Recent development:
NNLO heavy flavour jet algorithms

Alexander Mitov

Cavendish Laboratory

UNIVERSITY OF CAMBRIDGE

Introduction

- ✓ Experimentally, flavored jets are simple (conceptually):
  - ✓ Use an inclusive (i.e. flavor-blind) algorithm to cluster hadrons
    - ✓ In practice at the LHC anti-kT is used
  - ✓ Tag heavy flavors (B-mesons) inside the jets and determine their flavor
  - ✓ We distinguish net flavor, total flavor, total flavor modulo 2 etc.

- ✓ All this is very straightforward so where is the issue?

1. Questions like b-tagging efficiency, mis-tagging, etc are important but orthogonal to our discussion. We ignore those here.

2. In this talk we focus on bottom i.e. flavor = bottom. Extensions to charm is mostly straightforward.

- ✓ The issue with flavor jet identification is on the theory side.
  - ✓ Issues arise because we work with (fixed order) calculations based on partons, i.e. quarks and gluons, not hadrons.
  - ✓ Conceptually, one thinks of b-quarks as producing the observed B-hadrons.
    - ✓ Therefore, we can try to assign jet flavor based on the partonic flavor.
  - ✓ This approach is supported by parton-hadron duality in IR safe observables, which for jets, translates into: partonic jets ~ hadronic jets.
  - ✓ However, for this identification to work, we need IR safety in the presence of flavor.

As it turns out, ensuring this is neither easy nor straightforward.

And this is the goal of this talk.

✓ One might wonder:

Why would flavor be an IR safety concern if we have an IR safe jet algorithm (like anti-kT)?

✓ Here is a typical example:

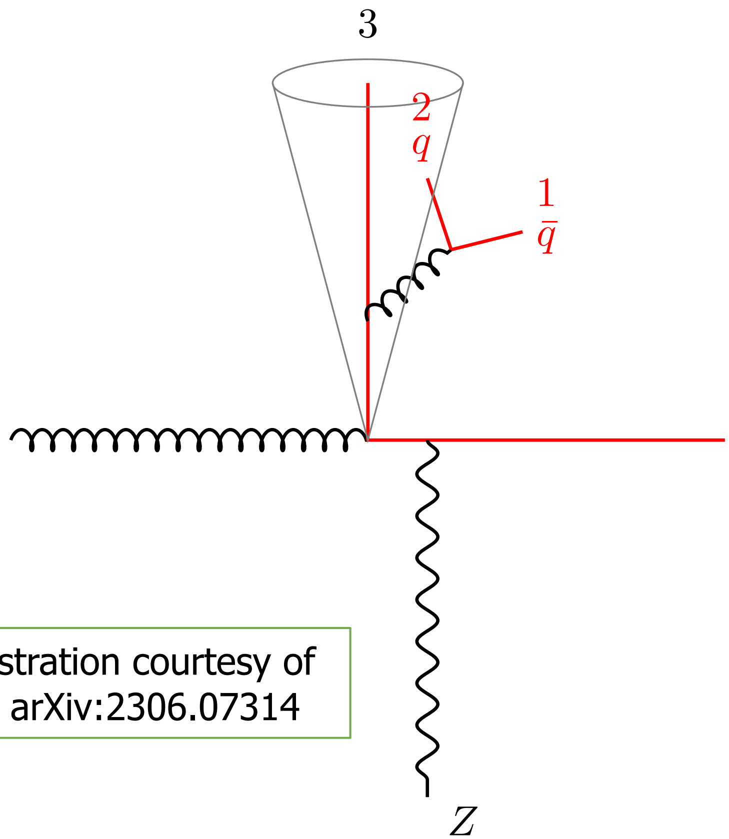

Illustration courtesy of ref. arXiv:2306.07314

✓ Consider the emission of a soft bb pair

- ✓ These are massless quarks; there is no limit to how soft as they can be

✓ If the pair happens to be clustered in one jet all is good – they look like a soft gluon so no issues with IR safety

✓ If one of them (2) is clustered in an unrelated hard jet (3), while the other one (1) forms its own jet, then the kinematics is unchanged but the flavor is.

- ✓ This is IR unsafe

✓ The problem first appears at NNLO

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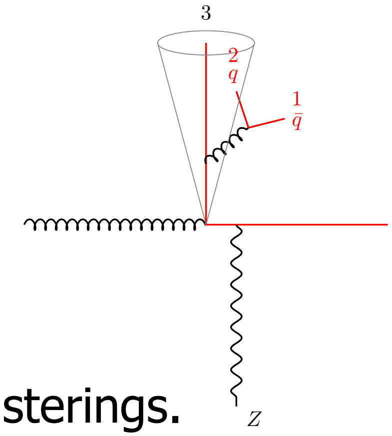


- ✓ We are interested in the anti-kT algorithm, which does not prioritize such clusterings.
- ✓ So, what to do?
- ✓ Clearly, one needs to somehow modify the jet algorithm in the presence of flavor.
- ✓ Let's call such an algorithm "flavor-anti-kT"
- ✓ It cannot be unique! We will see 4 such examples in this talk.
  - ✓ It must respect IR safety.
  - ✓ Ideally, it should also offer:
    - ✓ Easy implementation
    - ✓ Small deviation from the flavorless algorithm
    - ✓ Small unfolding corrections, i.e. the ratio

$$\frac{\text{anti-kT(hadron level)}}{\text{flavor-anti-KT (parton level)}} \sim 1$$

✓ Why the current interest in this subject?

See also Bayu Hartanto's talk

✓ The problem was recognized almost 20 years ago when the first "flavored" jet algorithm was proposed (flavor-kT)

Banfi, Salam, Zanderighi (2006)

✓ On the other hand, NNLO LHC calculations with heavy flavored jets started to proliferate very recently. They necessitate the use of IR safe flavor jet algorithms.

✓ First attempts before a flavor-anti-kT algorithm was proposed:

1. use flavor-kT, then unfold to anti-kT

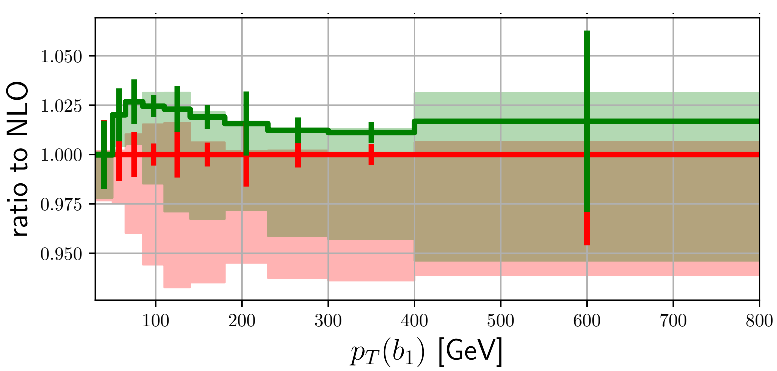
Gauld, Gehrmann-De Ridder, Glover, Huss, Majer (2020)

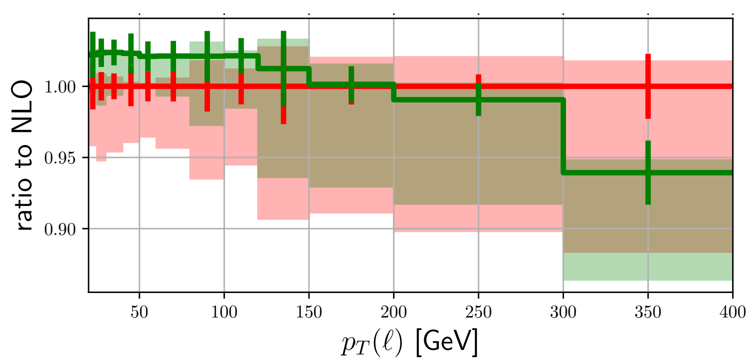
2. Use plain anti-kT with an IR regulator

Czakon, Mitov, Poncelet (2020)

Actually, this can be well motivated! Soft bb pair emissions which lead to singularities are a very small contribution to typical observables, so their impact is not large.

Used in tt production and decay where we have plenty of b-quarks already at LO. The comparison with proper IR safe flavor-anti-kT indeed shows relatively small impact:





✓ The current landscape of flavor-anti-kt jet algorithms

- Soft Drop Flavour (SDF) — Caletti, Larkoski, Marzani, Reichelt (2022)
- Flavour anti-kT (CMP) — Czakon, Mitov, Poncelet (2022)
- Flavour Dressing (GHS) — Gauld, Huss, Stagnitto (2022)
- Interleaved Flavour Neutralisation (IFN) — Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler (2023)

✓ Will not review here their definitions, and those can be found in the papers.

✓ However, a couple of comments:

  ✓ CMP ensures flavor IR safety by modifying the jet distance.

    ✓ This affects the kinematics relative to standard anti-kT.
    ✓ A tunable parameter is chosen in such a way that this effect is minimized.

  ✓ The other three algorithms preserve exact anti-kT kinematics; they modify what is meant by partonic flavor.

✓ Various subtleties were revealed by the detailed testing of flavor IR safety performed in

Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler (2023)

order relative to Born		anti-k_t	flav-k_t ($\alpha = 2$)	CMP	$\mathrm{GHS}_{\alpha,\beta}$ $(2,2)$	anti-k_t+IFN_α	C/A+IFN_α
α_s	FHC	✓	✓	✓	✓	✓	✓
	IHC	✓	✓	✓	✓	✓	✓
α_s^2	FDS	×II B	✓	✓	✓	✓	✓
	IDS	×II B	✓	✓	✓	✓	✓
	FHC×IHC	✓	✓	✓	✓	✓	✓
	IHC^2	✓	✓	×C 2	✓	✓	✓
	FHC^2	✓	✓	✓	×C 4	✓	✓
α_s^3	IHC×IDS		∼C 1	×C 3	∼C 1	✓	✓
	rest					✓	✓
α_s^4	IDS×FDS				×C 5	✓	✓
	rest					✓	✓
α_s^5						✓	✓
α_s^6						✓	✓

✓ Some of those can be removed by simple modifications

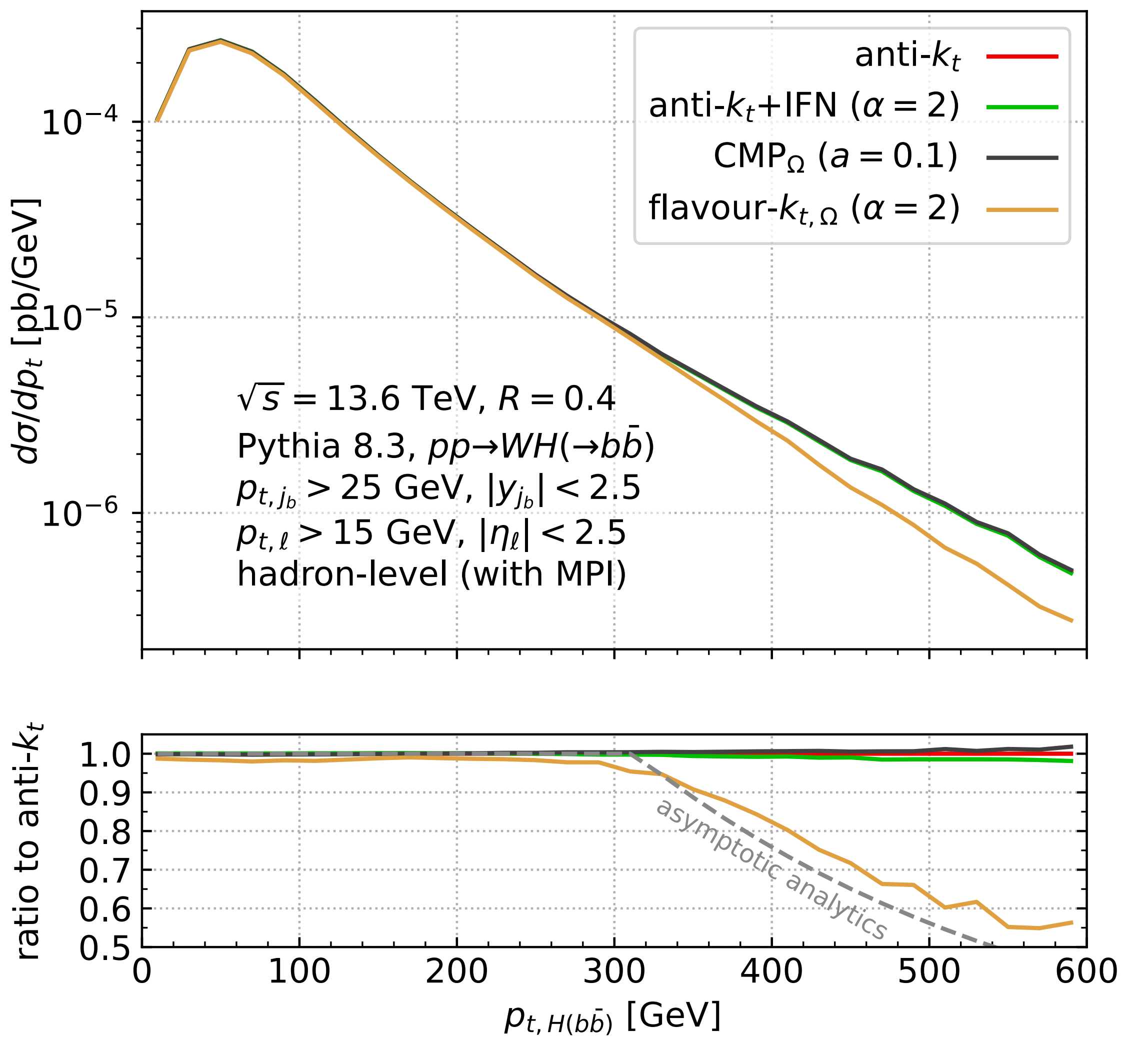
- Clearly an accord is needed to:
    - Compare the performance of all algorithms in as many processes as possible
    - Quantify the size of unfolding effects (i.e. how far each algorithm is from the experimental truth). This may be very observable dependent.
- The original papers published some of these already (next two slides)
- Last year's Les Houches started a concerted effort in this regard. Work is ongoing. Many participants!
- A common framework for the 4 algorithms has been created and made public https://github.com/jetflav
- Some selected results:

Compare algorithms

✓ Comparison of anti-kT flavored algorithms

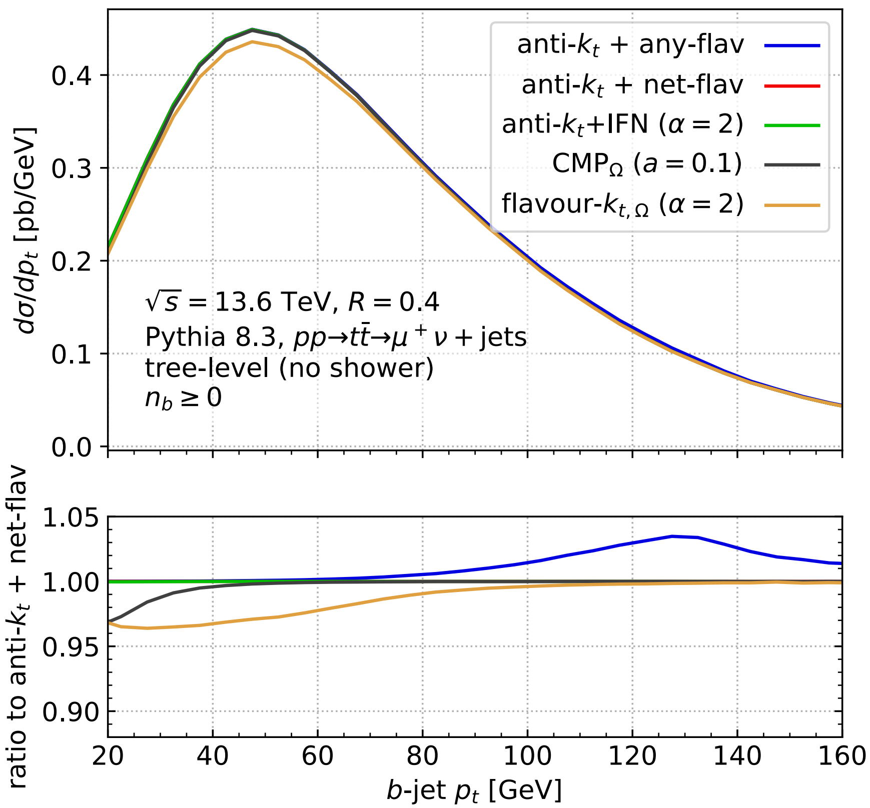
✓ H→bb

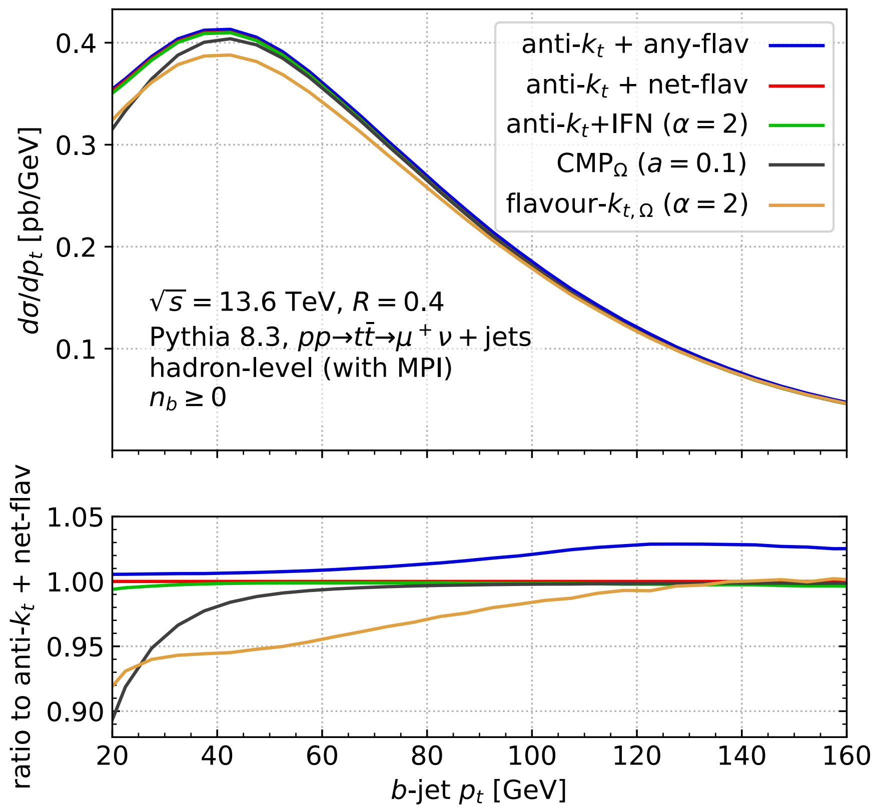
Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler (2023)



- ✓ Comparison of anti-kT flavored algorithms
  - ✓ pp->tt (→W+b)

Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler (2023)

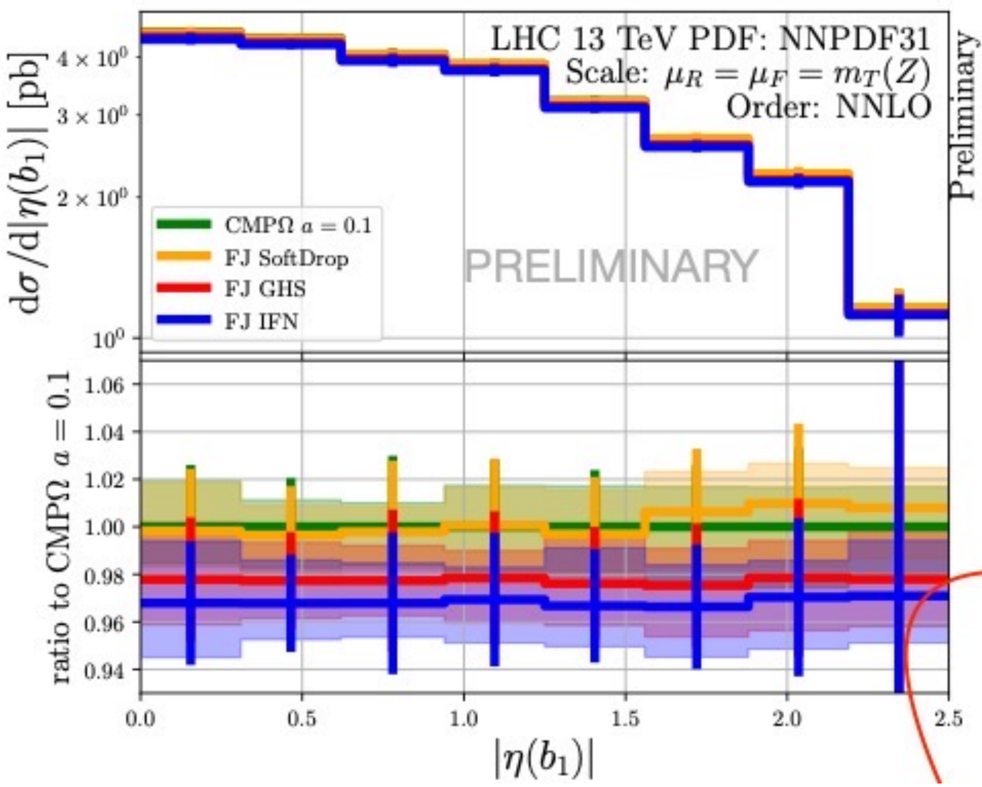


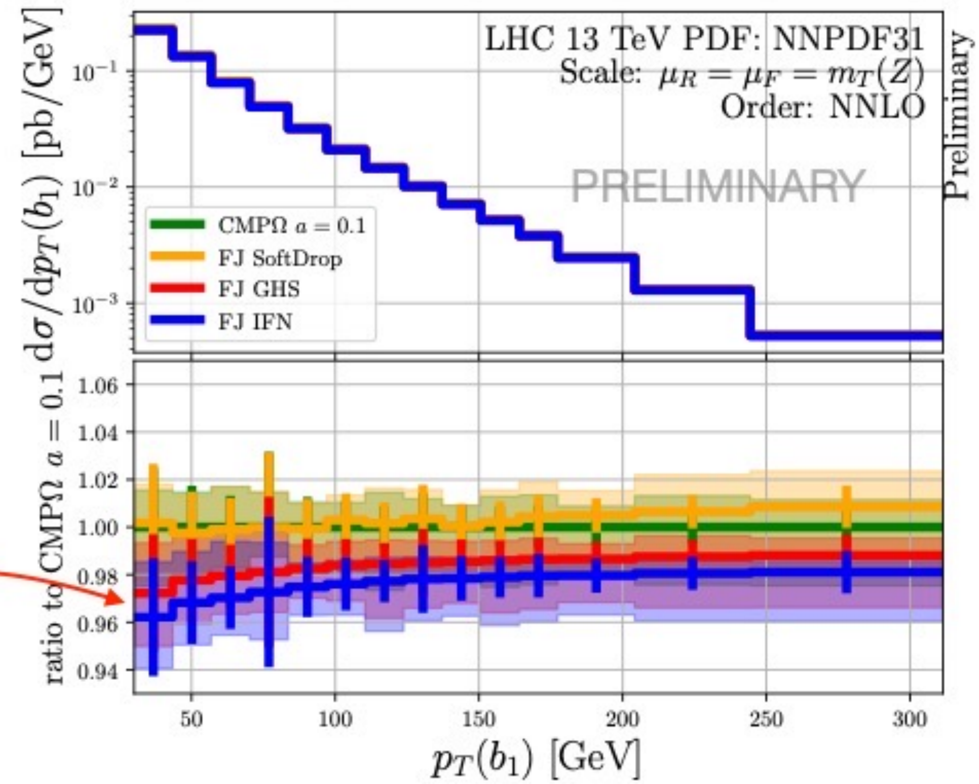


✓ Comparison of anti-kT flavored algorithms

✓ Z+bjet, NNLO

Plots produced for Les Houches
Thanks to the Workshop participants





- Comparison of anti-kT flavored algorithms
  - Z+bjet, NLO+PS

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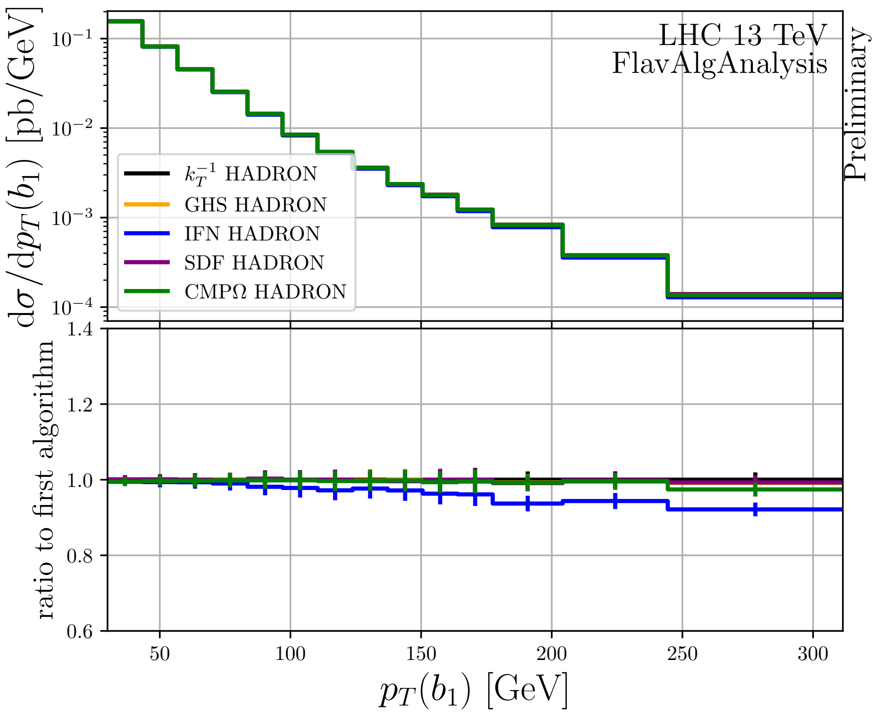

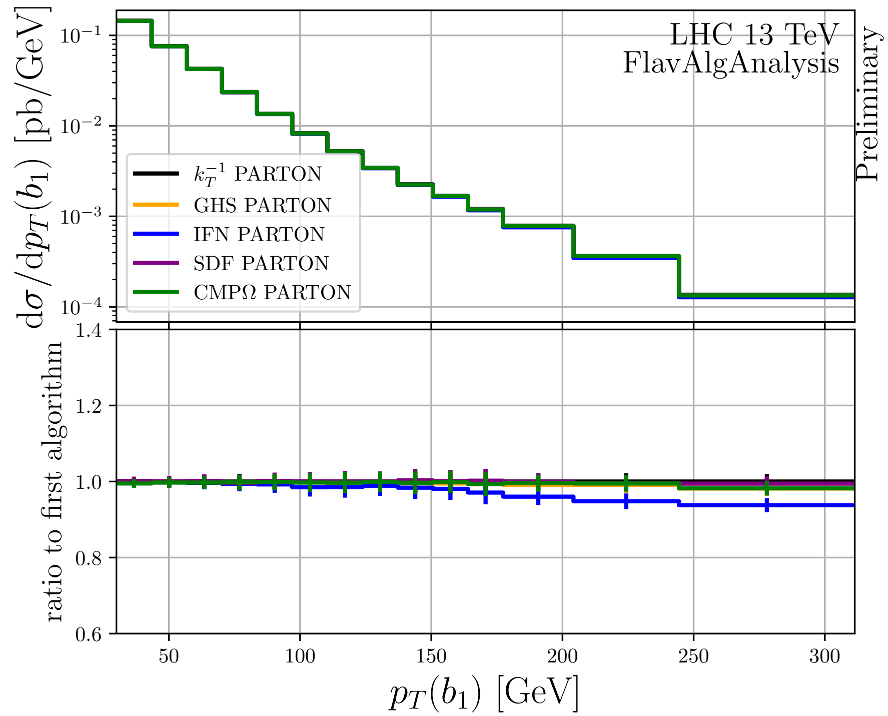

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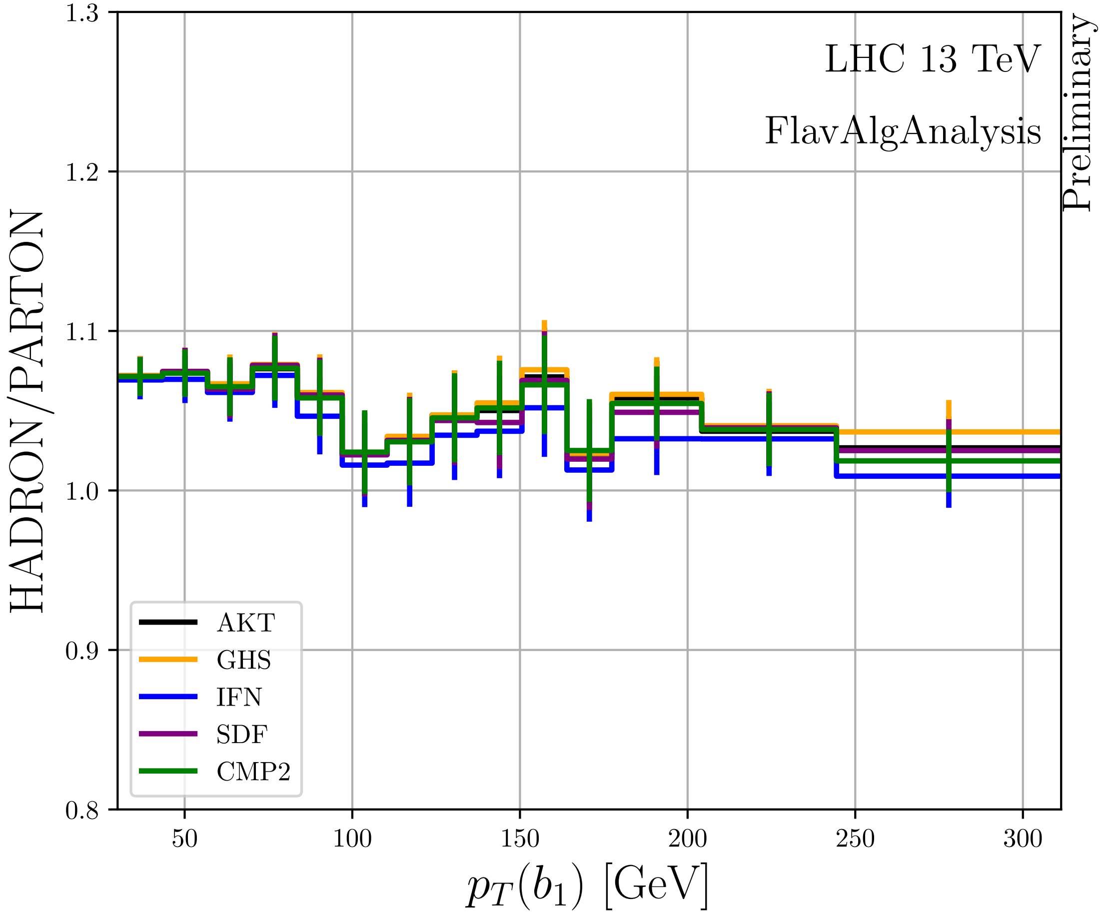

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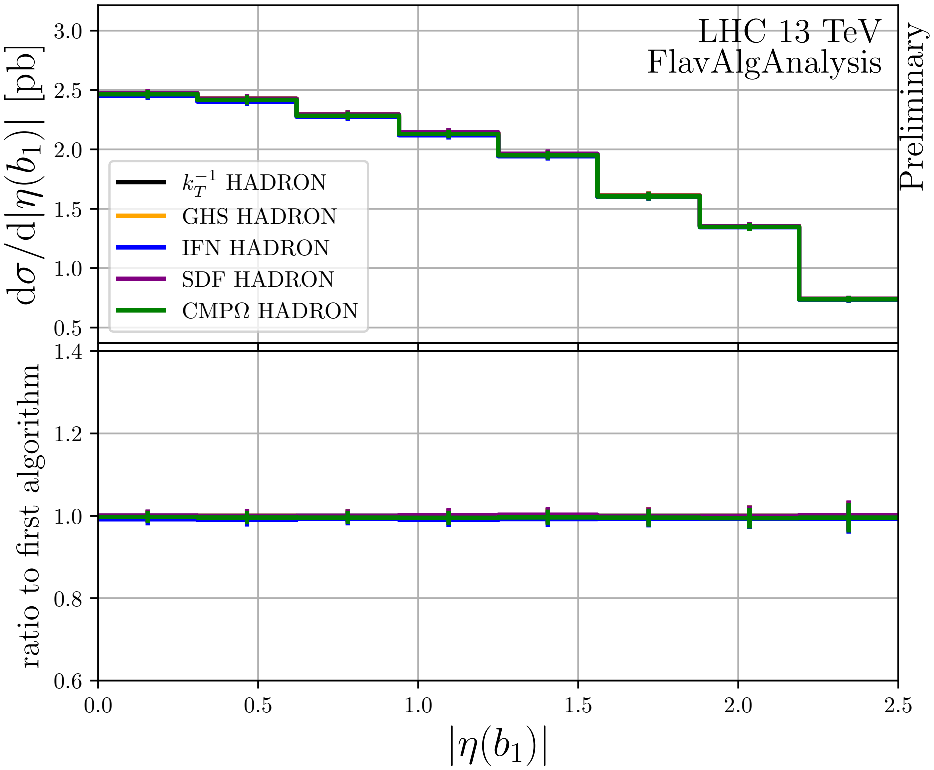

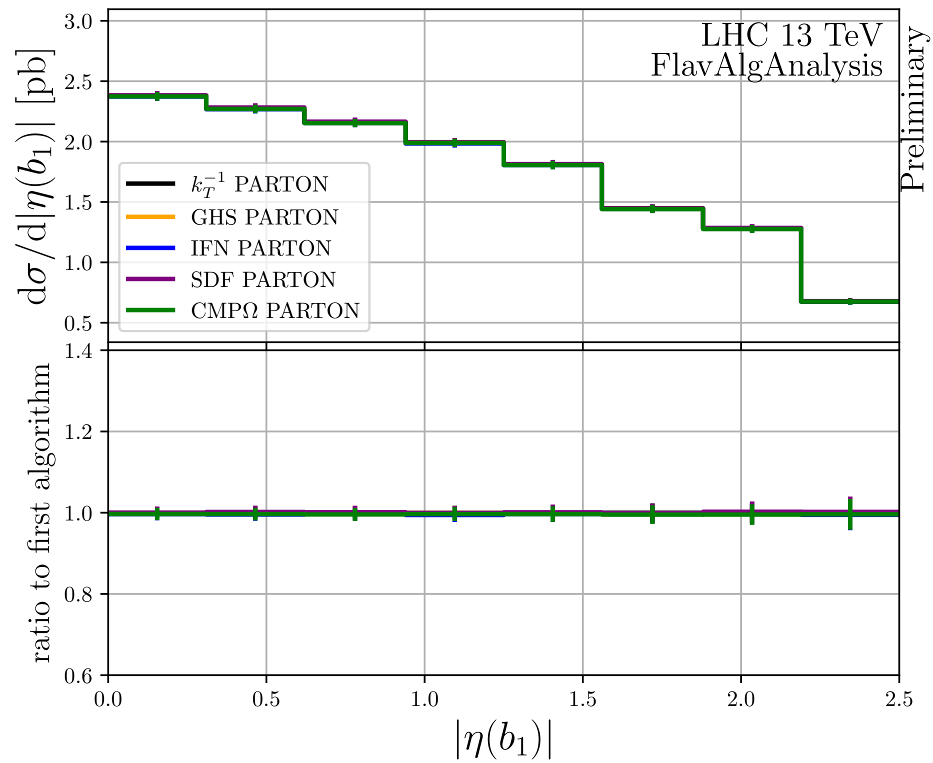

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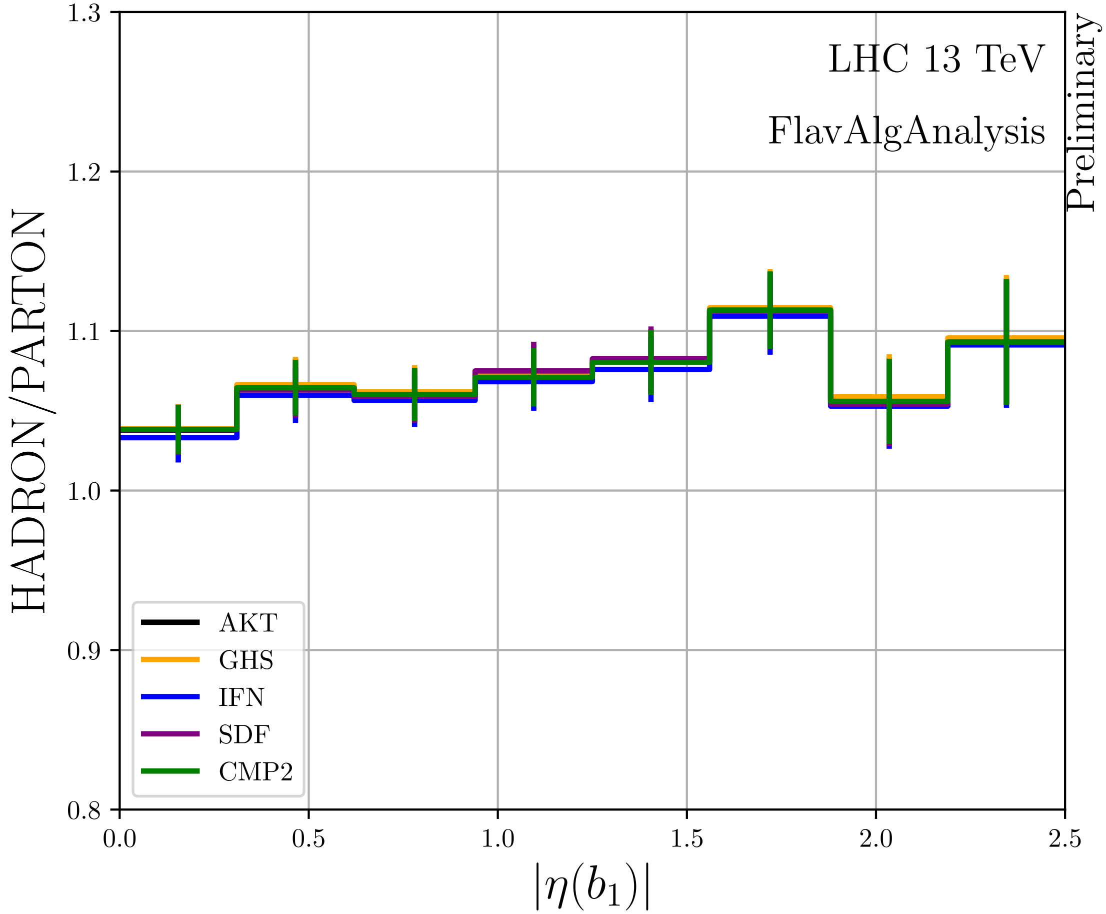

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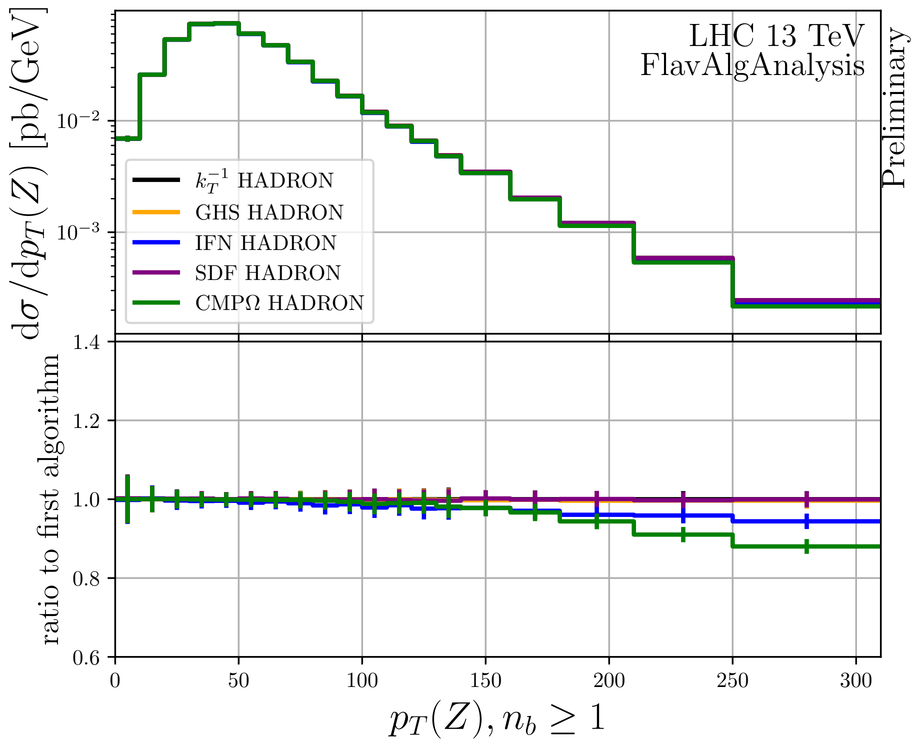

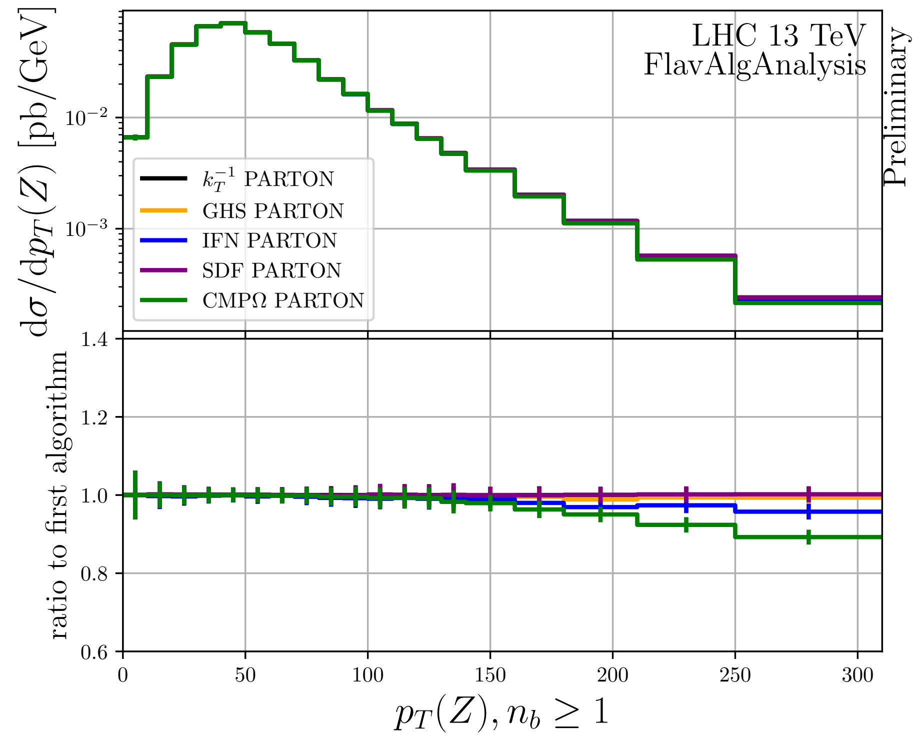

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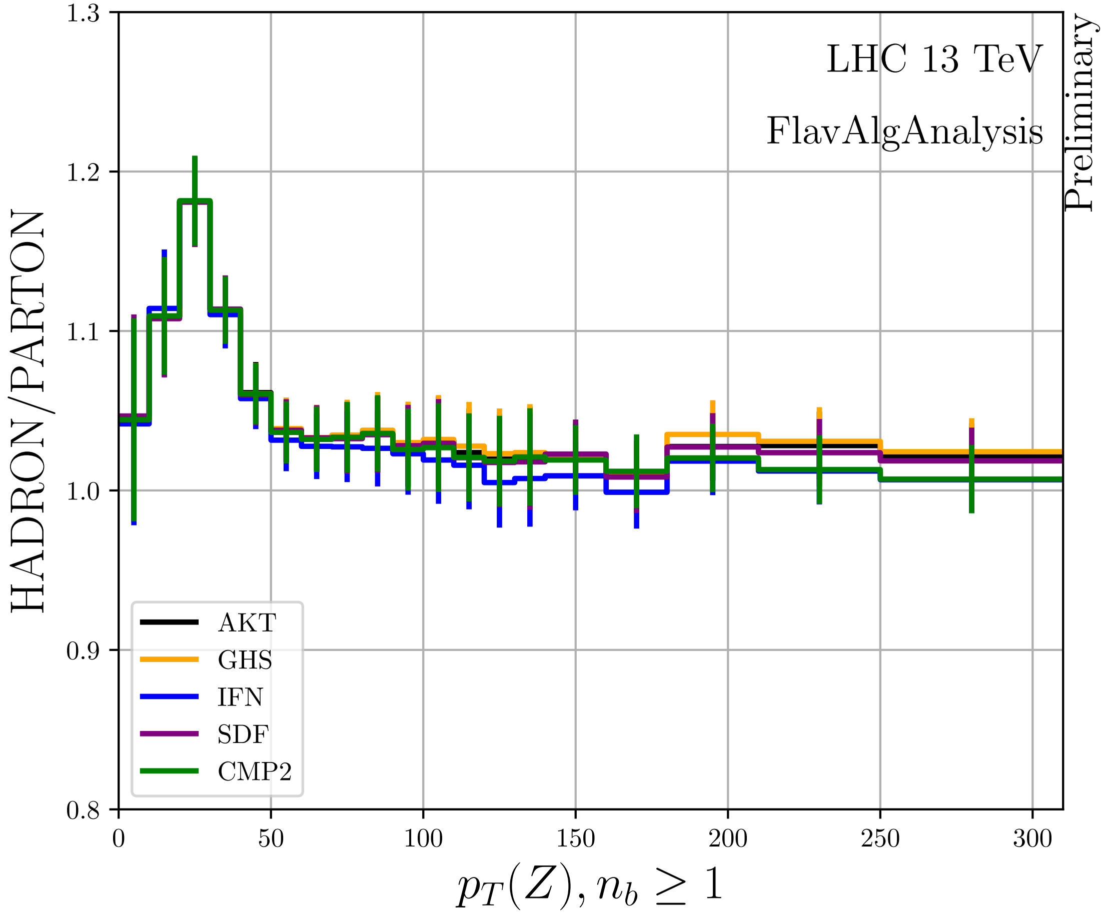

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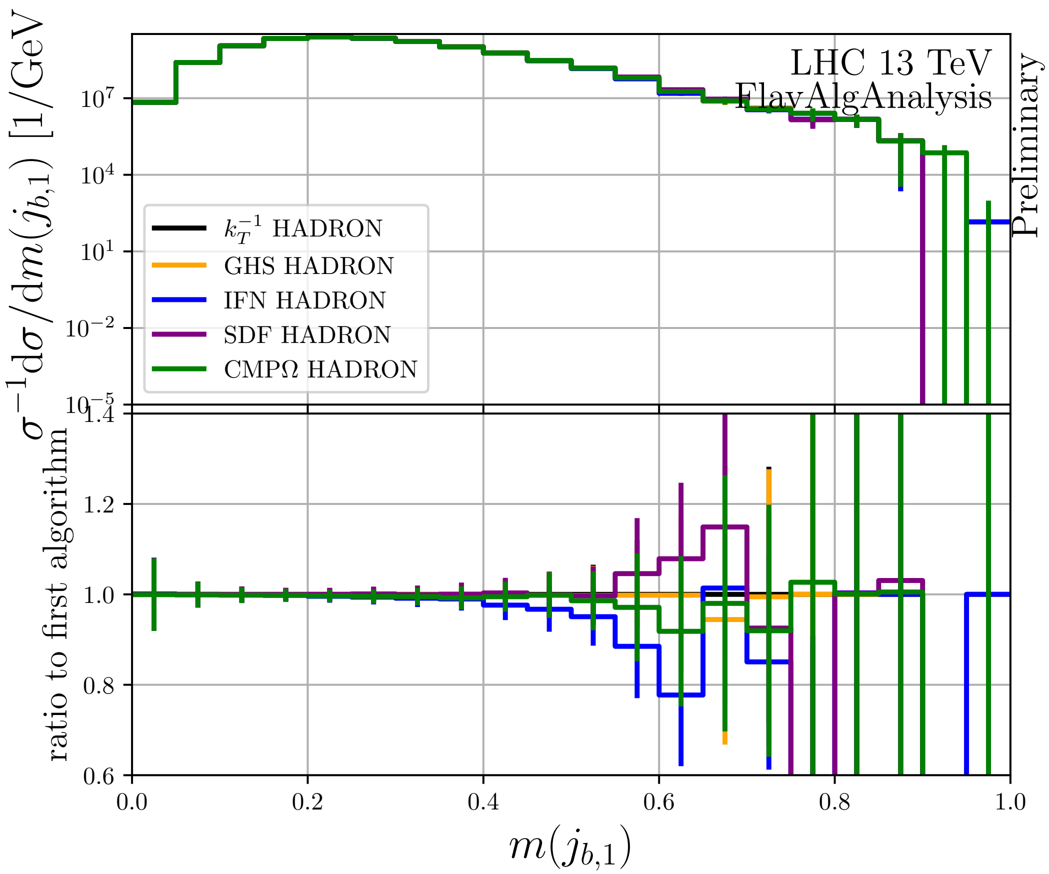

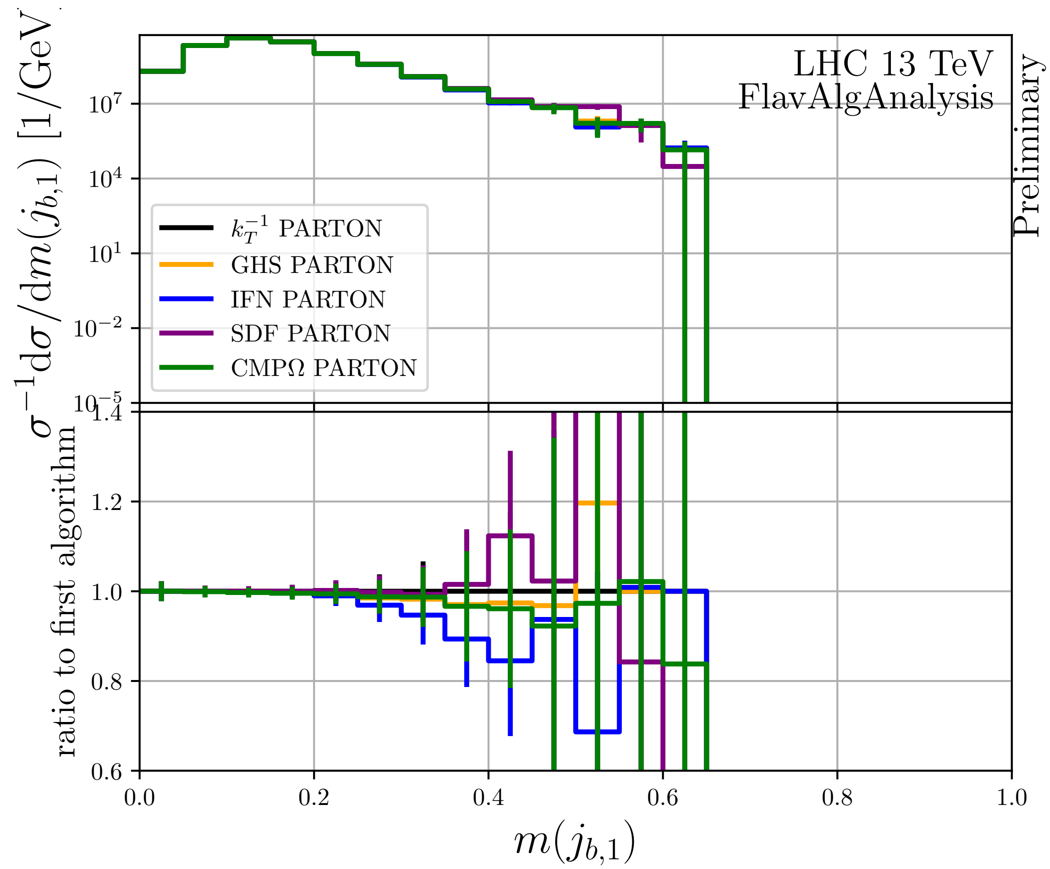

Conclusions

- ✓ Steady progress of NNLO calculations with heavy flavors for the LHC
- ✓ Consistent calculations with anti-kT jets is now possible (at NNLO and beyond)
- ✓ Four flavor-anti-kT algorithms proposed in the last two years
- ✓ Main points:
  - ✓ Main differences: some modify the kinematics of the standard anti-kT algorithm, while others do not.
  - ✓ Despite the differences the algorithms differ little from each other, both at parton and hadron levels. Differences w/r to anti-kT is generally also small.

The future

- ✓ A synchronized accord is needed for a consensus how to use these algorithms.
- ✓ A comprehensive comparison is needed across a number of observables, both at parton and hadron levels.
- ✓ Such work was started at Les Houches and is currently ongoing.
- ✓ Please join if interested ☺

Appendix

Some jet algorithm definitions

✓ Standard anti-kT distance:

$$d_{ij} = R_{ij}^2 \ \min(k_{T,i}^{-2}, k_{T,j}^{-2})$$

✓ CMP flavor-anti-kT distance:

$$d_{ij}^{(F)} \equiv d_{ij} \times \begin{cases} \mathcal{S}_{ij}\,, & \text{if both } i \text{ and } j \text{ have non-zero flavour of opposite sign,} \\ 1\,, & \text{otherwise.} \end{cases}$$

where:

$$\mathcal{S}_{ij} \equiv 1 - \theta\left(1-\kappa_{ij}\right)\cos\left(\frac{\pi}{2}\kappa_{ij}\right) \quad \text{with} \quad \kappa_{ij} \equiv \frac{1}{a}\frac{k_{T,i}^2 + k_{T,j}^2}{2k_{T,\max}^2}$$

✓ INF modification to CMP distance:

$$\mathcal{S}_{ij} \to \overline{\mathcal{S}}_{ij} = \mathcal{S}_{ij}\frac{\Omega_{ij}^2}{\Delta R_{ij}^2}$$

where:

$$\Delta R_{ij}^2 \equiv (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

$$\Omega_{ik}^2 \equiv 2\left[\frac{1}{\omega^2}\left(\cosh(\omega\Delta y_{ik}) - 1\right) - \left(\cos\Delta\phi_{ik} - 1\right)\right] \quad \text{with} \quad \omega > 1$$