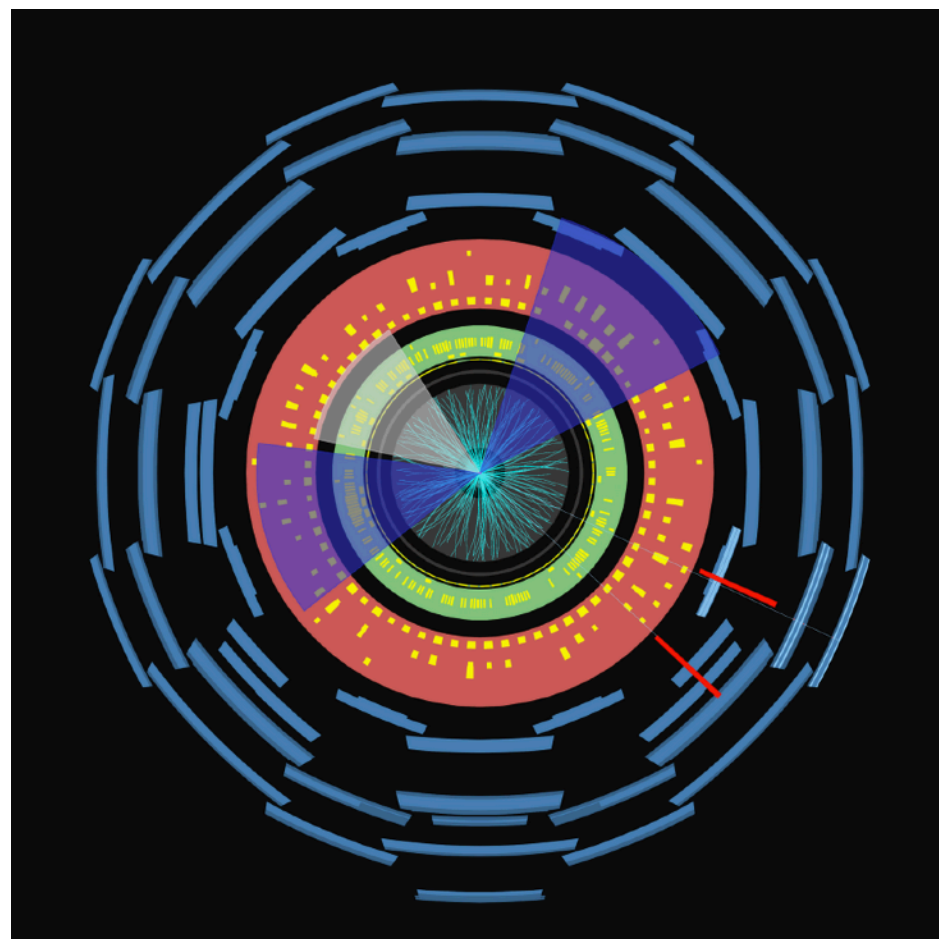
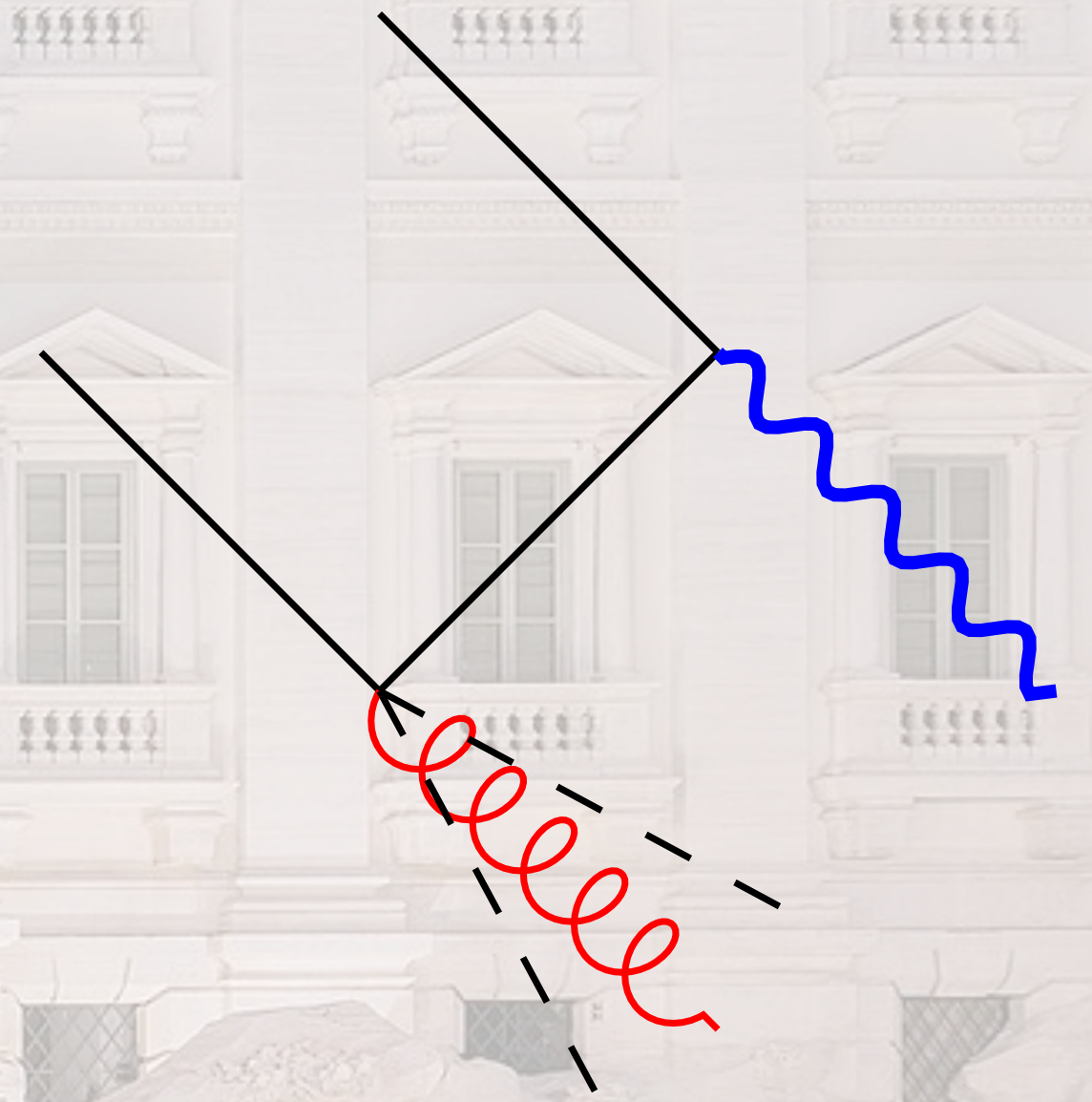
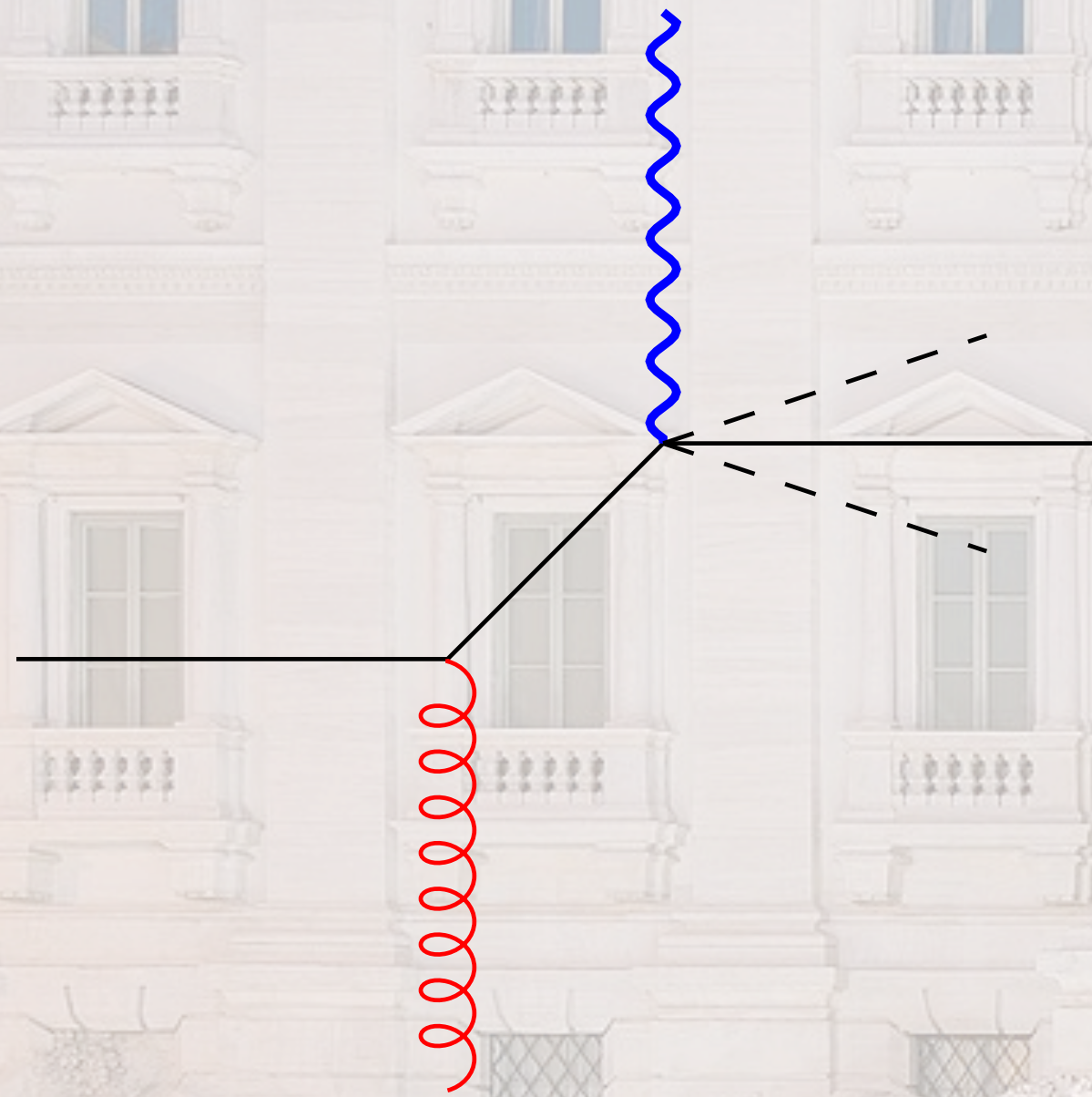
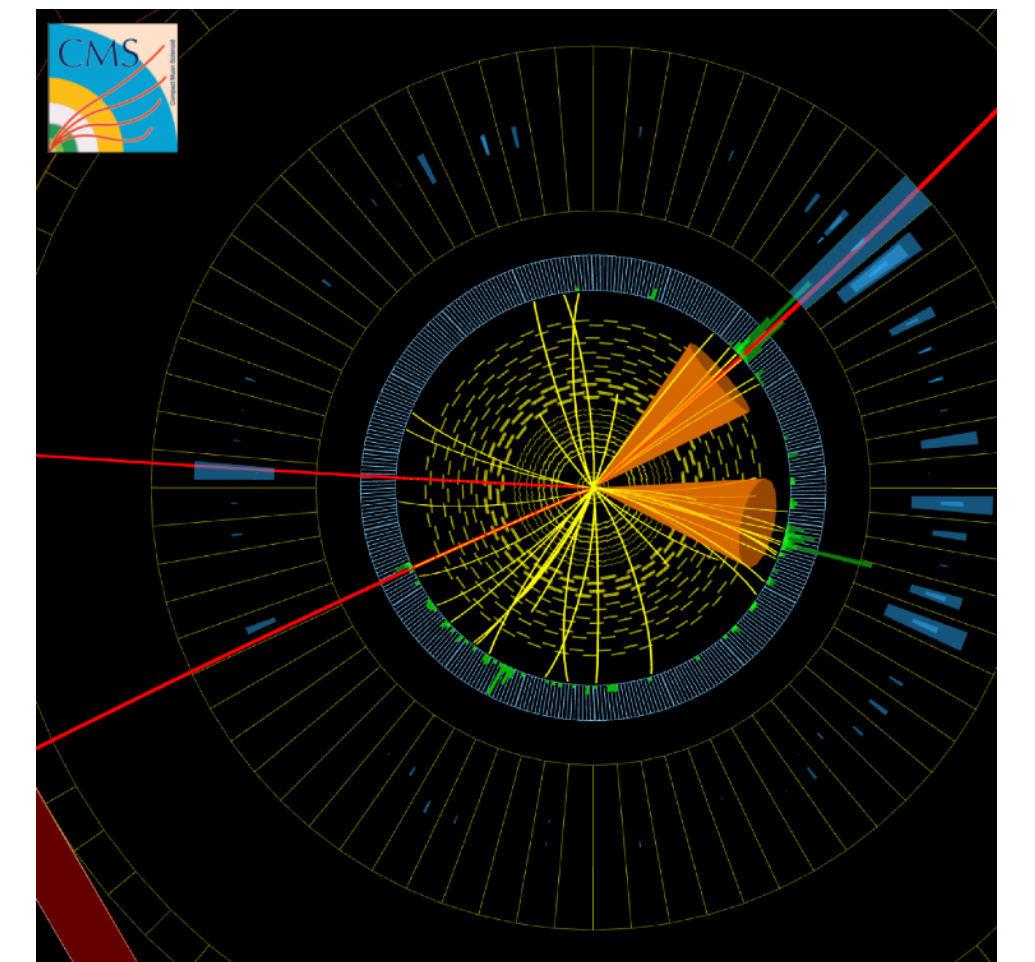


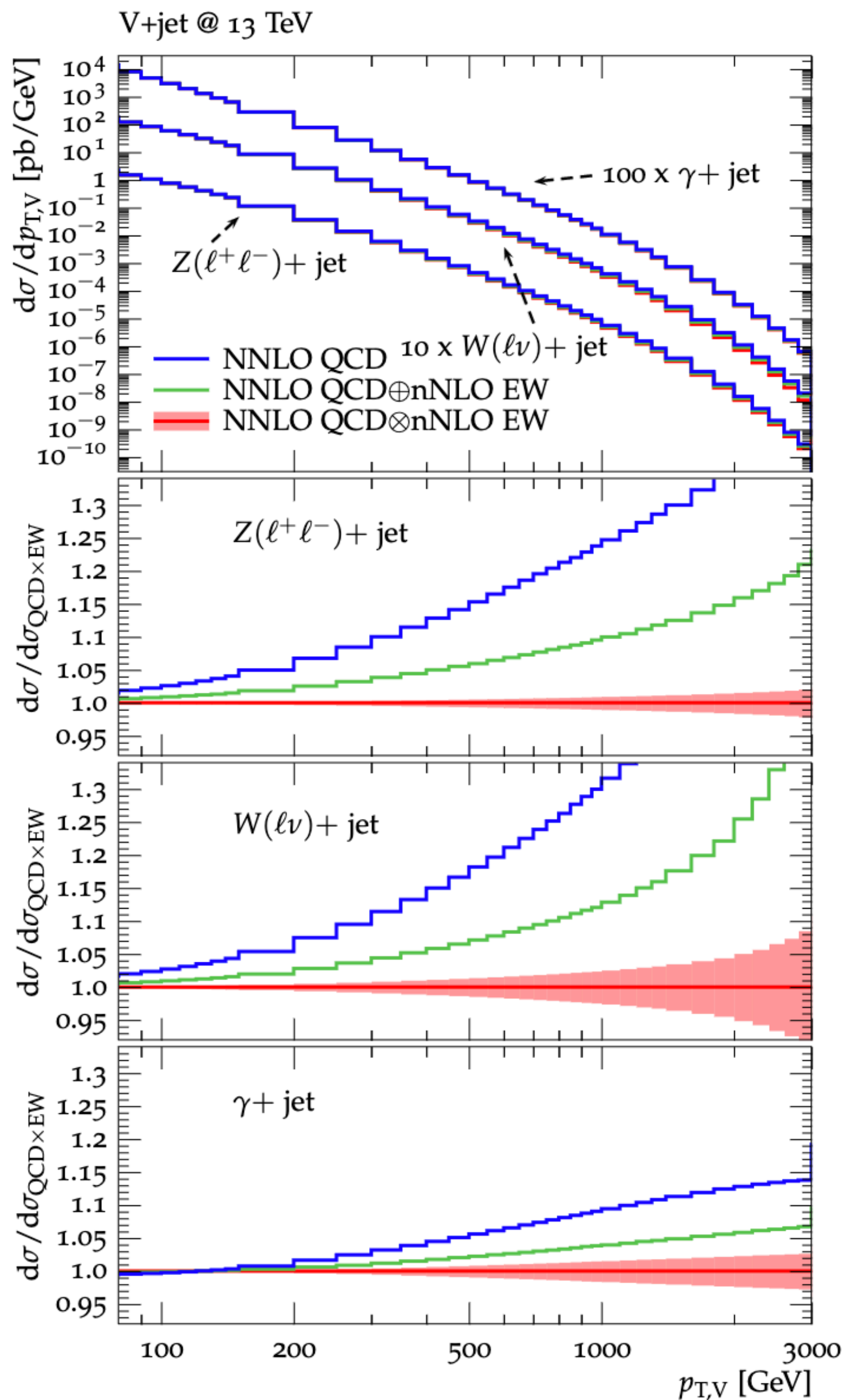
Precise predictions for V +jet production

Giovanni Stagnitto
(Milano Bicocca University & INFN)



SM@LHC 2777 *ab Urbe condita*



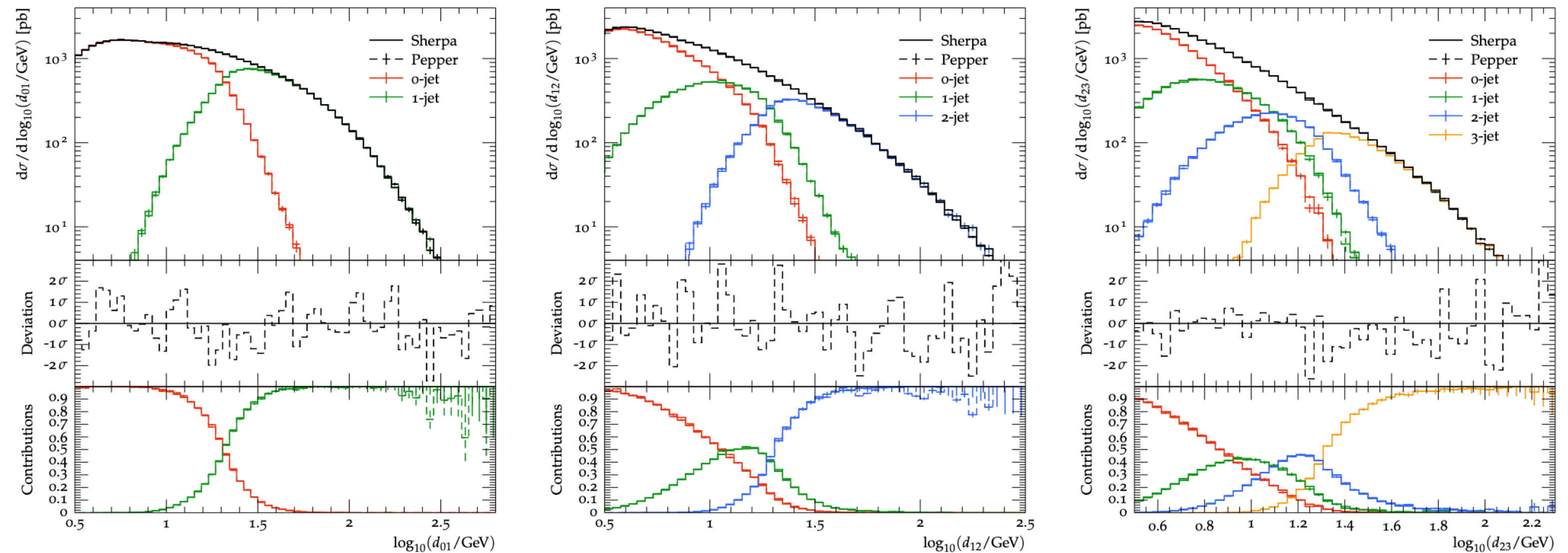


[Lindert et al. '17]

V+jets ($V = \gamma/Z, W^\pm$) events:
 ideal probe for testing QCD and EW interactions
 major source of backgrounds for new physics searches

V+jet fixed-order: NNLO QCD + NLO EW

V+jets MC samples: multi-jet merging at LO/NLO QCD



Progress towards new parton-level event generators
 capable of utilising modern hardware [Bothmann et al. '23]

Outlook of this talk

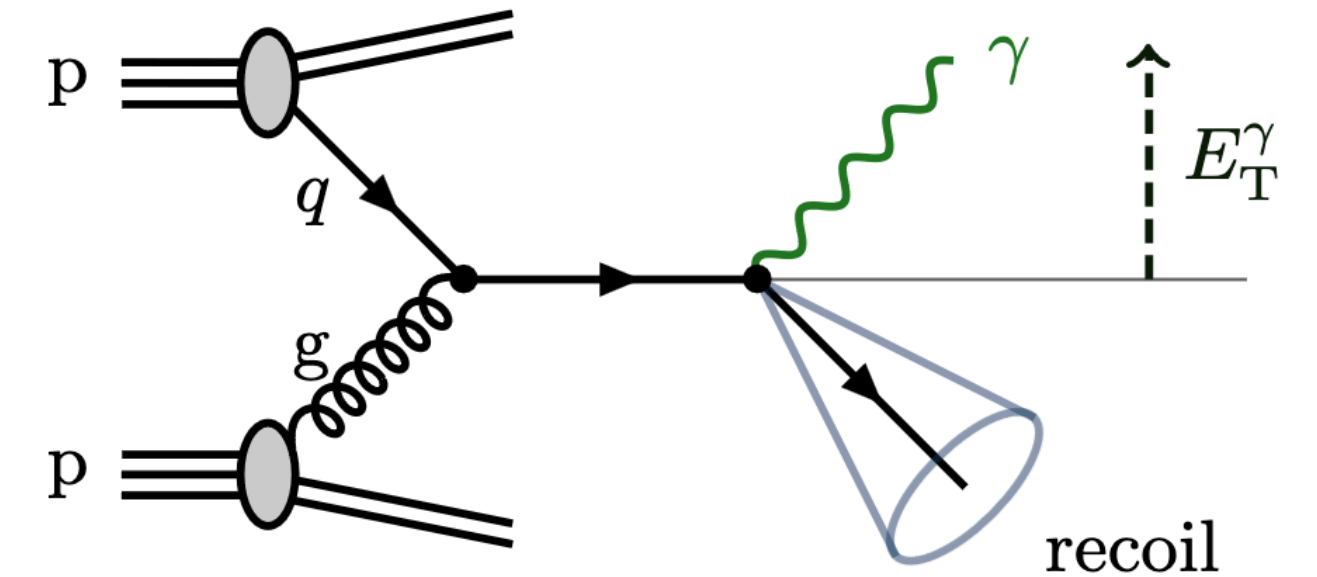
1. Two recent phenomenological results related to γ +jet(s):
 - Realistic photon isolation in photon-plus-jet events at NNLO
[Chen, Gehrmann, Glover, Höfer, Huss, Schürmann '22]
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[Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia '23]
2. Progress towards NNLO+PS for V +jet
3. Progress towards N³LO fixed-order for V +jet

*Personal selection of recent results that are representative for on-going progress.
Apologies for any relevant omission of references*

Outlook of this talk

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Realistic photon isolation in photon-plus-jet events at NNLO

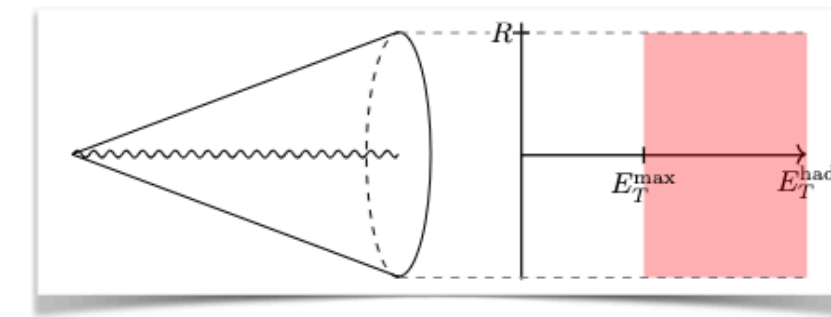
[Chen, Gehrmann, Glover, Höfer, Huss, Schürmann '22]

How to define an isolated photon?

Fixed-cone vs. dynamic-cone isolation has long been a systematic difference between theory and experiment

Hybrid-cone partially alleviate this inconsistency (correct R -dependence)

Inclusion of photon fragmentation [with $D_{p \rightarrow \gamma}(z)$] in theory predictions solve the mismatch: theory predictions with fixed-cone!

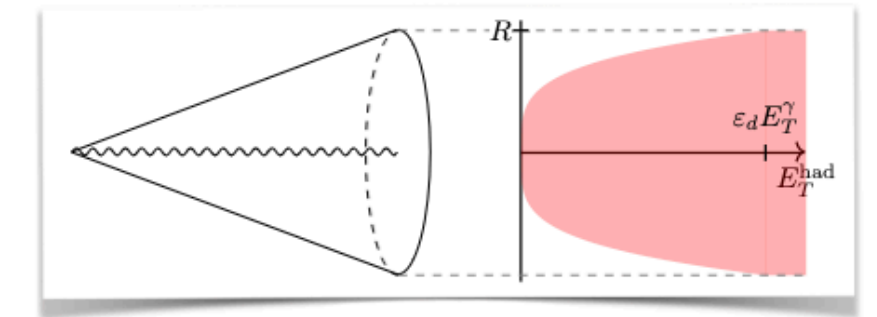


Fixed cone isolation

can choose simple linear dependence:

$$E_T^{\text{had.}}(R) < E_T^{\text{max}} = \epsilon E_T^\gamma + E_T^{\text{thresh.}}$$

- ✓ used in experiments
- ✗ sensitivity to fragmentation

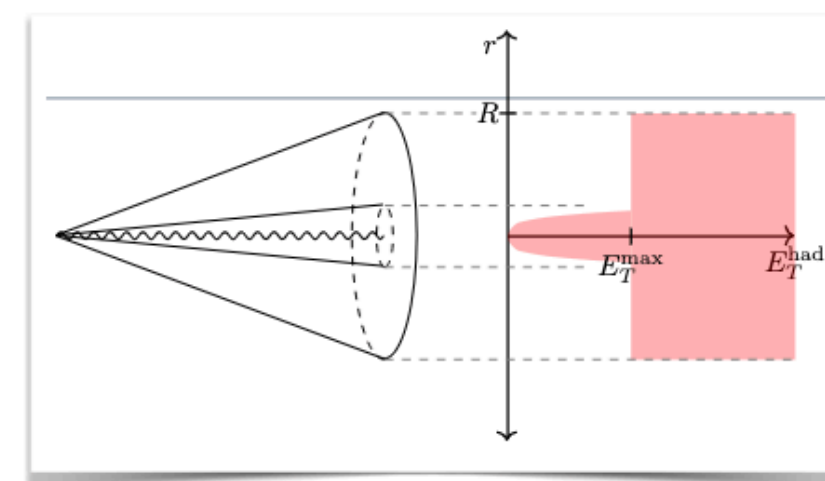


Dynamic cone isolation [Frixione '98]

smoothly get rid of collinear radiation:

$$E_T^{\text{had.}}(r) < \epsilon E_T^\gamma \left(\frac{1 - \cos r}{1 - \cos R} \right)^n \quad \forall r < R$$

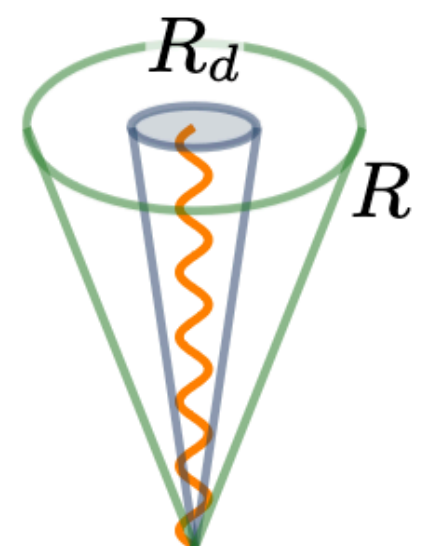
- ✓ eliminates fragmentation part
- ✗ no direct analogue in experiment

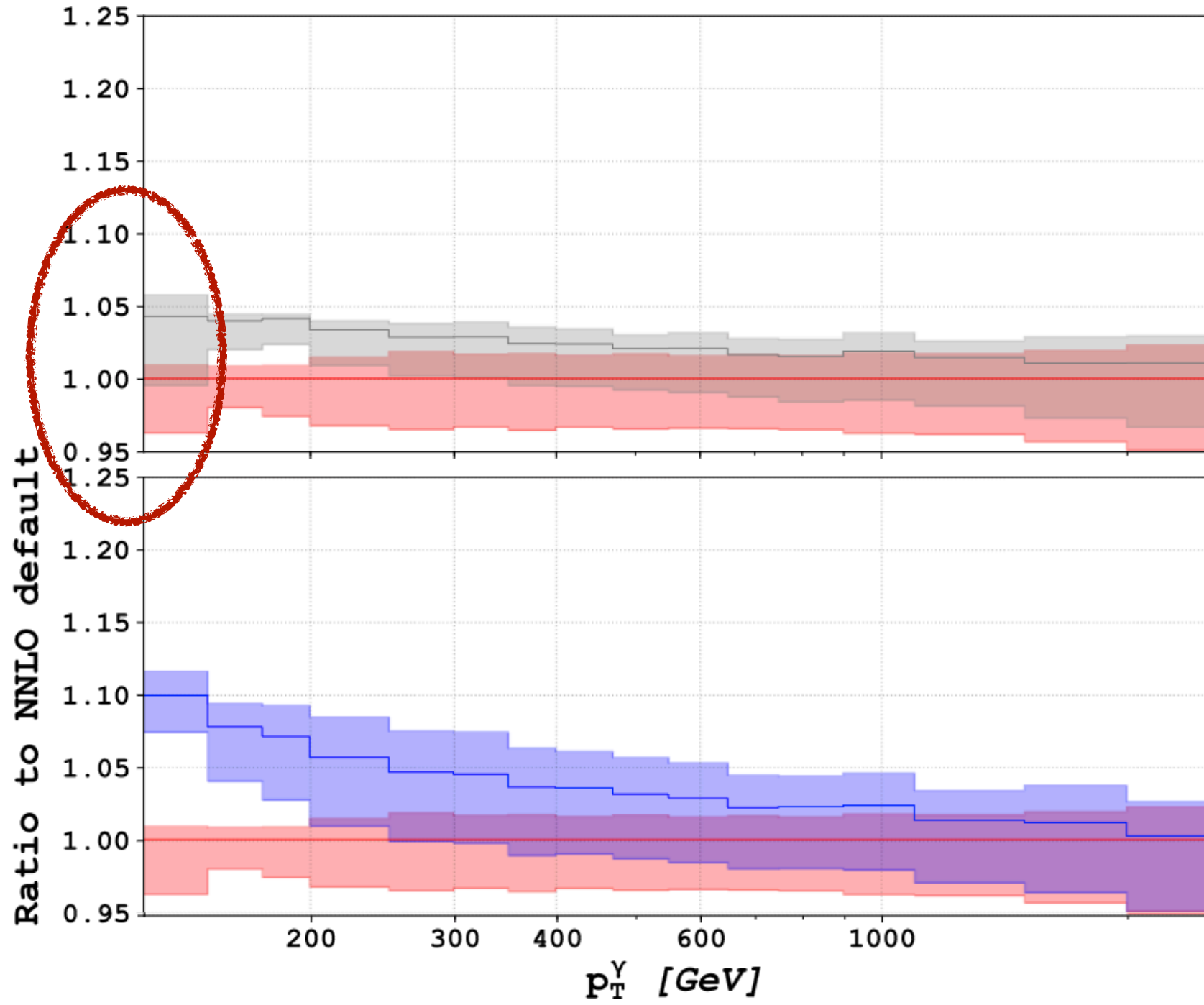
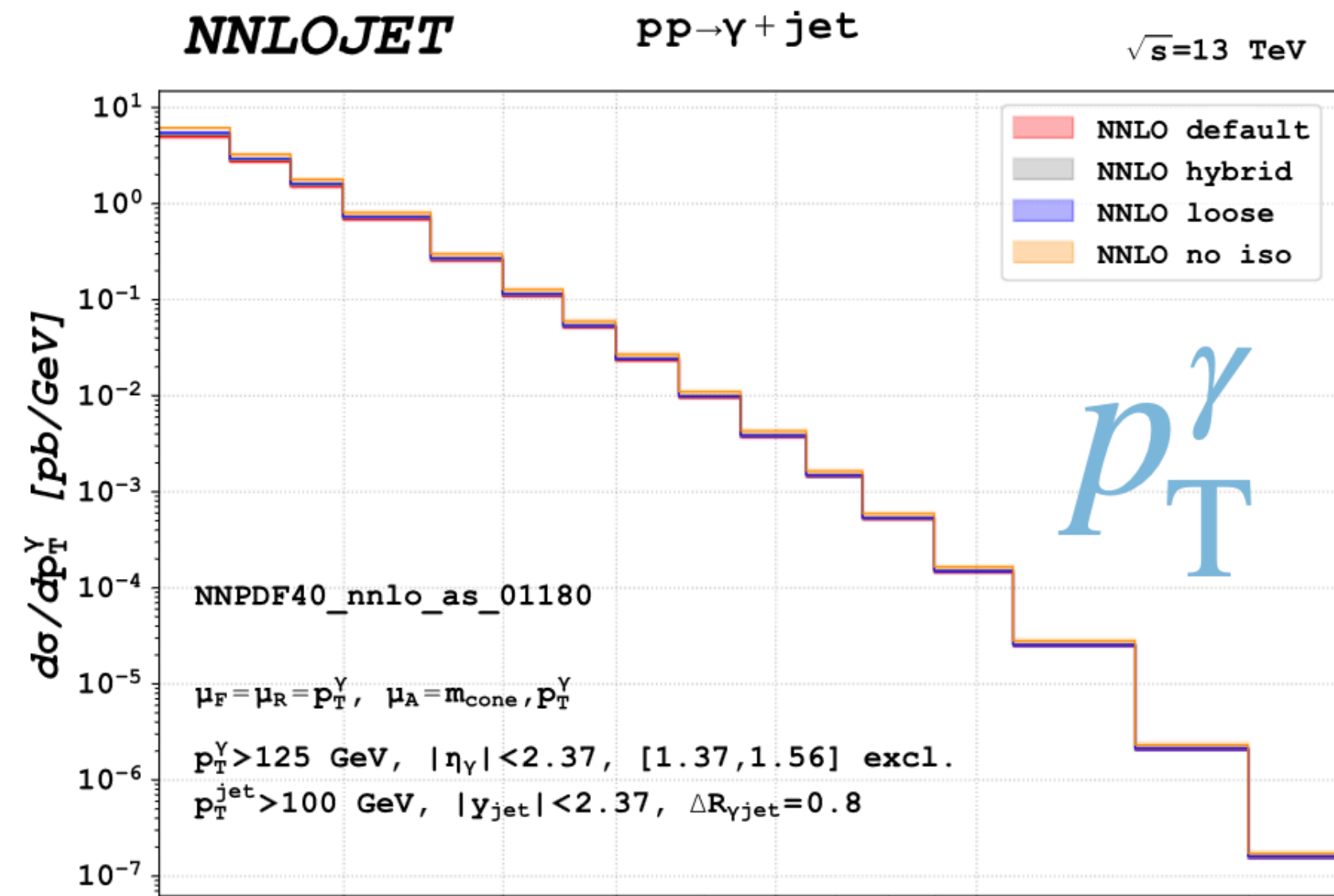


Hybrid cone isolation [Siegert '17]

[Siegert '17]

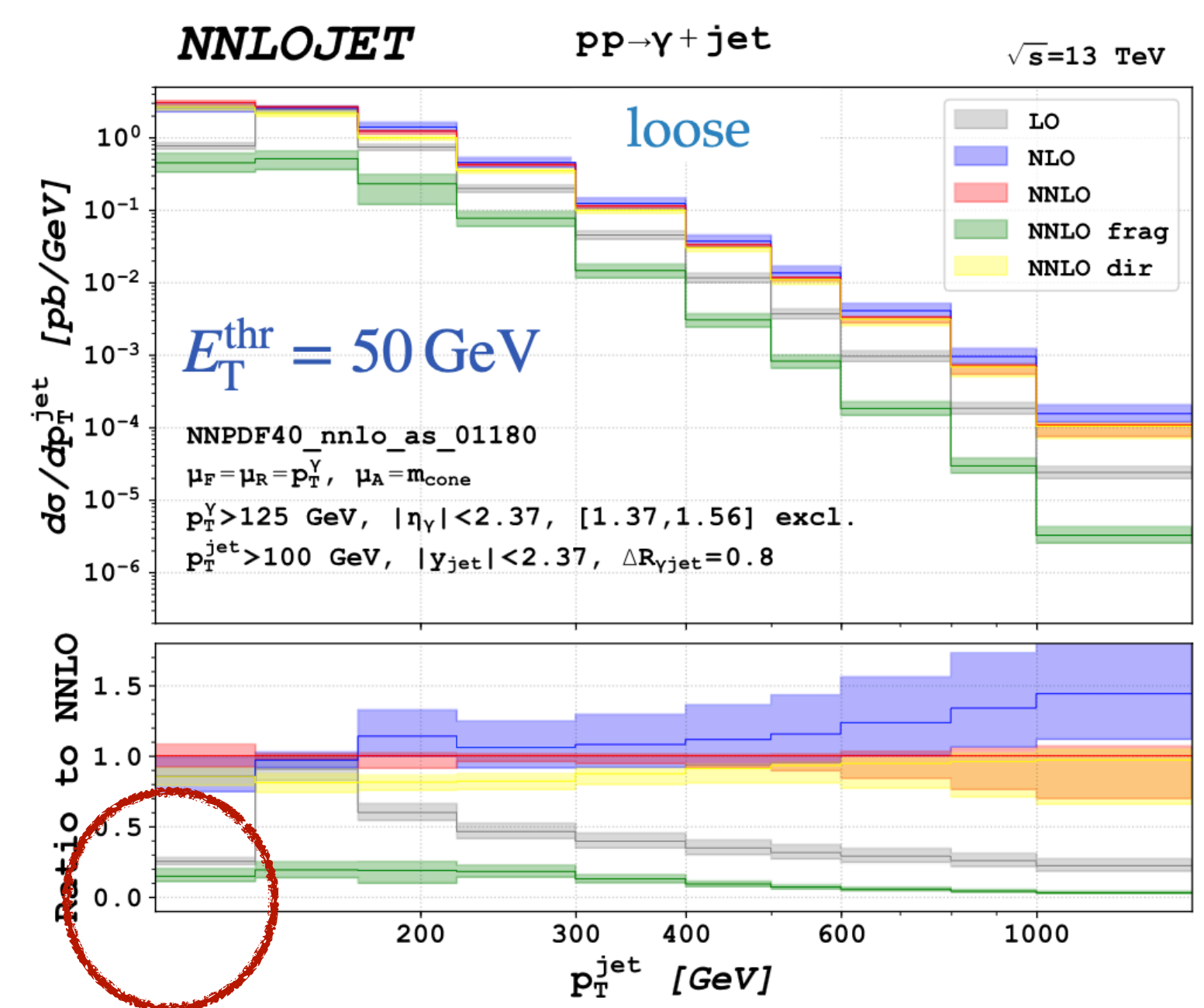
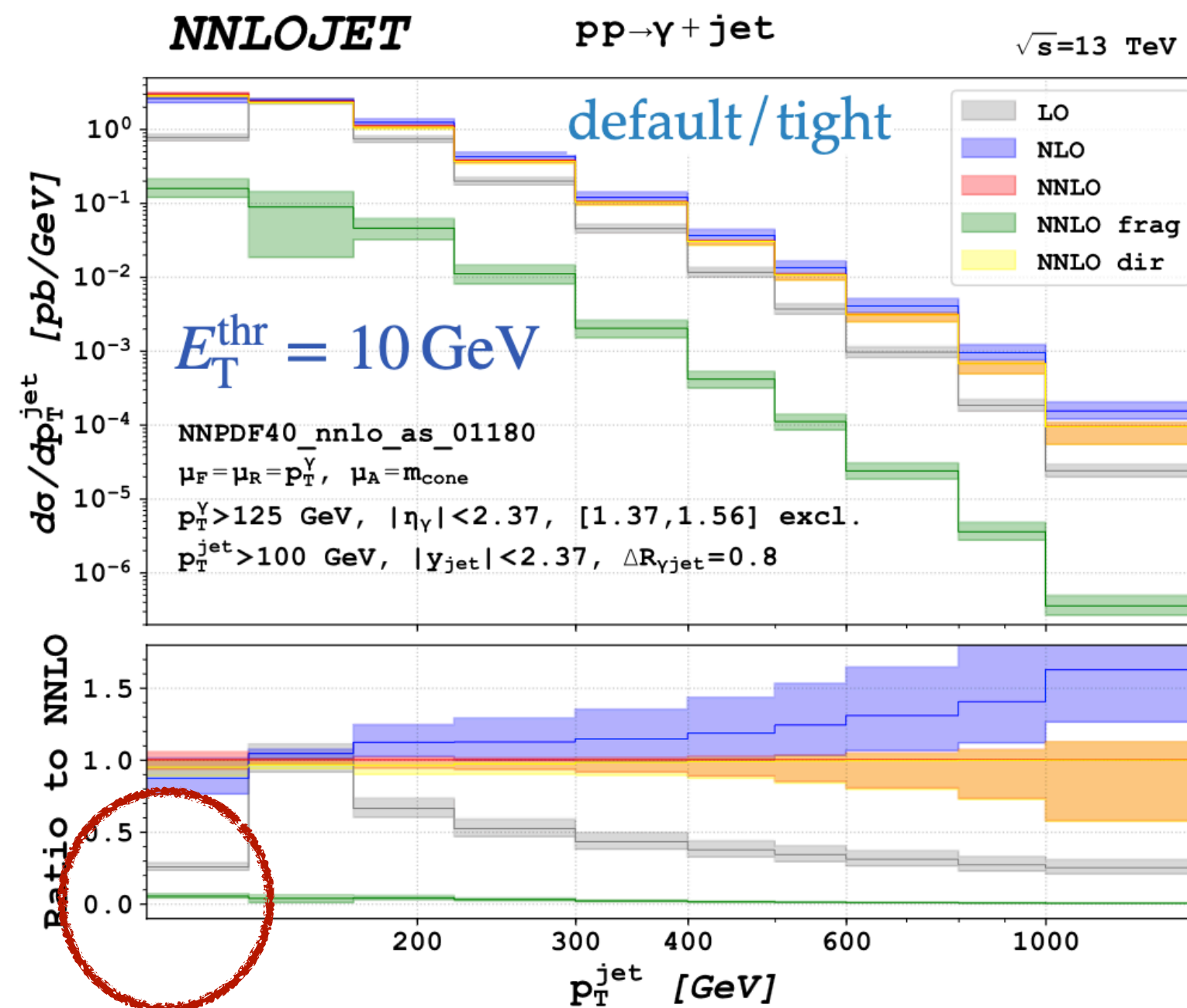
1. narrow dynamic cone $R_d < R$ (0.1)
 2. wider fixed cone R (0.4)
- ✓ eliminates fragmentation part
 - ✓ reduces mismatch to experiment
 - ✓ correct R dependence





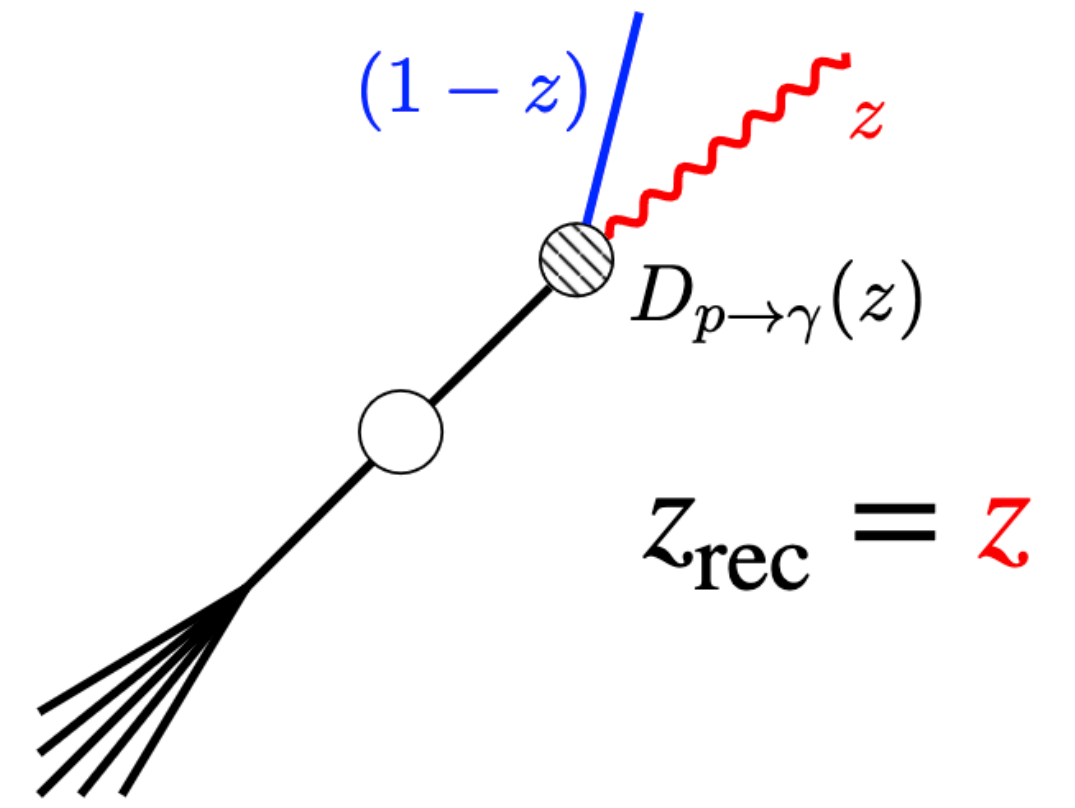
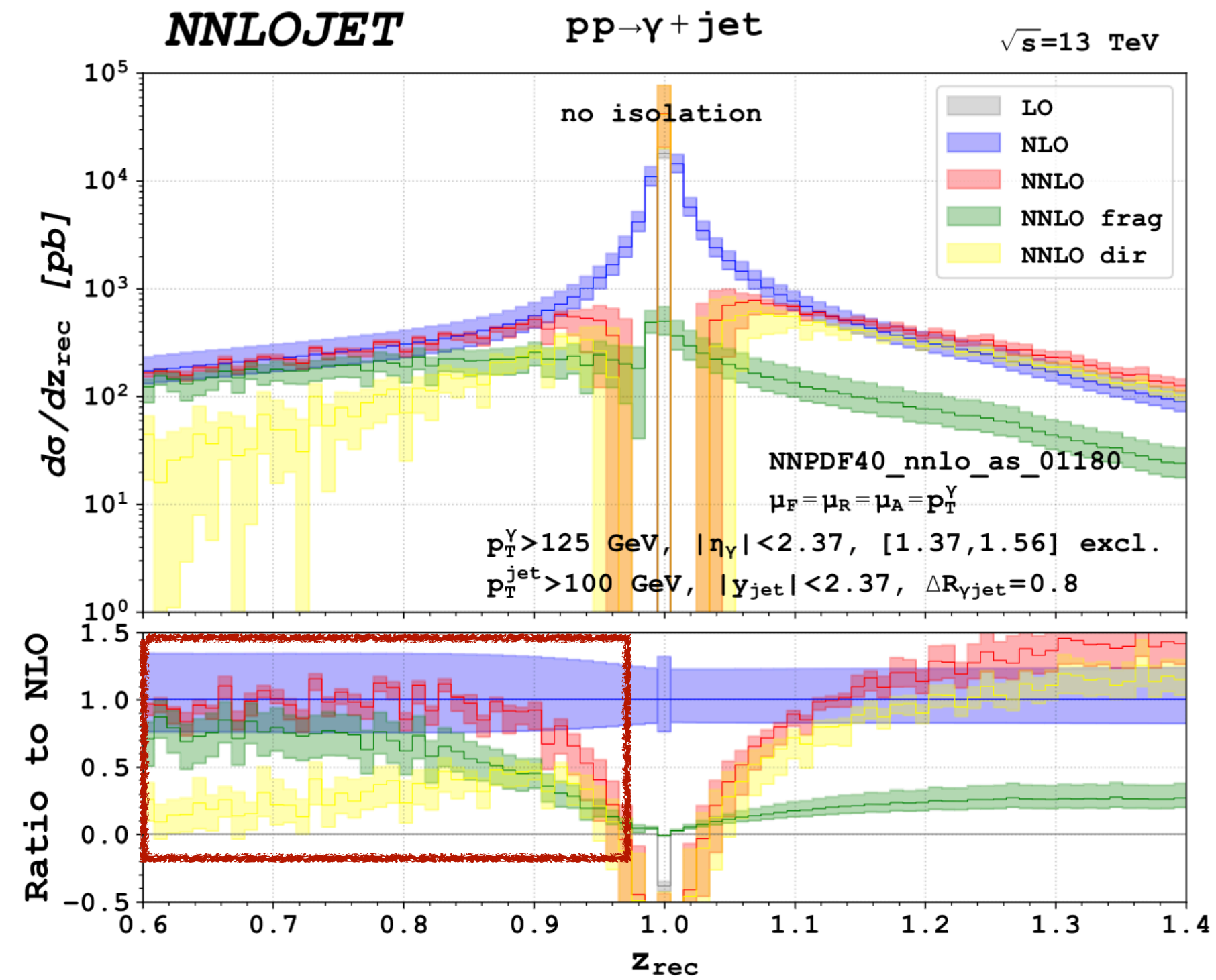
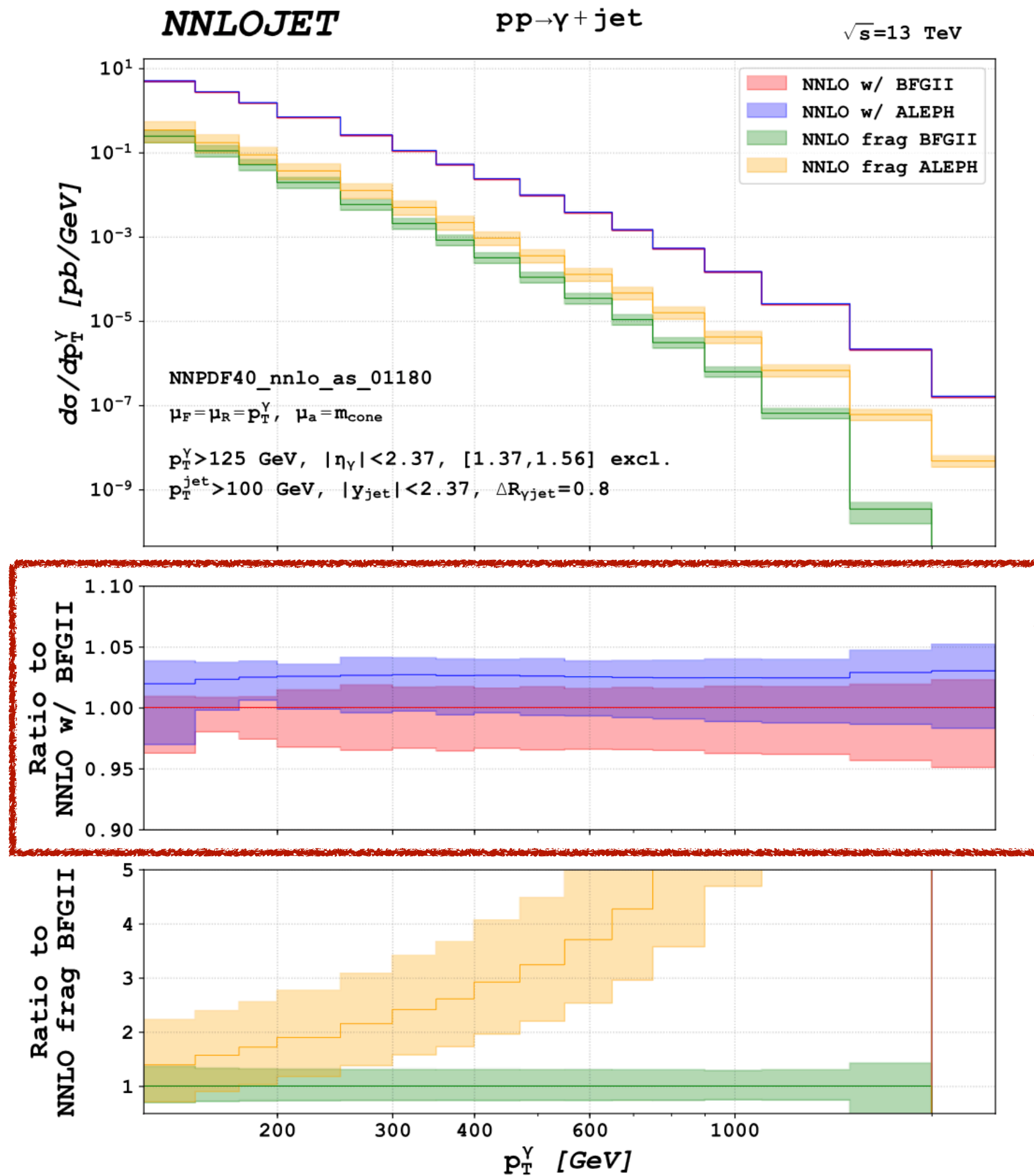
Fixed-cone with $R = 0.4, \epsilon = 0.0042$:
 $E_T^{\text{thrs.}} = 10 \text{ GeV}$ (default), $E_T^{\text{thrs.}} = 50 \text{ GeV}$ (loose)
 Hybrid-cone with $R_d = 0.1, R = 0.4$ and $\epsilon = 0.0042$

Hybrid vs. Fixed: 5% effect in the small- $p_T^{\gamma/\text{jet}}$ region
 Fragmentation component larger with looser isolation



Isolated photons at the LHC
 probe high- z ($z \gtrsim 0.93$), where
 $D_{p \rightarrow \gamma}(z)$ is poorly constrained

New observable $z_{\text{rec}} = p_T^\gamma / p_T^{\text{jet}}$ ($= z$ at LO)
 to extract $D_{p \rightarrow \gamma}(z)$ at the LHC



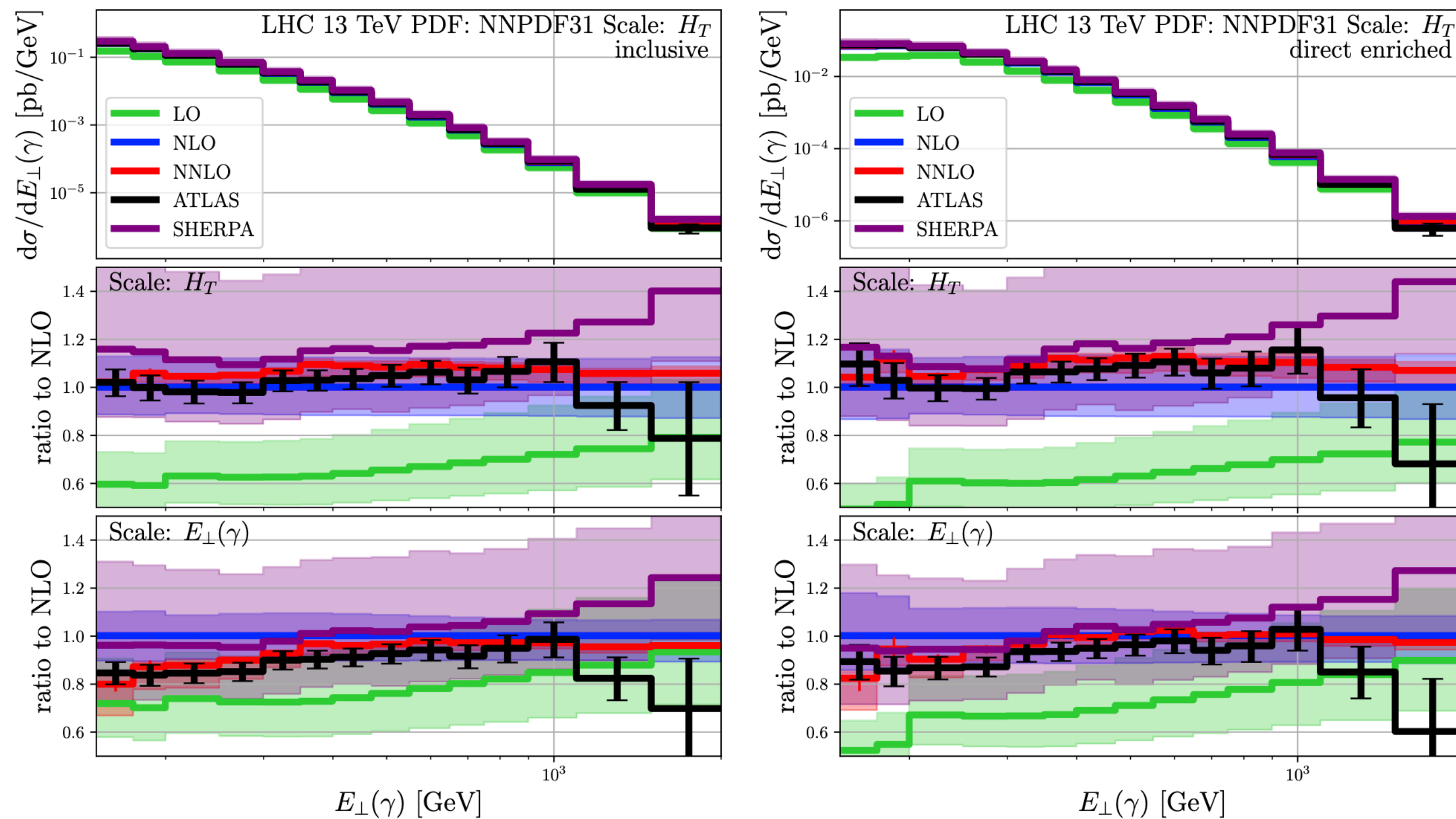
Best sensitivity to fragmentation component with
 no isolation (but experimentally challenging)

Isolated photon plus two jets at NNLO

[Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia '23]

Interesting process: access angular correlations between the photon and jets.

Hierarchy between $E_{\perp}(\gamma)$, $p_T(j_1)$ and $p_T(j_2)$: relative size of direct and fragmentation contributions e.g. *direct-enriched*: $E_{\perp}(\gamma) > p_T(j_1)$



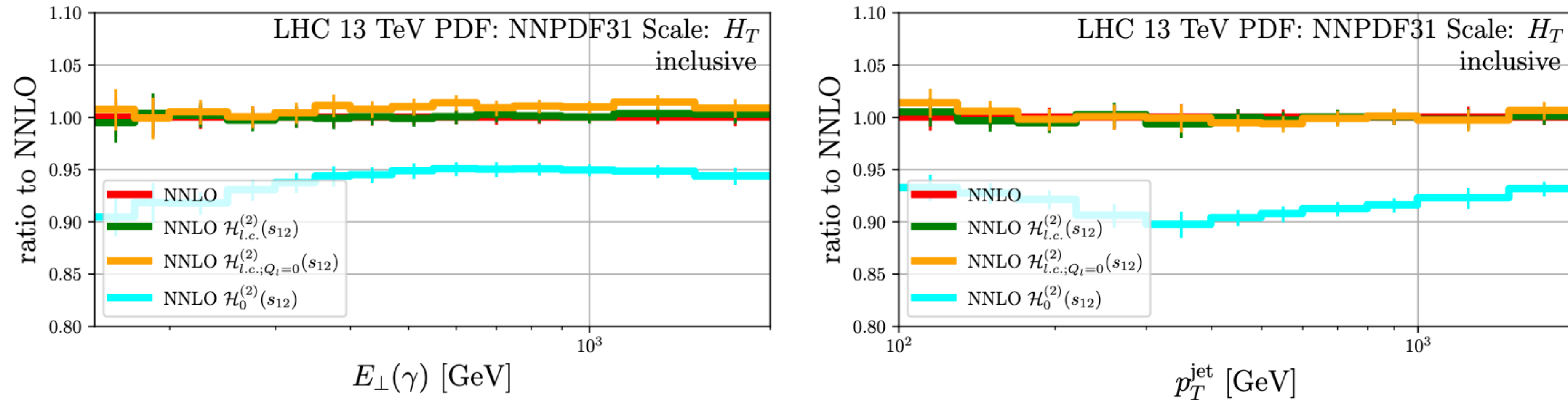
Good perturbative convergence
Improved agreement with ATLAS data

No fragmentation included, but expected to be small with hybrid-cone and in the direct-enriched region

In the tail, missing EW corrections, expected to be large and negative

First calculation for a $2 \rightarrow 3$ process with exact full colour 2-loop amplitude

Comparison of full NNLO, NNLO with two-loop finite-remainder at leading colour (“NNLO $\mathcal{H}_{l.c.}^{(2)}$ ”) and NNLO no two-loop finite-remainder (“NNLO $\mathcal{H}_0^{(2)}$ ”)



While the inclusion of finite-remainder is important (5-10% effect), the leading colour approximation seems to be good at cross section level

Similar observations in $pp \rightarrow \gamma\gamma\gamma$ [Abreu, De Laurentis, Ita, Klinkert, Page, Sotnikov '23]: differences between $\mathcal{H}^{(2)}$ and $\mathcal{H}_{l.c.}^{(2)}$ expected to be small at cross section level

N.B. process-dependent statement! Knowledge of full colour is generally important

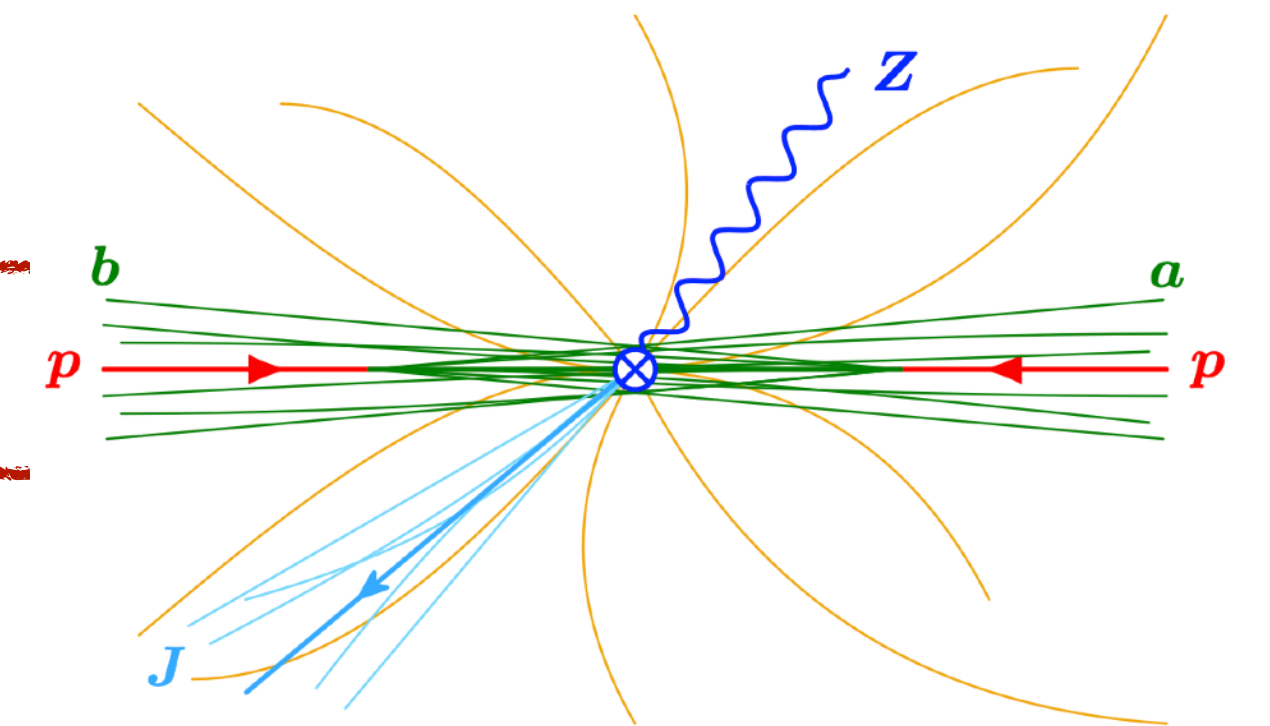
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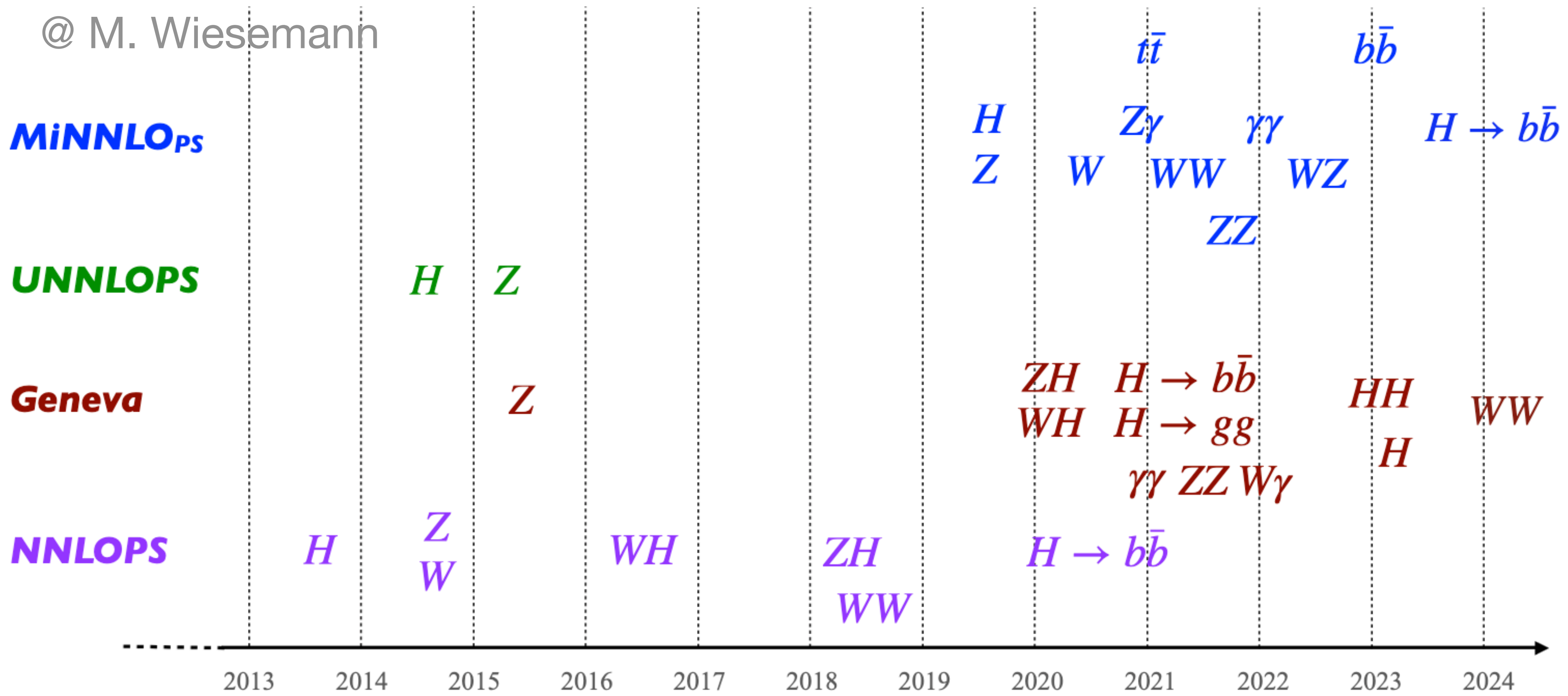
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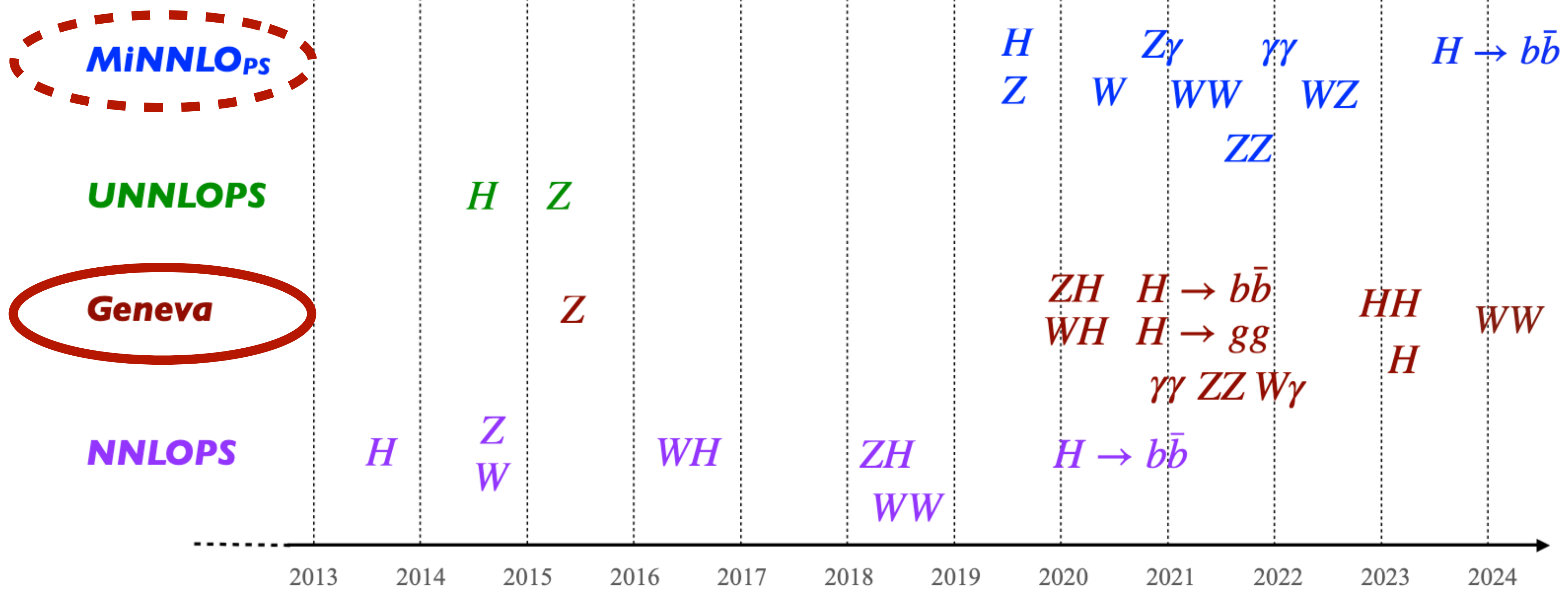
Apologies for any relevant omission of references

@ M. Wiesemann



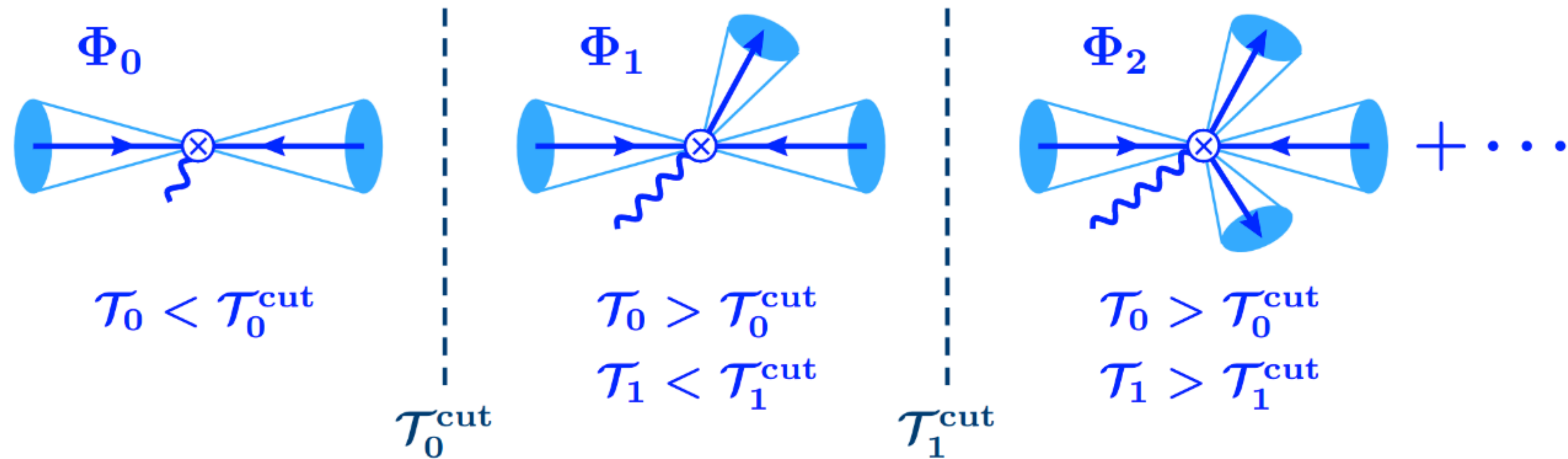
Impressive results in the recent years, but so far limited to processes with colour-singlets or heavy quarks in the final state

@ M. Wiesemann



Impressive results in the recent years, but so far limited to processes with colour-singlets or heavy quarks in the final state

GENEVA in a nutshell (for colour-singlet production)



Division into 0/1/2-jet events dictated by **resolution variable(s)** \mathcal{T}_N

Originally developed for N -jettiness \mathcal{T}_N , but later extended to colour-singlet q_T [Alioli, Bauer et al. '21] and leading-jet p_T [Gavardi, Lim et al. '23]

As \mathcal{T}_N s regulate IR divergences, large logarithms appear: resummation is required!

\mathcal{T}_0 resummed up to NNLL', \mathcal{T}_1 up to NLL

GENEVA in a nutshell (for colour-singlet production)

Example: 0/1-jet separation, dictated by $\mathcal{T}_0^{\text{cut}}$

$$\frac{d\sigma_0^{\text{MC}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) = \frac{d\sigma^{\text{NNLL}'}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) + \frac{d\sigma_0^{\text{nons}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}),$$

$$\frac{d\sigma_0^{\text{nons}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) = \frac{d\sigma_0^{\text{NNLO}_0}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) - \left[\frac{d\sigma^{\text{NNLL}'}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) \right]_{\text{NNLO}_0}$$

Below the cut, one adopts the *integrated* resummed cross section, with additive matching to fixed-order result (by requiring $\mathcal{T}_0 < \mathcal{T}_0^{\text{cut}}$)

$$\frac{d\sigma_{\geq 1}^{\text{MC}}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}) = \frac{d\sigma^{\text{NNLL}'}}{d\Phi_0 d\mathcal{T}_0} \mathcal{P}(\Phi_1) \theta(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}})$$

$$+ \frac{d\sigma_{\geq 1}^{\text{nons}}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}),$$

Above the cut, one adopts the *differential* resummed cross section, with additive matching to fixed-order result (by requiring $\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}$)

$$\int \frac{d\Phi_1}{d\Phi_0 d\mathcal{T}_0} \mathcal{P}(\Phi_1) = 1$$

Normalised “splitting” function $\mathcal{P}(\Phi_1)$ to make the resummed cross section differential in the higher multiplicity phase space

How to extend GENEVA to vector boson plus jet production?

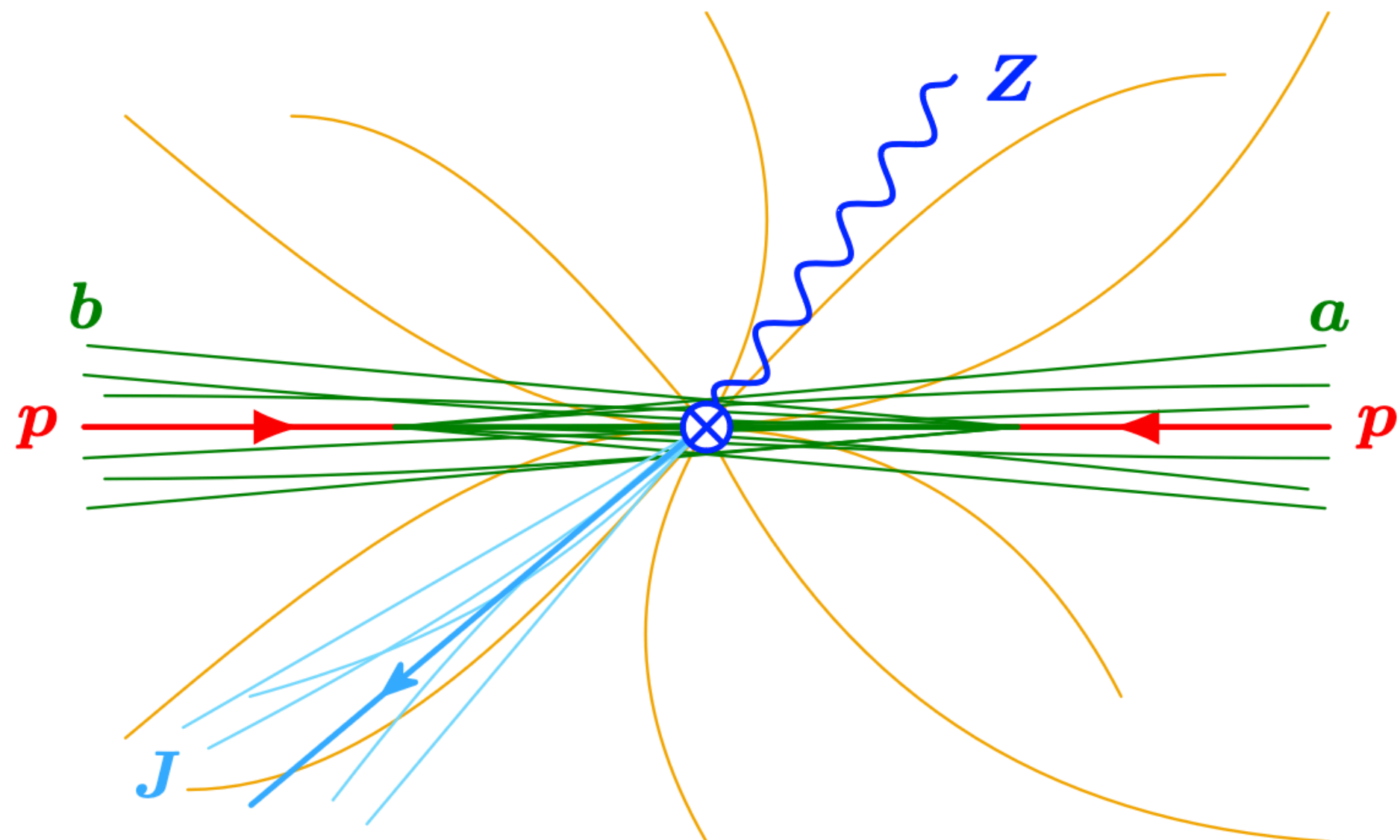
First step: resummation of one-jettiness \mathcal{T}_1 , performed up to N³LL

[Alioli, Bell, Billis, Broggio, Dehnadi, Lim, Marinelli, Nagar, Napoletano, Rahn '23]

$$\mathcal{T}_1 = \sum_k \min \left\{ \frac{2q_a \cdot k}{Q_a}, \frac{2q_b \cdot k}{Q_b}, \frac{2q_J \cdot k}{Q_J} \right\}$$

$$Q_i = 2\rho_i E_i$$

Freedom in precise definition of \mathcal{T}_1 :
 dependence on reference frame;
 dependence on definition of jet axis
 (e.g. obtained recursively with exclusive clustering or a priori with inclusive clustering)



frames	$\rho_{a,b}$	ρ_J
Lab	1	1
Color Singlet (CS)	$e^{\pm Y_V}$	$(e^{Y_V} p_J^- + e^{-Y_V} p_J^+) / E_J$
Underlying Born (UB)	$e^{\pm Y_{VJ}}$	$(e^{Y_{VJ}} p_J^- + e^{-Y_{VJ}} p_J^+) / E_J$

@ G. Billis

Resummation of one-jettiness \mathcal{T}_1 in SCET: analytical ingredients

Beam function: known up to N³LO for any \mathcal{T}_N
 [Ebert, Mistlberger, Vita '20]

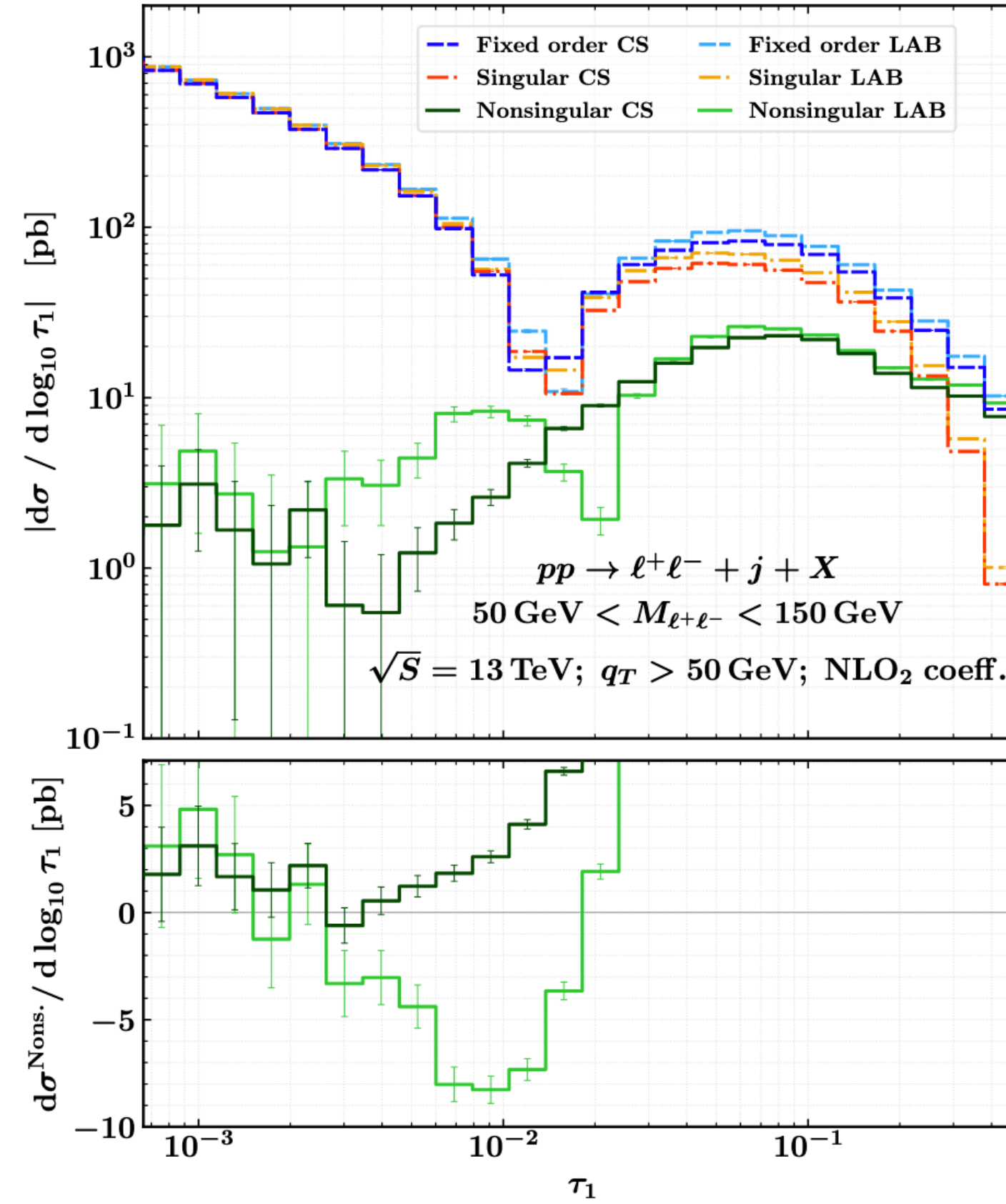
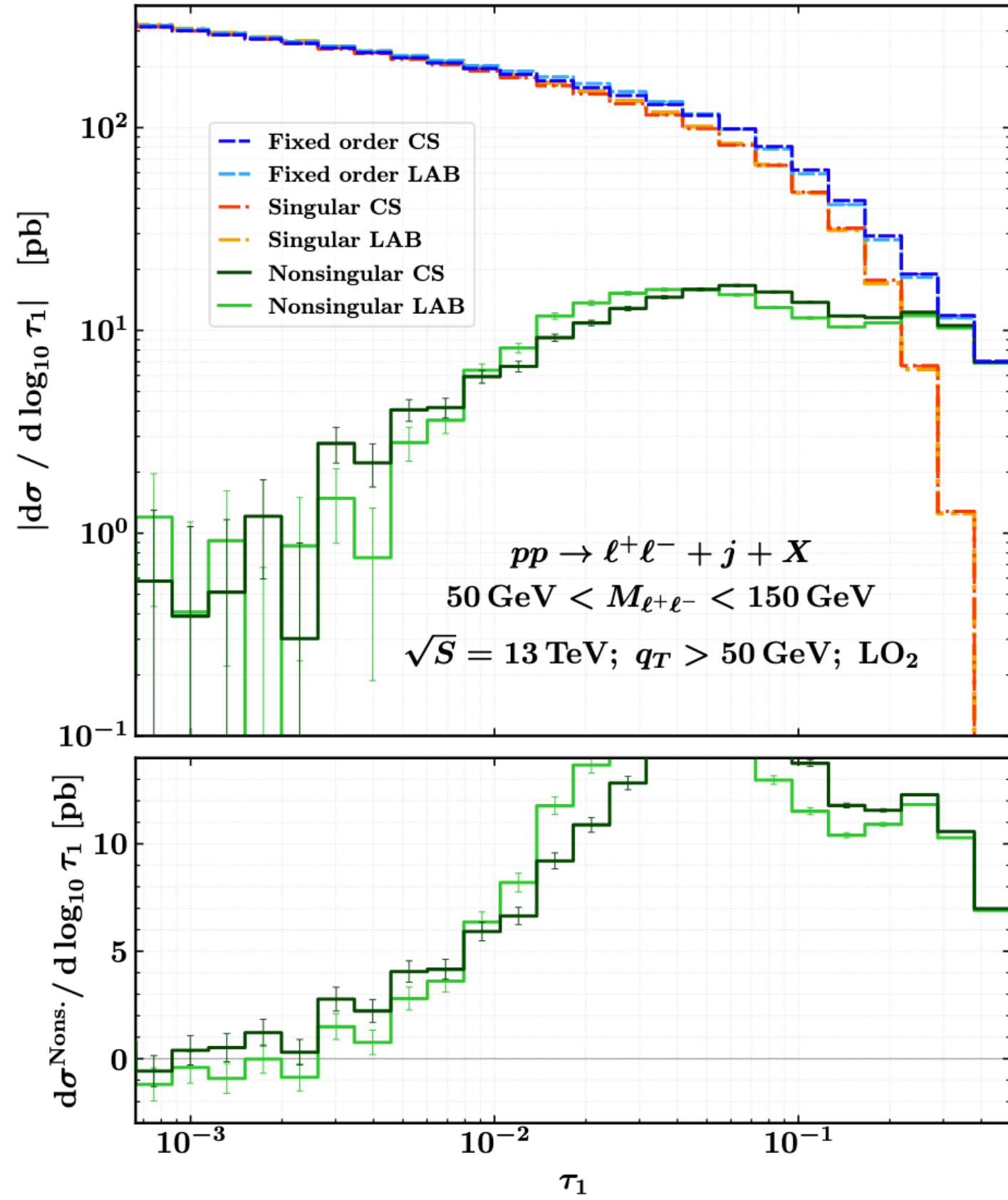
Jet function (universal): known up to N³LO
 [Brüser, Liu, Stahlhofen '18]
 [Banerjee, Dhani, Ravindran '18]

$$\frac{d\sigma}{d\mathcal{T}_1} = H_\kappa(\mu) \int dt_a dt_b ds_J B_a(t_a, \mu) B_b(t_b, \mu) J_c(s_J, \mu) \times S_\kappa(\mathcal{T}_1 - t_a/Q_a - t_b/Q_b - s_J/Q_J, \mu)$$

Hard function: extracted from two-loop amplitudes
 [Gehrmann, Tancredi et al. '12, '22]

Soft function: known for \mathcal{T}_1 up to NNLO [Campbell, Ellis, Mondini, Williams '17],
 but novel NNLO evaluations for any \mathcal{T}_N
 [Bell, Dehnadi, Mohrmann, Rahn '23] [Agarwal, Melnikov, Pedron '24]

Matching the resummation to fixed order: size of nonsingular



$$\frac{d\sigma^{\text{N}^3\text{LL}+\text{NLO}_2}}{d\Phi_1 d\mathcal{T}_1} = \frac{d\sigma^{\text{N}^3\text{LL}}}{d\Phi_1 d\mathcal{T}_1} + \frac{d\sigma^{\text{Nons.}}}{d\Phi_1 d\mathcal{T}_1},$$

$$\frac{d\sigma^{\text{Nons.}}}{d\Phi_1 d\mathcal{T}_1} = \left(\frac{d\sigma^{\text{NLO}_2}}{d\Phi_1 d\mathcal{T}_1} - \frac{d\sigma^{\text{N}^3\text{LL}}}{d\Phi_1 d\mathcal{T}_1} \Big|_{\mathcal{O}(\alpha_s^2)} \right) \theta(\mathcal{T}_1)$$

Nonsingular = Fixed order - Singular

In order to have a finite Born for Z +jet, one adopts a cut on q_T (or on \mathcal{T}_0 , see backup)

$$\tau_1 = \mathcal{T}_1 / m_T$$

$$m_T \equiv \sqrt{M_{\ell^+\ell^-}^2 + q_T^2}$$

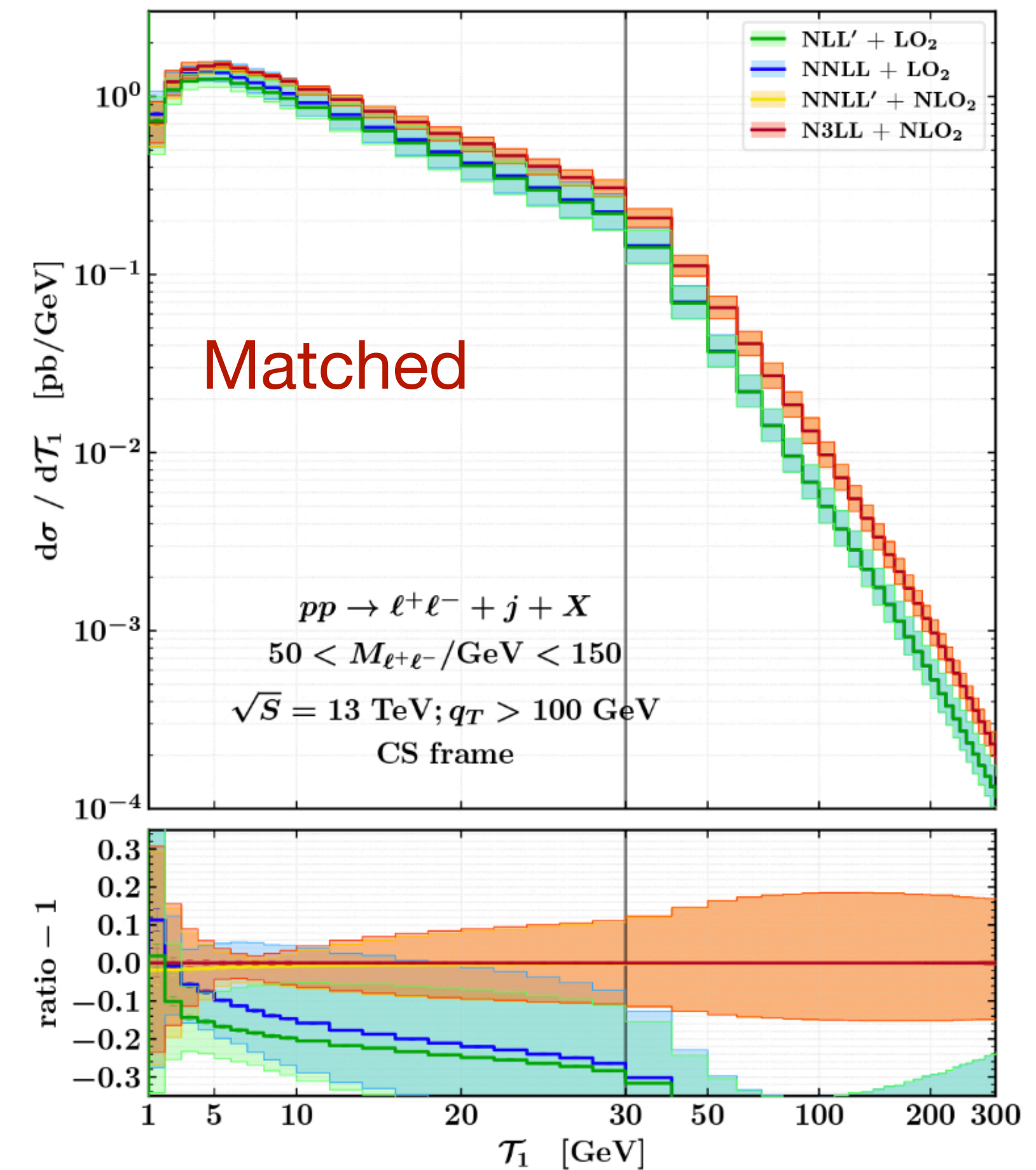
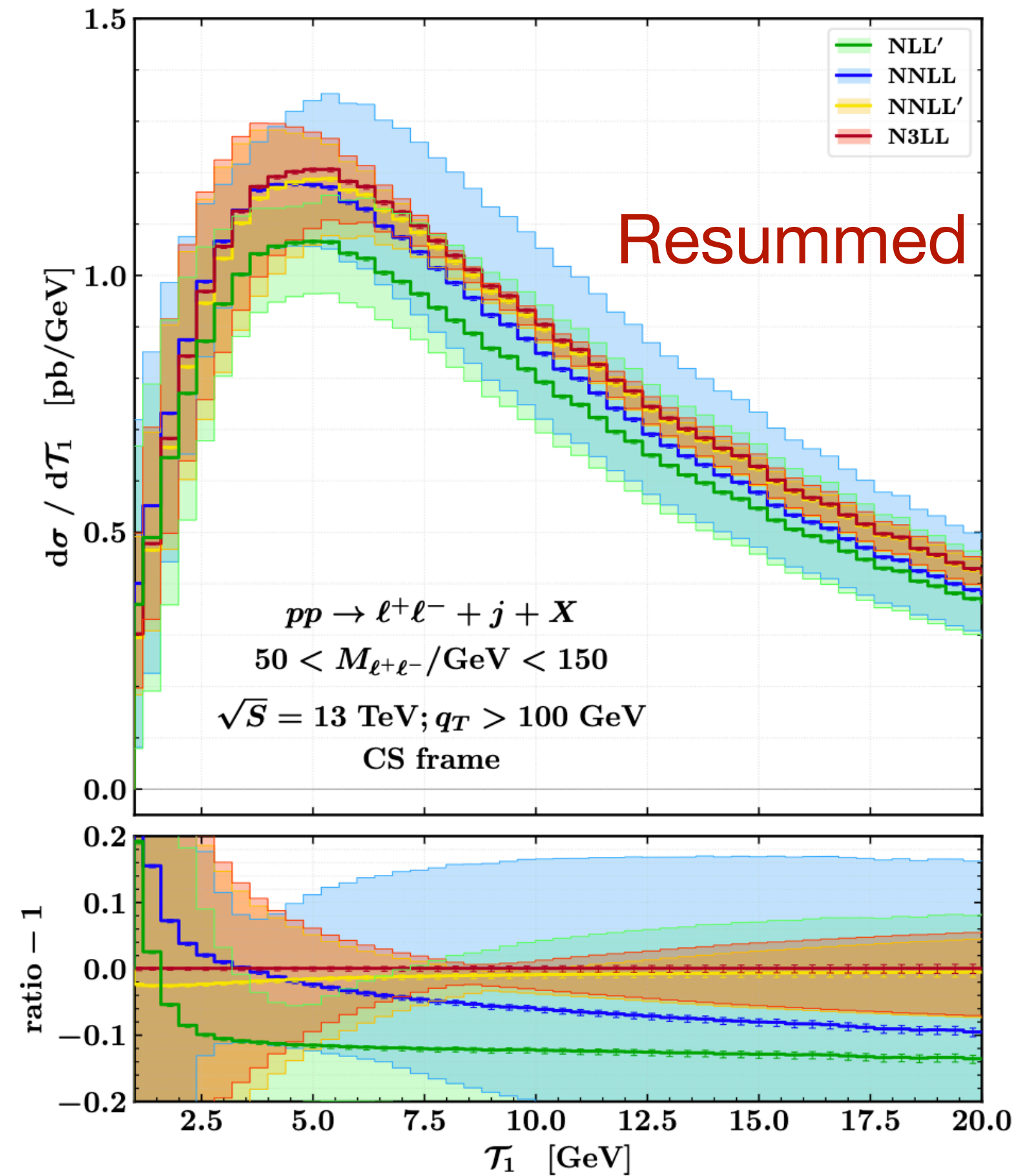
Fixed-order approaches singular as $\tau_1 \rightarrow 0$ (as expected)
 Power corrections seem to behave better in the CS frame

Results for resummed and matched result

NLL' \rightarrow NNLL \rightarrow NNLL' sizeable
 NNLL' \rightarrow N³LL minor effect

Large effect from NLO₂
 fixed-order (not surprising)

When decreasing q_T , larger
 differences between curves.
 Joint resummation would be
 required in that case.

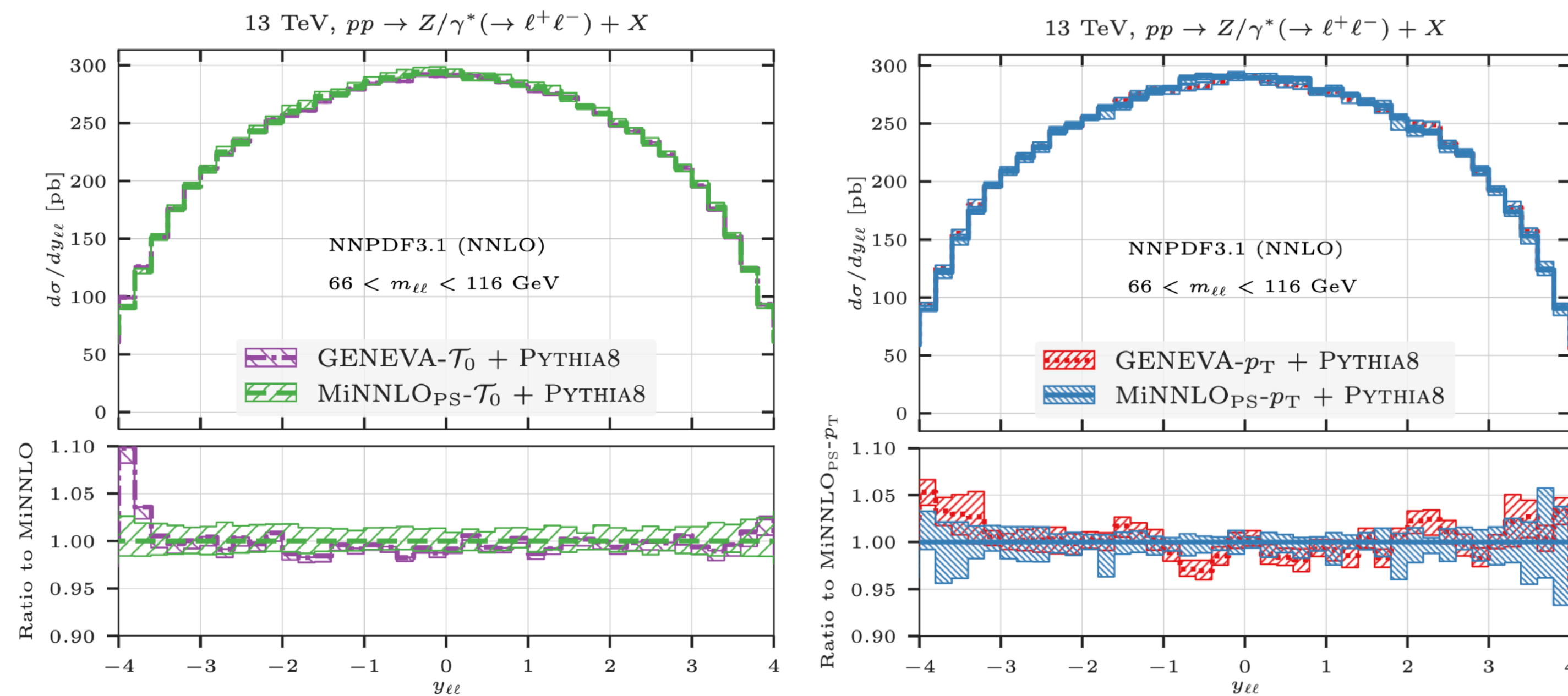


Next steps towards NNLO+PS: \mathcal{T}_1 -preserving mapping, splitting functions $\mathcal{P}_{2 \rightarrow 3}(\Phi_2)$,
 interface to PS, better understanding of different definitions of \mathcal{T}_1 ...

Jettiness-like variables in MiNNLO_{PS}

MiNNLO_{PS} is another powerful method to achieve NNLO+PS accuracy based on Sudakov factors to resum logarithmic dependence on resolution parameters and to a multiplicative-like matching to reach NNLO accuracy

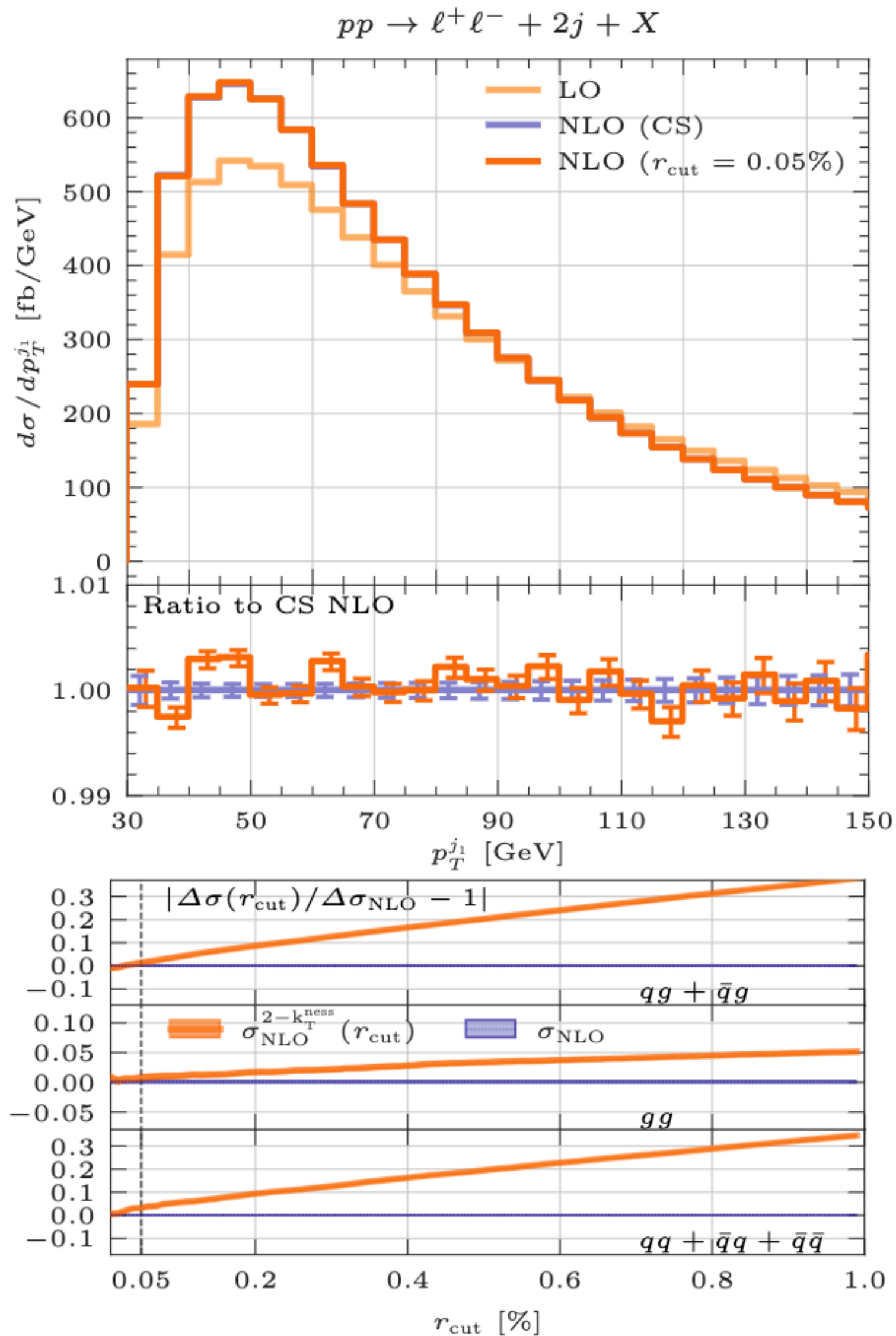
Originally developed using q_T -like observables, it has been recently extended to use jettiness-like variables [Ebert, Rottoli, Wiesemann, Zanderighi, Zanolini '24]



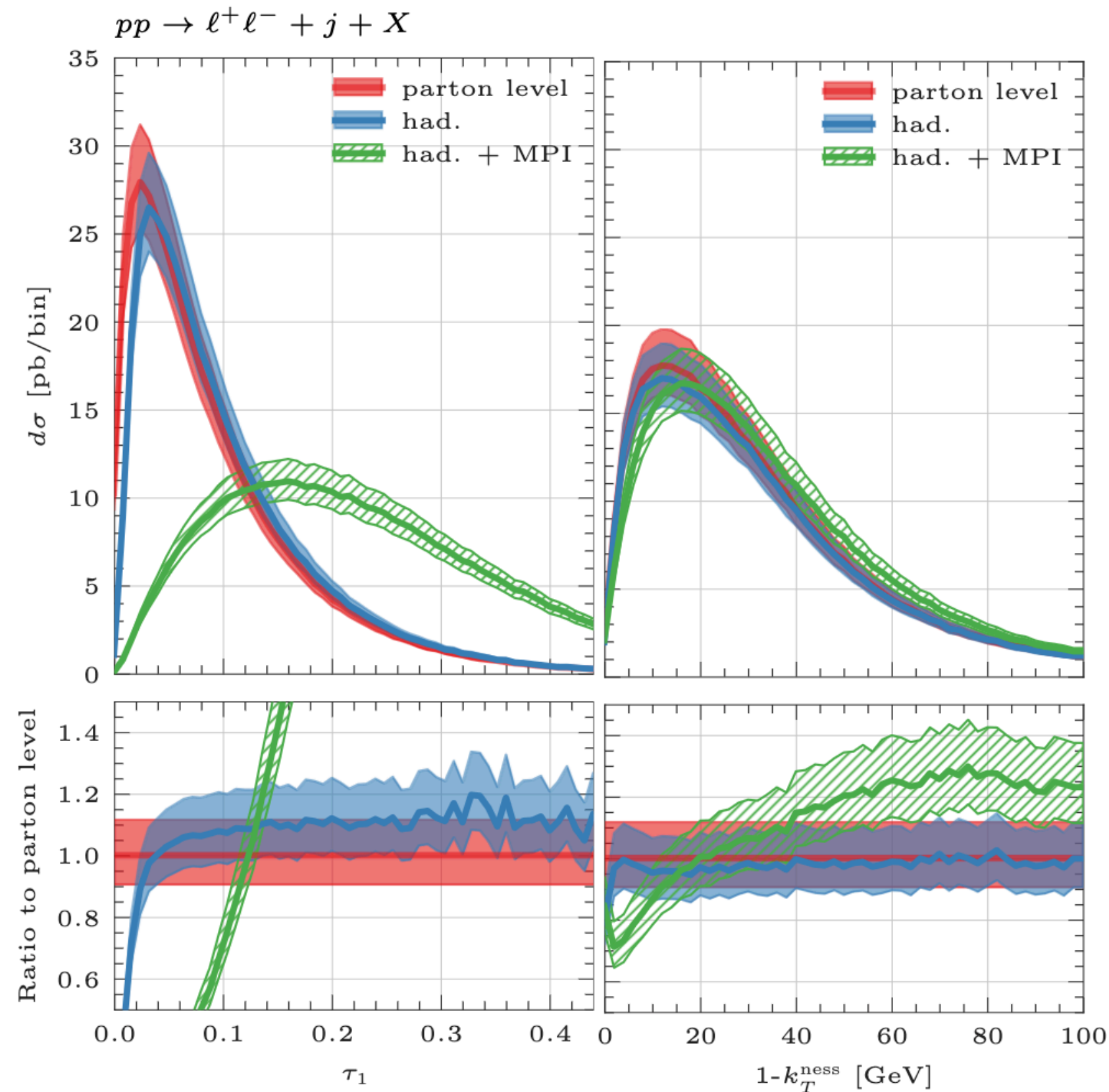
Formalism for \mathcal{T}_0 and \mathcal{T}_1 ,
phenomenological results for \mathcal{T}_0

Implementation of different
resolution variables in different
frameworks important to assess
systematic uncertainties

Transverse-momentum like observables for processes with final-state jets?



e.g. k_T^{ness} , based on exclusive k_T -clustering algorithm
 [Buonocore, Grazzini, Haag, Rottoli, C. Savoini '22, '23]



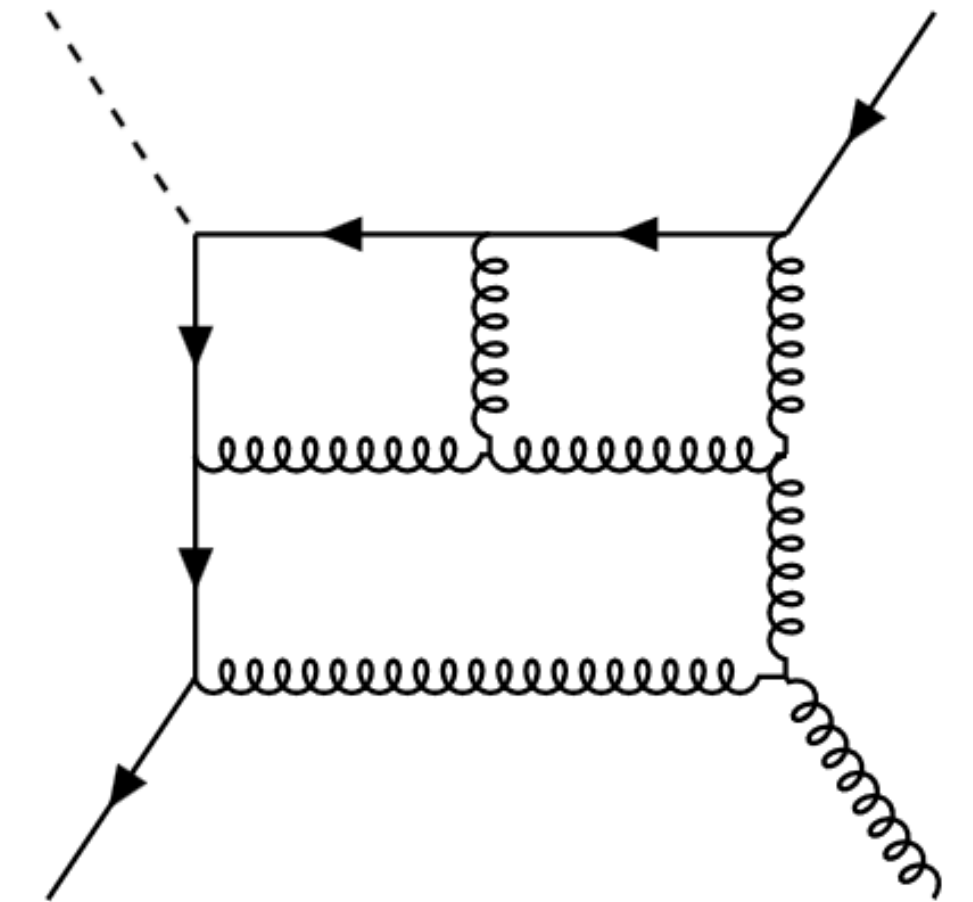
More stable than \mathcal{T}_1
 under had. and MPI effects

All ingredients at NLO,
 extension to NNLO in progress

Resummation up to NNLL'
 would also allow for usage in
 NNLO+PS frameworks

Outlook of this talk

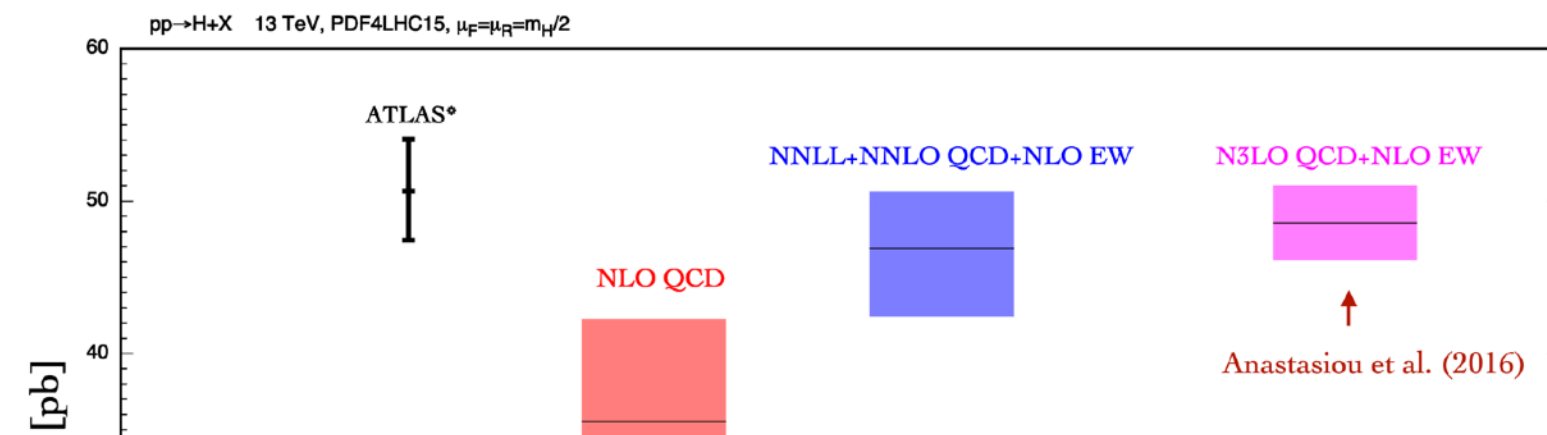
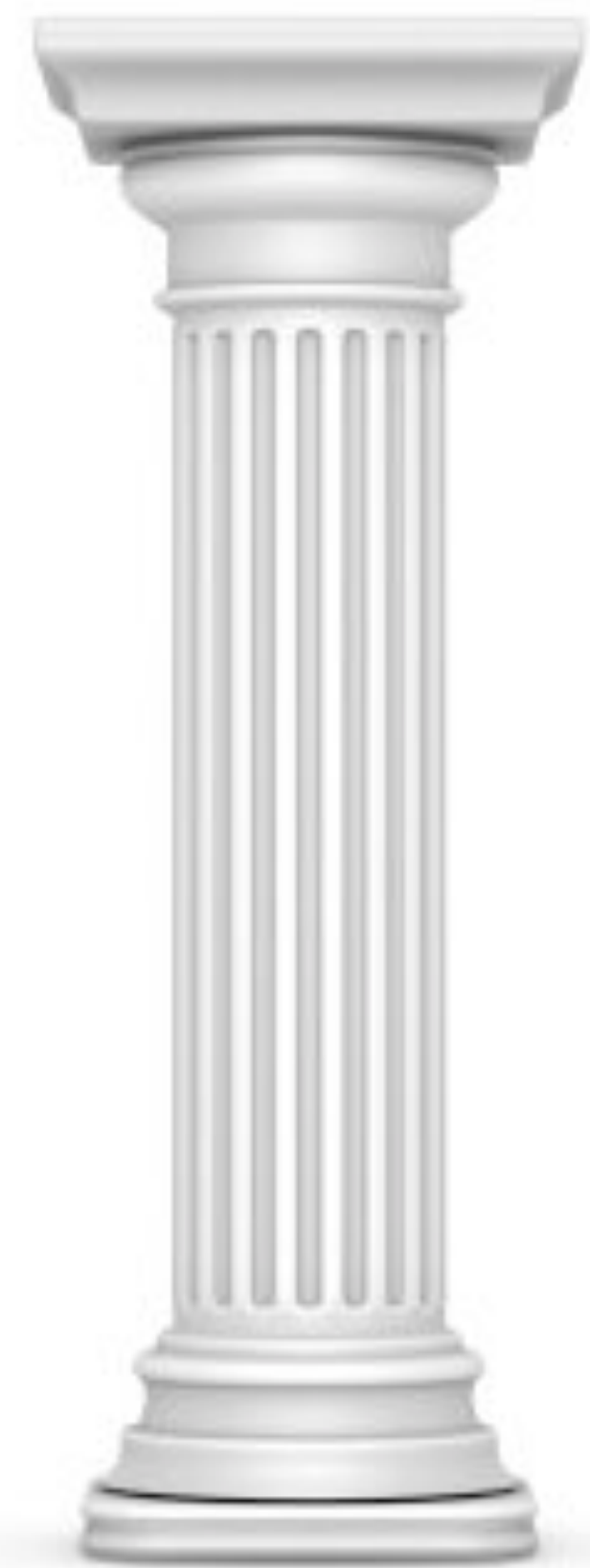
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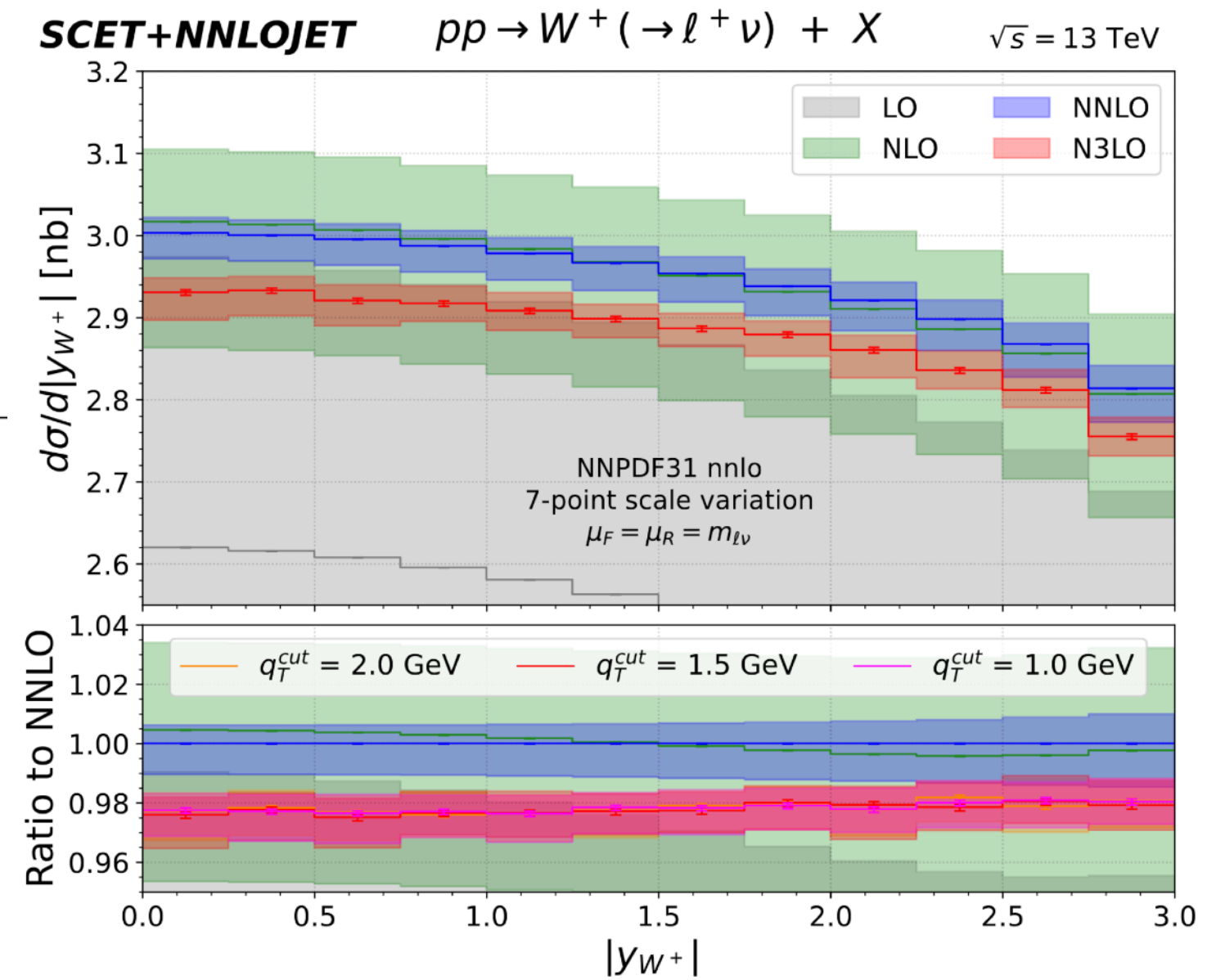
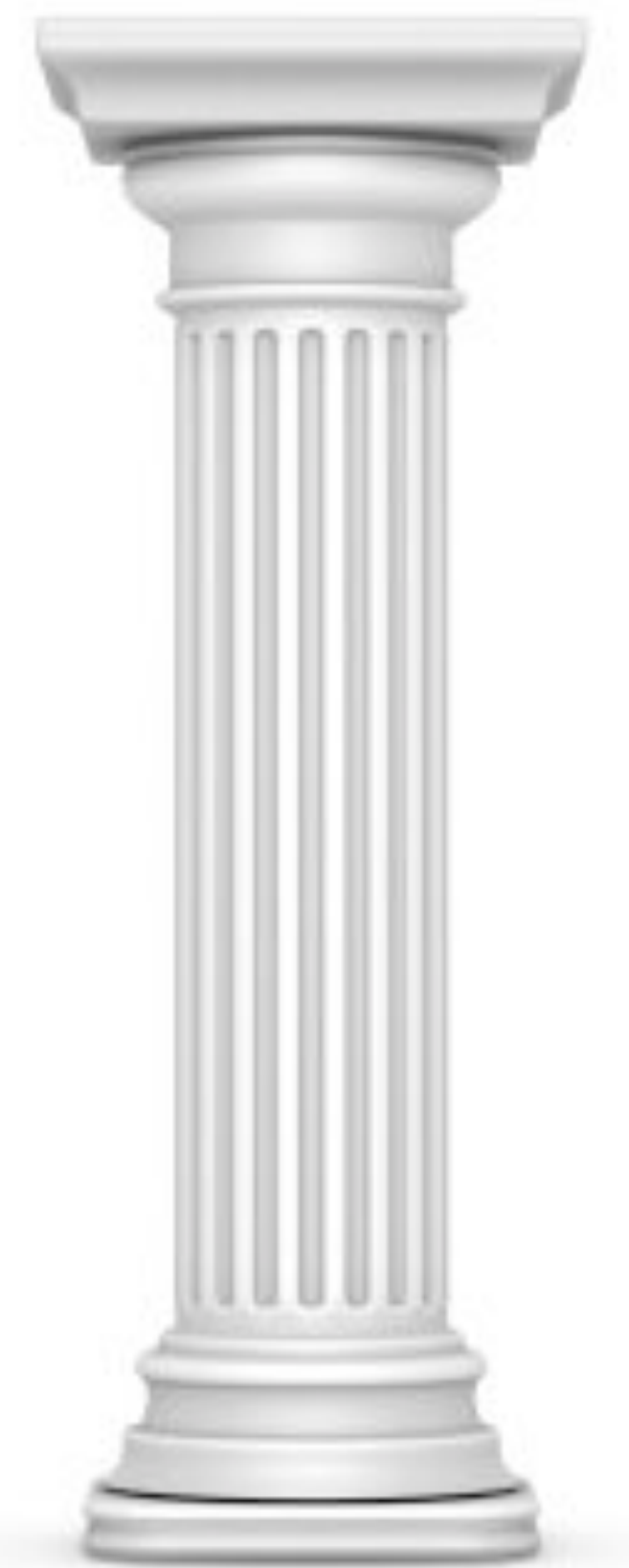
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The two pillars of fixed-order calculations

AMPLITUDES



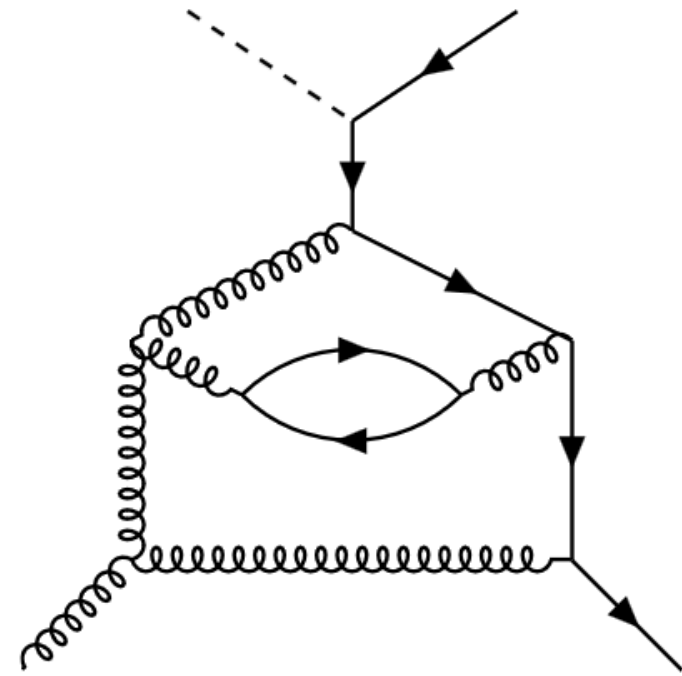
SUBTRACTION



Accuracy for standard candle processes at LHC (e.g. Higgs and DY) pushed to (fully differential) N³LO

→ see talk by P. Torrielli

Amplitudes



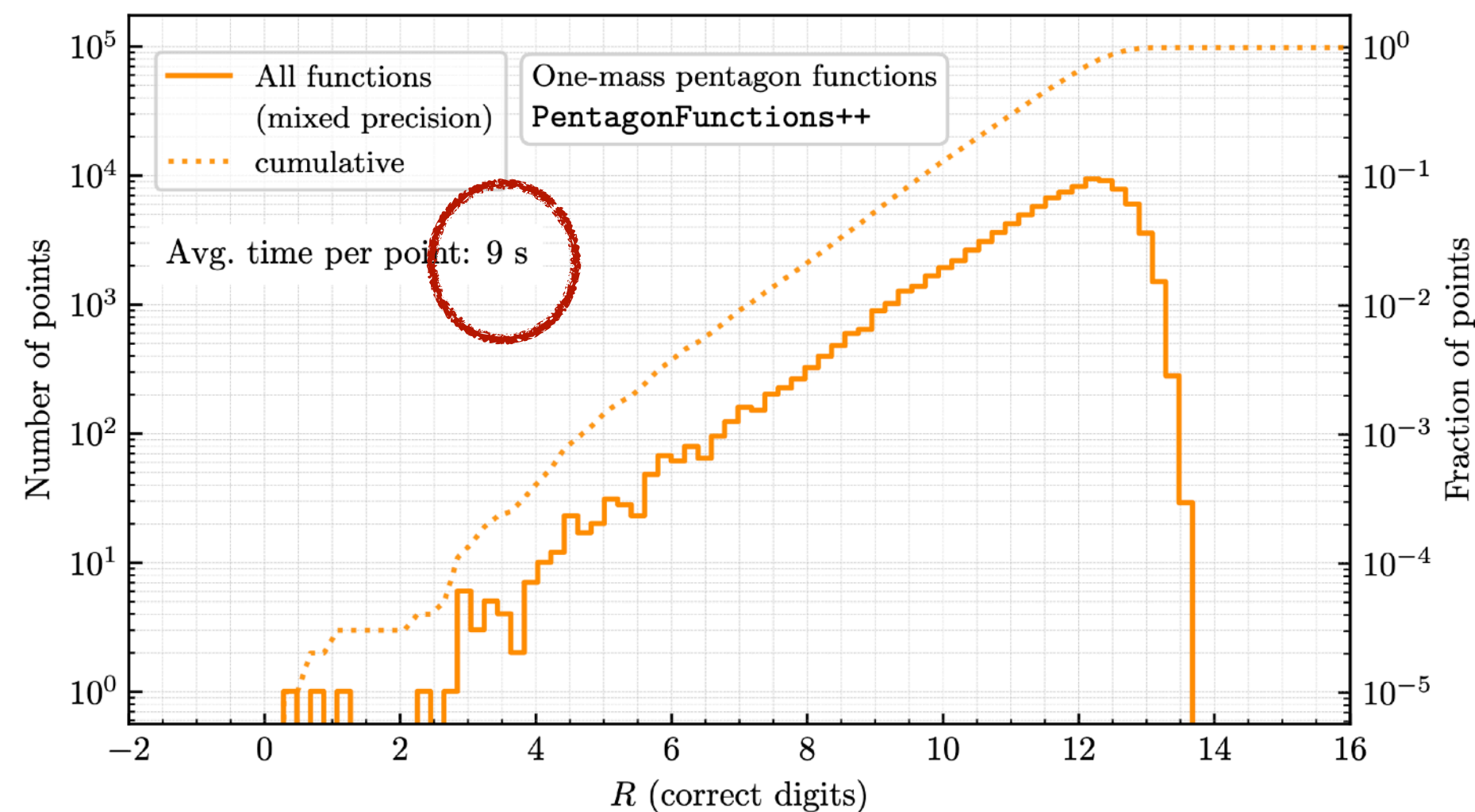
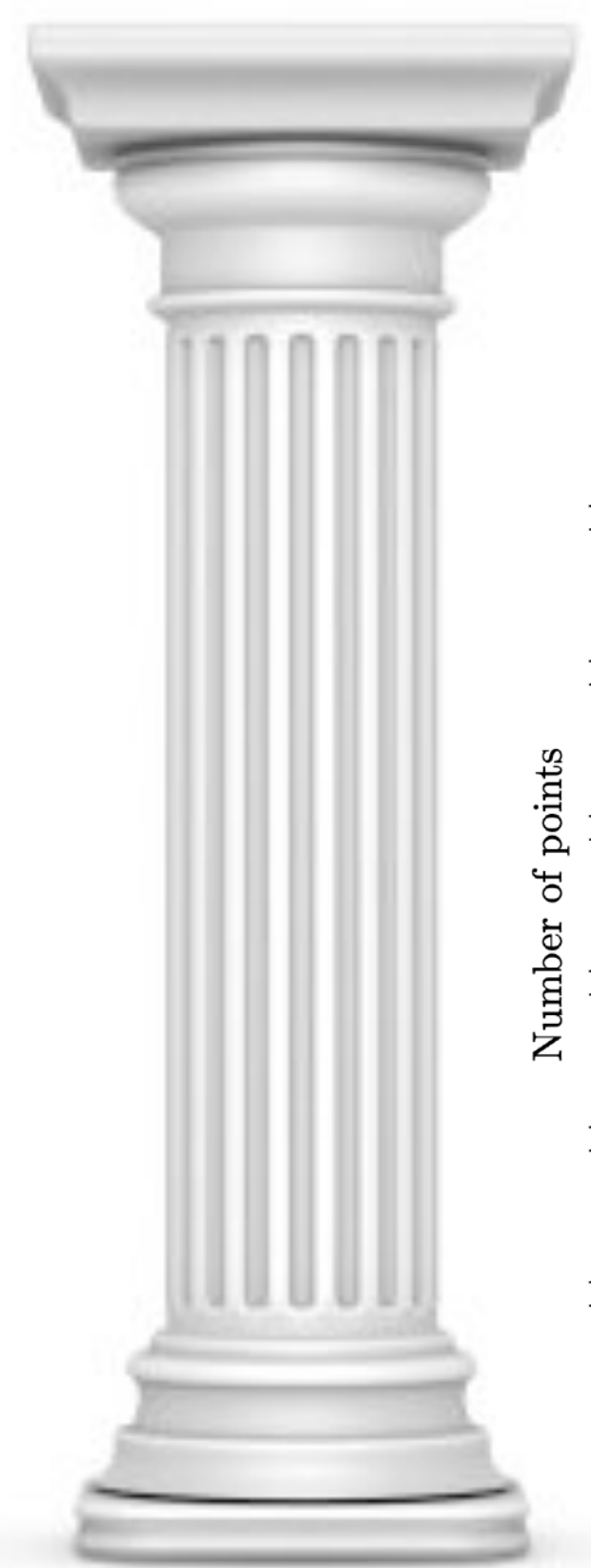
$V+3$ partons at three-loop (and two-loop to higher orders in ϵ)

[Gehrmann, Jakubcik, Mella, Syrrakos, Tancredi '22,'23]

(Planar) amplitudes in terms of GHPLs with simple alphabet:

$$\{x, y, 1 - x - y, 1 - x, 1 - y, x + y\}, x = s_{12}/m^2, y = s_{13}/m^2$$

Fast to evaluate



$V+4$ partons at two-loop [Abreu, Chicherin, Febres Cordero, Ita, Klinkert, Page, Sotnikov, Tschernow, Zoia '21,'23]

Amplitudes in terms of “(one-mass) pentagon functions”

Lot of recent progress to evaluate them efficiently

Subtraction

$$\begin{aligned} d\sigma_{N^3LO}^V &= d\sigma_{N^3LO}^V \Big|_{q_T < q_T^{\text{cut}}} + d\sigma_{N^3LO}^V \Big|_{q_T > q_T^{\text{cut}}} \\ &= \mathcal{H}_{N^3LO}^V \otimes d\sigma_{LO}^V + \left[d\sigma_{NNLO}^{V+\text{jet}} - d\sigma_{N^3LO}^{V,CT} \right]_{q_T > q_T^{\text{cut}}} \end{aligned}$$

Non-local (slicing) schemes : used for differential N³LO colour-singlet

For $V+\text{jet}$, one could use N -jettiness subtraction with \mathcal{T}_1

Beam, hard and jet functions are known (see above)

Missing ingredient: N³LO soft function for \mathcal{T}_1 , currently beyond reach

Calculation of N³LO soft function for \mathcal{T}_0 in progress

[Baranowski, Delto, Melnikov, Pikelner, Wang]

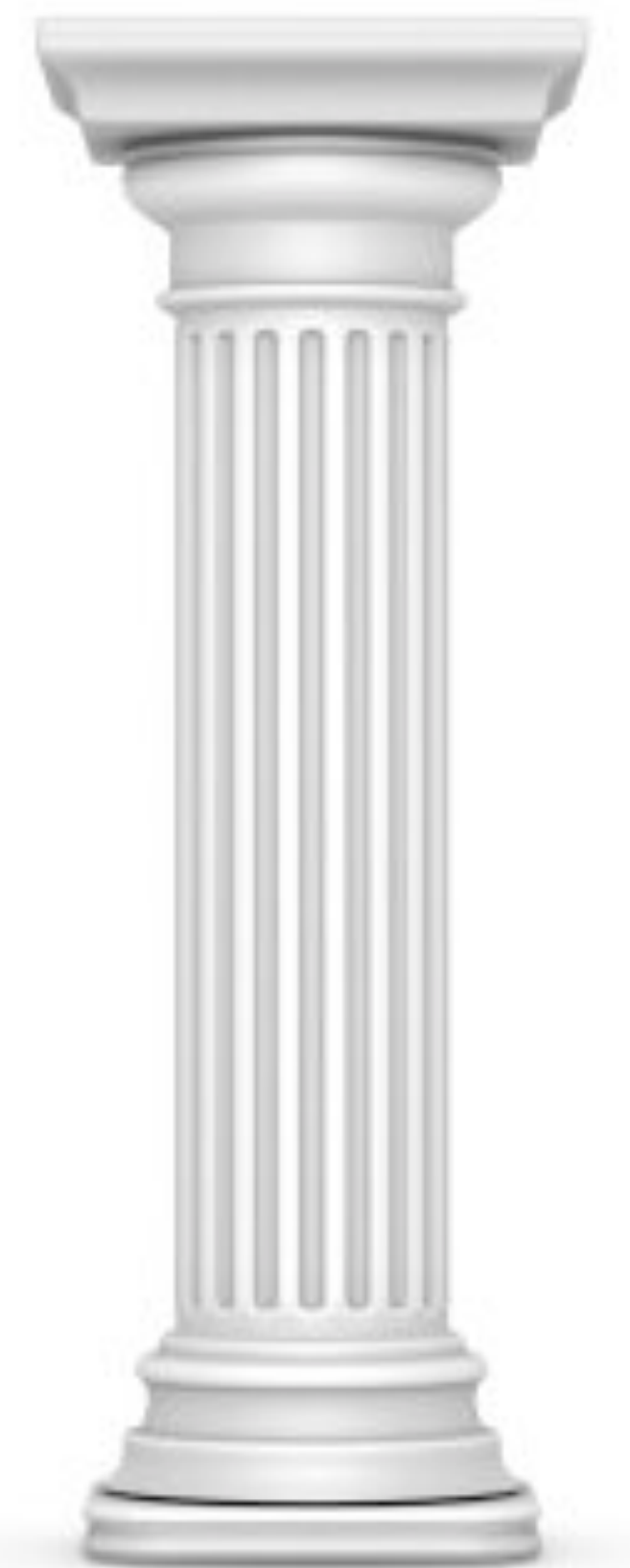
Local schemes : still in their infancy

Analytical ingredients for N³LO antenna subtraction in e^+e^- collisions

[Chen, Jakubcik, Marcoli, GS '22,'23]

Ideas for the N³LO extension of the local analytic subtraction method

[Magnea, Milloy, Signorile-Signorile, Torrielli '24]



Mixed $\mathcal{O}(\alpha_s\alpha)$ effects?

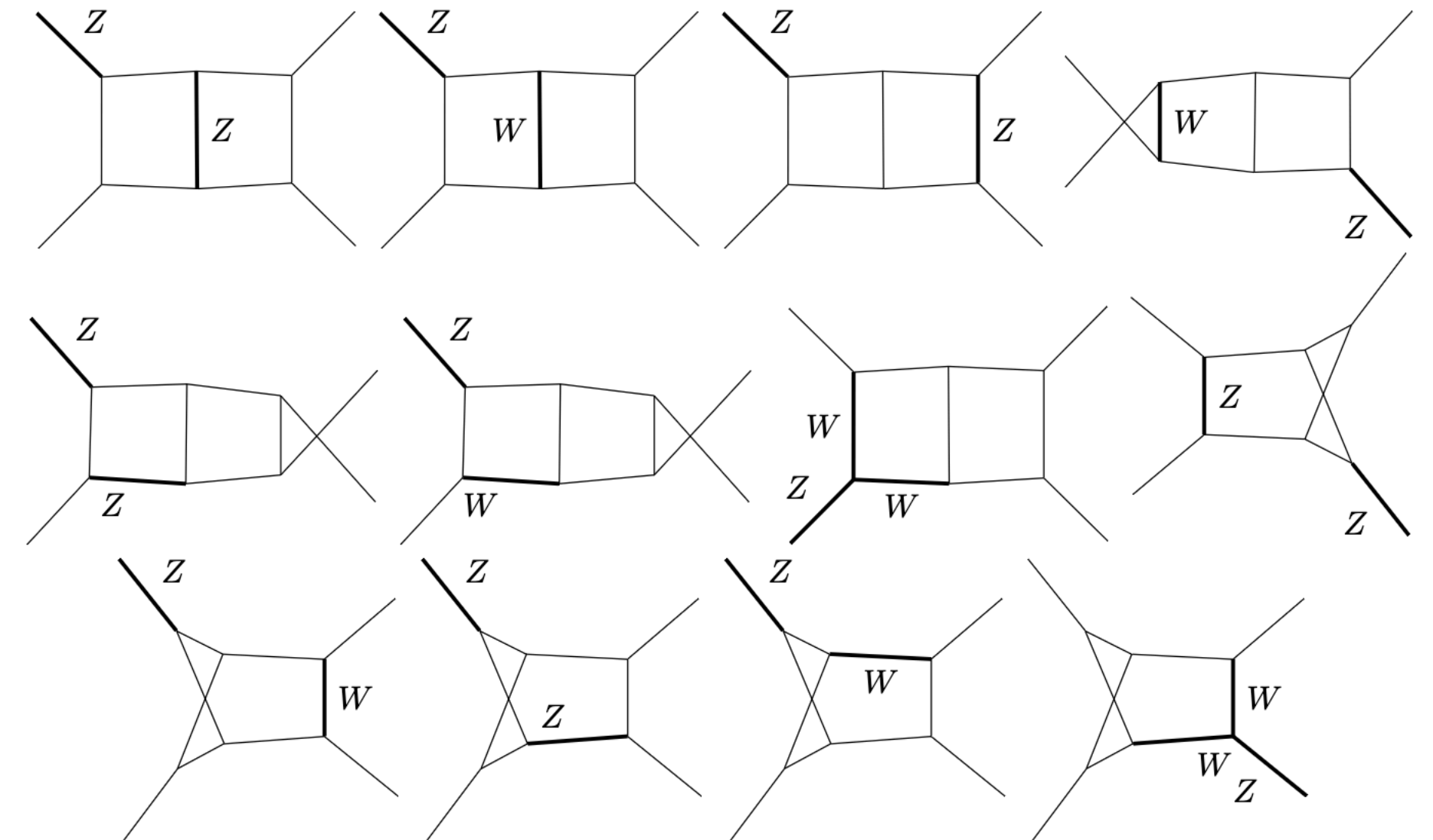
Full $\mathcal{O}(\alpha_s\alpha)$ corrections known for Drell-Yan

[Bonciani et al. '21] [Buccioni et al. '22] → see talk by A. Vicini

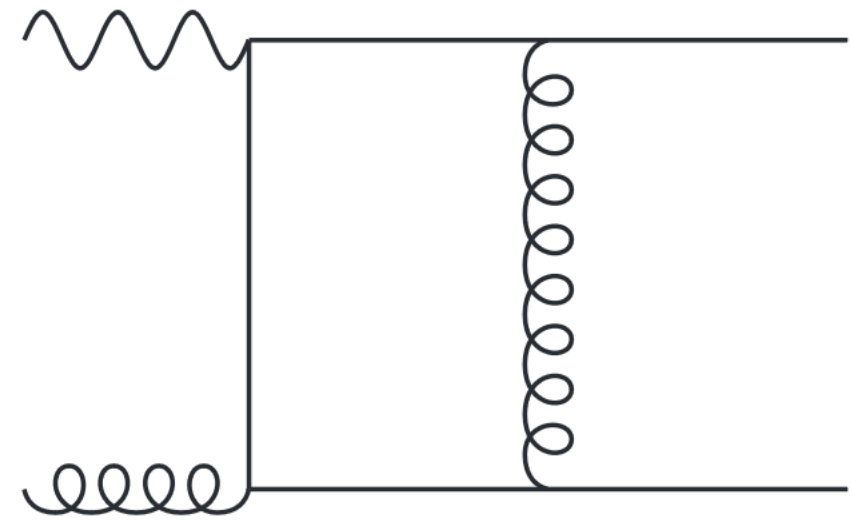
Still not known for V +jet. Estimation of size in [Lindert et al. '17]:
on multiplicative combination of NNLO QCD and NLO EW,
uncertainty of 10-20% for W/Z +jet and 40% for γ +jet

First step: bosonic (neglecting closed fermion loops) contribution to the two-loop mixed QCD-EW amplitudes for Z +jet
[Bargiela, Caola, Chawdhry, Liu '23]

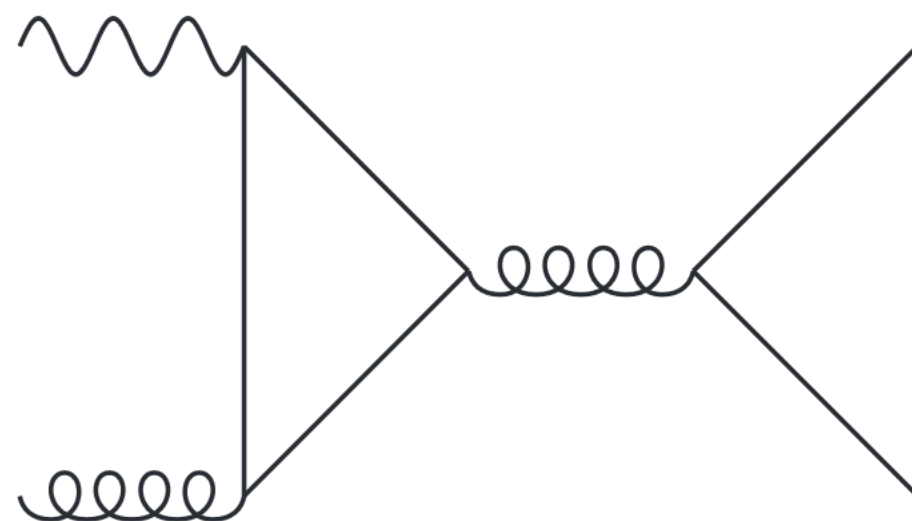
Appropriate IR subtraction schemes for mixed QCD-QED real-emission would be required



Axial-vector contributions?



Non-singlet



Pure-singlet
(vanishes for W^\pm)

Known exactly at one-loop

Non-singlet: vector = axial-vector

Pure-singlet: missing two-loop axial-vector contributions computed recently (with large m_t)
[Gehrmann, Peraro, Tancredi '22]

Phenomenological impact to be assessed: expected to be very small (per-mille correction) for sufficiently inclusive observable, but may be sizeable in e.g. angular correlations between leptons and jet

Related calculation is the three-loop quark form factor, entering NC DY @ N3LO:
exact top quark mass dependence in [Chen, Czakon, Niggetiedt '21]
Effect of exact axial-vector on total cross section is negligible [Duhr, Mistlberger '21]

Conclusions

Work to improve the theoretical description of V +jet well inserted in the overall effort of the community towards better SM predictions:

- push predictions for multi-leg final states to NNLO
- consider more exclusive final states e.g. with identified photons/hadrons (or flavoured jets → see talks by H. B. Hartanto and A. Mitov)
- improve generators (including accuracy of PS and matching to fixed-order)
- go to N³LO (likely with non-local subtraction methods in a first phase)
- start thinking about formally sub-dominant effects that may become relevant

I am grateful to S. Alioli, X. Chen, P. Jakubcik, A. Huss and L. Rottoli for discussions

BACKUP

$$\frac{d\sigma_0^{\text{MC}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) = \frac{d\sigma^{\text{NNLL}'}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) + \frac{d\sigma_0^{\text{nons}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}),$$

$$\begin{aligned} \frac{d\sigma_{\geq 1}^{\text{MC}}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}) &= \frac{d\sigma^{\text{NNLL}'}}{d\Phi_0 d\mathcal{T}_0} \mathcal{P}(\Phi_1) \theta(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}) \\ &+ \frac{d\sigma_{\geq 1}^{\text{nons}}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}), \end{aligned}$$

$$\frac{d\sigma_{\geq 1}^{\text{MC}}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}) = \frac{d\sigma^{\text{NNLL}'}}{d\Phi_0 d\mathcal{T}_0} \mathcal{P}(\Phi_1) \theta(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}})$$

$$\int \frac{d\Phi_1}{d\Phi_0 d\mathcal{T}_0} \mathcal{P}(\Phi_1) = 1$$

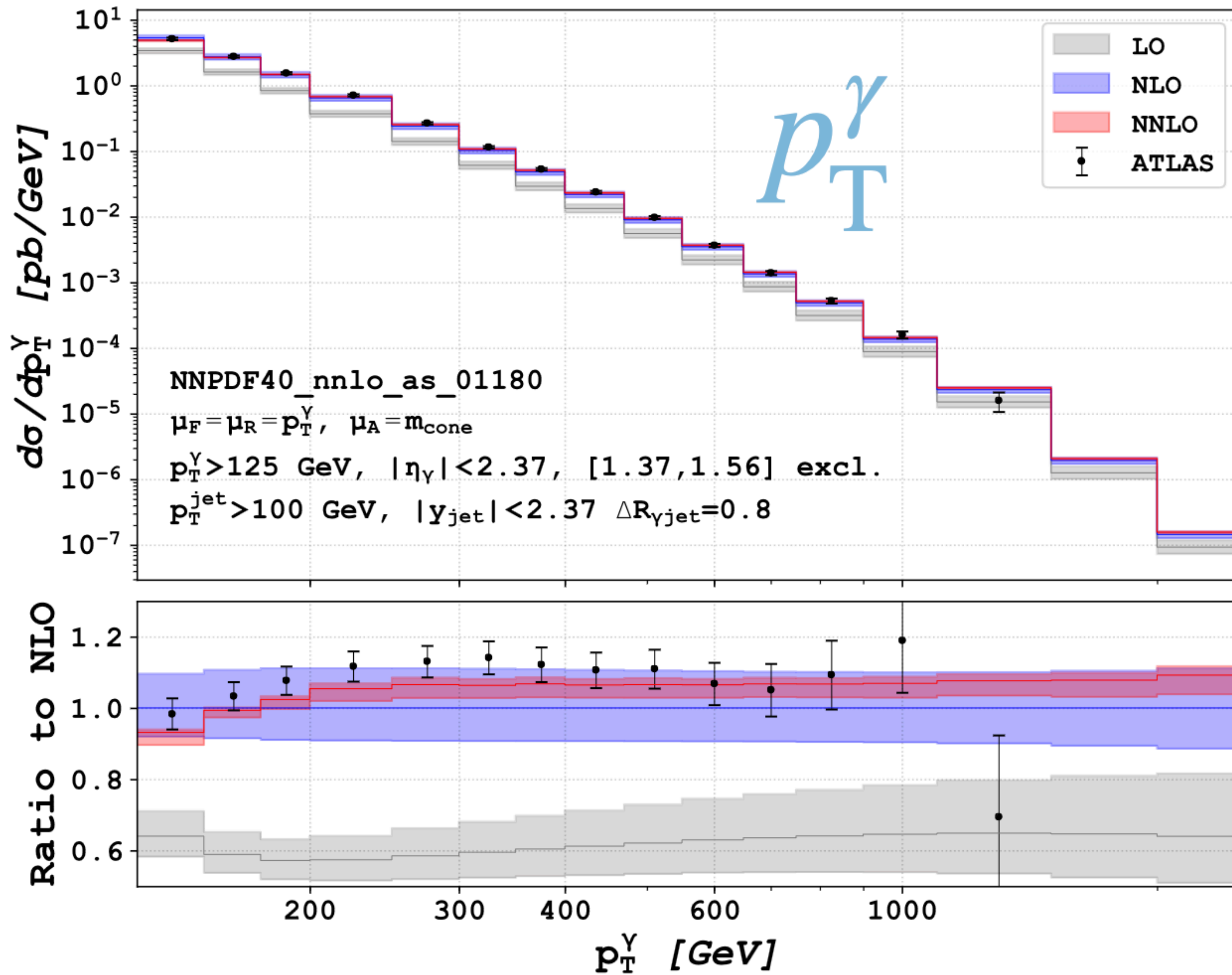
$$\begin{aligned} &- \left[\frac{d\sigma^{\text{NNLL}'}}{d\Phi_0 d\mathcal{T}_0} \mathcal{P}(\Phi_1) \right]_{\text{NLO}_1} \theta(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}) \\ &+ (B_1 + V_1)(\Phi_1) \theta(\mathcal{T}_0(\Phi_1) > \mathcal{T}_0^{\text{cut}}) \\ &+ \int \frac{d\Phi_2}{d\Phi_1^{\mathcal{T}}} B_2(\Phi_2) \theta(\mathcal{T}_0(\Phi_2) > \mathcal{T}_0^{\text{cut}}) \end{aligned}$$

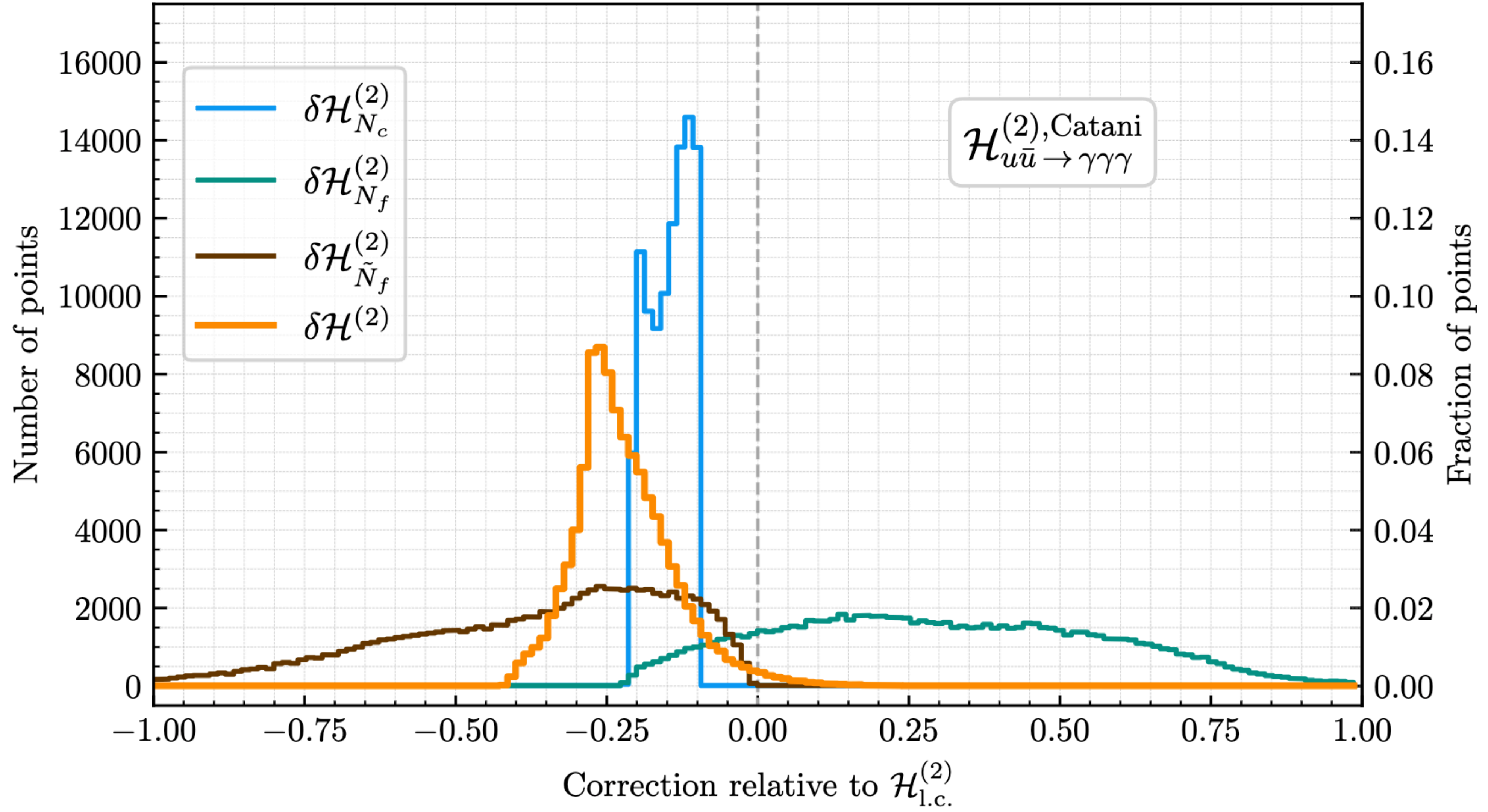
$$\mathcal{T}_0(\Phi_1^{\mathcal{T}}(\Phi_2)) = \mathcal{T}_0(\Phi_2)$$

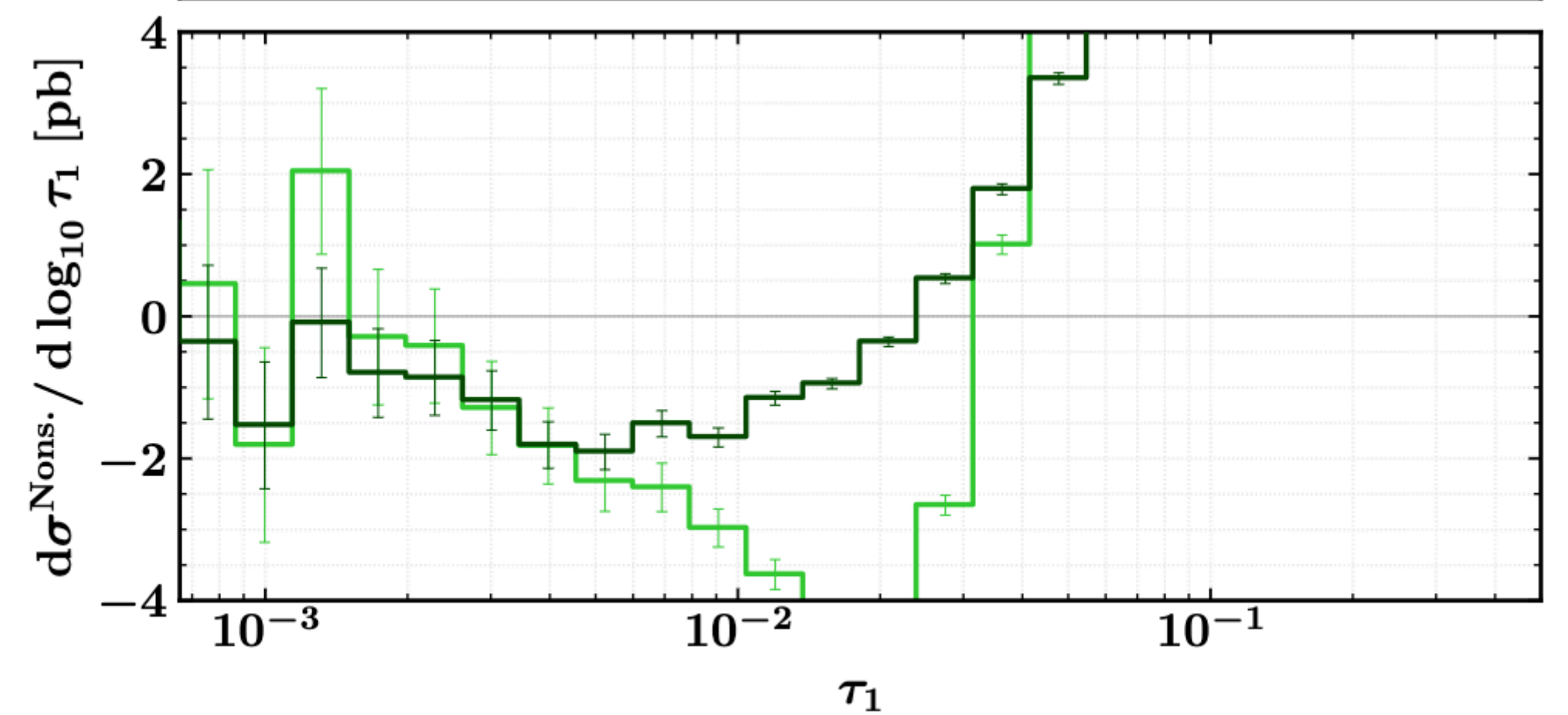
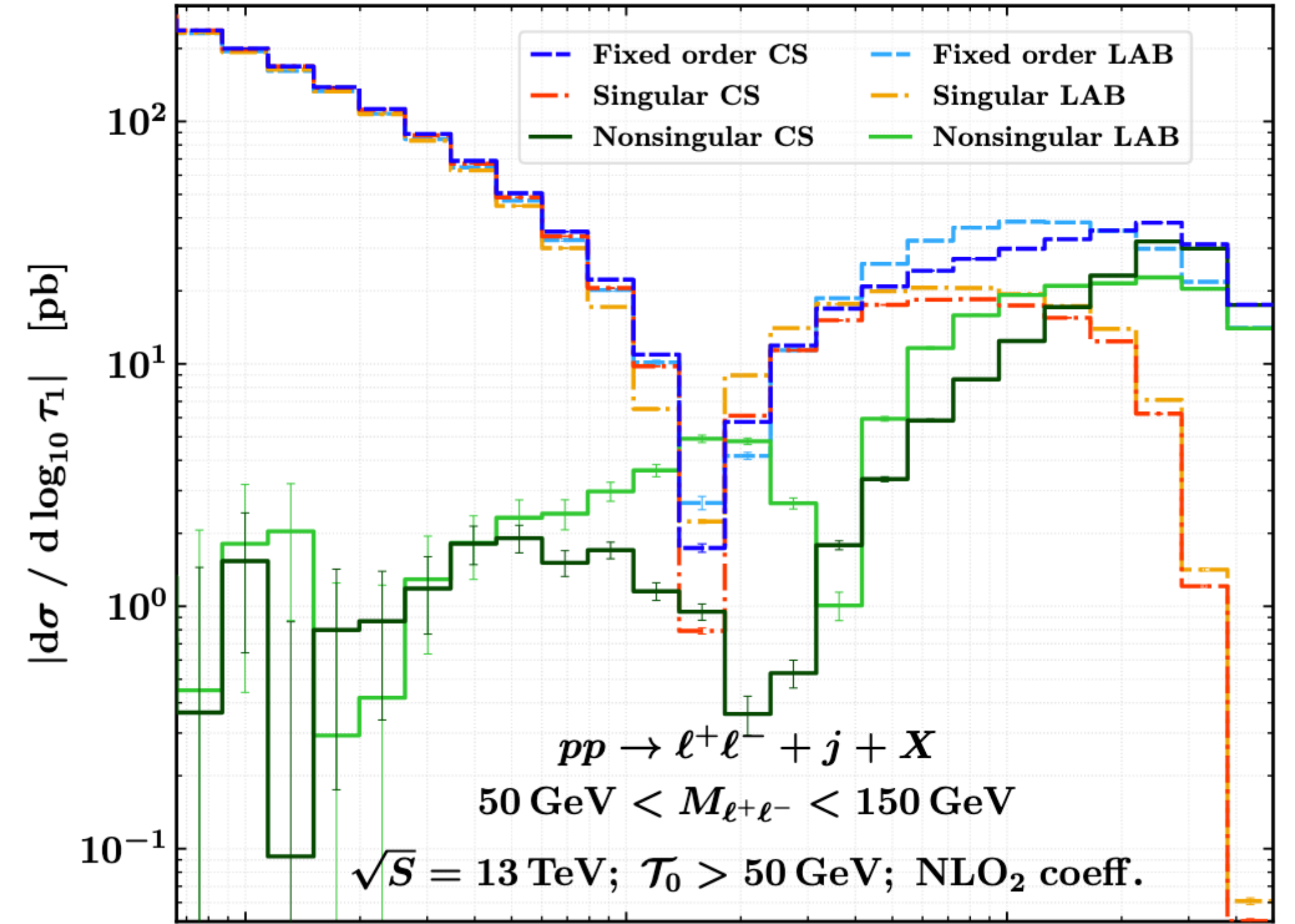
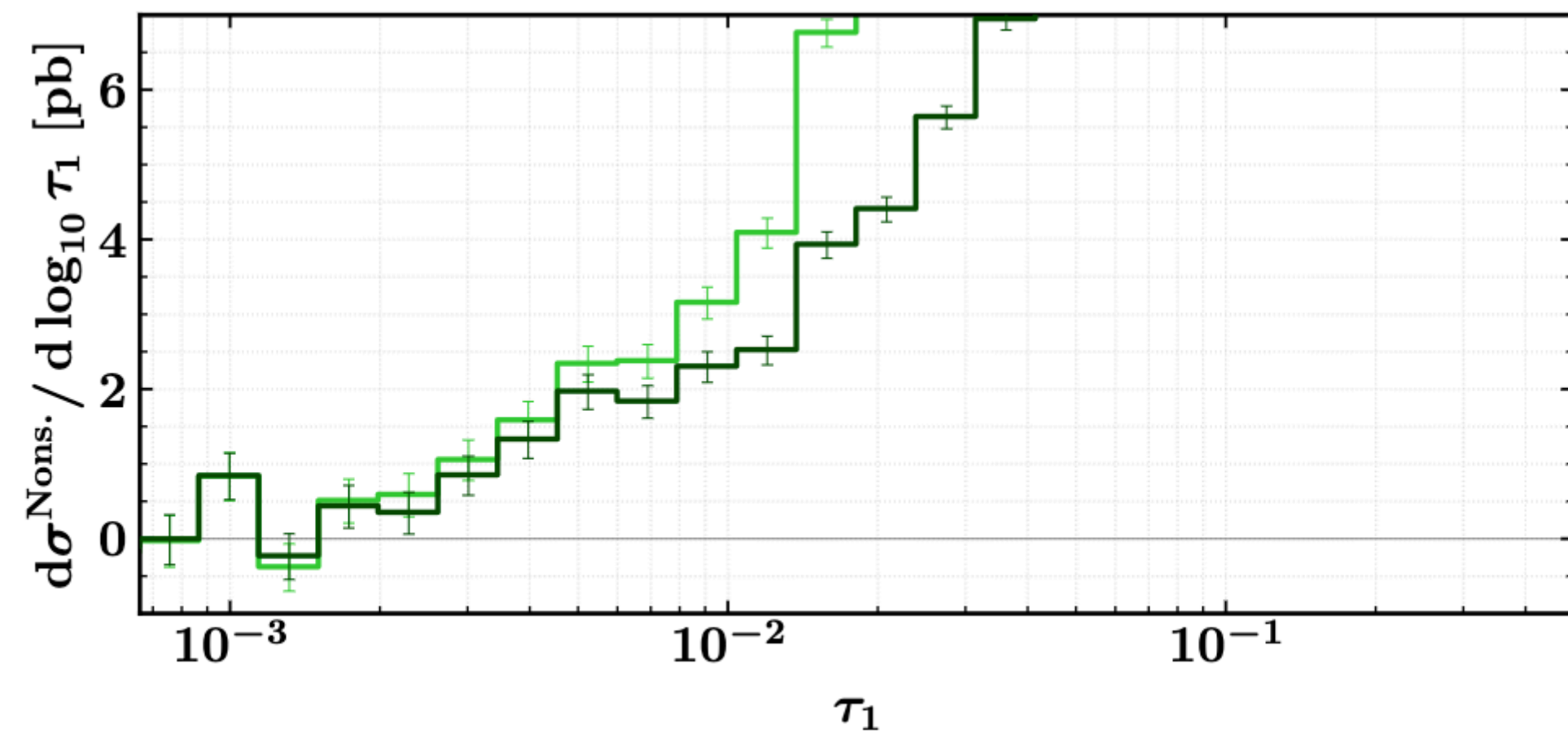
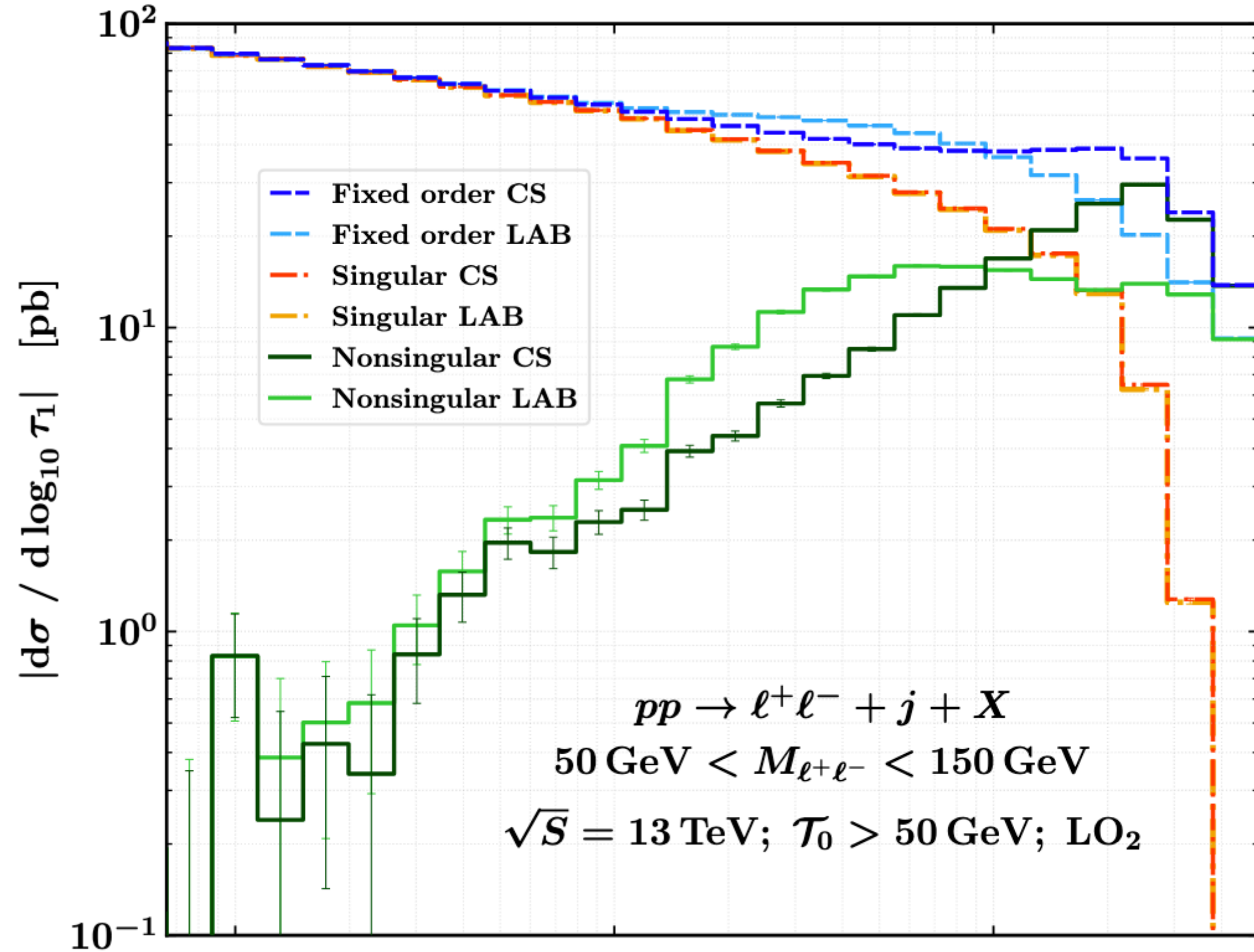
NNLOJET

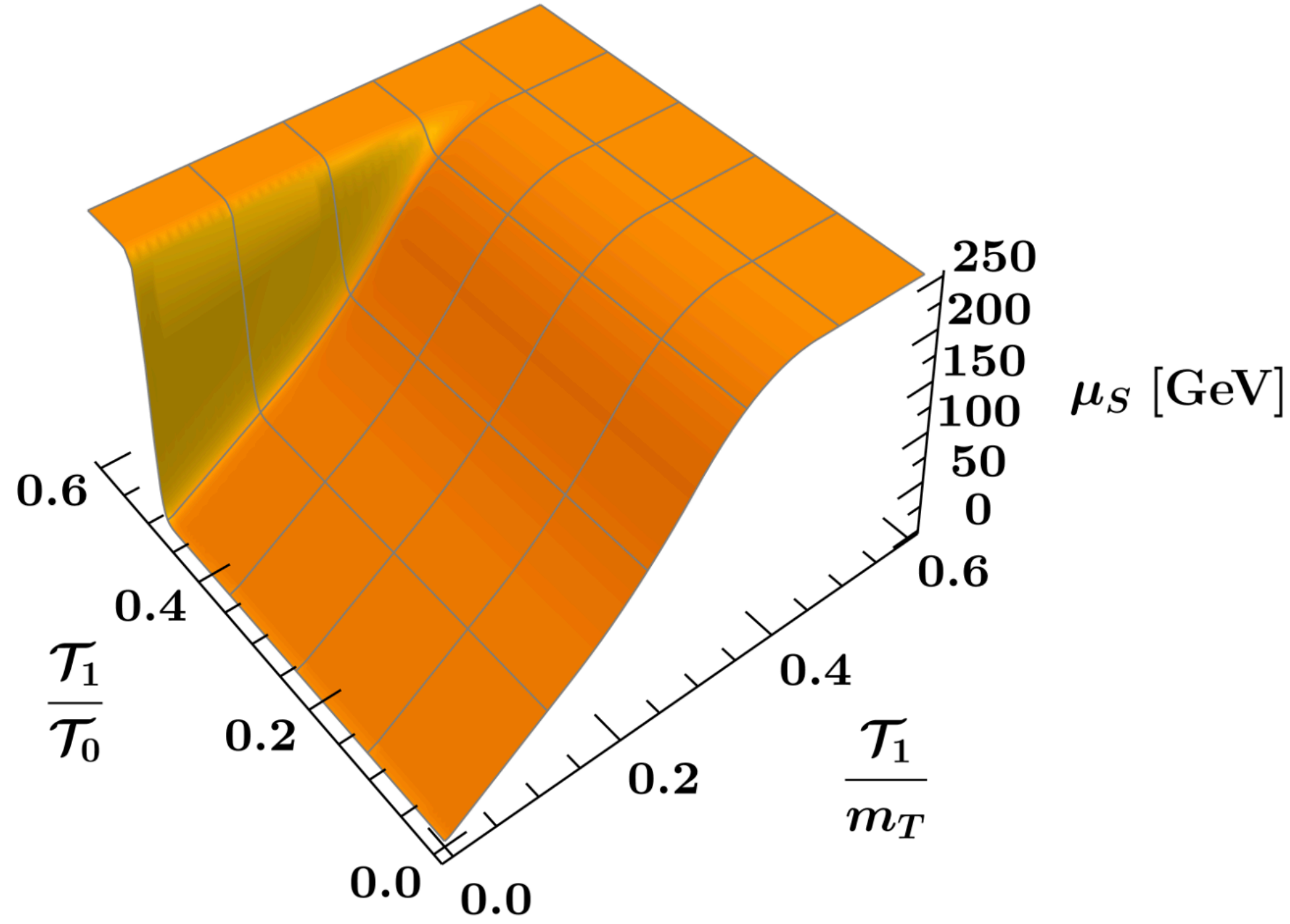
pp → γ + jet

√s = 13 TeV









$$\frac{\mathcal{T}_1(\Phi_N)}{\mathcal{T}_0(\Phi_N)} \leq \frac{N-1}{N} = \begin{cases} 1/2, & N=2 \\ 2/3, & N=3 \end{cases}$$

$$\mu_S(\mathcal{T}_1 \ll \mu_{\text{FO}}) \sim \mathcal{T}_1,$$

$$\mu_S(\mathcal{T}_1 \sim \mu_{\text{FO}}) \sim \mu_{\text{FO}},$$

$$\mu_S(\mathcal{T}_1/\mathcal{T}_0 \sim (N-1)/N) \sim \mu_{\text{FO}}.$$

$$\mu_B(\mathcal{T}_1/\mu_{\text{FO}}, \mathcal{T}_1/\mathcal{T}_0) = \sqrt{\mu_{\text{FO}} \mu_S(\mathcal{T}_1/\mu_{\text{FO}}, \mathcal{T}_1/\mathcal{T}_0)},$$

$$\mu_J(\mathcal{T}_1/\mu_{\text{FO}}, \mathcal{T}_1/\mathcal{T}_0) = \sqrt{\mu_{\text{FO}} \mu_S(\mathcal{T}_1/\mu_{\text{FO}}, \mathcal{T}_1/\mathcal{T}_0)},$$

$$\mu_H = \mu_{\text{FO}} = m_T \equiv \sqrt{m_{\ell^+\ell^-}^2 + q_T^2}.$$

