^{4}\left(M_{W_{2}}^{2} p_{1} \cdot p_{2}+\right. \\
& \left.\left.2 p_{2} \cdot k_{2} p_{1} \cdot k_{2}\right)+4\left(p_{1} \cdot k_{1}\right)^{2}\left(M_{W_{2}}^{2} p_{1} \cdot p_{2}+2 p_{2} \cdot k_{2} p_{1} \cdot k_{2}\right)\right]+ \\
& \left|C_{u}^{12}\right|^{2}\left[4 M_{W_{1}}^{2} M_{W_{2}}^{2} p_{1} \cdot k_{2}\left(p_{2} \cdot k_{2}-p_{1} \cdot p_{2}\right)+8 M_{W_{2}}^{2} p_{1} \cdot k_{2} p_{2} \cdot k_{1}\left(k_{1} \cdot k_{2}\right.\right. \\
& \left.-p_{1} \cdot k_{1}\right)-M_{W_{2}}^{4}\left(M_{W_{1}}^{2} p_{1} \cdot p_{2}+2 p_{2} \cdot k_{1} p_{1} \cdot k_{1}\right)+ \\
& \left.\left.4\left(p_{1} \cdot k_{2}\right)^{2}\left(M_{W_{1}}^{2} p_{1} \cdot p_{2}+2 p_{2} \cdot k_{1} p_{1} \cdot k_{1}\right)\right]\right\}
\end{aligned}
$$

## 2 Theoretical calculation

$$
\begin{aligned}
\mid \mathcal{M}\left(\mu^{ \pm} \mu^{ \pm} \rightarrow\right. & \left.W_{2}^{ \pm} W_{2}^{ \pm}\right)\left.\right|^{2} \approx \frac{g_{2}^{4}}{M_{W_{1}}^{4}}\left\{( | C _ { t } ^ { 2 2 } | ^ { 2 } + | C _ { u } ^ { 2 2 } | ^ { 2 } ) M _ { W _ { 2 } } ^ { 2 } \left(2 p_{1} \cdot k_{1} p_{1} \cdot k_{2}+\right.\right. \\
& \left.M_{W_{2}}^{2} p_{1} \cdot p_{2} / 2\right)+2\left|C_{t}^{22}\right|^{2} k_{1} \cdot k_{2}\left(p_{1} \cdot k_{1}\right)^{2}+2\left|C_{u}^{11}\right|^{2} k_{1} \cdot k_{2}\left(p_{1} \cdot k_{2}\right)^{2}+ \\
& \mathcal{R}\left(C_{t}^{22} C_{u}^{22 *}\right)\left[2 p_{1} \cdot p_{2}\left(k_{1} \cdot k_{2}\right)^{2}+M_{W_{2}}^{2}\left(3 M_{W_{2}}^{2} p_{1} \cdot p_{2}-4 p_{1} \cdot k_{1} p_{1} \cdot k_{2}\right)-\right. \\
& \left.2 k_{1} \cdot k_{2}\left(\left(p_{1} \cdot k_{1}\right)^{2}+\left(p_{1} \cdot k_{2}\right)^{2}\right)\right\}
\end{aligned}
$$

where

$$
\begin{aligned}
C_{t}^{11} & =\cos ^{2} \zeta\left(S_{2 j}\right)^{2} \frac{M_{N_{j}}}{t-M_{N_{j}}^{2}}+\frac{2 \sqrt{2} Y_{L, 22} v_{L}}{s-M_{\Delta_{L}}^{2}+i \Gamma_{\Delta_{L}} M_{\Delta_{L}}} \\
C_{u}^{11} & =\cos ^{2} \zeta\left(S_{2 j}\right)^{2} \frac{M_{N_{j}}}{u-M_{N_{j}}^{2}}+\frac{2 \sqrt{2} Y_{L, 22} v_{L}}{s-M_{\Delta_{L}}^{2}+i \Gamma_{\Delta_{L}} M_{\Delta_{L}}} \\
C_{t}^{12} & =\cos ^{2} \zeta\left(\frac{T_{2 j}^{*} U_{2 j}}{t-m_{v_{j}}^{2}}+\frac{V_{2 j}^{*} S_{2 j}}{t-M_{N_{j}}^{2}}\right) \\
C_{u}^{12} & =\cos ^{2} \zeta\left(\frac{T_{2 j}^{*} U_{2 j}}{u-m_{v_{j}}^{2}}+\frac{V_{2 j}^{*} S_{2 j}}{u-M_{N_{j}}^{2}}\right)
\end{aligned}
$$

## 2 Theoretical calculation

$$
\begin{aligned}
& C_{t}^{22}=\cos ^{2} \zeta\left(V_{2 j}^{*}\right)^{2} \frac{M_{N_{j}}}{t-M_{N_{j}}^{2}}+\frac{2 \sqrt{2} Y_{R, 22} v_{R}}{s-M_{\Delta_{R}}^{2}+i \Gamma_{\Delta_{R}} M_{\Delta_{R}}} \\
& C_{u}^{22}=\cos ^{2} \zeta\left(V_{2 j}^{*}\right)^{2} \frac{M_{N_{j}}}{u-M_{N_{j}}^{2}}+\frac{2 \sqrt{2} Y_{R, 22} v_{R}}{s-M_{\Delta_{R}}^{2}+i \Gamma_{\Delta_{R}} M_{\Delta_{R}}}
\end{aligned}
$$

$\left|\mathcal{M}\left(\mu^{ \pm} \mu^{ \pm} \rightarrow W_{1}^{ \pm} W_{1}^{ \pm}\right)\right|^{2}$ in TI-NP and TII-NP can be obtained by setting $Y_{L, 22}=0 ;\left|\mathcal{M}\left(\mu^{ \pm} \mu^{ \pm} \rightarrow W_{2}^{ \pm} W_{2}^{ \pm}\right)\right|^{2}$ in TI-NP can be obtained by setting $Y_{R, 22}=0$. The results of $\sigma\left(\mu^{ \pm} \mu^{ \pm} \rightarrow W_{1}^{ \pm} W_{1}^{ \pm}\right)$in TI-NP and TII-NP are similar; The results of $\sigma\left(\mu^{ \pm} \mu^{ \pm} \rightarrow W_{1}^{ \pm} W_{2}^{ \pm}\right)$in TII-NP and TIII-NP are similar.

## 3 Numerical results

## - Experimental constraints

For simplicity, the mass matrix of heavy neutrinos is assumed to be diagonal, the Yukawa coupling of charged leptons and doubly charged Higgs is assumed to be diagonal $Y_{L}=\operatorname{diag}\left(Y_{e e}, Y_{\mu \mu}, Y_{\tau \tau}\right)$. Considering the sensitivity of future HL-LHC, we take

$$
S_{\mu}^{2} \leq 0.01, \quad S_{\tau}^{2} \leq 0.01
$$

$Y_{e e}$ and $S_{e}^{2}$ suffer strict constraints from the nuclear $0 v 2 \beta$ decays experimentally,

$$
Y_{e e}<0.04, \quad S_{e}^{2} \leq 10^{-5}
$$

The experimental constraints on the right-handed W boson, doubly charged Higgs and neutrino masses

$$
M_{W_{2}}>4.8 \mathrm{TeV}, \quad M_{\Delta_{L}^{ \pm \pm}}>0.8 \mathrm{TeV}, M_{\Delta_{R}^{ \pm \pm}}>0.65 \mathrm{TeV}, \quad \sum_{i=1}^{3} m_{v_{i}}<0.12 \mathrm{eV}
$$

## 3 Numerical results

## - Numerical results for the LFV processes



Fig. 3. Taking $M_{N_{1}}=1.0 \mathrm{TeV}, M_{N_{3}}=3.0 \mathrm{TeV}, S_{e}^{2}=10^{-5}, S_{\tau}^{2}=0.01$ and $\sqrt{s}=5 \mathrm{TeV}, \sigma\left(\mu^{ \pm} \mu^{ \pm} \rightarrow l^{ \pm} l^{ \pm}\right)$versus $S_{\mu}^{2}$ are plotted, where (a) for $l=\tau$, (b) for $l=\mathrm{e}$. The solid, dashed and dotted curves denote the results for $M_{N_{2}}=1.0,2.0,3.0 \mathrm{TeV}$. The black curves denotes the results in TI-NP, the red curves denotes the results in TII-NP with $M_{W_{2}}=5 \mathrm{TeV}$, the blue curves denotes the results in TIII-NP with $M_{W_{2}}=5.0 \mathrm{TeV}, ~ M_{\Delta^{ \pm \pm}}=3.0 \mathrm{TeV}, Y_{e e}=0.04, Y_{\mu \mu}=1.0, Y_{\tau \tau}=1.0$.

## 3 Numerical results




Fig. 4. Taking $M_{N_{1}}=1.0 \mathrm{TeV}, M_{N_{2}}=2.0 \mathrm{TeV}, M_{N_{3}}=3.0 \mathrm{TeV}, S_{e}^{2}=10^{-5}, S_{\mu}^{2}=10^{-4}$ and $S_{\tau}^{2}=0.01, \sigma\left(\mu^{ \pm} \mu^{ \pm} \rightarrow l^{ \pm} l^{ \pm}\right)$versus $\sqrt{s}$ are plotted, where (a) for $l=\tau$, (b) for $l=\mathrm{e}$. The black curves denotes the results in TI-NP. The red solid, dashed, dotted curves denote the results in TII-NP with $M_{W_{2}}=5 \mathrm{TeV}$ for $M_{W_{2}}=5,7,9 \mathrm{TeV}$ respectively. The blue curves denote the results in TIII-NP with $M_{W_{2}}=5.0 \mathrm{TeV}, Y_{e e}=0.04, Y_{\mu \mu}=1.0, Y_{\tau \tau}=1.0$, where the solid curves denote the results for $M_{\Delta_{L}^{ \pm \pm}}=10.0 \mathrm{TeV}, M_{\Delta_{R}^{ \pm \pm}}=$ 11.0 TeV , the dashed curves denote the results for $M_{\Delta_{L}^{ \pm \pm}}=5.0 \mathrm{TeV}, M_{\Delta_{R}^{ \pm \pm}}=15.0 \mathrm{TeV}$, the dotted curves denote the results for $M_{\Delta_{L}^{ \pm \pm}}=2.0 \mathrm{TeV}, M_{\Delta_{R}^{ \pm \pm}}=4.0 \mathrm{TeV}$.

## 3 Numerical results



Fig. 5. $M_{N_{1}}=1.0 \mathrm{TeV}, M_{N_{2}}=2.0 \mathrm{TeV}, M_{N_{3}}=3.0 \mathrm{TeV}, S_{e}^{2}=10^{-5}, S_{\tau}^{2}=0.01$ and $\sqrt{s}=5.0 \mathrm{TeV}$, the angle distributions of the processes $\mu^{ \pm} \mu^{ \pm} \rightarrow l^{ \pm} l^{ \pm}$are plotted, where (a) for $l=\tau$, (b) for $l=$ e. The solid, dashed and dotted curves denote the results for $S_{\mu}^{2}=$ $10^{-6}, ~ 10^{-4}, ~ 10^{-2}$ respectively. The black curves denotes the results in TI-NP, the red curves denotes the results in TII-NP with $M_{W_{2}}=5.0 \mathrm{TeV}$, the blue curves denotes the results in TIII-NP with $M_{W_{2}}=5.0 \mathrm{TeV}, M_{\Delta_{L}^{ \pm \pm}}=M_{\Delta_{R^{ \pm}}^{ \pm}}=3.0 \mathrm{TeV}, Y_{e e}=0.04, Y_{\mu \mu}=$ 1.0 and $Y_{\tau \tau}=1.0$.

## 3 Numerical results

## - Numerical results for the LNV processes



Fig. 6. Taking $M_{N_{2}}=2.0 \mathrm{TeV}, \sqrt{s}=15.0 \mathrm{TeV}, \sigma\left(\mu^{ \pm} \mu^{ \pm} \rightarrow W_{i}^{ \pm} W_{j}^{ \pm}\right)$versus $S_{\mu}^{2}$ are plotted, where the black solid curve denotes the results in TI-NP for $W_{i} W_{j}=W_{1} W_{1}$, the red dashed and red dotted curves denote the results in TII-NP with $M_{W_{2}}=5.0 \mathrm{TeV}$ for $W_{i} W_{j}=W_{1} W_{2}$ and $W_{i} W_{j}=W_{2} W_{2}$ respectively, the blue solid and blue dotted curves denote the results in TIII-NP with $M_{W_{2}}=$ $5.0 \mathrm{TeV}, M_{\Delta_{L}^{ \pm \pm}}=10.0 \mathrm{TeV}, M_{\Delta_{R}^{ \pm \pm}}=11.0 \mathrm{TeV}, Y_{\mu \mu}=1.0$ for $W_{i} W_{j}=W_{1} W_{1}$ and $W_{i} W_{j}=W_{2} W_{2}$ respectively.

## 3 Numerical results



Fig. 7. Taking $M_{N_{2}}=2.0 \mathrm{TeV}, S_{\mu}^{2}=10^{-4}, \sigma\left(\mu^{ \pm} \mu^{ \pm} \rightarrow W_{i}^{ \pm} W_{j}^{ \pm}\right)$versus $\sqrt{s}$ are plotted, where the solid, dashed, dotted curves denote $W_{i} W_{j}=W_{1} W_{1}, W_{1} W_{j}=W_{1} W_{2}, W_{1} W_{j}=W_{2} W_{2}$ respectively. The black curves denotes the results in TI-NP. The red curves denotes the results in TII-NP with $M_{W_{2}}=5.0 \mathrm{TeV}$. The blue curves denotes the results in TIII-NP with $M_{W_{2}}=5.0 \mathrm{TeV}, Y_{\mu \mu}=1.0$, where (a) for $M_{\Delta_{L}^{ \pm \pm}}=10.0 \mathrm{TeV}, M_{\Delta_{R}^{ \pm \pm}}=11.0 \mathrm{TeV}$ and (b) for $M_{\Delta_{L}^{ \pm \pm}}=5.0 \mathrm{TeV}, M_{\Delta_{R}^{ \pm \pm}}=15.0 \mathrm{TeV}$.

## 3 Numerical results



Fig. 8. Taking $M_{N_{2}}=2.0 \mathrm{TeV}, S_{\mu}^{2}=10^{-4}, M_{W_{2}}=5.0 \mathrm{TeV}$, the angle distributions of the LNV processes are plotted. (a): the angle distributions of $\mu^{ \pm} \mu^{ \pm} \rightarrow W_{1}^{ \pm} W_{1}^{ \pm}$with $\sqrt{s}=5.0 \mathrm{TeV}$, the black curve denotes the results in TI-NP, the blue solid, blue dashed, blue dotted curves denote the results in TIII-NP with $Y_{\mu \mu}=1.0$ for $M_{\Delta_{L}^{ \pm}}=M_{\Delta_{R}^{ \pm \pm}}=3,5,7 \mathrm{TeV}$ respectively. (b): the angle distributions of $\mu^{ \pm} \mu^{ \pm} \rightarrow$ $W_{1}{ }^{ \pm} W_{2}^{ \pm}$with $\sqrt{s}=7.0 \mathrm{TeV}$, where the red curve denotes the results in TII-NP. (c): the angle distributions of $\mu^{ \pm} \mu^{ \pm} \rightarrow W_{2}^{ \pm} W_{2}^{ \pm}$with $\sqrt{s}=$ 12.0 TeV , the red curve denotes the results in TII-NP, the blue solid, blue dashed, blue dotted curves denote the results in TIII-NP with $Y_{\mu \mu}=$ 1.0 for $M_{\Delta_{L}^{ \pm \pm}}=M_{\Delta_{R}^{ \pm \pm}}=7,12,15 \mathrm{TeV}$ respectively。

## 4 Summary

1. The contributions in the three types of NP to the LFV and LNV processes can be identified by observing the cross sections of these processes at the same-sign muon colliders.
2. The angle distributions of LFV processes are flat in all three types of NP, and observing the angle distributions of LNV processes at the same-sign muon colliders can help to identify the contributions to these LNV processes come from Majorana neutrinos or doubly charged Higgs.
3. The high-energy same-sign muon colliders are effective to observe the doubly charged Higgs through the LFV and LNV processes.

## THANKS

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