Rare decays of B_c Mesons

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Motivation

- Flavour changing neutral currents occur at electroweak loop level in the Standard Model (SM).
- Several anomalies reported for FCNC $b \to s$ decays.
- $R_{K^{(*)}}$ and some other observables are deviating by $2-3\sigma$ from SM.
- However, recent simultaneous measurements by LHCb on $R_{K^{(*)}}$ in low and central q^2 range shows very good agreement with SM prediction at 0.2σ[1].
- Similarly, $b \to d\ell\ell$ can also serve as important probe as they also follow the same FCNC at quark level.
- Bell[2] and LHCb[3] have provided some important data for the channels $B \to (\rho, \omega, \pi, \eta)\ell^+\ell^-$ and $B_s^0 \to \bar{K}^{*0}\mu^+\mu^-$.
- LHCb also provided the relative ratios for $(b \to d\ell^+\ell^-)/(b \to s\ell^+\ell^-)$ transition in the channels $\mathcal{B}(B^+ \to \pi^+\mu^+\mu^-)/\mathcal{B}(B^+ \to K^+\mu^+\mu^-)$ and $\mathcal{B}(B_s^0 \to \bar{K}^{*0}\mu^+\mu^-)/\mathcal{B}(\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-)$ [4].
- These anomalies can also be tested in rare B_c decays.

[1] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. D **108**, 032002 (2023).
[2] I. Adachi *et al.* (Belle-II and Belle Collaboration), arXiv:2404.08133 [hep-ph].
[3] R. Aaij *et al.* (LHCb Collaboration), JHEP **04**, 029 (2017).
[4] R. Aaij et al. (LHCb Collaboration), JHEP **07**, 125 (2012), JHEP **07**, 020 (2018).

Effective Hamiltonian and Hadronic Matrix Element for $b \to q\ell\ell$

$$\mathcal{H}_{eff}^{SM} = -\frac{4G_F}{\sqrt{2}} V_{tq}^* V_{tb} \left\{ \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu) + \frac{V_{ub}^* V_{uq}}{V_{tb}^* V_{tq}} \sum_{i=1}^{2} C_i(\mu)[\mathcal{O}_i(\mu) - \mathcal{O}_i^u(\mu)] \right\}$$

$$\begin{aligned} \mathcal{M}(B_c \to D_{(s)}^{(*)}\ell^+\ell^-) \;=\; & \frac{G_F\alpha}{\sqrt{2}\pi} V_{tq}^* V_{tb} \Big\{ C_9^{\rm eff} \langle D_{(s)}^{(*)}|\bar{q}\gamma_\mu P_L b|B_c\rangle(\bar{\ell}\gamma^\mu\ell) \\ & + C_{10}\langle D_{(s)}^{(*)}|\bar{q}\gamma_\mu P_L b|B_c\rangle(\bar{\ell}\gamma^\mu\gamma_5\ell) \\ & - \frac{2\bar{m}_b}{q^2} C_7^{\rm eff}\langle D_{(s)}^{(*)}|\bar{q}i\sigma^{\mu\nu}q_\nu P_R b|B_c\rangle(\bar{\ell}\gamma^\mu\ell) \Big\} \end{aligned}$$

where $q = d$ for $B_c \to D^{(*)}\ell^+\ell^-$ and $q = s$ for $B_c \to D_s^{(*)}\ell^+\ell^-$.

Wilson Coefficients

$$C_9^{\rm eff}(\mu) = \xi_1 + (V_{ub}^* V_{uq})/(V_{tb}^* V_{tq})\xi_2,$$

with

$$\begin{aligned}
\xi_1 &= C_9 + C_0 h^{\rm eff}(\hat{m}_c, \hat{s}) - \frac{1}{2} h(1, \hat{s})(4C_3 + 4C_4 + 3C_5 + C_6) \\
&\quad - \frac{1}{2} h(0, \hat{s})(C_3 + 3C_4) + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6) \\
\xi_2 &= \Big[h^{\rm eff}(\hat{m}_c, \hat{s}) - h^{\rm eff}(\hat{m}_u, \hat{s})\Big](3C_1 + C_2)
\end{aligned}$$

where $C_0 \equiv 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$.

Table: SM Wilson coefficients [5]

C_1	C_2	C_3	C_4	C_5	C_6	$C_7^{\rm eff}$	C_9	C_{10}
-0.2632	1.0111	-0.0055	-0.0806	0.0004	0.0009	-0.2923	4.0749	-4.3085

[5] S. Descotes-Genon, T. Hurth, J. Matias, and J. Virto, JHEP **05**, 137 (2013).

Quark loop function

$$
\begin{aligned}
h(\hat{m}_q, \hat{s}) &= -\frac{8}{9}\ln \hat{m}_q + \frac{8}{27} + \frac{4}{9}x \\
&\quad - \frac{2}{9}(2+x)|1-x|^{1/2} \begin{cases} \left(\ln\left|\frac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right| - i\pi\right), & \text{for } x \equiv \frac{4\hat{m}_q^2}{\hat{s}} < 1, \\ 2\arctan\frac{1}{\sqrt{x-1}}, & \text{for } x \equiv \frac{4\hat{m}_q^2}{\hat{s}} > 1, \end{cases} \\
h(0, s) &= \frac{8}{27} - \frac{4}{9}\ln s + \frac{4}{9} i\pi,
\end{aligned}
$$

further the functions,

$$
\begin{aligned}
h^{\text{eff}}(\hat{m}_c, \hat{s}) &= h(\hat{m}_c, \hat{s}) + \frac{3\pi}{\alpha^2 C_0} \sum_{V=J/\psi,\psi(2S)} \frac{m_V \mathcal{B}(V \to \ell^+\ell^-)\Gamma_V}{m_V^2 - q^2 - i m_V \Gamma_V}, \\
h^{\text{eff}}(\hat{m}_u, \hat{s}) &= h(\hat{m}_u, \hat{s}) + \frac{3\pi}{\alpha^2 C_0} \sum_{V=\rho^0,\omega,\phi} \frac{m_V \mathcal{B}(V \to \ell^+\ell^-)\Gamma_V}{m_V^2 - q^2 - i m_V \Gamma_V}
\end{aligned}
$$

Covariant Confined Quark Model of Hadrons

- Interaction Lagrangian

$$L_{int} = g_M M(x) \int dx_1 \int dx_2 F_M(x; x_1, x_2) \cdot \bar{q}^a_{f_1}(x_1) \Gamma_M q^a_{f_2}(x_2) + H.c.$$

- Vertex function

$$F_M(x, x_1, x_2) = \delta^{(4)}\left(x - \sum_{i=1}^{2} w_i x_i\right) \Phi_M\left((x_1 - x_2)^2\right)$$

where

$$\tilde{\Phi}_M(-K^2) = exp\left(k^2/\Lambda_M^2\right)$$

- IR confinement[6]

$$\Pi^c = \int_0^{1/\lambda^2} dt t^{n-1} \int_0^1 \delta\left(1 - \sum_{i=1}^{n} \alpha_i\right) F(t\alpha_1, ..., t\alpha_n)$$

[6] T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner and V. E. Lyubovitskij, Phys. Rev. D **81**, 034010 (2010).

Model parameters

Quark masses m_{qi}, the infrared cutoff parameter λ and the size parameters Λ_{H_i} (all in GeV)

m_u	m_s	m_c	m_b	λ
0.241	0.428	1.67	5.05	0.181

Λ_π	Λ_K	Λ_D	Λ_{D_s}	Λ_B	Λ_{B_s}	Λ_{B_c}	Λ_ρ
0.87	1.04	1.47	1.57	1.88	1.95	2.42	0.61

Λ_ω	Λ_ϕ	$\Lambda_{J/\psi}$	Λ_{K^*}	Λ_{D^*}	Λ_{D_s*}	Λ_{B*}	Λ_{B_s*}
0.47	0.88	1.48	0.72	1.16	1.17	1.72	1.71

Transition form factors

$$
\begin{aligned}
\langle D_{(s)}(p_2) \quad | \quad & \bar{q} O^\mu b \mid B_c(p_1)\rangle \\
= \quad & N_c g_{B_c} g_{D_{(s)}} \int \frac{d^4k}{(2\pi)^4 i} \tilde{\phi}_{B_c}(-(k+w_{13}p_1)^2)\tilde{\phi}_{D_{(s)}}(-(k+w_{23}p_2)^2) \\
& \times \mathrm{tr}[O^\mu S_1(k+p_1)\gamma^5 S_3(k)\gamma^5 S_2(k+p_2)] \\
= \quad & F_+(q^2)P^\mu + F_-(q^2)q^\mu \ ,
\end{aligned}
$$

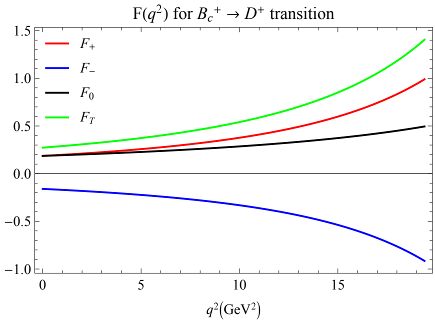
$$
\begin{aligned}
\langle D_{(s)}(p_2) \quad | \quad & \bar{q} \sigma^{\mu\nu}(1-\gamma^5) b \mid B_c(p_1)\rangle \\
= \quad & N_c g_{B_c} g_{D_{(s)}} \int \frac{d^4k}{(2\pi)^4 i} \tilde{\phi}_{B_c}(-(k+w_{13}p_1)^2)\tilde{\phi}_{D_{(s)}}(-(k+w_{23}p_2)^2) \\
& \times \mathrm{tr}[\sigma^{\mu\nu}(1-\gamma^5) S_1(k+p_1)\gamma^5 S_3(k)\gamma^5 S_2(k+p_2)] \\
= \quad & \frac{iF_T(q^2)}{m_1+m_2}(P^\mu q^\nu - P^\nu q^\mu + i\varepsilon^{\mu\nu Pq}).
\end{aligned}
$$

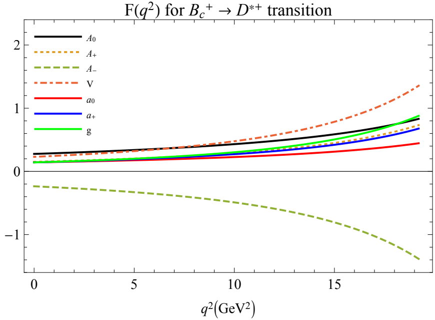
Transition form factors

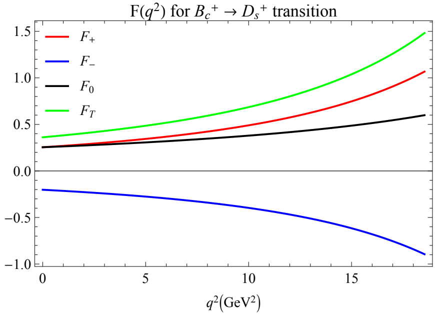
$$
\begin{aligned}
\langle D^*_{(s)}(p_2,\epsilon) \quad | \quad & \bar{q} O^{\mu} b \mid B_c(p_1)\rangle \\
= \quad & N_c g_{B_c} g_{D^*_{(s)}} \int \frac{d^4k}{(2\pi)^4 i} \tilde{\phi}_{B_c}(-(k+w_{13}p_1)^2)\tilde{\phi}_{D^*_{(s)}}(-(k+w_{23}p_2)^2) \\
& \times \, \mathrm{tr}[O^{\mu} S_1(k+p_1)\gamma^5 S_3(k) \not{\epsilon}^{\dagger}_{\nu} S_2(k+p_2)] \\
= \quad & \frac{\epsilon^{\dagger}_{\nu}}{m_1+m_2}[-g^{\mu\nu} P\cdot q A_0(q^2) + P^{\mu}P^{\nu}A_+(q^2) + q^{\mu}P^{\nu}A_-(q^2) \\
& + i\varepsilon^{\mu\nu\alpha\beta}P_{\alpha}q_{\beta}V(q^2)] \; ,
\end{aligned}
$$

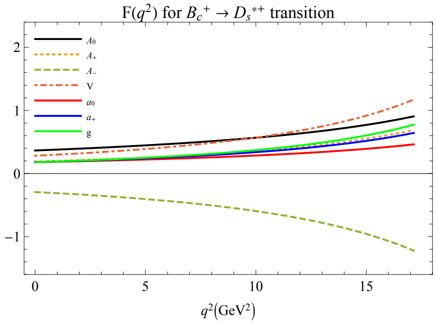
$$
\begin{aligned}
\langle D^*_{(s)}(p_2,\epsilon) \quad | \quad & \bar{q}\sigma^{\mu\nu} q_{\nu}(1+\gamma^5) b \mid B_c(p_1)\rangle \\
= \quad & N_c g_{B_c} g_{D^*_{(s)}} \int \frac{d^4k}{(2\pi)^4 i} \tilde{\phi}_{B_c}(-(k+w_{13}p_1)^2)\tilde{\phi}_{D^*_{(s)}}(-(k+w_{23}p_2)^2) \\
& \times \, \mathrm{tr}[\sigma^{\mu\nu} q_{\nu}(1+\gamma^5) S_1(k+p_1)\gamma^5 S_3(k) \not{\epsilon}^{\dagger}_{\nu} S_2(k+p_2)] \\
= \quad & \epsilon^{\dagger}_{\nu}[-(g^{\mu\nu} - q^{\mu}q^{\nu}/q^2) P\cdot q a_0(q^2) + i\varepsilon^{\mu\nu\alpha\beta}P_{\alpha}q_{\beta}g(q^2) \\
& + (P^{\mu}P^{\nu} - q^{\mu}P^{\nu}P\cdot q/q^2) a_+(q^2)\Big].
\end{aligned}
$$

Transition form factors









Differential branching fractions

$$\frac{d\Gamma(B_c \to D_{(s)}^{(*)}\ell^+\ell^-)}{dq^2} = \frac{G_F^2}{(2\pi)^3}\left(\frac{\alpha V_{tb}^* V_{tq}}{2\pi}\right)^2 \frac{|\mathbf{p_2}|q^2\beta_\ell}{12 m_{B_c}^2}\mathcal{H}_{\rm tot}$$

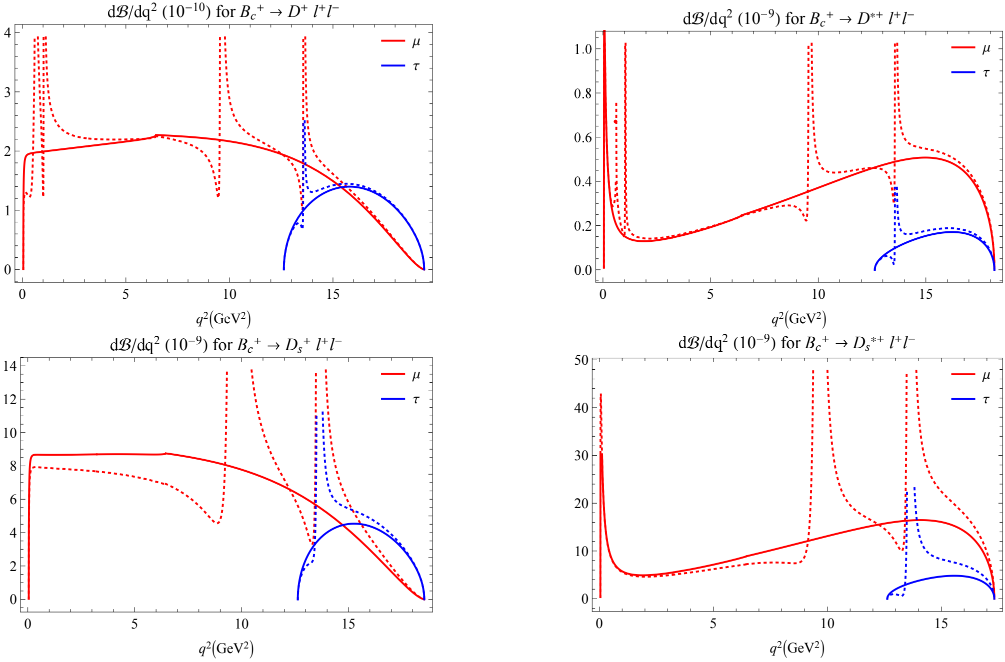
where

$$\begin{aligned}\mathcal{H}_{\rm tot} &= \frac{1}{2}(\mathcal{H}_U^{11} + \mathcal{H}_U^{22} + \mathcal{H}_L^{11} + \mathcal{H}_L^{22}) \\ &+ \delta_{\ell\ell}\left(\frac{1}{2}\mathcal{H}_U^{11} - \mathcal{H}_U^{22} + \frac{1}{2}\mathcal{H}_L^{11} - \mathcal{H}_L^{22} + \frac{3}{2}\mathcal{H}_S^{22}\right).\end{aligned}$$

Here $H's$ are the helicity structure functions which depends on form factors.

- $\delta_{\ell\ell} = 2m_\ell^2/q^2$
- $\beta_\ell = \sqrt{1 - 4m_\ell^2/q^2}$
- $|p_2| = \lambda^{1/2}(m_{B_c}^2, m_{D_{(s)}^{(*)}}^2, q^2)/2m_{B_c}$
- $v = 1 - m_\ell^2/q^2$

Differential branching fractions



Branching fractions

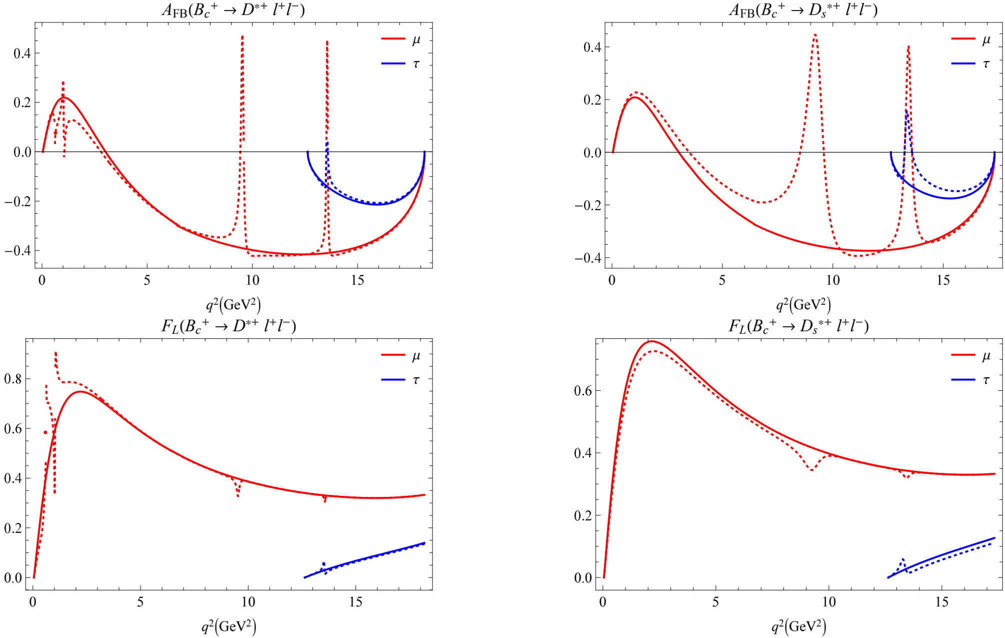
Channel	Present	LFQM [7]	CQM [7]	pQCD[8]	RQM [9]
$10^9\mathcal{B}(B_c^+ \to D^+ e^+ e^-)$	2.640 ± 0.210	4.100	4.000	–	–
$10^9\mathcal{B}(B_c^+ \to D^+ \mu^+ \mu^-)$	2.634 ± 0.210	4.100	4.000	3.790	3.700
$10^9\mathcal{B}(B_c^+ \to D^+ \tau^+ \tau^-)$	0.502 ± 0.085	1.300	1.200	1.030	1.500
$10^8\mathcal{B}(B_c^+ \to D^+ \nu^+ \nu^-)$	1.375 ± 0.136	2.770	2.740	3.130	2.160
$10^9\mathcal{B}(B_c^+ \to D^{*+} e^+ e^-)$	4.960 ± 0.251	10.100	7.900	–	–
$10^9\mathcal{B}(B_c^+ \to D^{*+} \mu^+ \mu^-)$	3.882 ± 0.160	10.100	7.900	12.100	8.100
$10^9\mathcal{B}(B_c^+ \to D^{*+} \tau^+ \tau^-)$	0.518 ± 0.025	1.800	1.400	1.600	1.900
$10^8\mathcal{B}(B_c^+ \to D^{*+} \nu^+ \nu^-)$	2.566 ± 0.119	7.640	5.990	11.000	5.120
$10^7\mathcal{B}(B_c^+ \to D^{*+} \gamma)$	1.213 ± 0.052	–	–	–	–

[7] C. Q. Geng, C.-W. Hwang, and C. C. Liu, Phys. Rev. D **65**, 094037 (2002).
[8] W.-F. Wang, X. Yu, C.-D. Lü, and Z.-J. Xiao, Phys. Rev. D **90**, 094018 (2014).
[9] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D **82**, 034032 (2010).

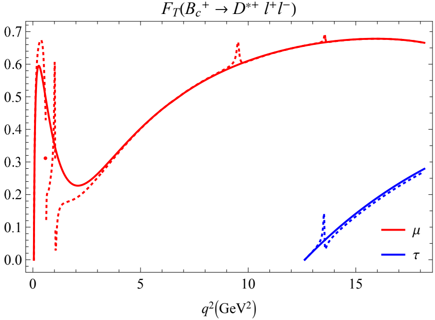
Branching fractions

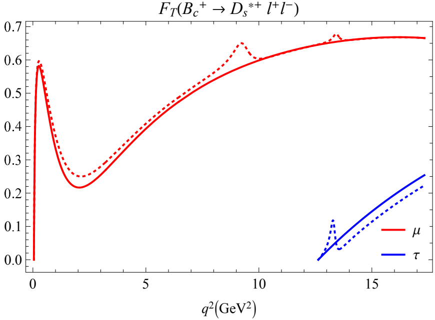
Channel	Present	LFQM[7]	CQM[7]	pQCD[8]	RQM[9]
$10^7\mathcal{B}(B_c^+ \to D_s^+ e^+ e^-)$	0.792 ± 0.076	1.360	1.330	–	–
$10^7\mathcal{B}(B_c^+ \to D_s^+ \mu^+ \mu^-)$	0.788 ± 0.076	1.360	1.330	1.560	1.160
$10^7\mathcal{B}(B_c^+ \to D_s^+ \tau^+ \tau^-)$	0.136 ± 0.025	0.340	0.370	0.380	0.330
$10^7\mathcal{B}(B_c^+ \to D_s^+ \nu^+ \nu^-)$	4.954 ± 0.591	9.200	9.200	0.129	6.500
$10^7\mathcal{B}(B_c^+ \to D_s^{*+} e^+ e^-)$	1.550 ± 0.141	4.090	2.810	–	–
$10^7\mathcal{B}(B_c^+ \to D_s^{*+} \mu^+ \mu^-)$	1.158 ± 0.065	4.090	2.810	4.400	2.120
$10^7\mathcal{B}(B_c^+ \to D_s^{*+} \tau^+ \tau^-)$	0.144 ± 0.009	0.510	0.410	0.520	0.350
$10^7\mathcal{B}(B_c^+ \to D_s^{*+} \nu^+ \nu^-)$	8.314 ± 0.526	31.200	21.200	40.400	13.500
$10^6\mathcal{B}(B_c^+ \to D_s^{*+} \gamma)$	4.412 ± 0.254	–	–	–	–

Other physical observables



Other physical observables





q^2 averages of polarization observables

Observables	$B_c^+ \to D^{*+}\ell^+\ell^-$			$B_c^+ \to D_s^{*+}\ell^+\ell^-$		
	e^+e^-	$\mu^+\mu^-$	$\tau^+\tau^-$	e^+e^-	$\mu^+\mu^-$	$\tau^+\tau^-$
$-\langle A_{FB}\rangle$	0.186	0.239	0.188	0.131	0.178	0.134
$\langle F_L\rangle$	0.345	0.430	0.095	0.322	0.419	0.080
$\langle F_T\rangle$	0.634	0.536	0.200	0.655	0.544	0.163
$-\langle P_1\rangle$	0.336	0.506	0.779	0.348	0.557	0.856
$-\langle P_2\rangle$	0.195	0.297	0.627	0.134	0.218	0.548
$10^4 \times \langle P_3\rangle$	2.176	2.298	4.801	9.085	14.211	4.754
$\langle P'_4\rangle$	0.813	1.050	1.333	0.781	1.049	1.362
$-\langle P'_5\rangle$	0.365	0.496	0.948	0.249	0.362	0.808
$10^2 \times \langle P'_8\rangle$	2.257	2.436	-0.105	0.933	1.034	0.088
$-\langle S_3\rangle$	0.107	0.136	0.078	0.114	0.152	0.070
$\langle S_4\rangle$	0.190	0.252	0.092	0.179	0.250	0.078

Conclusion

We have studied

- $B_c \to D_{(s)}^{(*)} \ell\ell$ transition form factors and branching fractions which are corresponding to the quark channels $b \to s\ell^+\ell^-$ and $b \to d\ell^+\ell^-$.
- Different physical observables
- Brief comparison with other theoretical predictions

References

- Recent publication on $B_{(s)}$ semileptonic decays:
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  - N. R. Soni, A. Issadykov, A. N. Gadaria, Z. Tyulemissov, J. J. Patel and J. N. Pandya, *Form factors and branching fraction calculations for $B_s \to D_s^{(*)}\ell^+\nu_\ell$ in view of LHCb observation*, Eur. Phys. J. Plus **138**, 163 (2023).
  - N. R. Soni, A. Issadykov, A. N. Gadaria, J. J. Patel and J. N. Pandya, *Rare $b \to d$ decays in covariant confined quark model*, Eur. Phys. J. A **58**, 39 (2022).

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Thank You