Quantum tomography at the LHC: top quark production

Luca Mantani

In collaboration with:
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+ insights from the community

European Research Council
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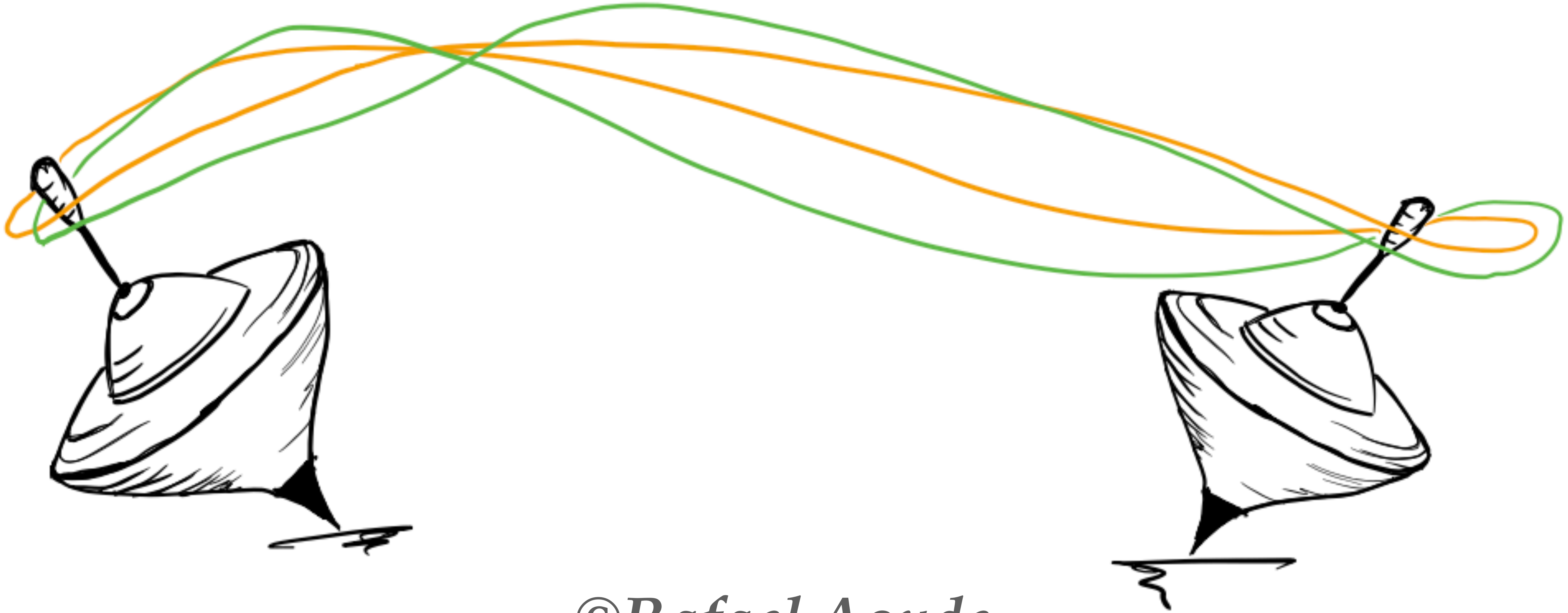
UNIVERSITY OF CAMBRIDGE

Outline

* Motivation
* Quantum Information Theory: the basics
* Why the top quark?
* Entanglement at the LHC



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Motivation

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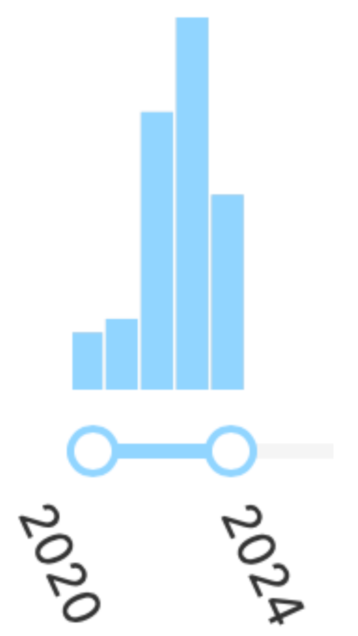
- Test Quantum Mechanics at the TeV scale
  - Quantum Information at the LHC: relativistic, fundamental particles
  - Learn from QI: fundamental interactions structure, interpretation
- Challenge: can we actually do it in a “dirty” environment?
- Rise in interest in the community: new ideas, methods, exp. strategies



Citations to the Afik & de Nova seminal paper

Quantum Information Theory

Entanglement in bipartite systems

Given a bipartite system, with Hilbert space $\mathscr{H} = \mathscr{H}_1 \otimes \mathscr{H}_2$

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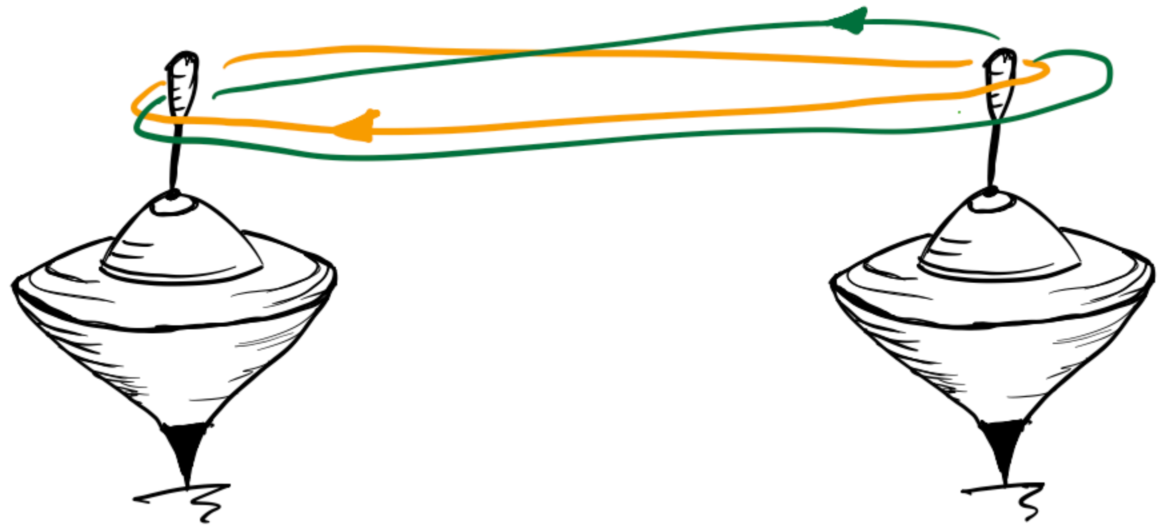
This is not always the case, e.g.: **Maximally entangled states: spin 1/2**



$$|\Phi^\pm\rangle = \frac{|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle}{\sqrt{2}} \qquad |\Psi^\pm\rangle = \frac{|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle}{\sqrt{2}}$$

Density matrix

* $\rho = \sum_k p_k \rho_k$

entangled if $\rho_k \neq \rho_1 \otimes \rho_2$

The fundamental object in QM is the density matrix *

$$\rho = \frac{1}{d}\mathbb{I} + \sum_{i=1}^{d^2-1} a_i \lambda_i$$

One particle of spin s:

d=2s+1

Generalised Gell-Mann matrix

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Two particles, each of spin s:

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The parameters completely characterise the quantum spin state of the system

How do we build the density matrix?

$$R^I_{\alpha_1\alpha_2,\beta_1\beta_2} \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors}\\ \text{a,b spins}}} \mathcal{M}^*_{\alpha_2\beta_2}\,\mathcal{M}_{\alpha_1\beta_1}$$

Sum over initial state only

Matrix-element $\mathcal{M}_{\alpha\beta} \equiv \langle t(k_1,\alpha)\bar{t}(k_2,\beta)|\mathcal{T}|a(p_1)b(p_2)\rangle$

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$$R = \tilde{A}\,\mathbb{I}\otimes\mathbb{I} + \sum_{i=1}^{d^2-1}\tilde{a}_i\,\lambda_i\otimes\mathbb{I} + \sum_{j=1}^{d^2-1}\tilde{b}_j\,\mathbb{I}\otimes\lambda_j + \sum_{i=1}^{d^2-1}\sum_{j=1}^{d^2-1}\tilde{c}_{ij}\,\lambda_i\otimes\lambda_j \quad \Rightarrow \quad \rho = \frac{R}{tr(R)}$$

Quantum observables

Concurrence $$\mathcal{C}(\rho) = \inf\left[\sum_i p_i c(|\psi_i\rangle)\right]$$ **Entangled if > 0**

2-qubits: $$C[\rho] \equiv \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

with λ_i eigenvalues of $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$

$\tilde{\rho} = (\sigma_2 \otimes \sigma_2)\,\rho^*\,(\sigma_2 \otimes \sigma_2)$

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$$P(\rho) \equiv \mathrm{tr}[\rho^2]$$

Pure if P=1

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Bell inequality

$$\langle \mathcal{B} \rangle_{\max} = \max_{U,V}\left(\mathrm{Tr}\left(\rho\,(U^\dagger \otimes V^\dagger)\,\mathcal{B}\,(U \otimes V)\right)\right) \geq 2$$

$$\mathcal{B} = -\frac{2}{\sqrt{3}}\,(S_x \otimes S_x + S_y \otimes S_y) + \lambda_4 \otimes \lambda_4 + \lambda_5 \otimes \lambda_5$$

The ideal candidate: the top quark

Why the top?

* *Aoude et al. 2307.09675*
Ashby-Pickering et al. 2209.13990
Fabbrichesi et al. 2302.00683

Weak bosons (*) and top quarks are the ideal candidates:
EW interactions allow for spin reconstruction from decay (no hadronisation)

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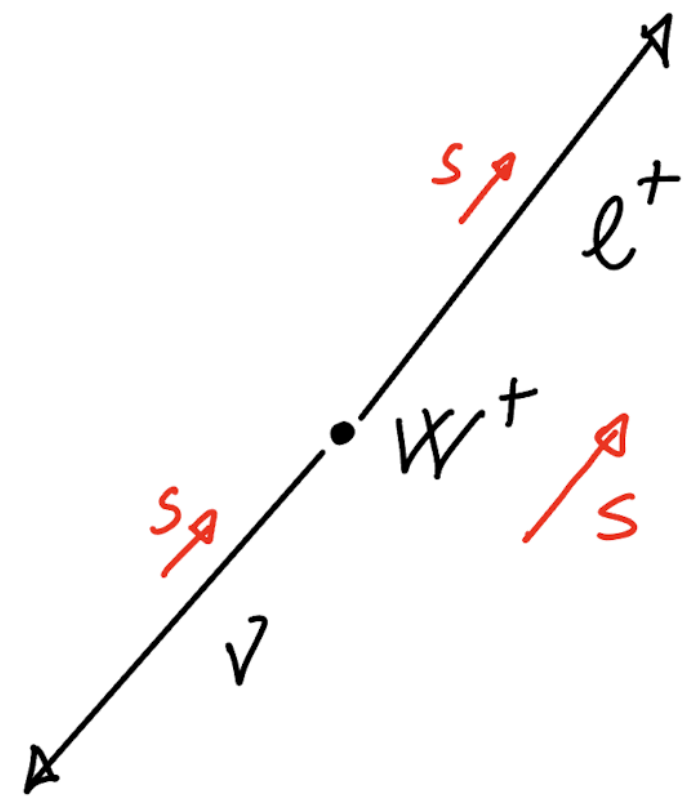
W decay:
lepton decays along W spin



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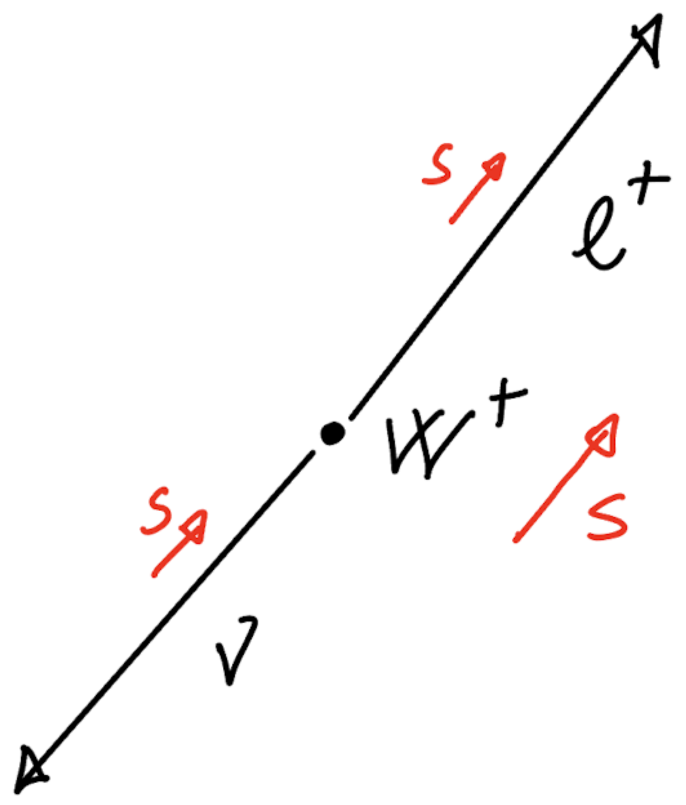

Top decay:
lepton decay correlated with top spin

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\phi} = \frac{1+\cos\phi}{2}$$

ϕ angle between lepton and spin

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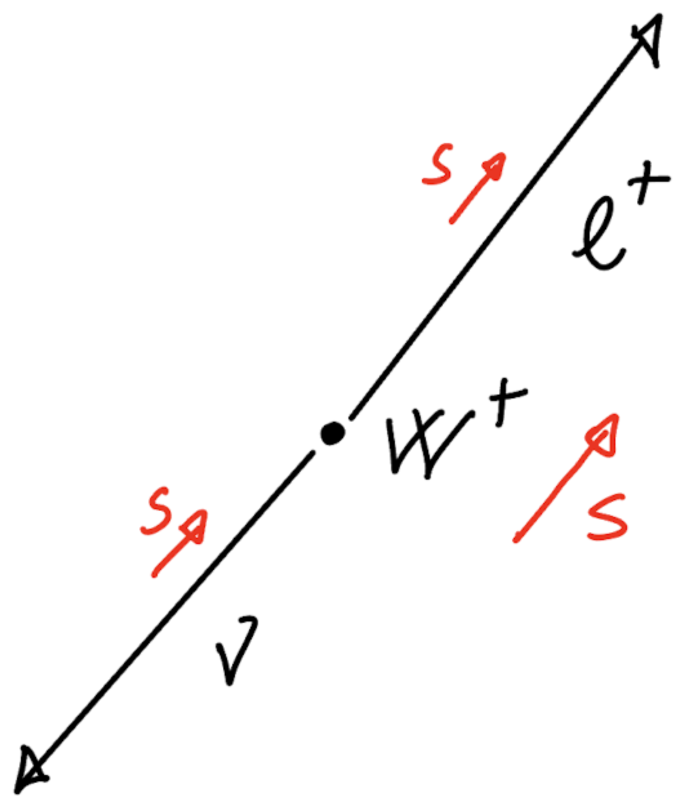

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Z boson more complicated but doable: spin can be reco if right/left asymmetry

Spin 1/2 density matrix

The R matrix can be decomposed in the spin space

$$R = \tilde{A}\,\mathbb{1}_2 \otimes \mathbb{1}_2 + \tilde{B}_i^+ \sigma^i \otimes \mathbb{1}_2 + \tilde{B}_i^- \mathbb{1}_2 \otimes \sigma^i + \tilde{C}_{ij}\,\sigma^i \otimes \sigma^j$$

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Cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega \mathrm{d}\hat{s}} = \frac{\alpha_s^2 \beta}{\hat{s}^2} \tilde{A}\left(\hat{s}, \boldsymbol{k}\right)$$

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Degree of top and anti-top polarisation

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If normalised, we define the density matrix

$$\rho = \frac{\mathbb{1}_2 \otimes \mathbb{1}_2 + B_i^{+}\sigma^i \otimes \mathbb{1}_2 + B_i^{-}\mathbb{1}_2 \otimes \sigma^i + C_{ij}\,\sigma^i \otimes \sigma^j}{4}.$$

Quantum tomography

How do we reconstruct the spin density matrix at colliders?

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Measure angular distributions of the decay products

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For example, for the density matrix of a W boson

Ashby-Pickering et al.
2209.13990

$$\Phi_1^{P\pm} = \sqrt{2}(5\cos\theta \pm 1)\sin\theta\cos\phi$$
$$\Phi_2^{P\pm} = \sqrt{2}(5\cos\theta \pm 1)\sin\theta\sin\phi$$
$$\Phi_3^{P\pm} = \frac{1}{4}(\pm 4\cos\theta + 15\cos 2\theta + 5)$$
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$$a_j = \frac{1}{2}\int \mathrm{d}\Omega_{\hat{\mathbf{n}}}\, p(\ell^{\pm}_{\hat{\mathbf{n}}};\rho)\,\Phi_j^{P\pm}$$

$$c_{ij} = \left(\frac{1}{2}\right)^2 \iint \mathrm{d}\Omega_{\hat{\mathbf{n}}_1}\,\mathrm{d}\Omega_{\hat{\mathbf{n}}_2}\, p\left(\ell^{+}_{\hat{\mathbf{n}}_1}, \ell^{-}_{\hat{\mathbf{n}}_2};\rho\right)\Phi_i^P\left(\hat{\mathbf{n}}_1\right)\Phi_j^P\left(\hat{\mathbf{n}}_2\right)$$

Expectation value of the Wigner P functions

Quantum tomography: top pair

Afik & De Nova
2003.02280

In the case of top pair things are simpler

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_+\mathrm{d}\Omega_-} = \frac{1 + \mathbf{B}^+ \cdot \hat{\mathbf{q}}_+ - \mathbf{B}^- \cdot \hat{\mathbf{q}}_- - \hat{\mathbf{q}}_+ \cdot \mathbf{C} \cdot \hat{\mathbf{q}}_-}{(4\pi)^2}$$

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Interestingly, at threshold, a specific angular distributions is **directly proportional to the entanglement:** entangled tops produce *small angular separation*

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$$

Angle between leptons

$$D = \frac{\mathrm{tr}[\mathbf{C}]}{3} \qquad C[\rho] = \max(-1-3D, 0)/2$$

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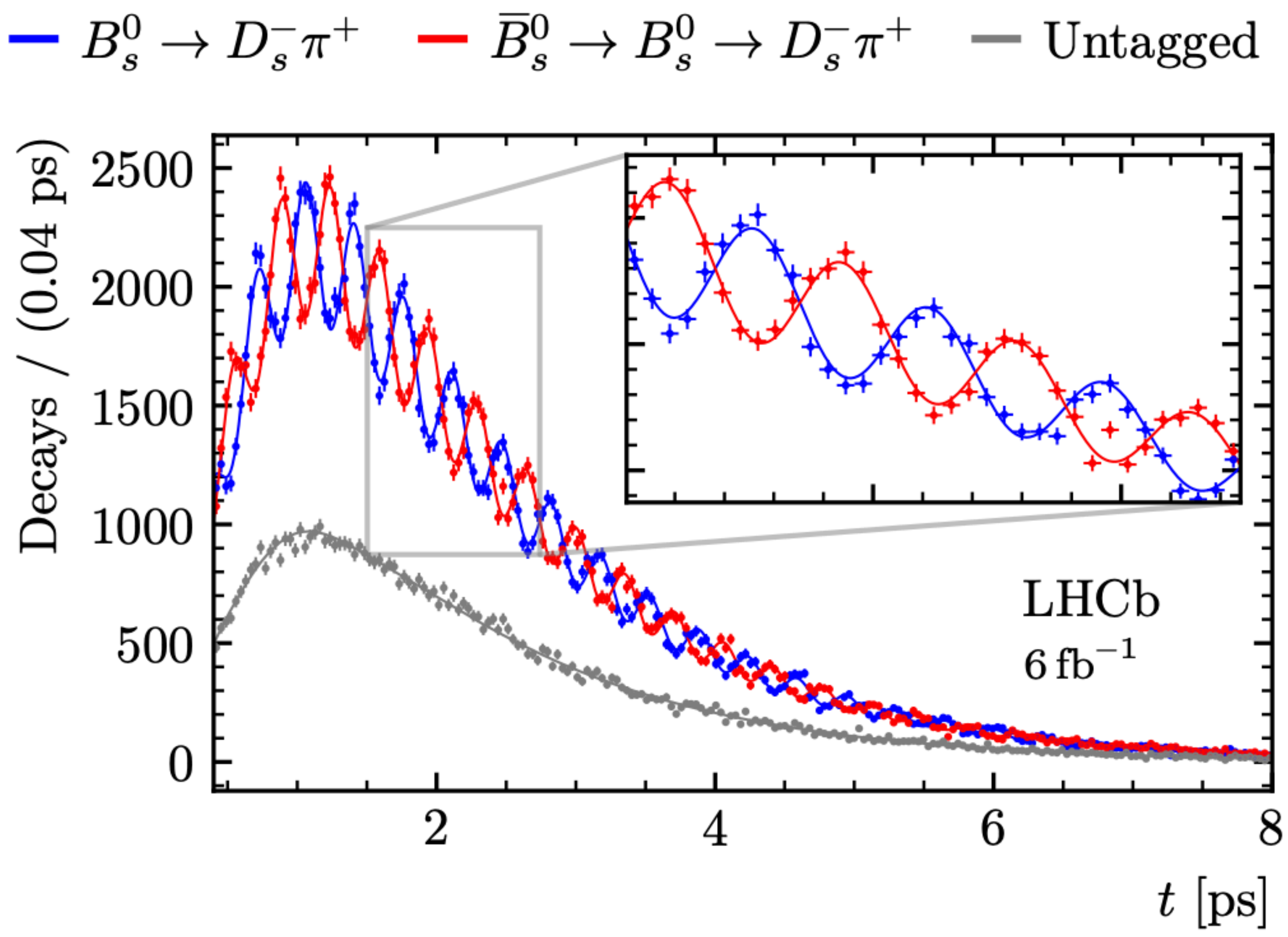
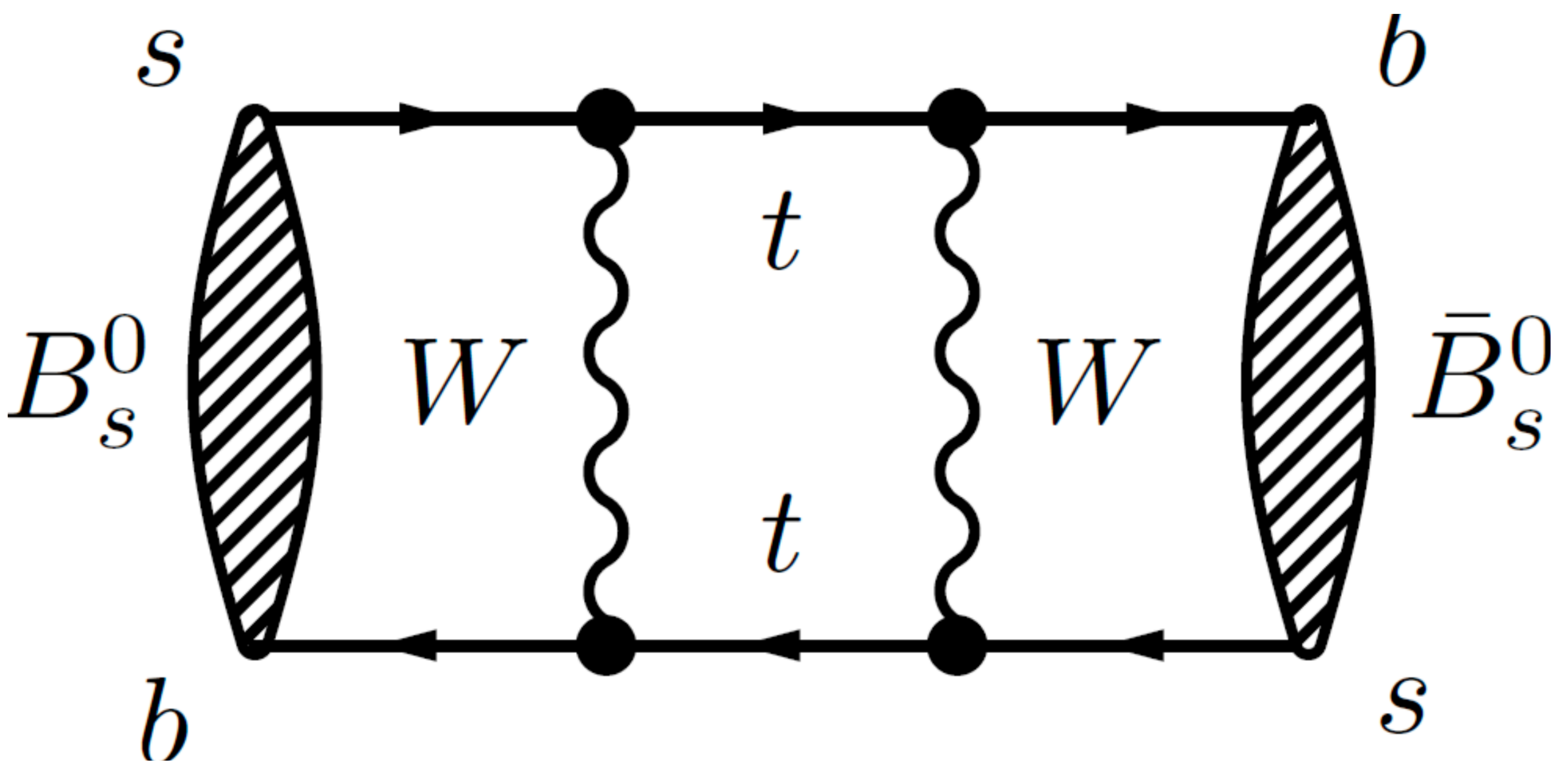
$$D < -\frac{1}{3}$$

Entanglement at the LHC

Quantum measurements at the LHC

LHCb
2104.04421





$$|B_{L,H}\rangle = p\,|B_q^0\rangle + q\,|\bar{B}_q^0\rangle$$

The R matrix at the LHC

Aoude et al.
2203.05619

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At LO in QCD

$$I = gg, q\bar{q}$$

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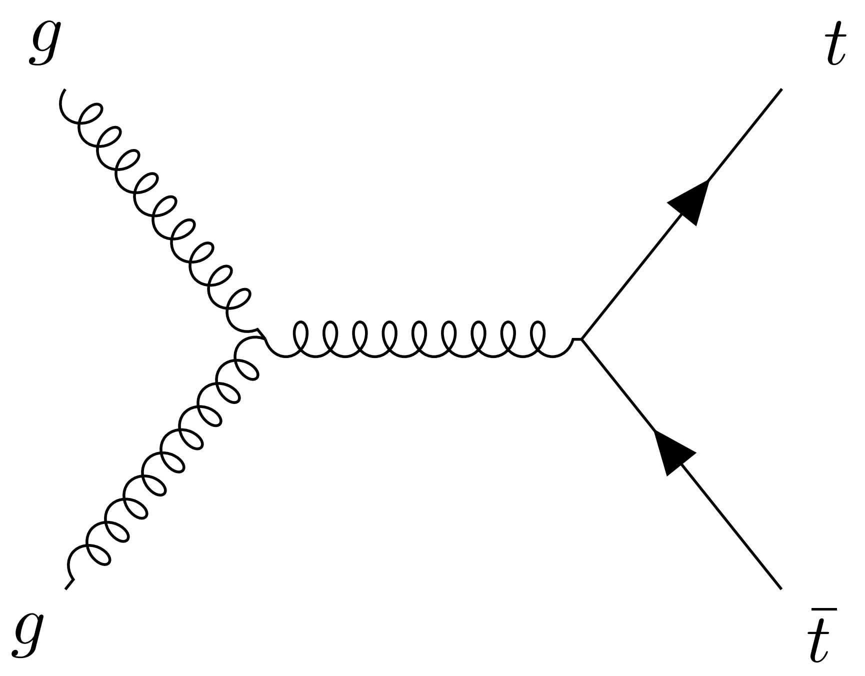
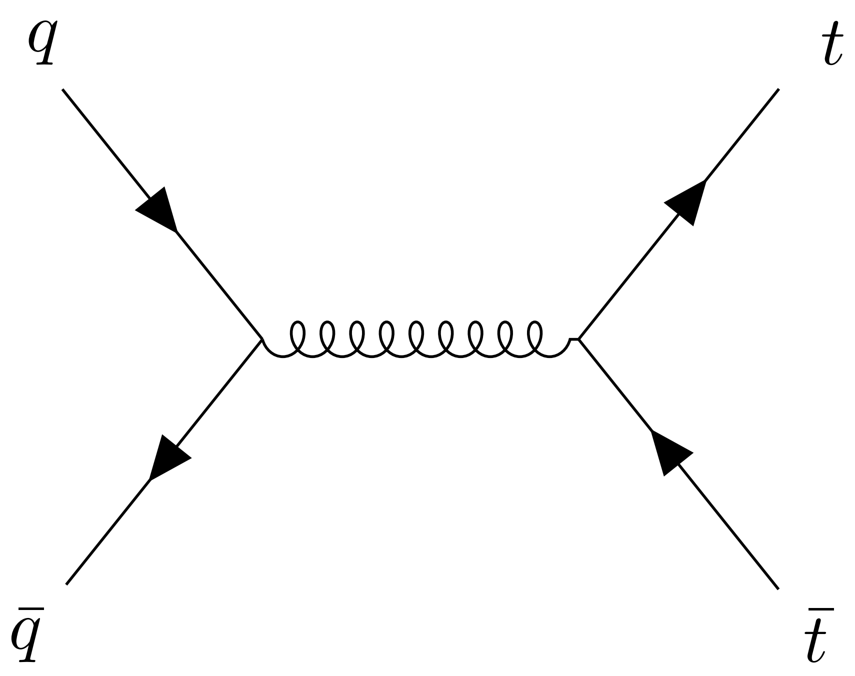
We collide protons



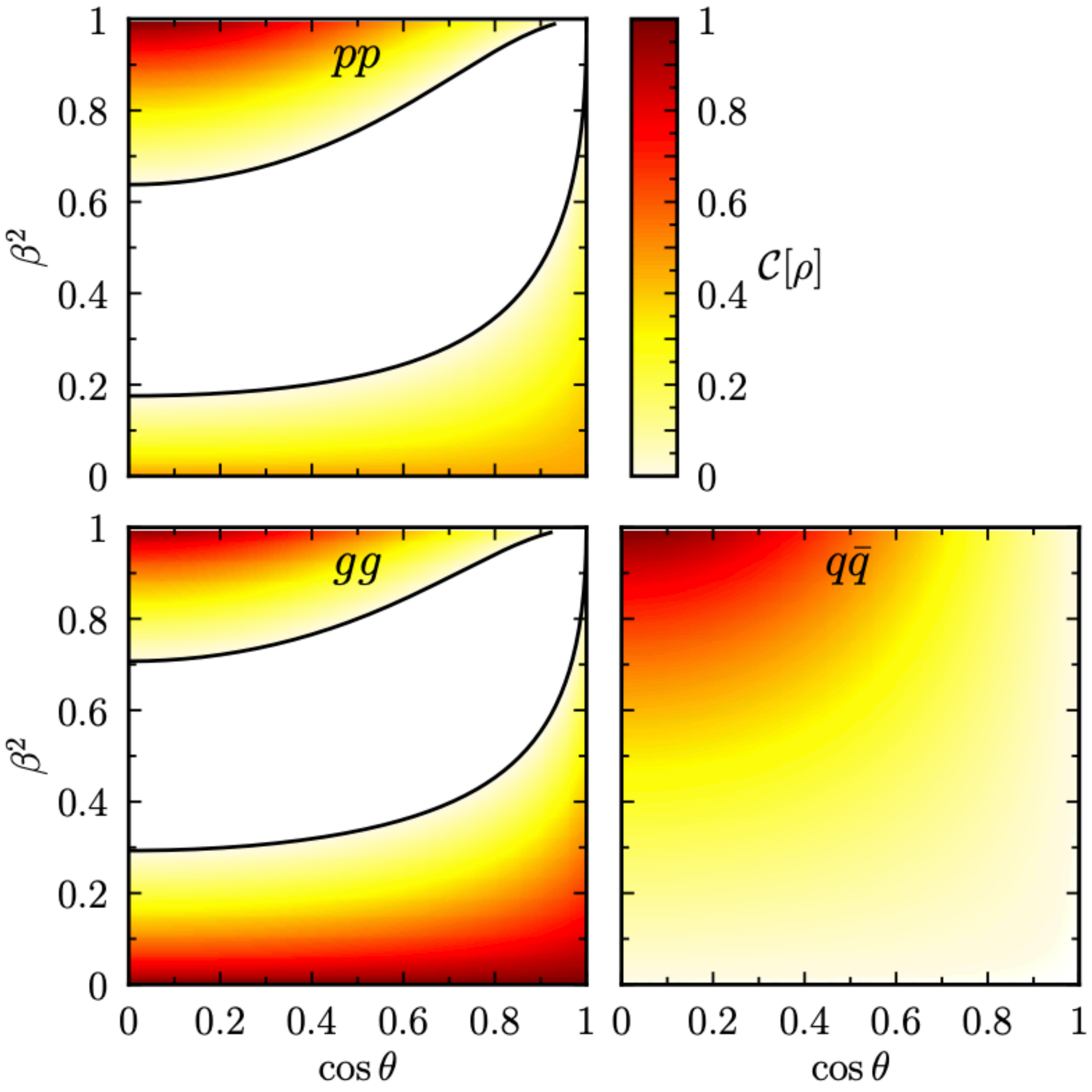
$$R(\hat{s},\boldsymbol{k}) = \sum_I L^I(\hat{s}) R^I(\hat{s},\boldsymbol{k})$$



Full density matrix is mixed state, weighted by parton luminosity

SM entanglement

Aoude et al.
2203.05619



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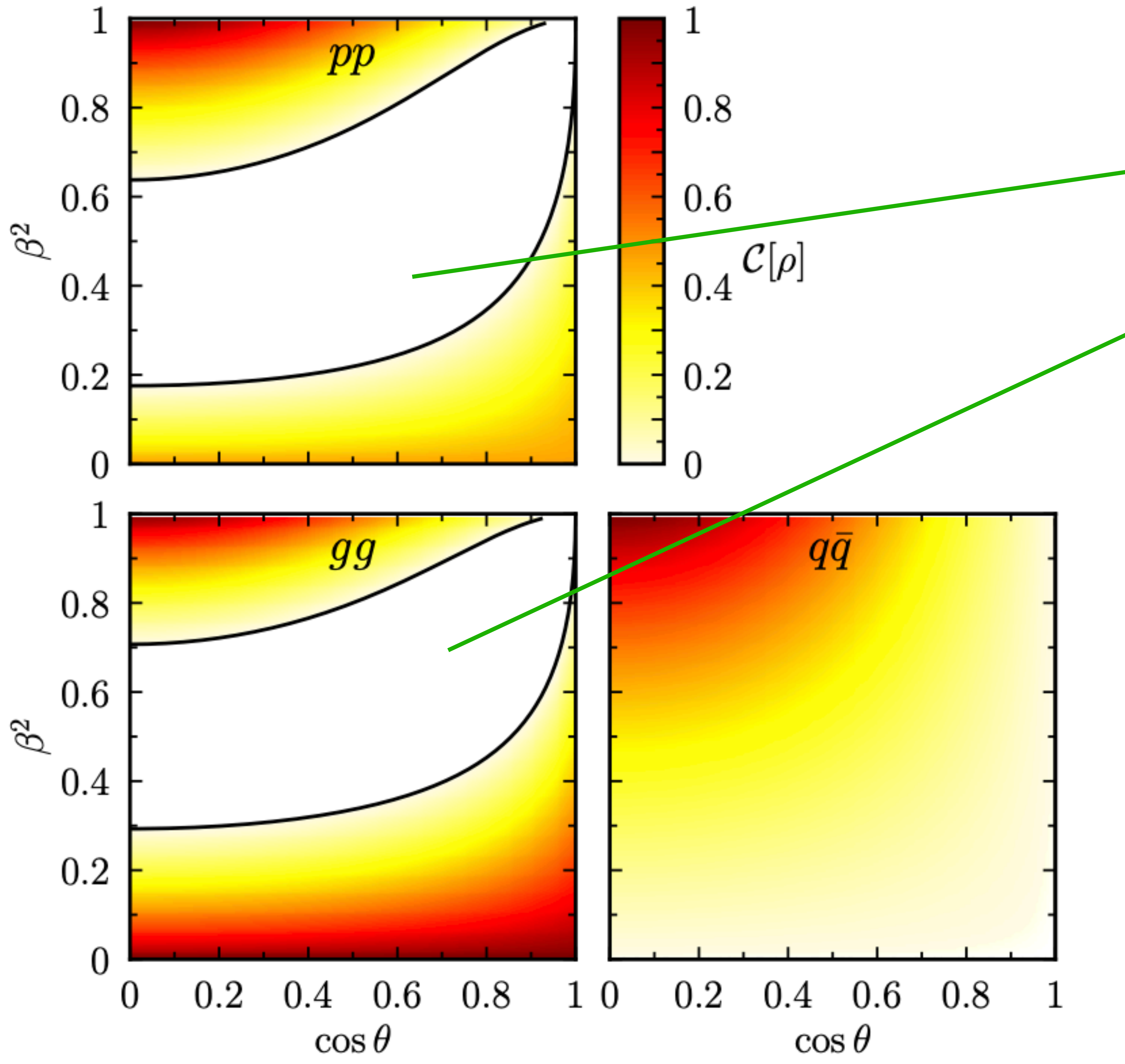

No entanglement

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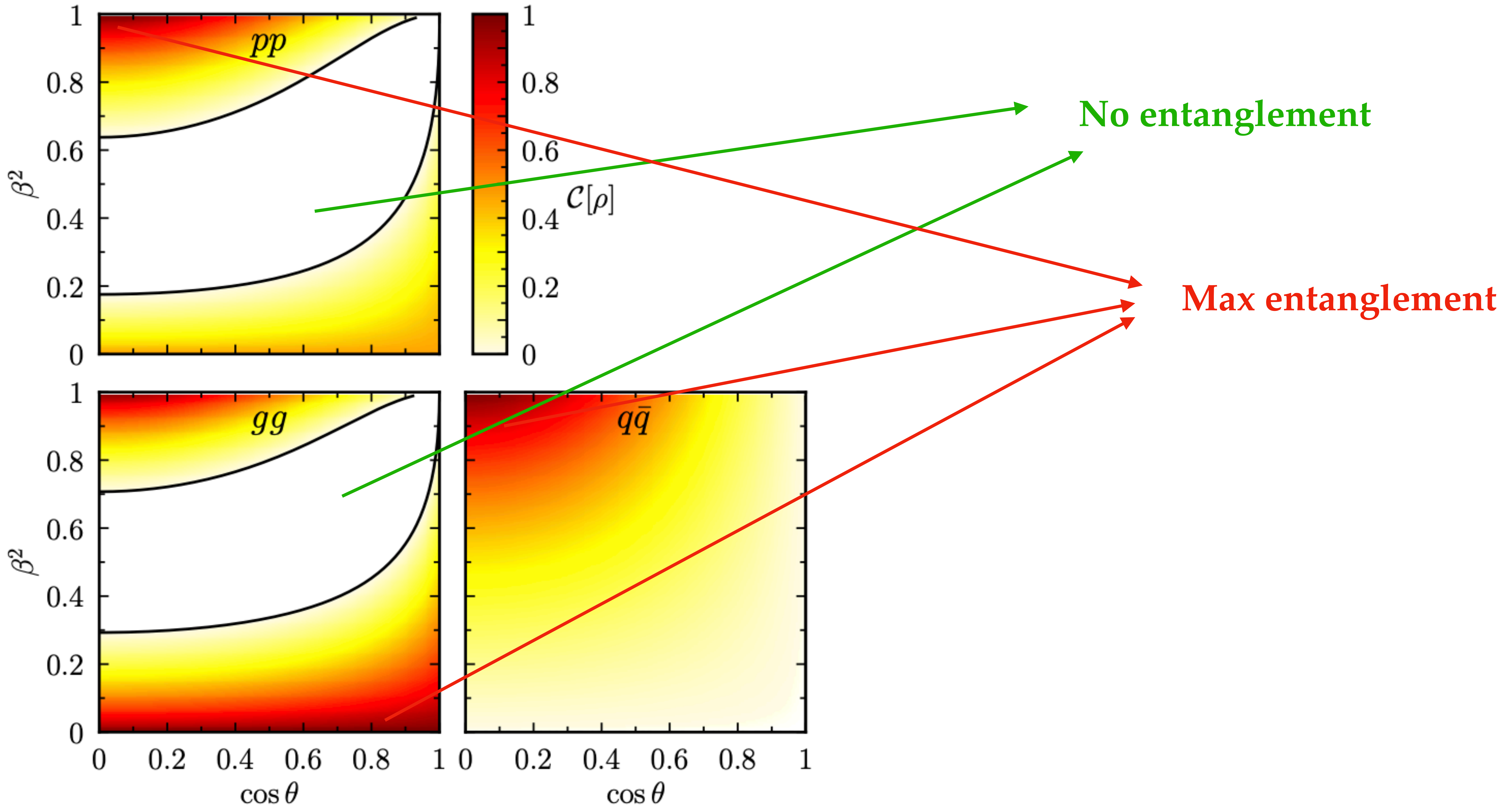
Aoude et al.
2203.05619



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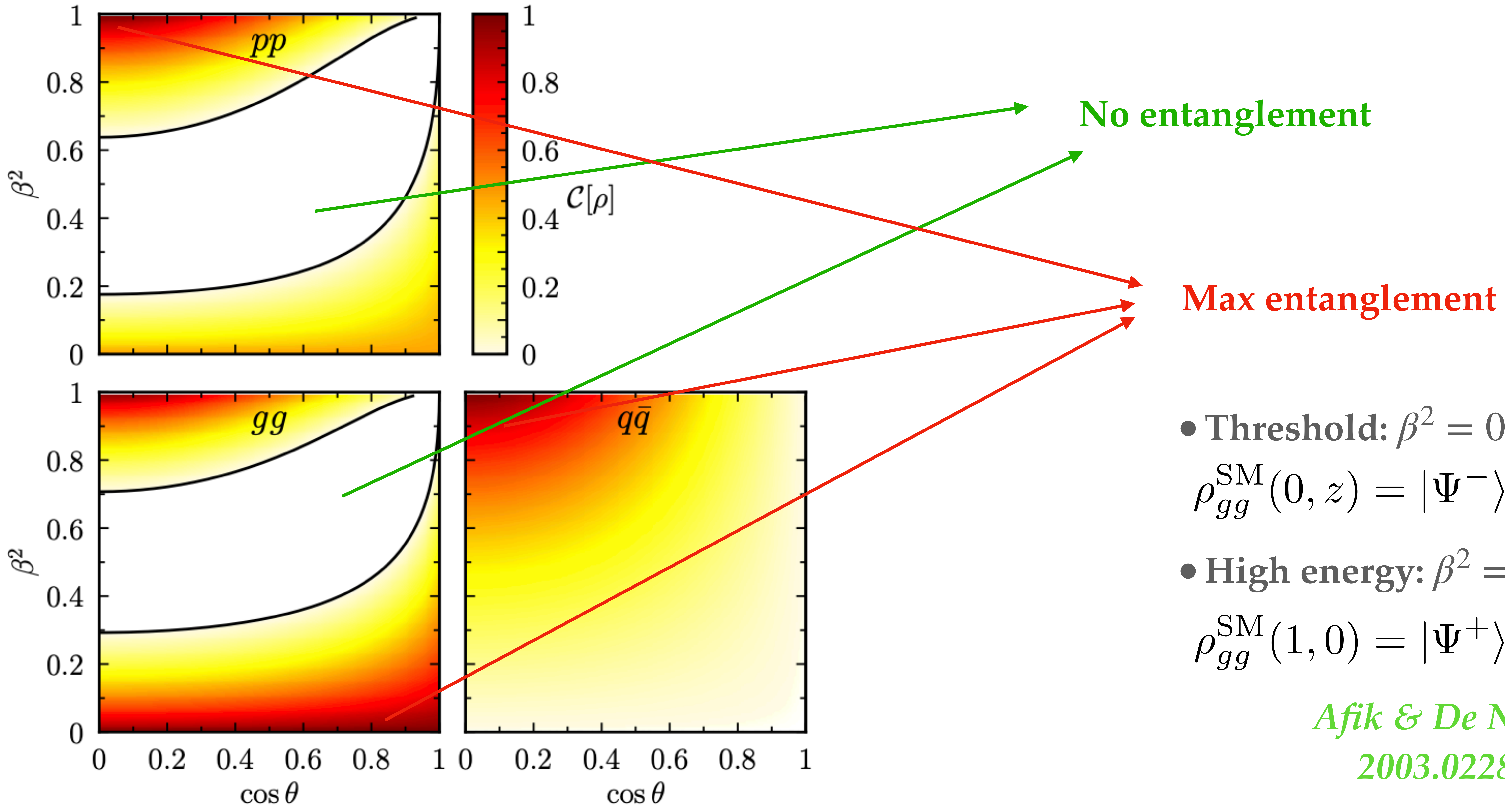

- **Threshold:** $\beta^2 = 0$

$$\rho_{gg}^{\text{SM}}(0,z) = |\Psi^-\rangle_{\boldsymbol{n}}\langle\Psi^-|_{\boldsymbol{n}}$$

- **High energy:** $\beta^2 = 1\,, \cos\theta = 0$

$$\rho_{gg}^{\text{SM}}(1,0) = |\Psi^+\rangle_{\boldsymbol{n}}\langle\Psi^+|_{\boldsymbol{n}}$$

Afik & De Nova
2003.02280

Scouting for entanglement

$$\bar{R} = (4\pi)^{-1} \int \mathrm{d}\Omega \, R(\hat{s}, \boldsymbol{k}) \,,$$

$$\delta \equiv -C_z + |2C_\perp| - 1 > 0$$

$$C[\rho] = \max(\delta/2, 0)$$

$$D = -3\langle \cos\varphi \rangle = -\frac{1+\delta}{3}$$

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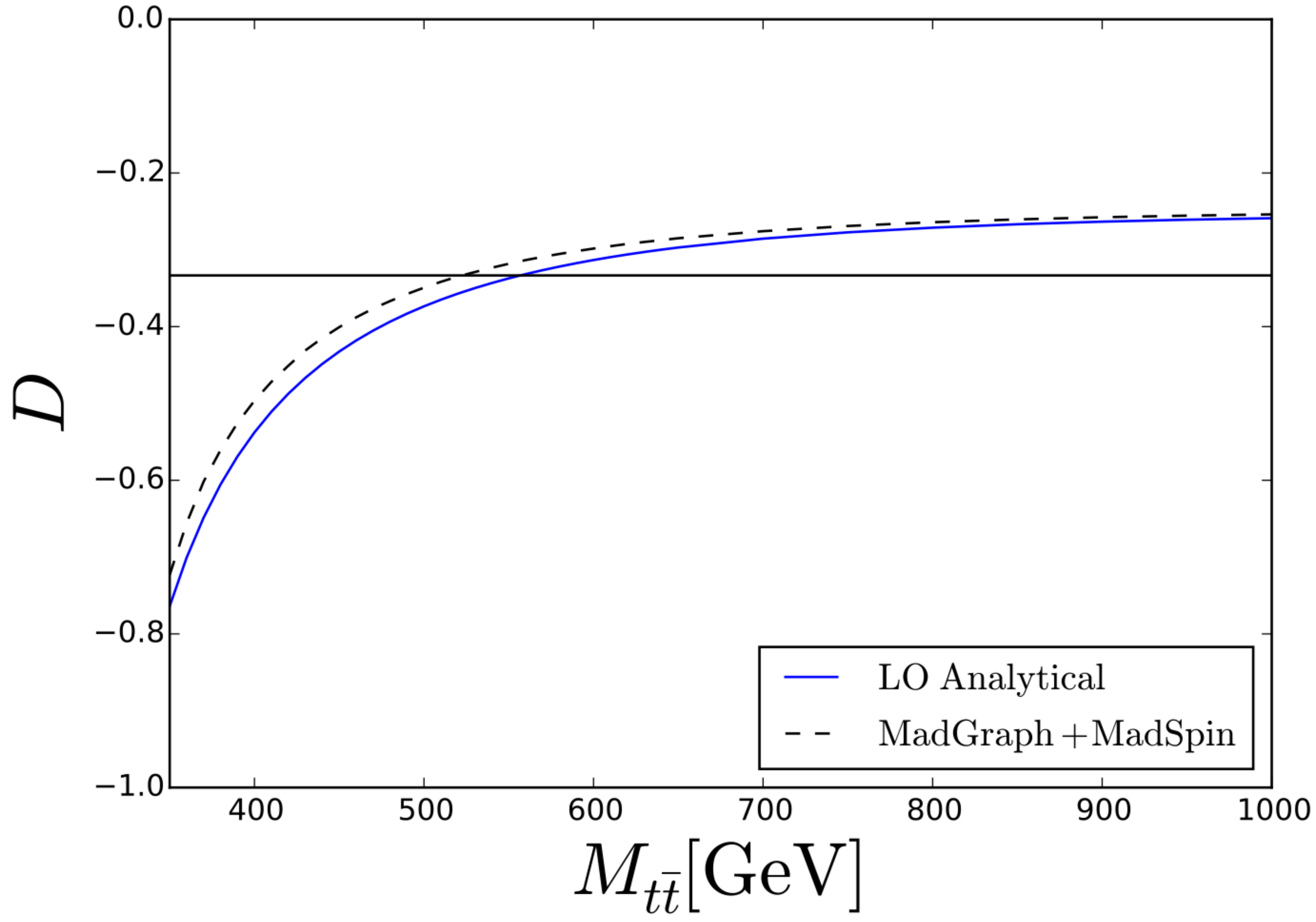

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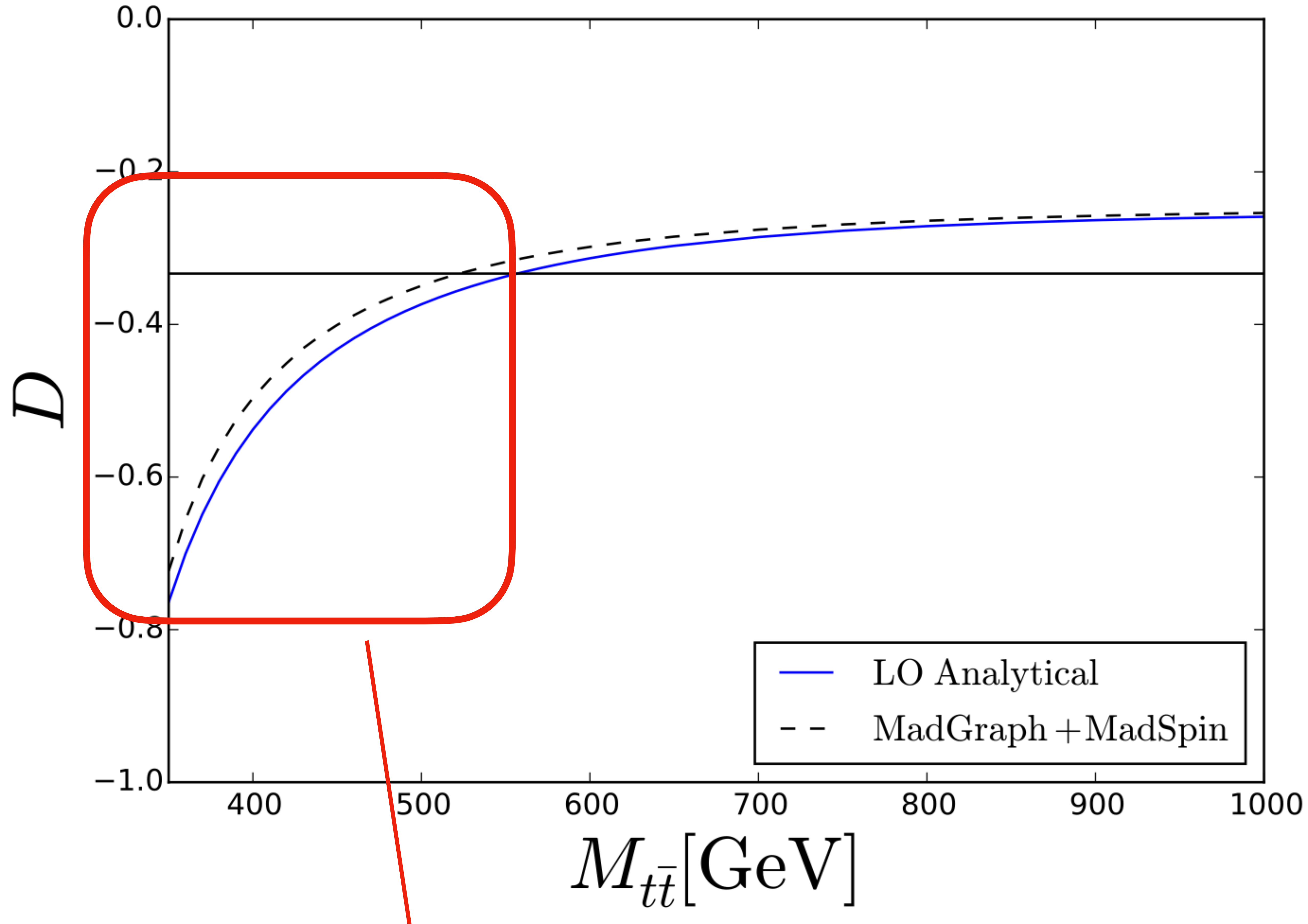
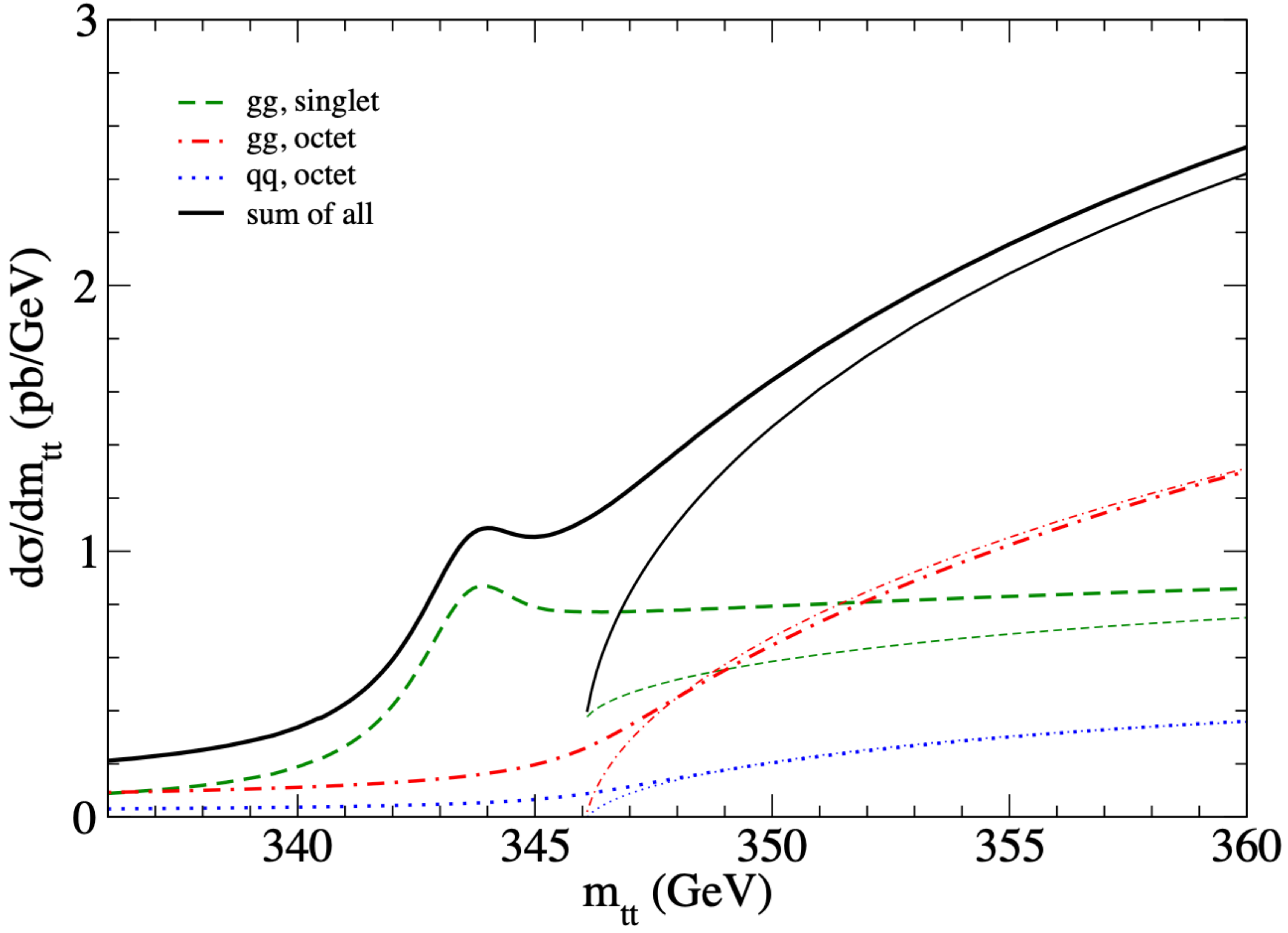


Need for a narrow bin close to threshold

Open new windows

Hagiwara, Sumino & Yokoya
0804.1014



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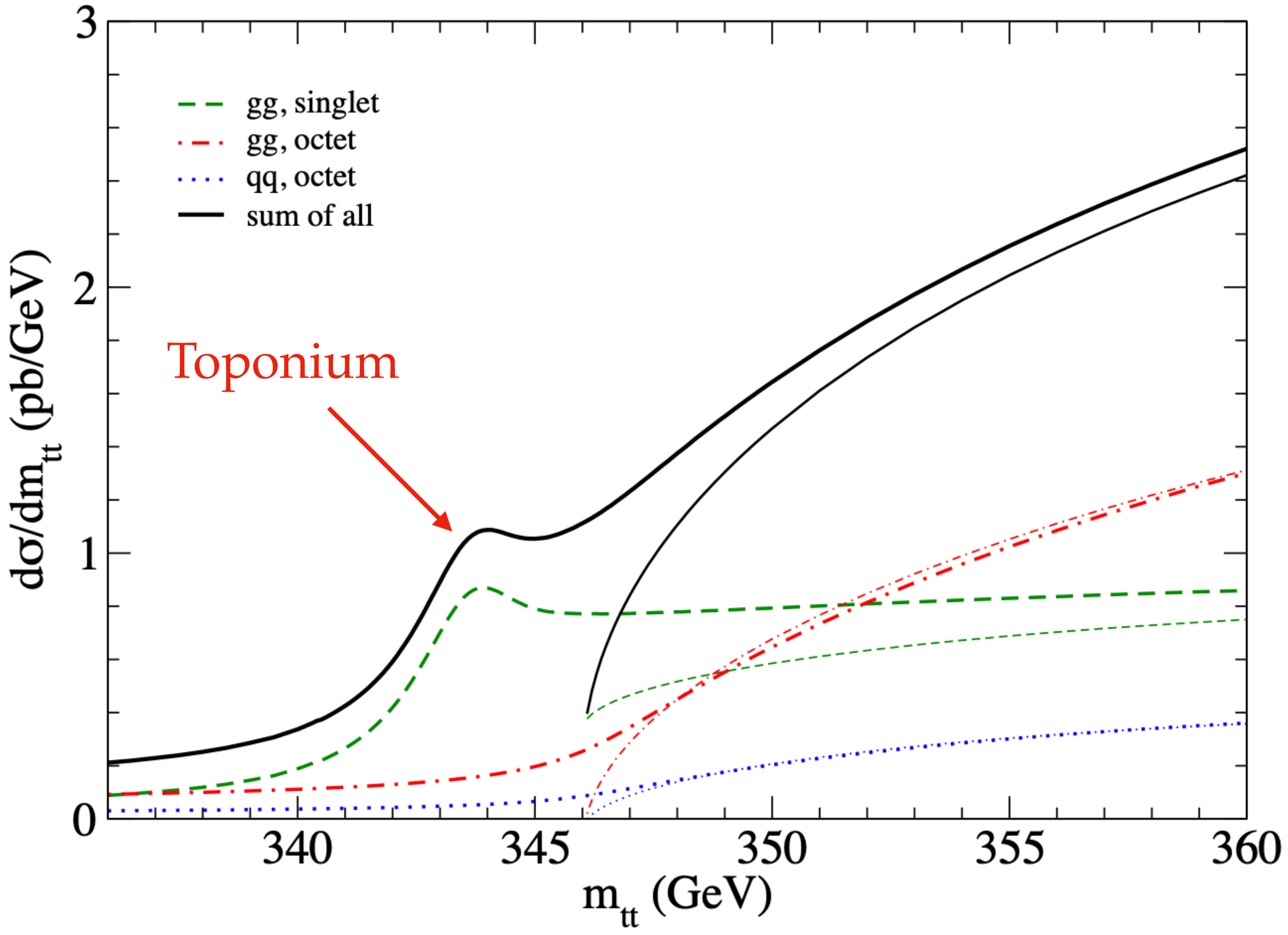

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Hagiwara, Sumino & Yokoya
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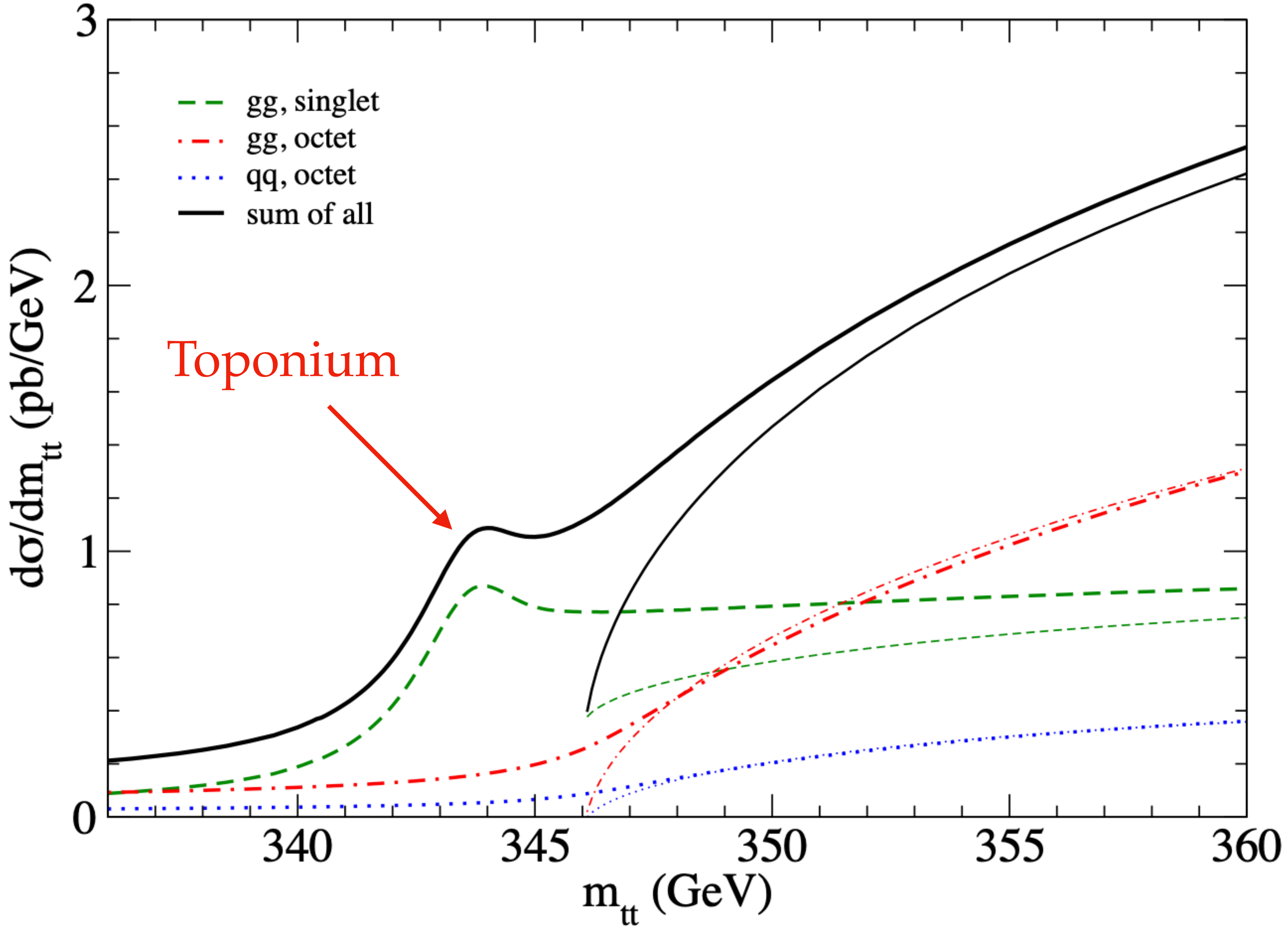
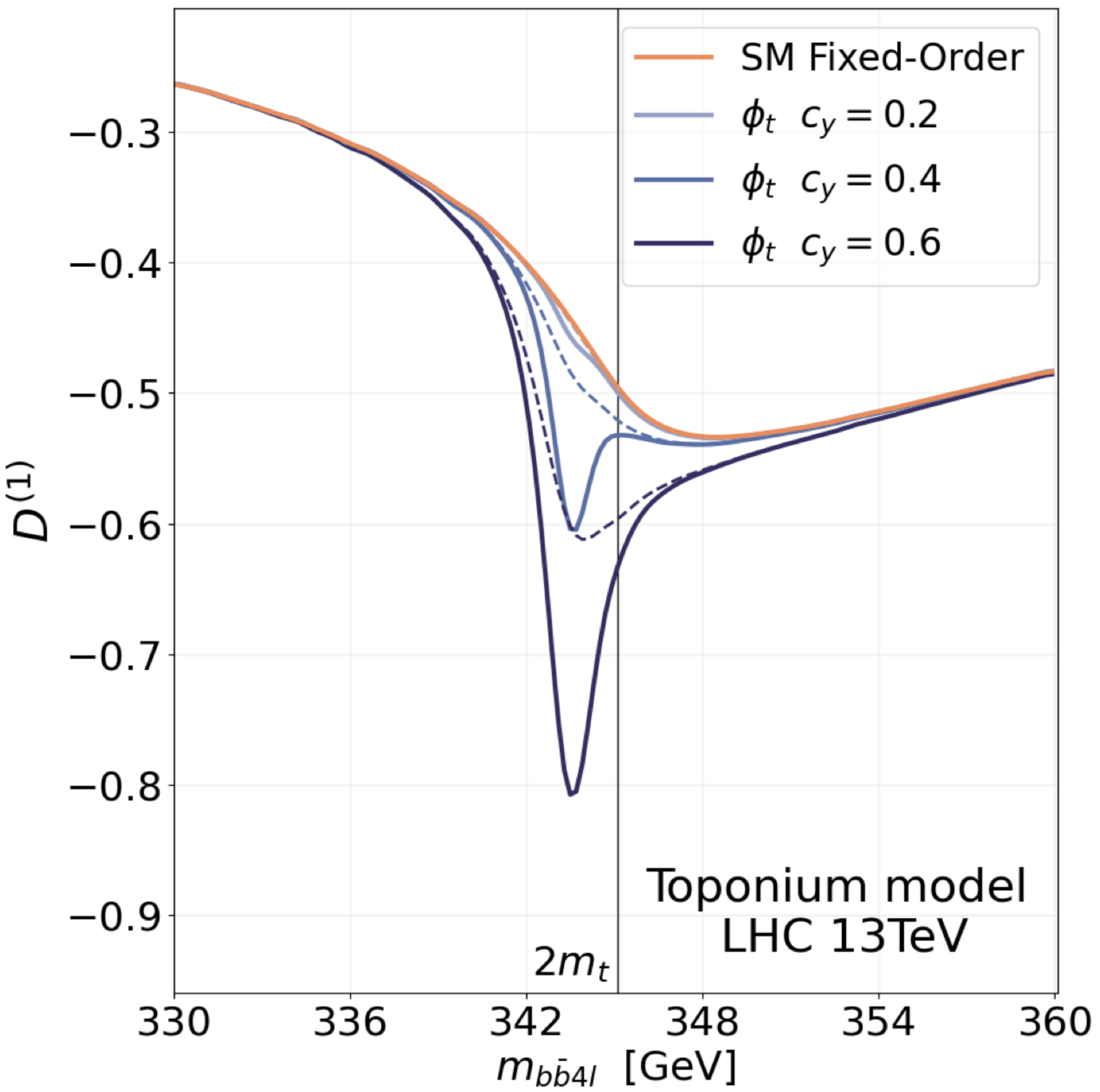


Maltoni et al.
2401.08751



New physics

The density matrix opens the window to new sensitivities

$$e^+e^- \to W^+W^-$$

$(\lambda_1\lambda_2\|\alpha\,\beta)$	SM	EFT Λ^{-2} : c_{WWW}
$+-00$	$-2\sqrt{2}G_F m_Z^2 \sin\theta$	-
$+--+$	$2\sqrt{2}G_F m_W^2 \sin\theta$	-
$+-+-$	$-\frac{1}{\sqrt{2}}G_F m_W^2 \sin^3\theta \csc^4(\theta/2)$	-
$+-\pm\pm$	-	$3\cdot 2^{1/4}\sqrt{G_F}m_W \sin\theta\,(4m_W^2x^2 - m_Z^2)$
$+-0\pm$	-	$-3\cdot 2^{3/4}\sqrt{G_F}m_W^3(\pm 1+\cos\theta)\,x$
$+-\pm 0$	-	$-3\cdot 2^{3/4}\sqrt{G_F}m_W^3(\mp 1+\cos\theta)\,x$
$-+00$	$2\sqrt{2}G_F(m_Z^2 - m_W^2)\sin\theta$	-
$-+\pm\pm$	-	$6\cdot 2^{1/4}\sqrt{G_F}m_W(m_Z^2 - m_W^2)\sin\theta$

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Cross section

$$\tilde{A}(\mathcal{O}_W) \sim 0$$

New physics

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$$\rho = \begin{bmatrix} \mathcal{M}_{++}\mathcal{M}^*_{++} & \mathcal{M}_{++}\mathcal{M}^*_{+-} & \cdots \\ \mathcal{M}_{+-}\mathcal{M}^*_{++} & \mathcal{M}_{+-}\mathcal{M}^*_{+-} & \cdots \\ \vdots & \ddots & \end{bmatrix}$$

New physics

$(\lambda_1\lambda_2\|\alpha\,\beta)$	SM	EFT Λ^{-2} : c_{WWW}
$+-00$	$-2\sqrt{2}G_F m_Z^2 \sin\theta$	-
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New physics

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$$\tilde{a}_1(\mathcal{O}_W) \simeq \tilde{b}_1(\mathcal{O}_W) \simeq \bar{c}_W\, 2^{5/4}\, x\;\cos^4(\theta/2)(\cos\theta+3)\csc\theta\,,$$

Resurrected sensitivity: energy growth!

Conclusions

- Measurement of entanglement between tops is highest energy evidence ever.
- In the SM, specific spin configurations are expected, dictated by interactions.
  - High degree of entanglement present at threshold and high energy (+ Bell violation)
  - Need to design measurements in corners of phase space.
- QI observables probe complementary directions to the cross-section:
  e.g. toponium and resurrect the EFT interference.


Thank you

Back-up

Isn't this just buzz words?

We are discussing correlations: observables that allow for information theory statements

Spin correlations are the generic aspect:
are they "classical" or "quantum"? Different regime.

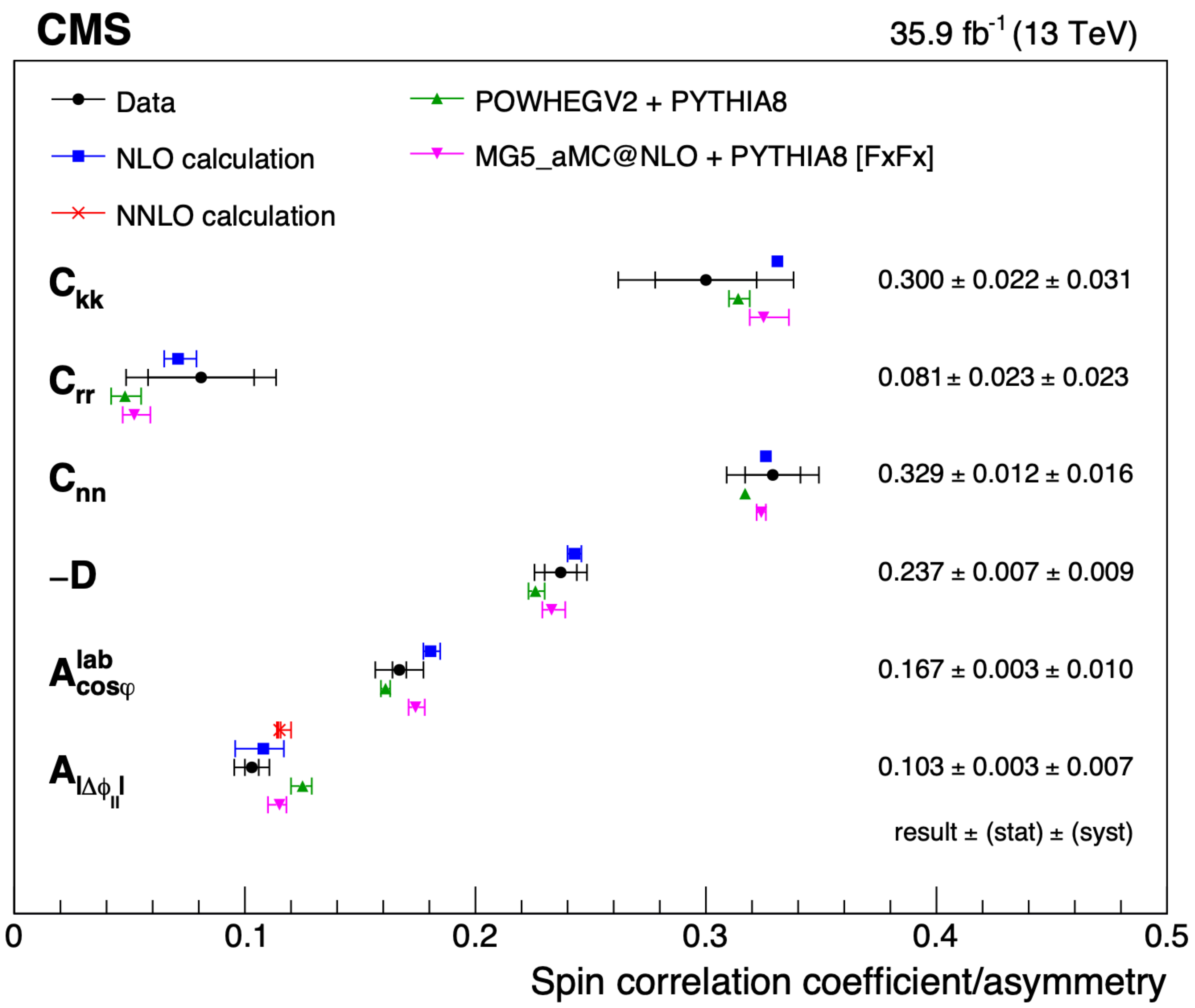
We are forced to look at corners of phase space,
where rather extreme conditions on correlations are met

Explore the world we know with the eyes of QI:
new methods, new frameworks

First workshop gathering the new community

Spin correlation measurement

Inclusive measurement *CMS 1907.03729*



Spin correlation measurement

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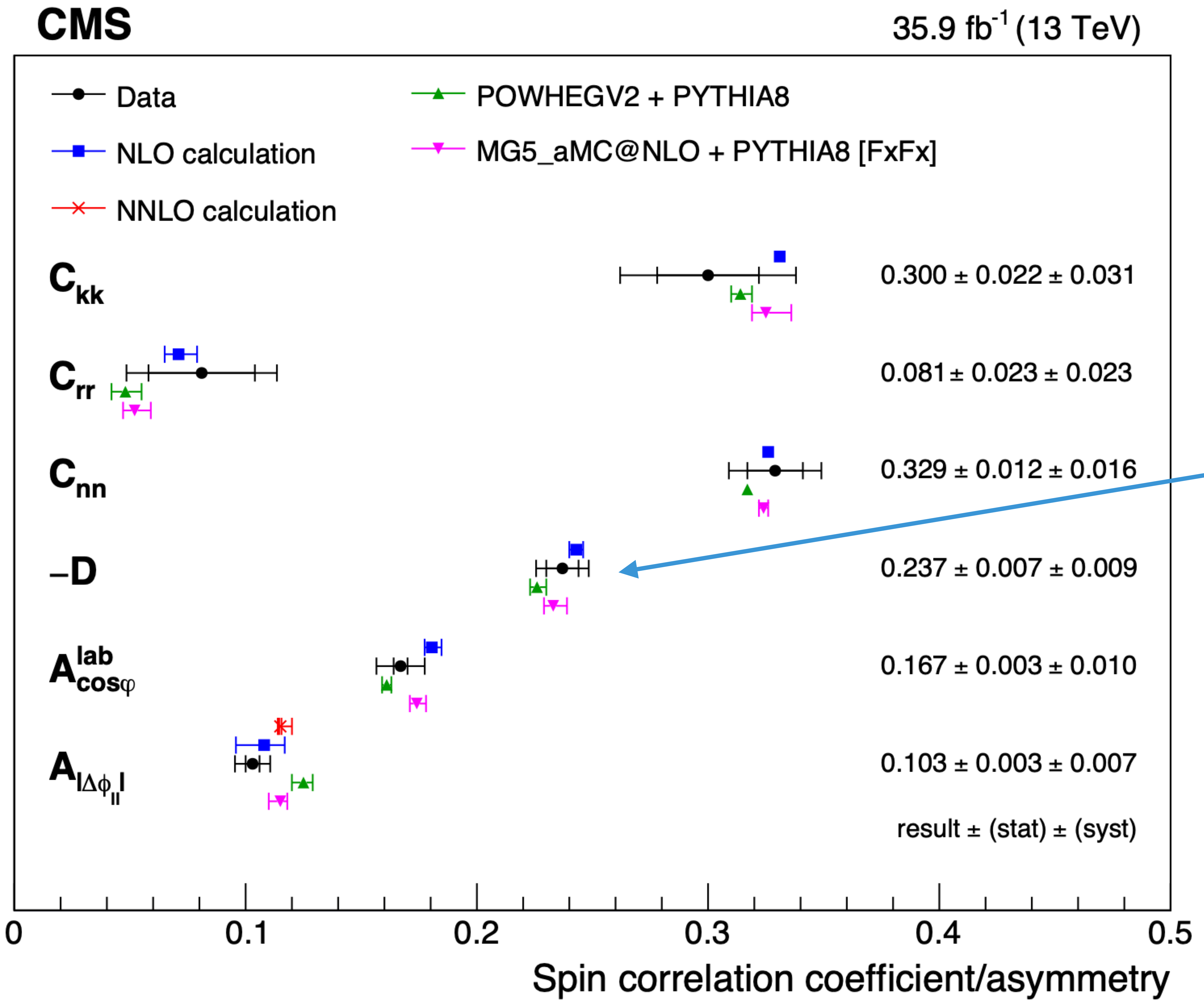
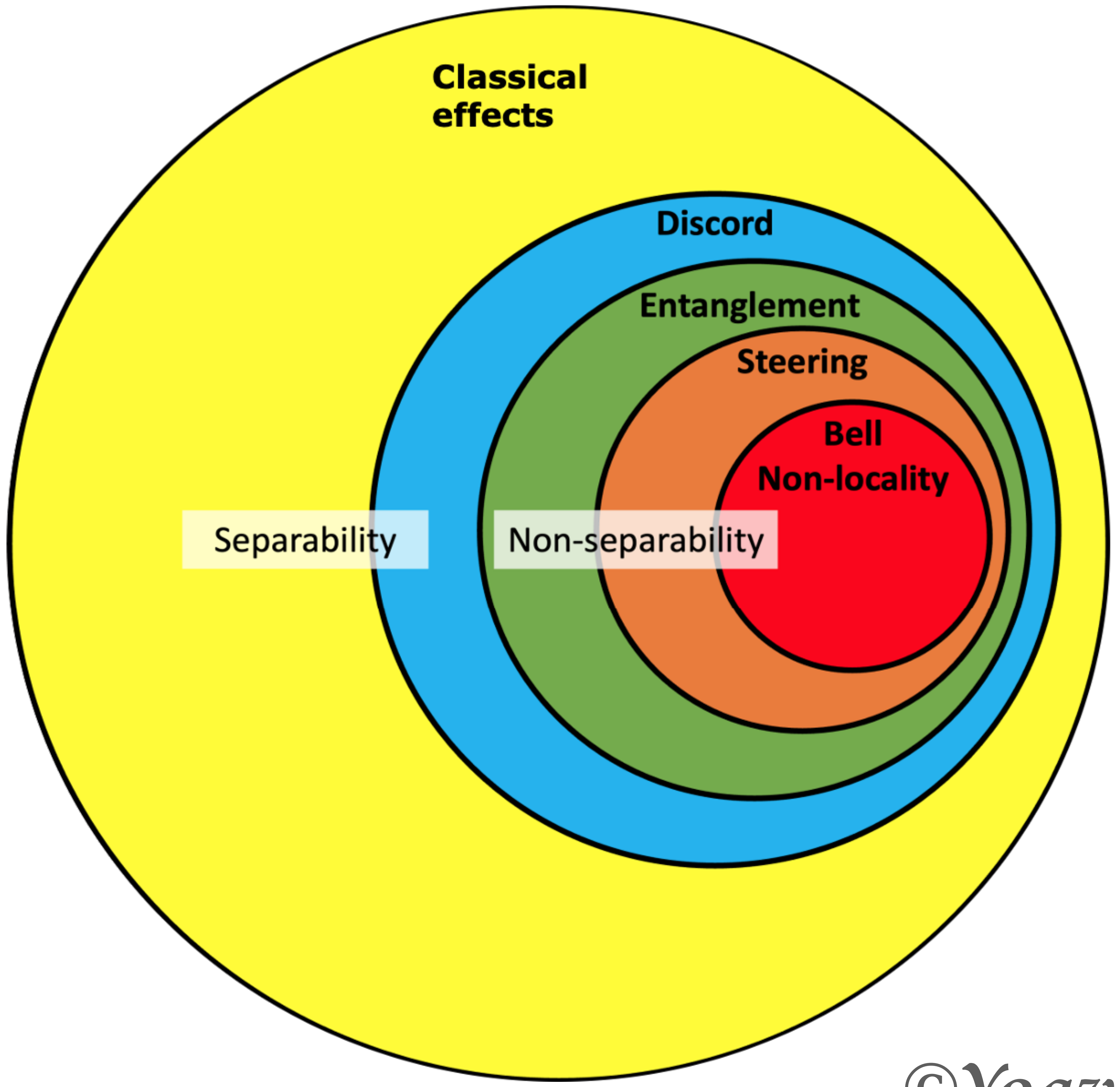
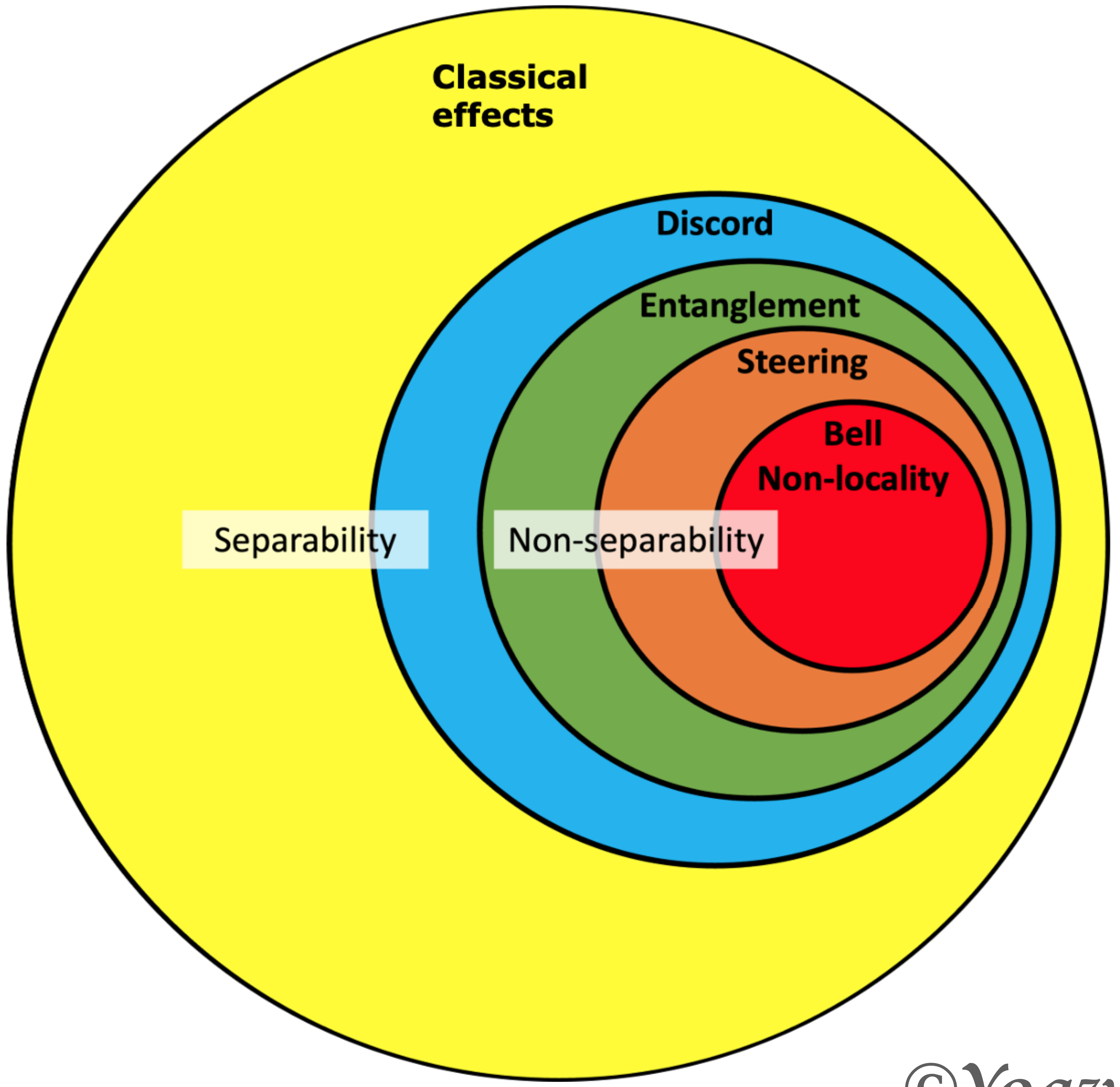


Not enough to see entanglement!

Hierarchy of quantumness



Hierarchy of quantumness



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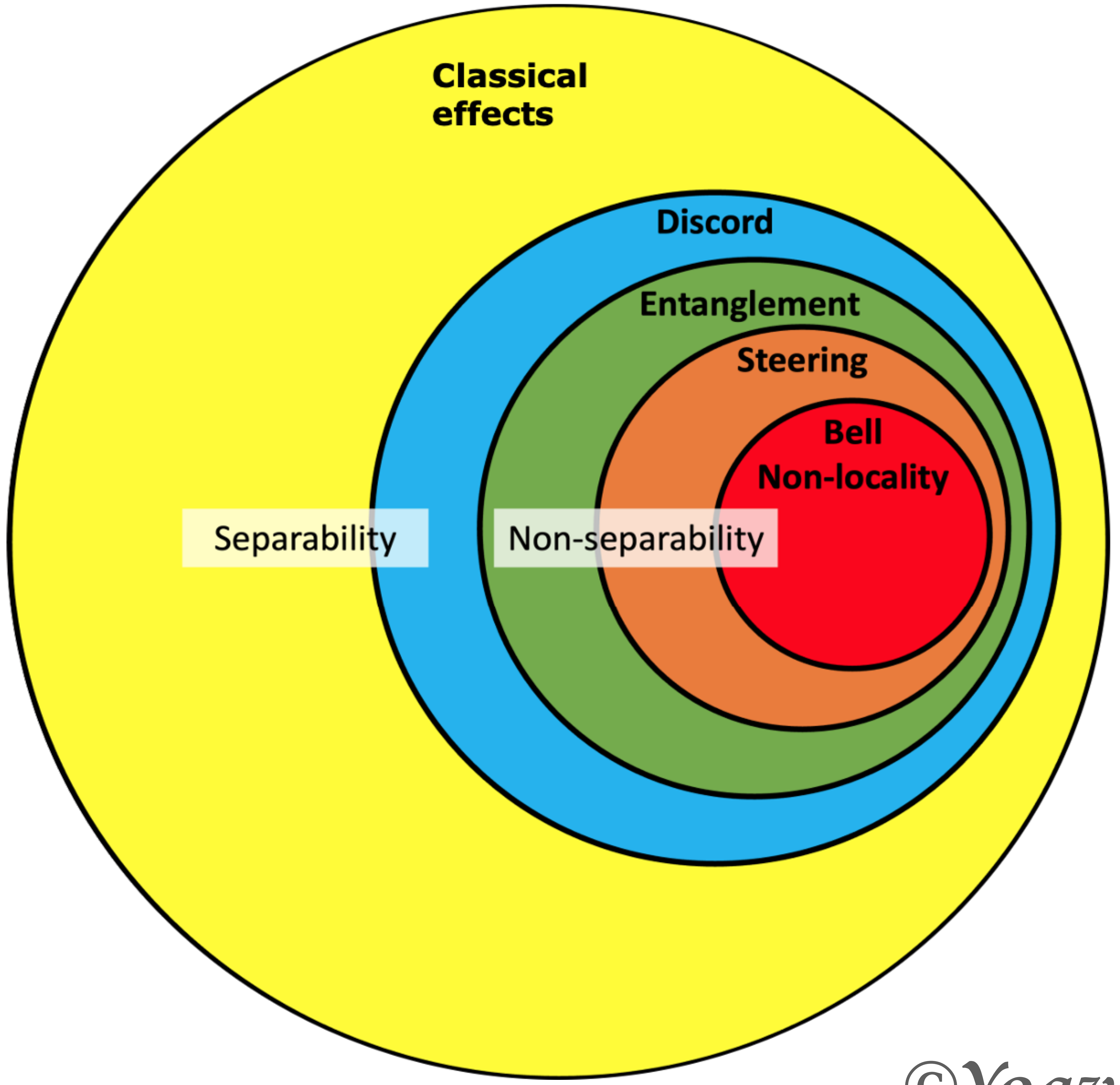
* Quantum discord: shared information.

Hierarchy of quantumness



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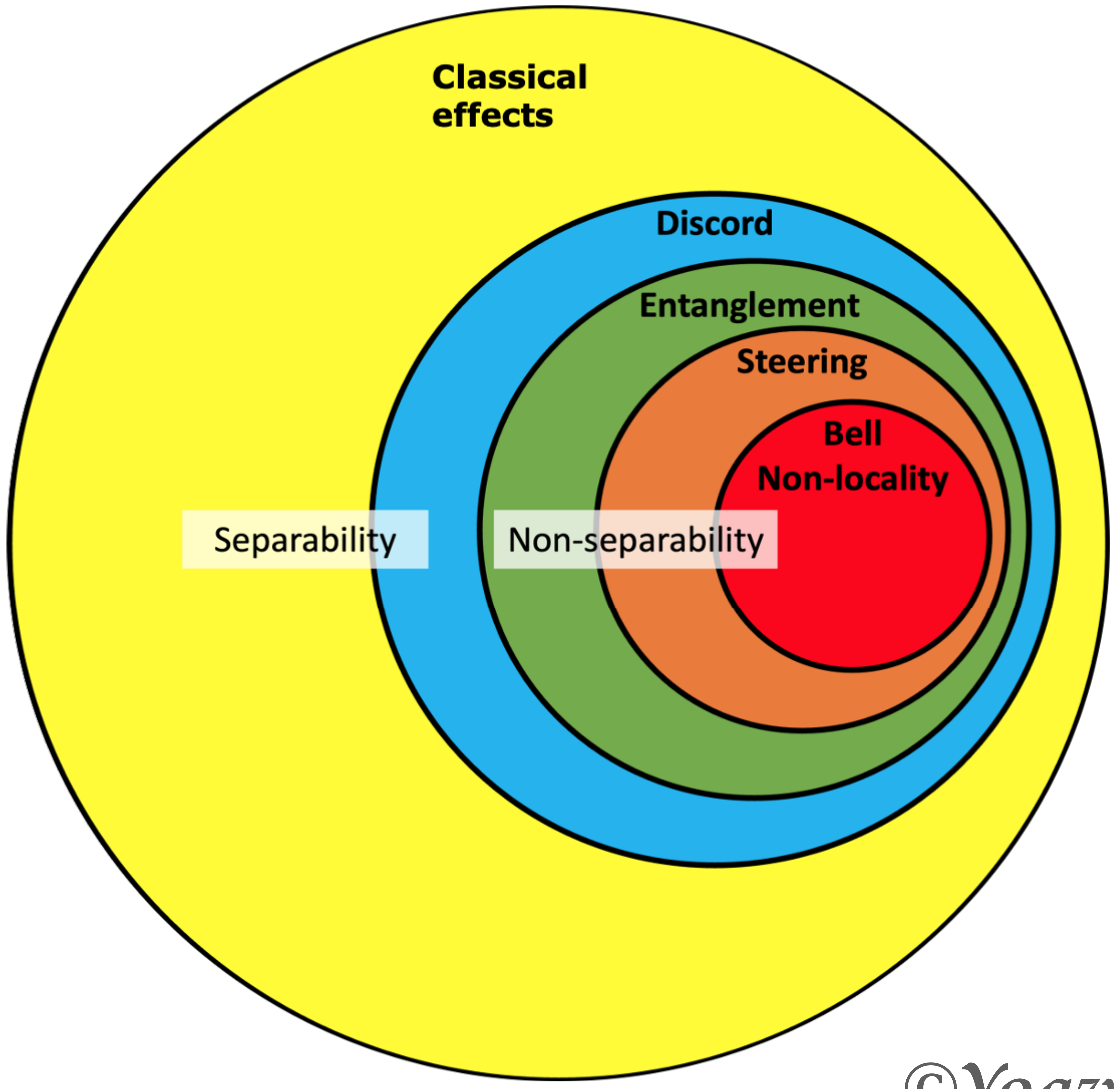
* Quantum discord: shared information.
* Entanglement: non-separability.

Hierarchy of quantumness



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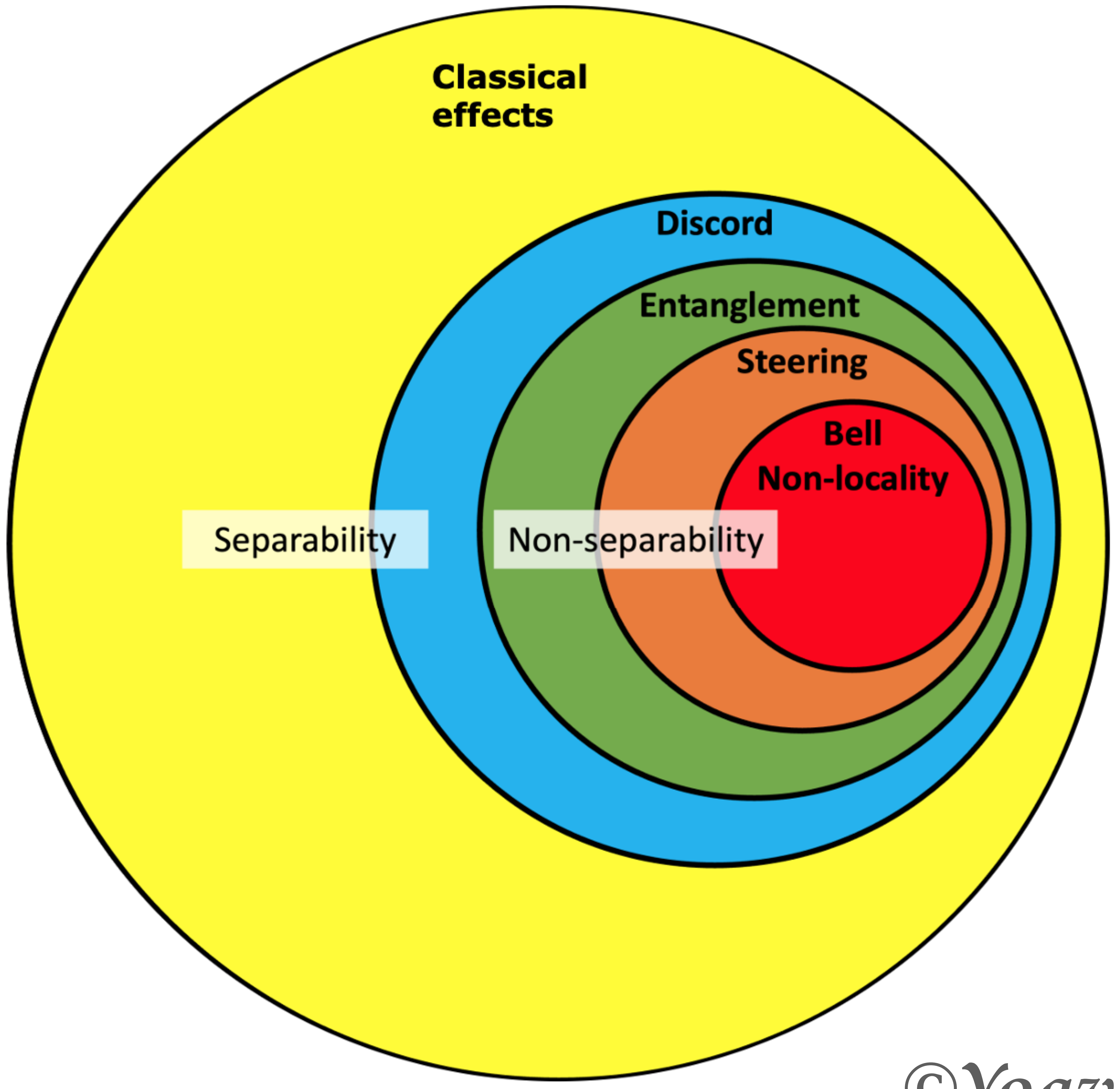
* Quantum discord: shared information.
* Entanglement: non-separability.
* Steering: “spooky action at a distance”

Hierarchy of quantumness



©*Yoav Afik*

- Quantum discord: shared information.
- Entanglement: non-separability.
- Steering: "spooky action at a distance"
- Bell non-locality: very strong correlations: Non local.

SMEFT relative effects

$$\Delta = -C_{nn} + |C_{kk} + C_{rr}| - 1 > 0$$

$$\Delta_1 \equiv \Delta - \Delta_0$$

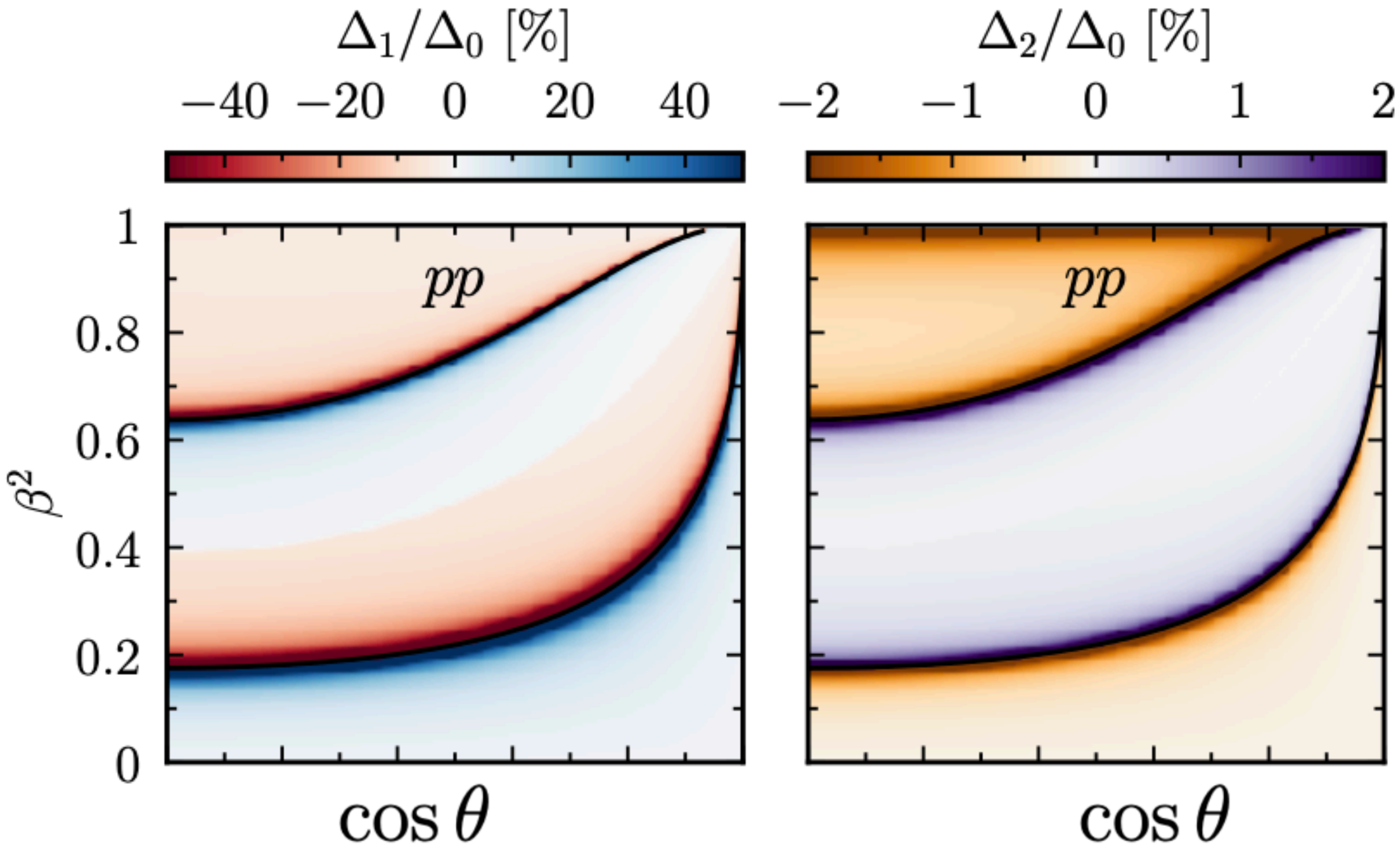
Δ computed up to $\mathcal{O}(1/\Lambda^2)$

$$\Delta_2 \equiv \Delta - \Delta_1 - \Delta_0$$

Δ computed up to $\mathcal{O}(1/\Lambda^4)$

SMEFT relative effects

$$\Delta = -C_{nn} + |C_{kk} + C_{rr}| - 1 > 0$$



$$c_{tG}/\Lambda^2 = 0.1\,\mathrm{TeV}^{-2}$$

$$\Delta_1 \equiv \Delta - \Delta_0$$

Δ computed up to $\mathcal{O}(1/\Lambda^2)$

$$\Delta_2 \equiv \Delta - \Delta_1 - \Delta_0$$

Δ computed up to $\mathcal{O}(1/\Lambda^4)$

Average concurrence

$$\bar{R} = (4\pi)^{-1} \int \mathrm{d}\Omega \, R(\hat{s}, \boldsymbol{k}) \,,$$

→

$$\delta \equiv -C_z + |2C_\perp| - 1 > 0$$

$$C[\rho] = \max(\delta/2, 0)$$

Average concurrence

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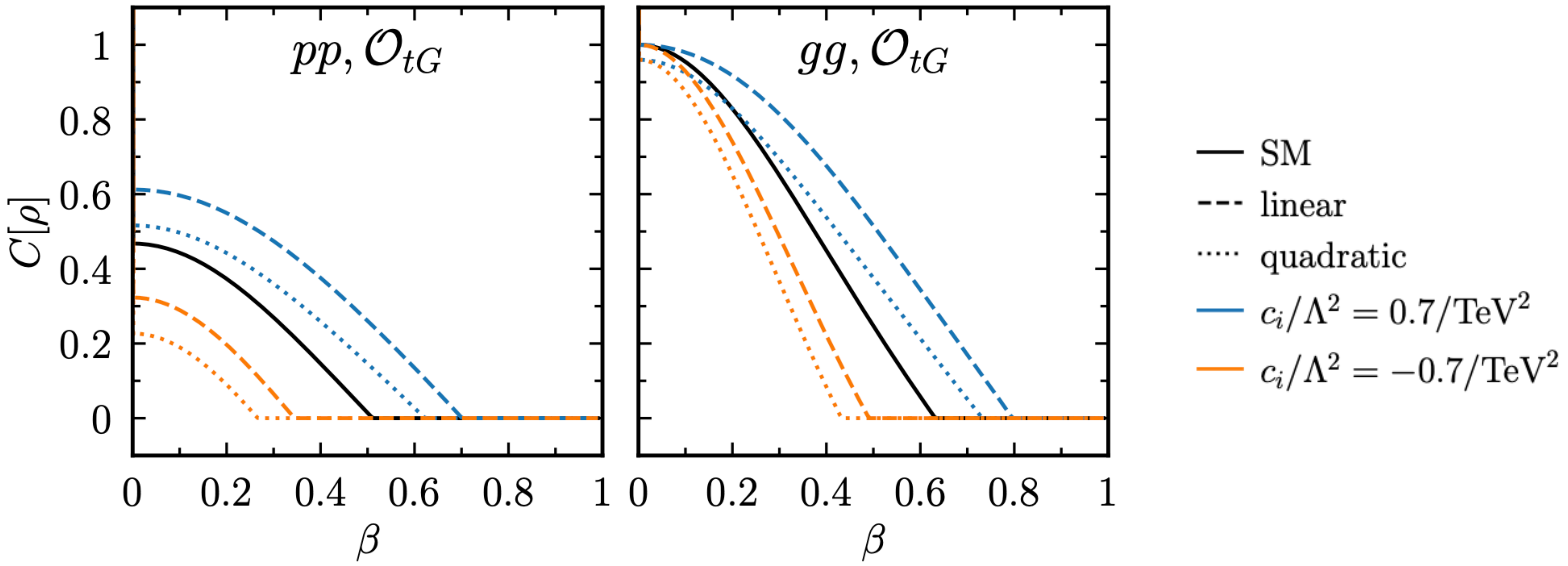

Quantum state in the EFT

gg-induced

$$\rho_{gg}^{\mathrm{EFT}}(0,z) = p_{gg}|\Psi^+\rangle_{\boldsymbol{p}}\langle\Psi^+|_{\boldsymbol{p}} + (1-p_{gg})|\Psi^-\rangle_{\boldsymbol{p}}\langle\Psi^-|_{\boldsymbol{p}}$$

$$p_{gg} = \frac{72}{7\Lambda^4} m_t^2 (3\sqrt{2} m_t\, c_G + v\, c_{tG})^2$$

Only quadratic effects!

Quantum state in the EFT

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qq-induced

$$\rho_{q\bar{q}}^{\mathrm{EFT}}(0,z) = p_{q\bar{q}}\,|\uparrow\uparrow\rangle_{\boldsymbol{p}}\,\langle\uparrow\uparrow|_{\boldsymbol{p}} + (1-p_{q\bar{q}})\,|\downarrow\downarrow\rangle_{\boldsymbol{p}}\,\langle\downarrow\downarrow|_{\boldsymbol{p}}$$

$$p_{q\bar{q}} = \frac{1}{2} - 4\frac{c_{VA}^{(8),u}}{\Lambda^2} + \frac{8m_t^4}{\Lambda^4}\left(\frac{v\sqrt{2}}{m_t} c_{VA}^{(8),u} c_{tG} - 9 c_{VA}^{(1),u} c_{VV}^{(1),u} + 2 c_{VA}^{(8),u} c_{VV}^{(8),u}\right)$$

Quantum state in the EFT

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