

# STATE-OF-THE-ART PREDICTIONS FOR $TT+H/TT+W/(TT+Z)$

ANNA KULESZA (UNIVERSITY OF MÜNSTER)

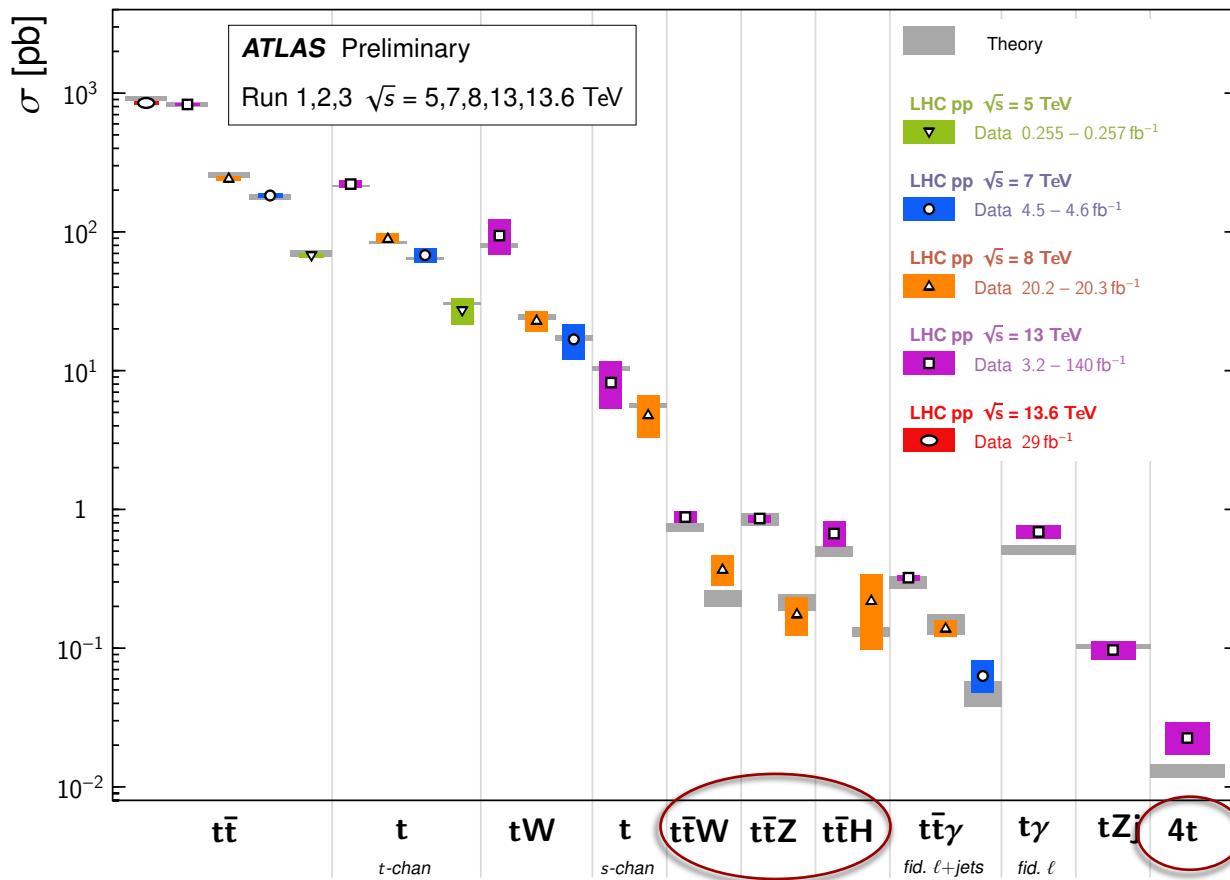


STANDARD MODEL AT THE LHC, 08.05.2024, ROME

# ASSOCIATED TTBAR PRODUCTION

## Top Quark Production Cross Section Measurements

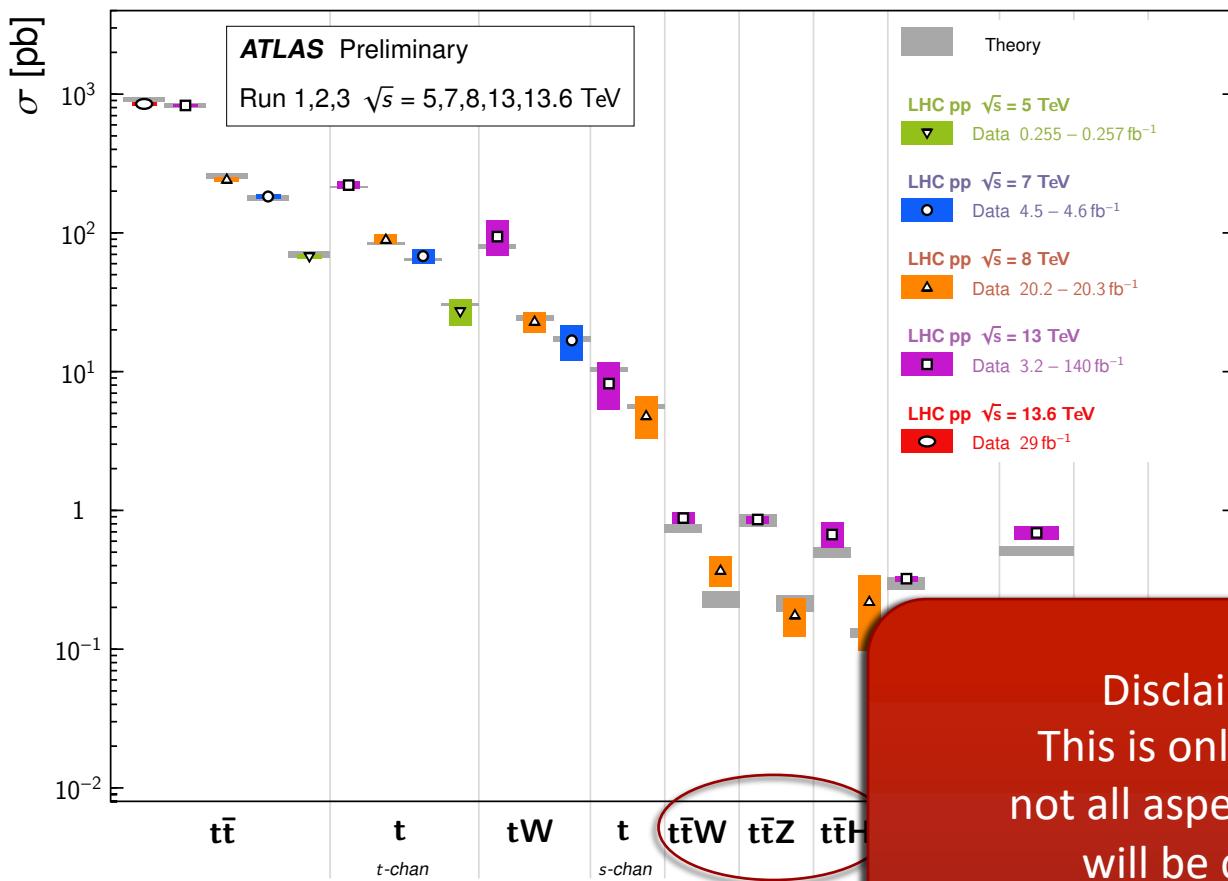
Status: April 2024



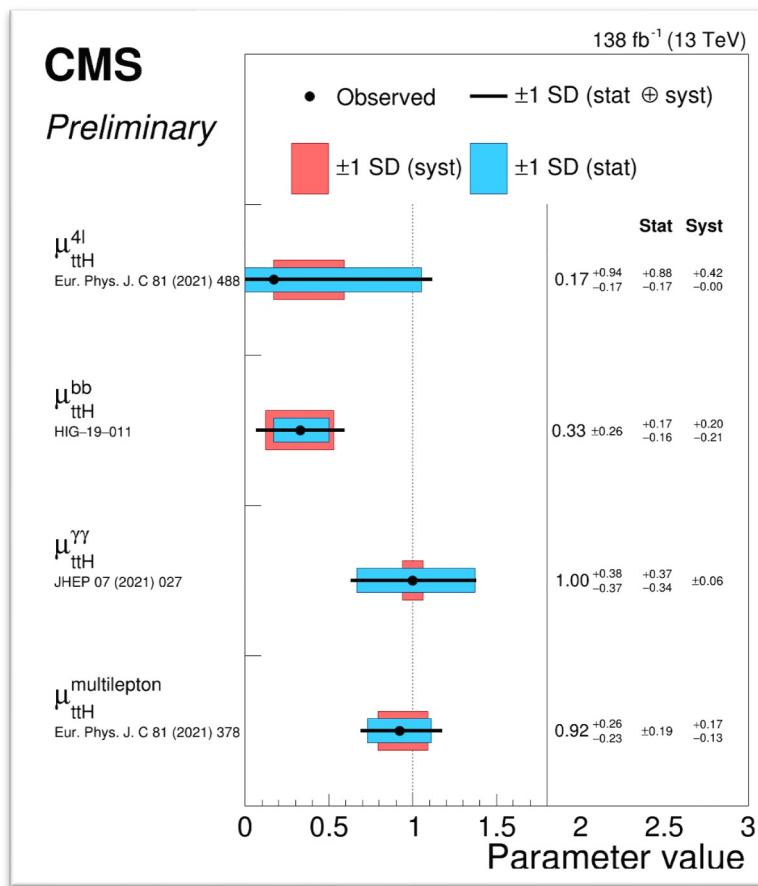
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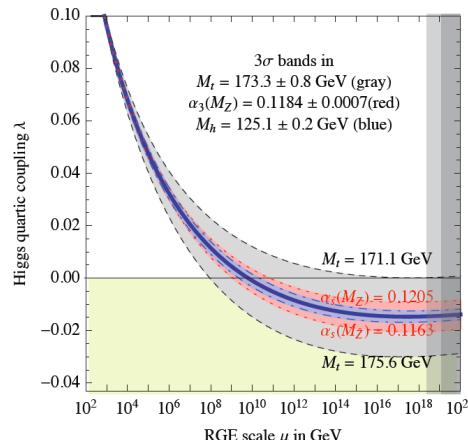
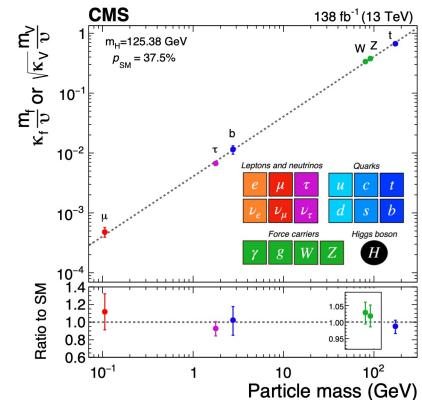
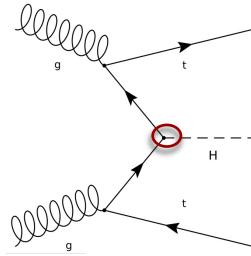
Disclaimer and apology:  
This is only a limited selection;  
not all aspects/publications/work  
will be covered in this talk



*see also talks by  
M. Schröder,  
M. Grazzini and T. Vitos*

# ASSOCIATED HIGGS PRODUCTION WITH TOP QUARKS

- ↗ Direct probe of the strength of the top-Yukawa coupling without making any assumptions regarding its nature
- ↗ Yukawa coupling proportional to mass -> top-Higgs is the strongest interaction of the Yukawa type between fundamental SM particles
- ↗ Far-reaching consequences: stability of our Universe
- ↗ HL-LHC: expected 3-4% precision for the top-quark Yukawa coupling determination

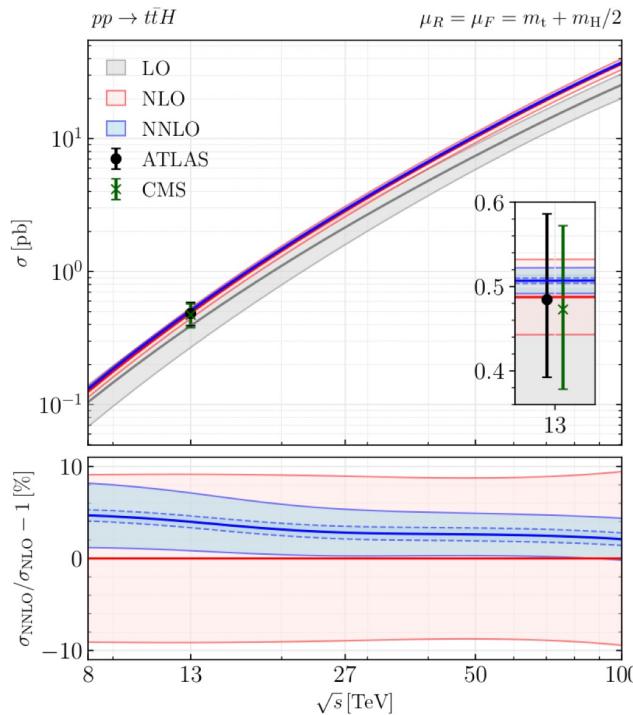


# A BRIEF HISTORY OF $t\bar{t}H$ THEORY

- ↗ NLO QCD available > 20 years [*Beenakker, Dittmaier, Krämer, Plumper, Spira, Zerwas '01-'02*][*Reina, Dawson'01*][*Reina, Dawson, Wackeroth'02*][*Dawson, Orr, Reina, Wackeroth'03*] [*Dawson, Jackson, Orr, Reina, Wackeroth'03*]
- ↗ NLO matched with parton showers [*Garzelli, Kardos, Papadopoulos, Trocsanyi'11*] [*Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli'11*] [*Hartanto, Jäger, Reina, Wackeroth'15*][*Maltoni, Pagani, Tsinikos'15*] [*Pagani, Vitos, Zaro'23*]
- ↗ QCD and EW@NLO [*Frixione, Hirschi, Pagani, Shao, Zaro'14-'15*][*Zhang, Ma, Zhang, Chen, Guo'14*][*Biedermann, Bräuer, Denner, Pellen, Schumann, Thompson'17*]
- ↗ Off-shell effects at NLO QCD [*Denner, Feger'15*] [*Stremmer, Worek'21*] and EW [*Denner, Lang, Pellen, Uccirati'16*]
- ↗ NNLL+NLO resummation in direct QCD [*AK, Motyka, Stebel, Theeuwes'15*], [*AK, Motyka, Stebel, Theeuwes'17*] [*AK, Motyka, Schwartländer, Stebel, Theeuwes'20*] [*Ju and Yang'19*] and in SCET [*Broggio, Ferroglia, Pecjak, Signer, Yang'15*] [*Broggio, Ferroglia, Pecjak, Yang'16*] [*Broggio, Ferroglia, Frederix, Pagani, Pecjak, Tsinikos'19*]

# NNLO ADVANCES: TTH

- ↗ Off-diagonal partonic channels [Catani, Fabre, Grazzini, Kallweit'21]
- ↗ Coefficients of the two-loop infrared singularities [Chen, Ma, Wang, Yang, Ye'22]
- ↗ Soft Higgs approximation [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini'22]



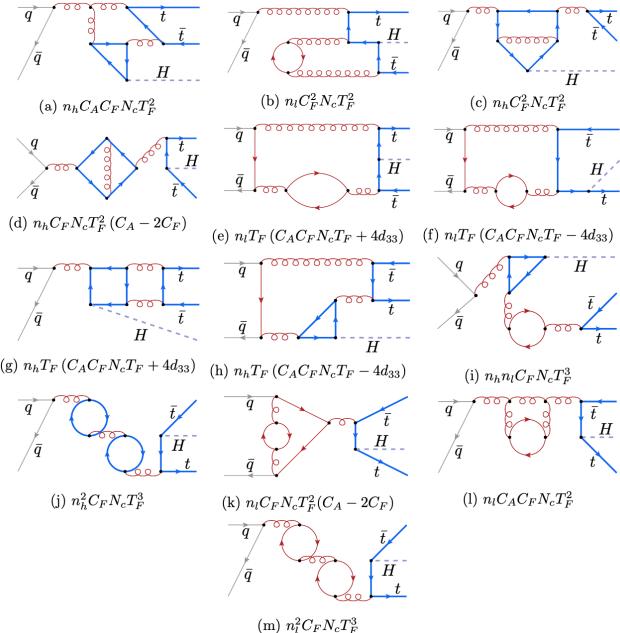
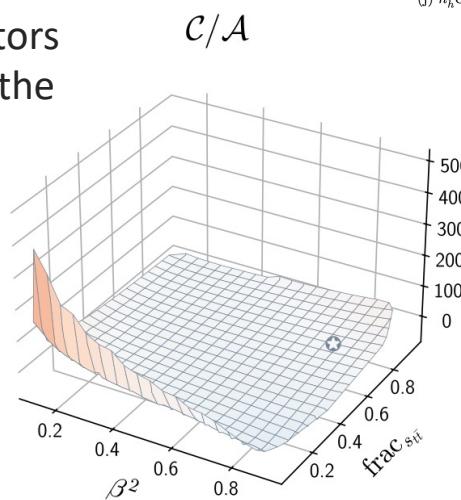
- ↗ Full computation using the  $q_T$  subtraction framework, apart from the two-loop amplitude
- ↗ Two-loop contribution estimated using the soft Higgs approximation: in the limit  $p_H \rightarrow 0$ ,  $m_H \ll m_t$ , Higgs emission is factorized out, in analogy to soft gluon emission
- ↗ Two-loop contributions provide  $\sim 1\%$  of the NNLO cross section, introducing  $O(+/- 0.6\%)$  systematic error on NNLO
- ↗ 4% correction from NNLO QCD at 13 TeV + reduction of scale uncertainties

# NNLO ADVANCES: ttH

- ↗ Off-diagonal partonic channels [*Catani, Fabre, Grazzini, Kallweit'21*]
- ↗ Coefficients of the two-loop infrared singularities [*Chen, Ma,Wang,Yang,Ye'22*]
- ↗ Soft Higgs approximation [*Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini'22*]
- ↗ Analytic results for two-loop master integrals with a light-quark loop in the leading colour approximation [*Febres Cordero, Figueiredo, Kraus, Page, Reina '23*]
- ↗ Semi-numerical calculation of the  $gg \rightarrow ttH$  one-loop amplitude to to  $O(\epsilon^2)$  [*Buccioni, Kreer, Liu, Tancredi '23*]
- ↗ Two-loop amplitudes in the high-energy (boosted) limit,  $|s_{ij}| \gg m_t^2$  [*Wang, Xia, Yang,Ye'24*]
- ↗ Numerical results for the  $N_f$  part of the two-loop  $qq\bar{q} \rightarrow ttH$  virtual amplitude [*Agarwal, Heinrich, Jones, Kerner, Klein, Lang, Magerya, Olsson'24*]

# 2-LOOP FOR ttH FRONTIER

- ↗  $N_f$  part of the two-loop  $qq \rightarrow ttH$  virtual amplitude [Agarwal, Heinrich, Jones, Kerner, Klein, Lang, Magerya, Olsson'24]
- ↗ Numerical reduction to master integrals for individual phase-space points, as well as master integral evaluation
- ↗ Proof of concept for calculation of two-loop pentagon amplitudes with internal massive propagators and three massive particles in the final state

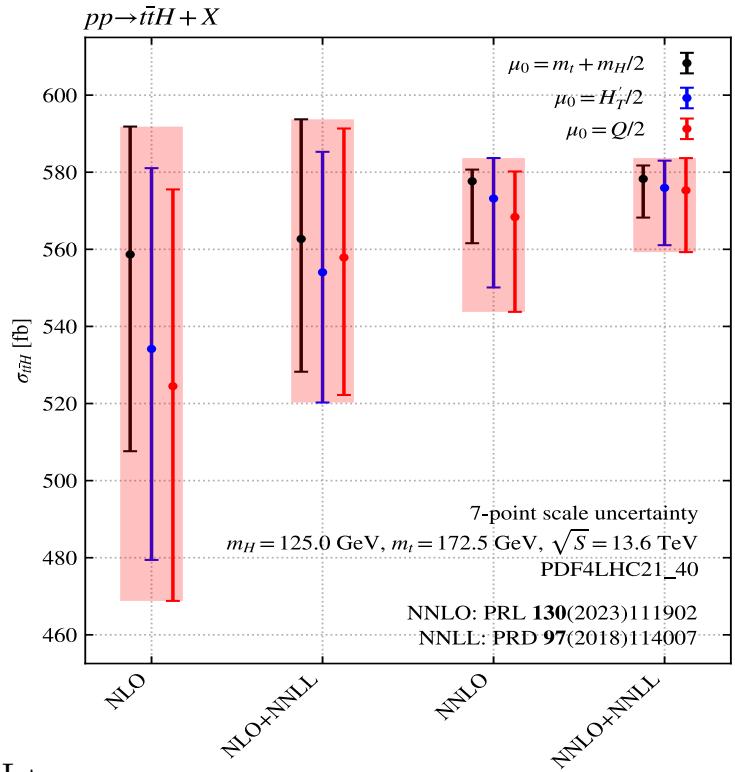


# TTH@NNLO+NNLL

[Balsach, AK, Motyka, Stebel]

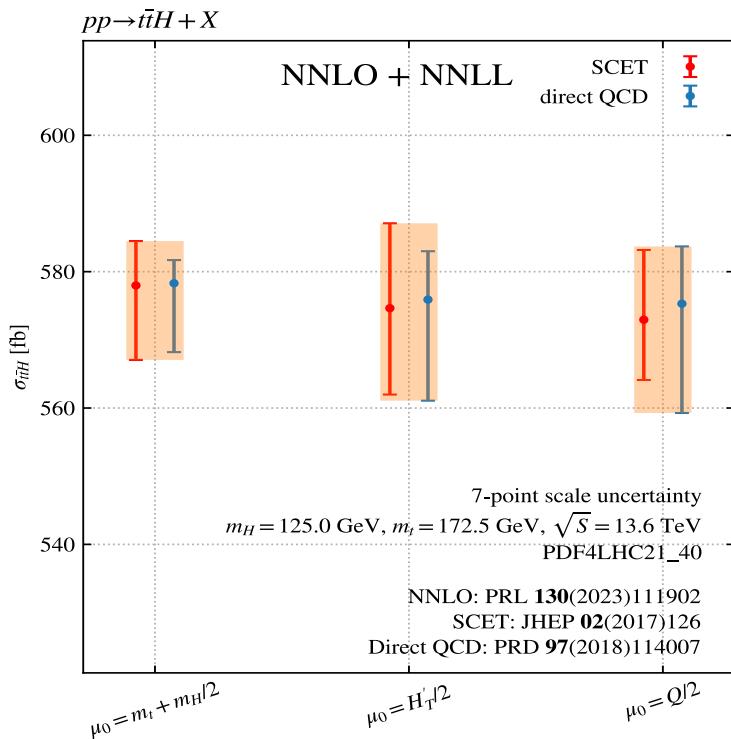
- The precision of the fixed-order predictions can be further improved by matching the NNLO cross section [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini'22] with NNLL soft gluon resummation [AK, Motyka, Stebel, Theeuwes'17]
- In addition to NNLO, logarithmic terms of the form  $\alpha_s^n \left( \frac{\log^m (1-\hat{\rho})}{1-\hat{\rho}} \right)_+$ ;  $\hat{\rho} = Q^2/\hat{s}$  are accounted for

$$d\sigma^{N(N)LO+NNLL} = d\sigma^{N(N)LO} + d\sigma^{NNLL} - d\sigma^{NNLL}|_{N(N)LO}$$



# TTH@NNLO+NNLL

- Comparison with the NNLL+NNLO result based on SCET [Broggio, Ferroglio, Pecjak, Yang'16] within the framework of the ttH LHCHWG subgroup

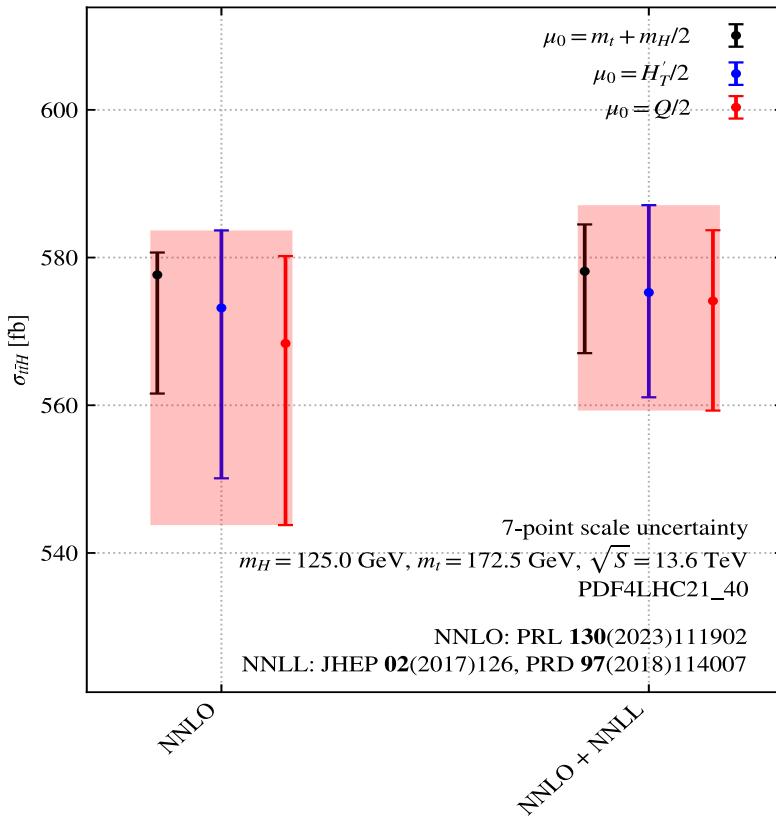


- Two **very different frameworks**: perturbative “full” theory (QCD) vs effective theory (SCET)
- Analytical formulas agree at NNLL
- Different subsets of subleading terms are included beyond NNLL  $\rightarrow$  small numerical differences
- Results for central scale choices agree within a few permille

[NNLL dQCD: Balsach, AK, Motyka, Stebel] [NNLL SCET: Broggio, Ferroglio, Pecjak] [NNLO: Devoto, Grazzini, Kallweit, Mazzitelli, Savoini]

# TTH@NNLO+NNLL

$pp \rightarrow t\bar{t}H + X$

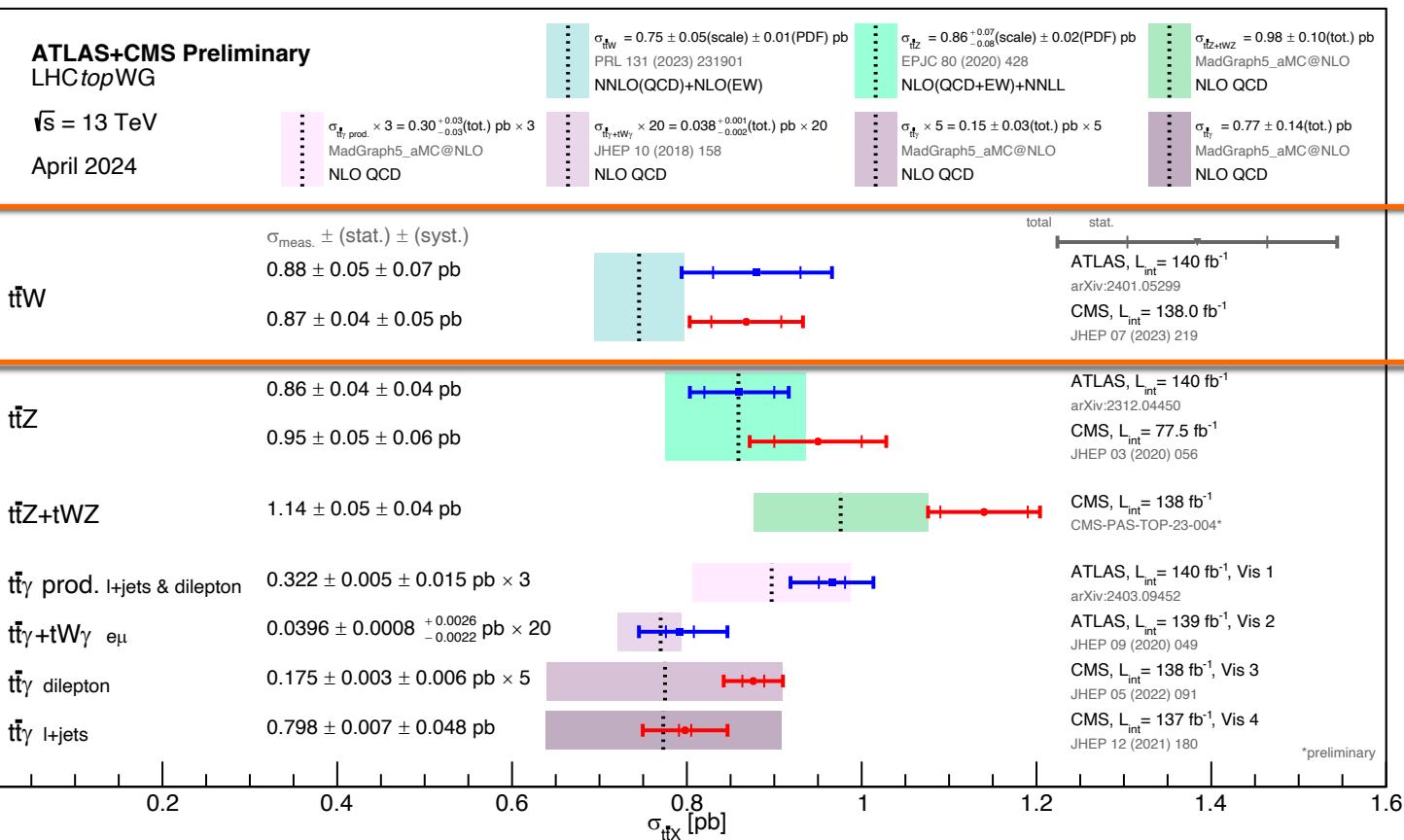


- ↗ Combination of the (NNLO+)NNLL results obtained in direct QCD and in SCET
  - ↗ Central value taken as average of the central values in the two approaches
  - ↗ Uncertainties determined from the envelope over the dQCD and SCET scale variation error bands
  - ↗ In this way, the uncertainties do not only account for **scale variation**, but also for  **$O(N^3 LL)$**  intrinsic differences between the two formalisms

[NNLL dQCD: Balsach, AK, Motyka, Stebel] [NNLL SCET: Broggio, Ferroglio, Pecjak] [NNLO: Devoto, Grazzini, Kallweit, Mazzitelli, Savoini]

**ATLAS+CMS Preliminary**  
LHC top WG

$\sqrt{s} = 13 \text{ TeV}$   
April 2024

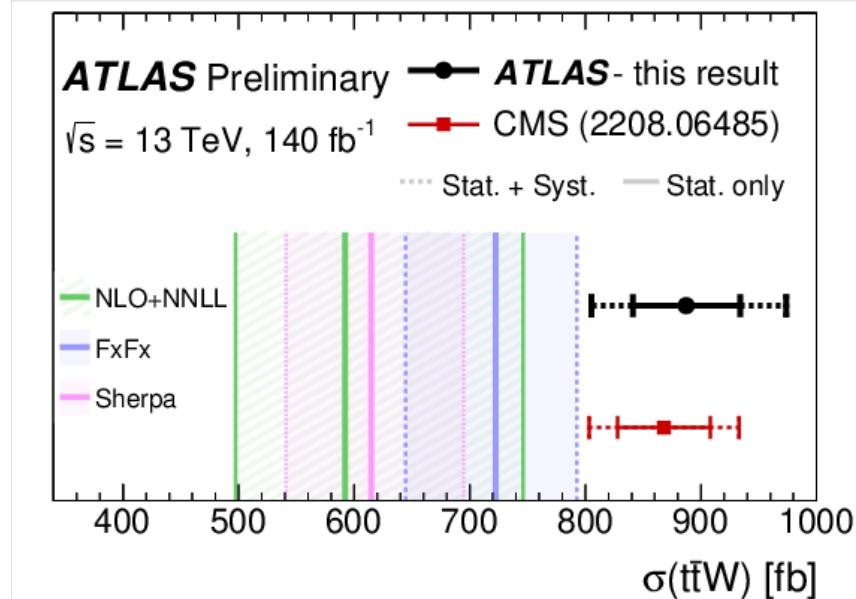


see also talks by

J. Jamieson, M. Schröder  
and M. Grazzini

- ↗ Probe of top-quark couplings to EW bosons
- ↗ Sensitive to BSM contributions (SUSY, BSM Higgs, vector-like quarks, heavy top quark partners, extra dimensions, ..)
- ↗ Dominant backgrounds to searches and SM precision measurements ( $t\bar{t}H$  included)
- ↗ Additional handle on the top charge asymmetry at the LHC [Maltoni, Mangano, Tsinikos, Zaro, '14]
- ↗ A lot of interest due to tension between theory and data, both in direct and indirect ( $t\bar{t}H$ , 4t) measurements

ATLAS-CONF-2023-019



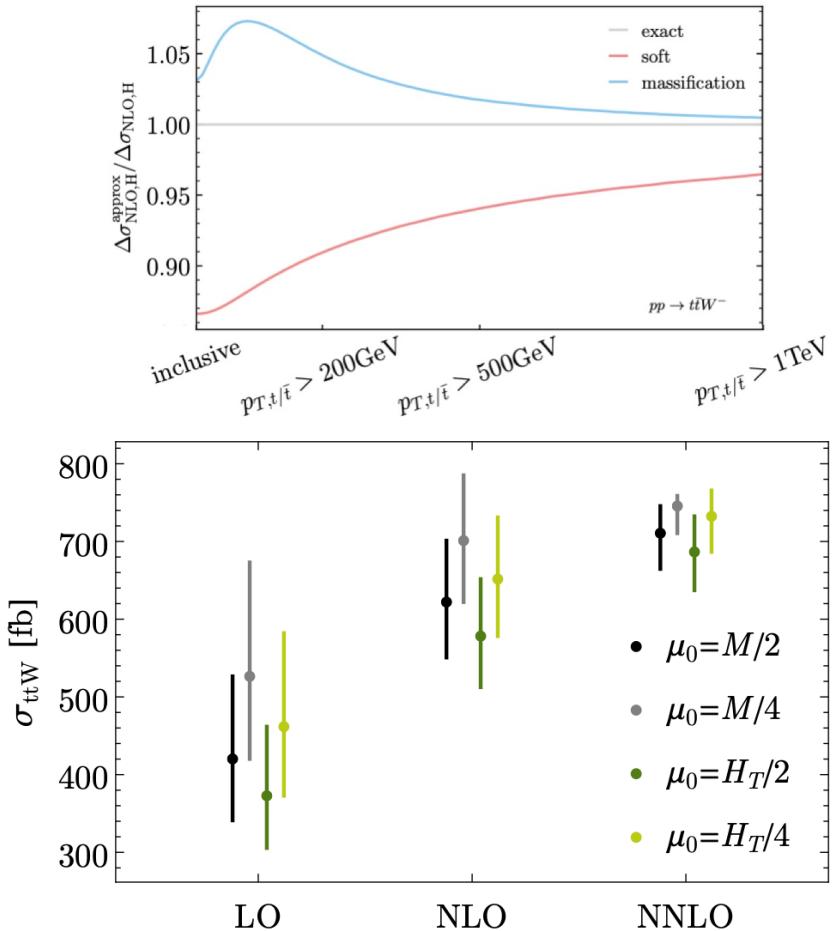
# THEORY FOR $t\bar{t}W$ OVER THE YEARS

- ↗ NLO QCD production and decay [Campbell, Ellis'12]
- ↗ NLO interfaced to parton showers [Garzelli, Kardos, Papadopoulos, Trocsanyi'12][Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattalaer, Shao Stelzer, Torrieli, Zaro,'14] [Maltoni, Mangano, Tsinikos, Zaro'14] [Maltoni, Pagani, Tsinikos'15]
- ↗ EW corrections [Frixione, Hirschi, Pagani, Shao, Zaro'14-15][Dror, Farina, Salvioni, Serra'16][Frederix, Pagani, Zaro'18], with matching to parton showers [Frederix, Tsinikos'20] [Febres Cordero, Kraus, Reina'21] and jet merging [von Buddenbrock, Ruiz, Mellado'20][Frederix, Tsinikos'21]
- ↗ NNLL+NLO resummation [Li, Li, Li'14], [Broggio, Ferroglia, Ossola, Pecjak'16] [AK, Motyka, Schwartländer, Stebel, Theeuwes'18] [Broggio, Ferroglia, Frederix, Pagani, Pecjak, Tsinikos'19] [AK, Motyka, Schwartländer, Stebel, Theeuwes'20]
- ↗ Off-shell effects at NLO QCD [Bevilacqua, Bi, Hartanto, Kraus, Worek'20][Bevilacqua, Bi, Hartanto, Kraus, Nasufi, Worek'21] [Denner, Pelliccioli'20] [Bevilacqua, Bi, Febres Cordero, Hartanto, Kraus, Nasufi, Reina, Worek'22] and together with NLO EW [Denner, Pelliccioli'21]

# TTW@NNLO

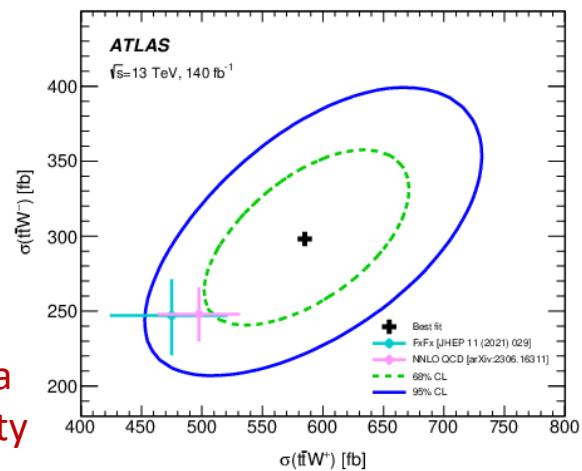
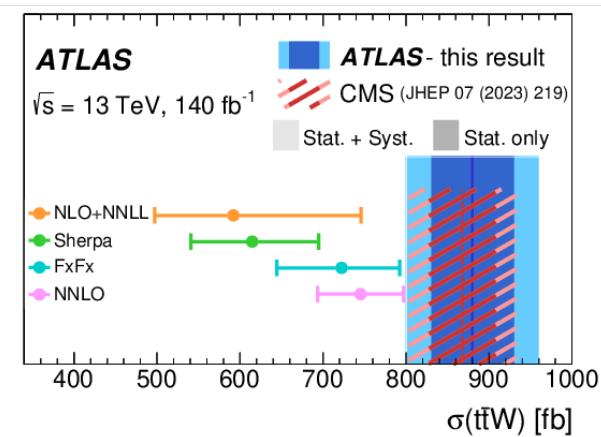
[Buonocore, Devoto, Grazzini, Kallweit, Mazzitelli, Rottoli, Savoini'23]

- ↗ Full computation in the  $q_T$  subtraction framework, apart from the two-loop amplitude
- ↗ Two approximations used to estimate the two-loop contributions:
  - ↗ soft W approximation ( $p_W$  small,  $m_W \ll m_t$ ), two-loop ttbar as input
  - ↗ massification procedure ( $m_t \ll Q_{\text{ttW}}$ ), two-loop W+4 parton as input
- ↗ Two-loop contributions provide 6-7% of the NNLO cross section, translating into  $O(+/- 2\%)$  systematic error on NNLO
- ↗ 15% correction from NNLO QCD
- ↗ Additional 5% correction from NLO EW



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Theory-data compatibility  
at  $1.4\sigma$

# TOP DECAYS IN $t\bar{t}W$

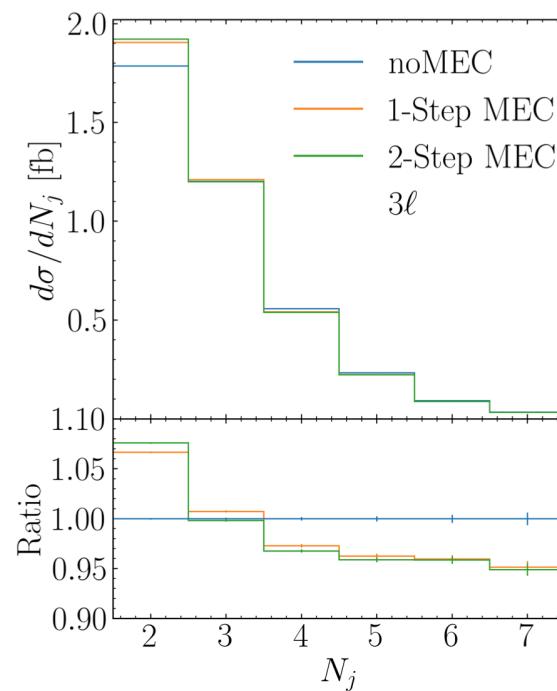
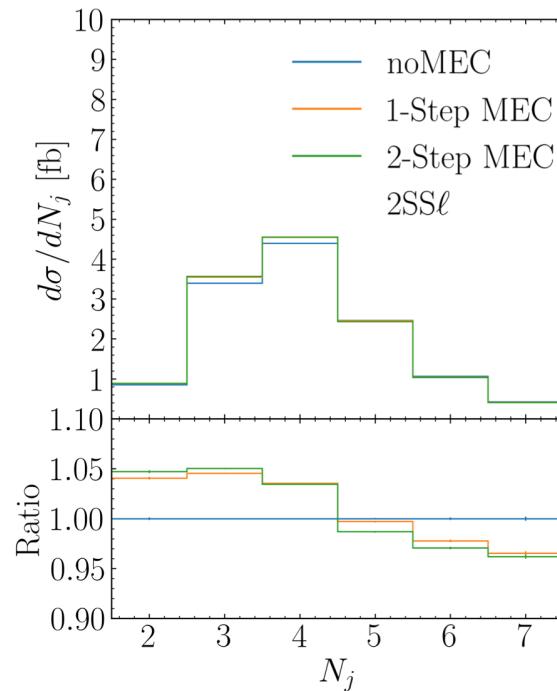
[Frederix, Gellersen, Nasufi'24]

- Resolving incompatibility between matrix element corrections (MEC) in PYTHIA8 and aMC@NLO-style matching enables to account, through MEC, for decay of tops at NLO in the PYTHIA shower

$$\begin{array}{ll} \text{no MEC} & \sigma_{2SS\ell}^{t\bar{t}W^\pm} = 12.84 \text{ fb} \\ \text{with MEC} & \sigma_{2SS\ell}^{t\bar{t}W^\pm} = 13.12 \text{ fb} \end{array}$$

$$\begin{array}{l} \sigma_{3\ell}^{t\bar{t}W^\pm} = 3.92 \text{ fb} \\ \sigma_{3\ell}^{t\bar{t}W^\pm} = 4.02 \text{ fb} \end{array}$$

2-3% correction to  
the integrated  
fiducial cross  
section



**ATLAS+CMS Preliminary**  
LHC topWG

$\sqrt{s} = 13 \text{ TeV}$   
April 2024

$t\bar{t}W$

$$\sigma_{\text{meas.}} \pm (\text{stat.}) \pm (\text{syst.}) \\ 0.88 \pm 0.05 \pm 0.07 \text{ pb} \\ 0.87 \pm 0.04 \pm 0.05 \text{ pb}$$

$$\sigma_{t\bar{t}W} = 0.75 \pm 0.05(\text{scale}) \pm 0.01(\text{PDF}) \text{ pb} \\ \text{PRL 131 (2023) 231901} \\ \text{NNLO(QCD)+NLO(EW)}$$

$$\sigma_{t\bar{t}\gamma} = 0.86^{+0.07}_{-0.08}(\text{scale}) \pm 0.02(\text{PDF}) \text{ pb} \\ \text{EPJC 80 (2020) 428} \\ \text{NLO(QCD+EW)+NNLL}$$

$$\sigma_{t\bar{t}Z+WZ} = 0.98 \pm 0.10(\text{tot.}) \text{ pb} \\ \text{MadGraph5\_aMC@NLO} \\ \text{NLO QCD}$$

$$\sigma_{t\bar{t}+\text{W}_Y} \times 20 = 0.038^{+0.001}_{-0.002}(\text{tot.}) \text{ pb} \times 20 \\ \text{JHEP 10 (2018) 158} \\ \text{NLO QCD}$$

$$\sigma_{t\bar{t}\gamma} \times 5 = 0.15 \pm 0.03(\text{tot.}) \text{ pb} \times 5 \\ \text{MadGraph5\_aMC@NLO} \\ \text{NLO QCD}$$

$$\sigma_{t\bar{t}\gamma} = 0.77 \pm 0.14(\text{tot.}) \text{ pb} \\ \text{MadGraph5\_aMC@NLO} \\ \text{NLO QCD}$$

$t\bar{t}Z$

$$0.86 \pm 0.04 \pm 0.04 \text{ pb} \\ 0.95 \pm 0.05 \pm 0.06 \text{ pb}$$

$$\text{total stat.} \\ \text{ATLAS, } L_{\text{int}} = 140 \text{ fb}^{-1} \\ \text{arXiv:2401.05299} \\ \text{CMS, } L_{\text{int}} = 138.0 \text{ fb}^{-1} \\ \text{JHEP 07 (2023) 219}$$

$$\text{ATLAS, } L_{\text{int}} = 140 \text{ fb}^{-1} \\ \text{arXiv:2312.04450} \\ \text{CMS, } L_{\text{int}} = 77.5 \text{ fb}^{-1} \\ \text{JHEP 03 (2020) 056}$$

$t\bar{t}Z+tWZ$

$$1.14 \pm 0.05 \pm 0.04 \text{ pb}$$



$$\text{CMS, } L_{\text{int}} = 138 \text{ fb}^{-1} \\ \text{CMS-PAS-TOP-23-004*}$$

$t\bar{t}\gamma$  prod. I+jets & dilepton

$$0.322 \pm 0.005 \pm 0.015 \text{ pb} \times 3$$

$$\text{ATLAS, } L_{\text{int}} = 140 \text{ fb}^{-1}, \text{ Vis 1} \\ \text{arXiv:2403.09452}$$

$t\bar{t}\gamma + tW_Y$  e $\mu$

$$0.0396 \pm 0.0008^{+0.0026}_{-0.0022} \text{ pb} \times 20$$

$$\text{ATLAS, } L_{\text{int}} = 139 \text{ fb}^{-1}, \text{ Vis 2} \\ \text{JHEP 09 (2020) 049}$$

$t\bar{t}\gamma$  dilepton

$$0.175 \pm 0.003 \pm 0.006 \text{ pb} \times 5$$

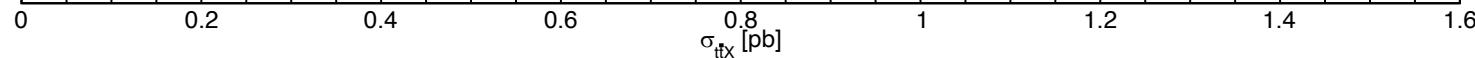
$$\text{CMS, } L_{\text{int}} = 138 \text{ fb}^{-1}, \text{ Vis 3} \\ \text{JHEP 05 (2022) 091}$$

$t\bar{t}\gamma$  I+jets

$$0.798 \pm 0.007 \pm 0.048 \text{ pb}$$

$$\text{CMS, } L_{\text{int}} = 137 \text{ fb}^{-1}, \text{ Vis 4} \\ \text{JHEP 12 (2021) 180}$$

\*preliminary

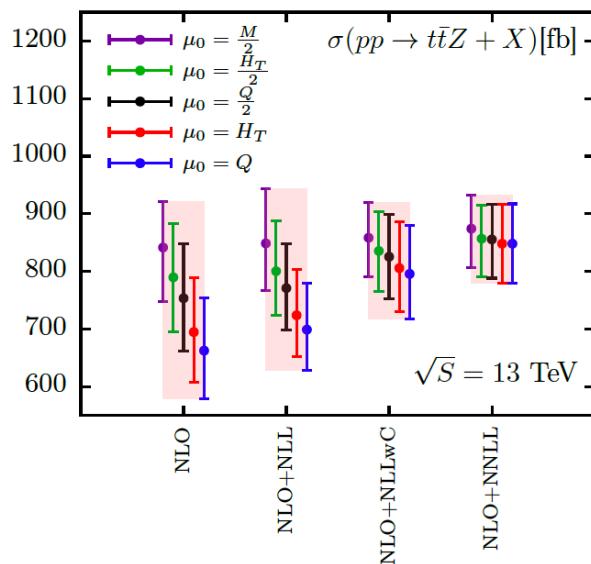


- ↗ NLO QCD [*Lazopoulos, Melnikov, Petriello'07*] [*Lazopoulos, McElmurry, Melnikov, Petriello'08*] [*Kardos, Trocsanyi, Papadopoulos'12*] with decays at NLO [*Roentsch, Schulze'14-'15*]
- ↗ NLO interfaced to parton showers [*Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattalaer, Shao Stelzer, Torrieli, Zaro,'14*] [*Maltoni, Pagani, Tsinikos'15*] [*Garzelli, Kardos, Papadopoulos, Trocsanyi'11-12*][*Ghezzi, Jaeger, Chavez, Reina, Wackerloth'15*]
- ↗ EW corrections [*Frixione, Hirschi, Pagani, Shao, Zaro'15*][*Frederix, Frixione, Hirschi, Pagani, Shao, Zaro'18*]
- ↗ NNLL resummation [*AK, Motyka, Schwartländer, Stebel, Theeuwes'18-'20* ][*Broggio, Ferroglio, Ossola, Pecjak, Samoshima'17*]
- ↗ Off-shell effects at NLO QCD [*Bevilacqua, Hartanto, Kraus, Weber, Worek'19*][*Bevilacqua, Bi, Hartanto, Kraus, Nasufi, Worek'22*] and NLO EW [*Denner, Lombardi, Pelliccioli'23*]

# TTZ ON SHELL

[AK, Motyka, Schwartländer, Stebel, Theeuwes'18-'20]

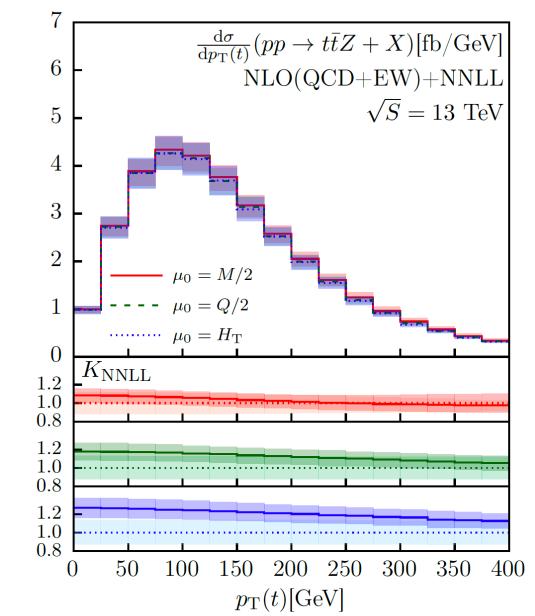
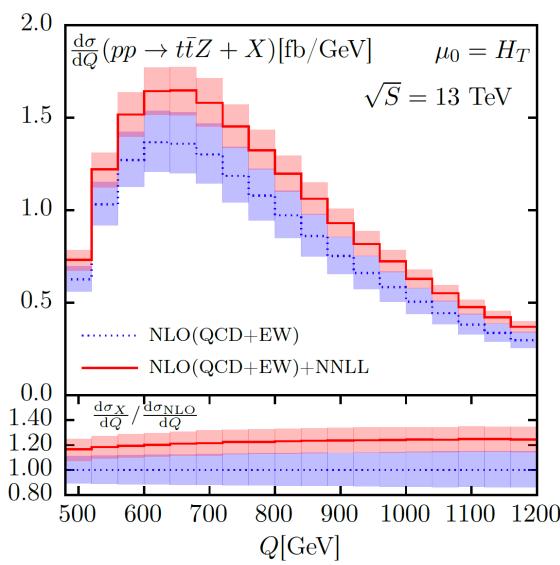
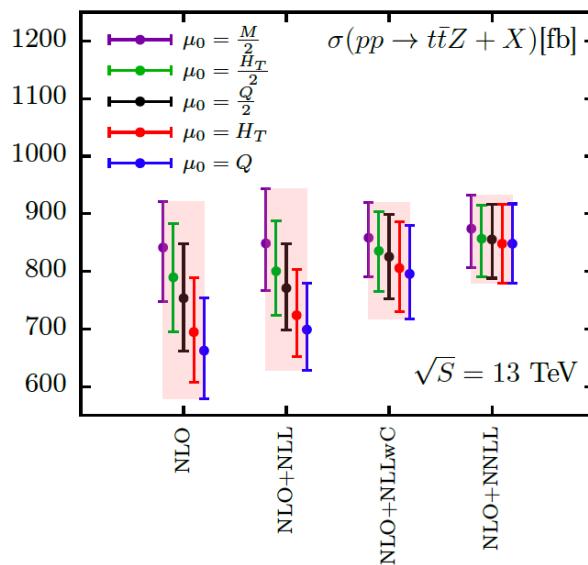
## ↗ NLO(QCD+EW) +NNLL resummation



# TTZ ON SHELL

[AK, Motyka, Schwartländer, Stebel, Theeuwes'20]

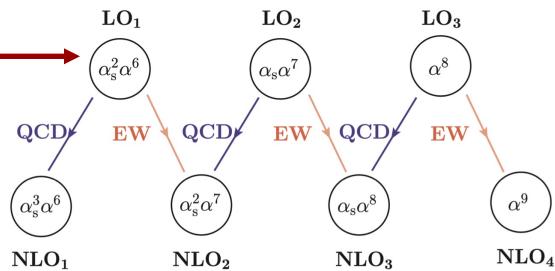
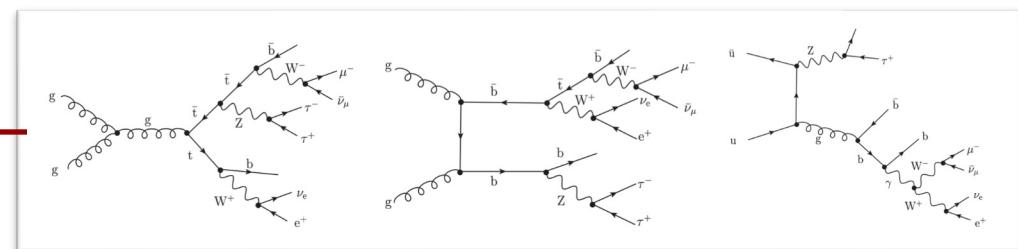
- ↗ NLO(QCD+EW) +NNLL resummation, also for differential distributions



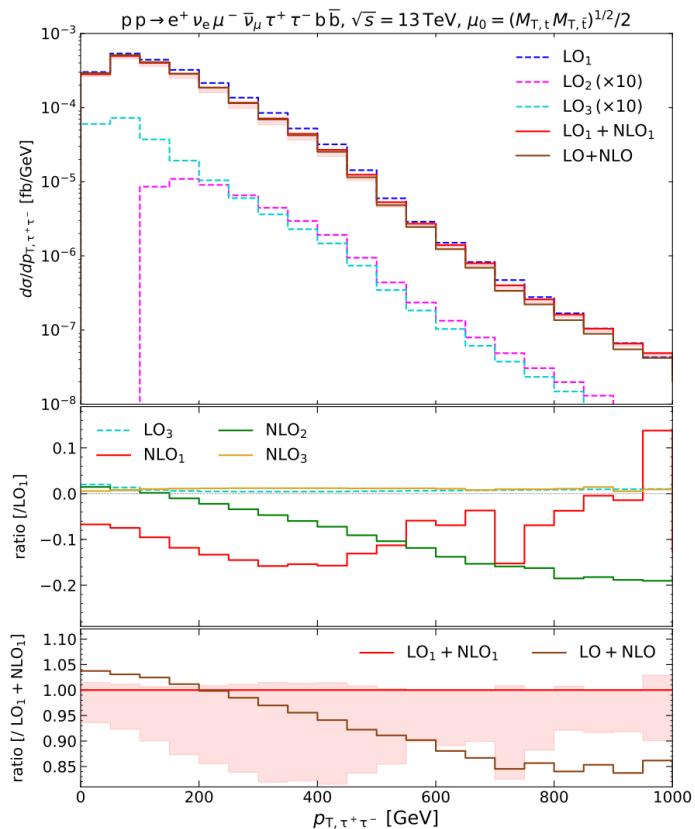
# TTZ OFF-SHELL

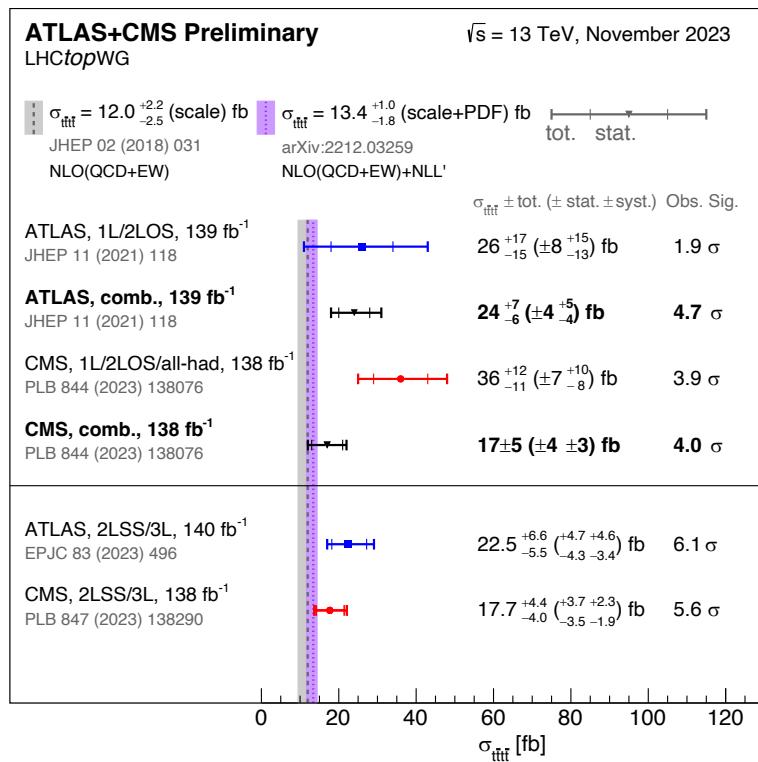
[Denner, Lombardi, Pelliccioli'23]

- First calculation of the off-shell production of a top–antitop pair in association with a Z boson in the multilepton decay channel accurate both at NLO QCD and NLO EW (full matrix elements)



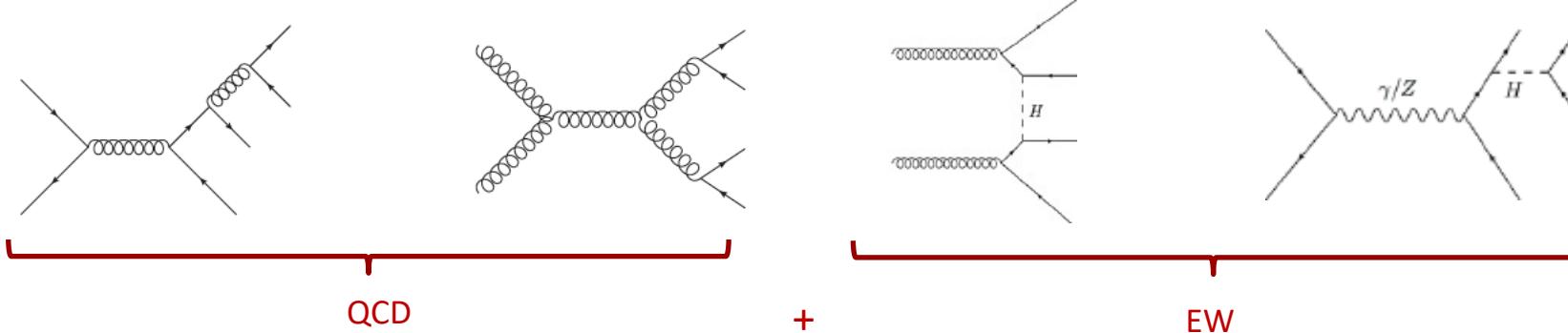
- At the inclusive level, subleading LO and NLO corrections amount to less than 1%





see also talk by  
Tae Jeong Kim

# 4 TOPS THEORY



- ↗ First calculations of NLO QCD corrections in [*Bevilacqua, Worek'12*]
- ↗ Matched with parton shower and studied in aMC@NLO [*Alwall et al. '14*][*Maltoni, Pagani, Tsinikos'15*]
- ↗ Full set of EW corrections added in [*Frederix, Pagani, Zaro'17*]
- ↗ Spin correlations in LO top quark decays within the framework of Powheg Box [*Jezo, Krauss'21*]

ATLAS, 2303.15061:

$$\sigma_{t\bar{t}t\bar{t}} = 22.5^{+6.6}_{-5.5} \text{ fb}$$

CMS , 2305.13439:

$$\sigma(t\bar{t}t\bar{t}) = 17.7^{+4.4}_{-4.0} \text{ fb}$$

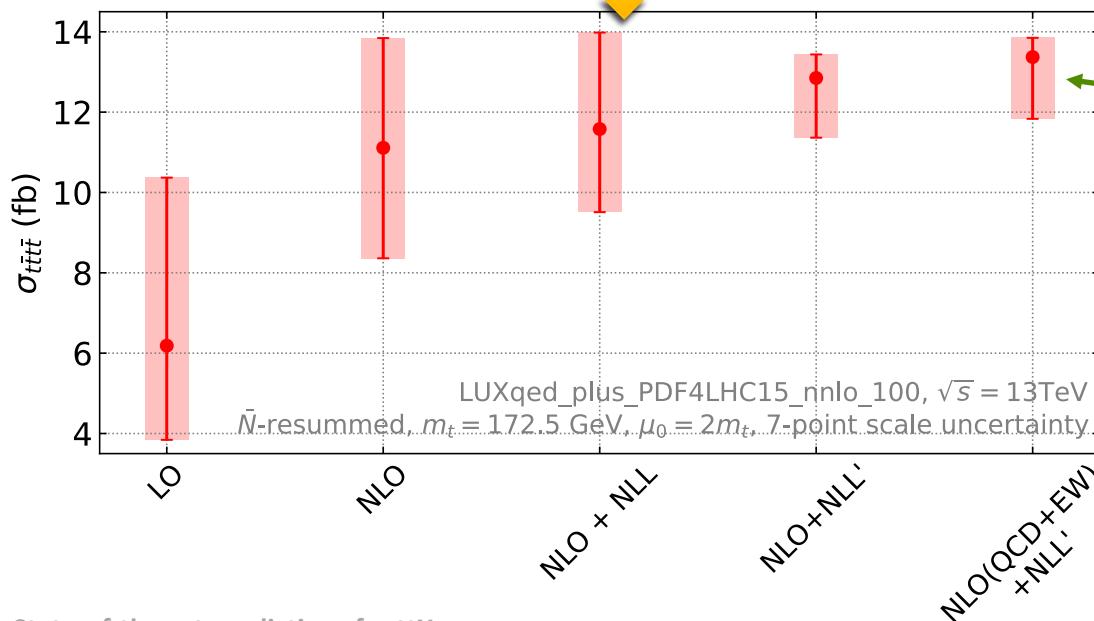
- On-shell production:

$\sqrt{s}$ (TeV)	NLO
13	11.00(2) <sup>+25.2%</sup> <sub>-24.5%</sub> fb
$\sqrt{s}$ (TeV)	NLO(QCD+EW)
13	11.64(2) <sup>+23.2%</sup> <sub>-22.8%</sub> fb

*[van Beekveld, AK, Moreno Valero'22]*

↗ On-shell production:

$\sqrt{s}$ (TeV)	NLO	NLO+NLL	NLO+NLL'	$K_{NLL'}$
13	$11.00(2)^{+25.2\%}_{-24.5\%}$ fb	$11.46(2)^{+21.3\%}_{-17.7\%}$ fb	$12.73(2)^{+4.1\%}_{-11.8\%}$ fb	1.16
$\sqrt{s}$ (TeV)	NLO(QCD+EW)	NLO(QCD+EW)+NLL	NLO(QCD+EW)+NLL'	$K_{NLL'}$
13	$11.64(2)^{+23.2\%}_{-22.8\%}$ fb	$12.10(2)^{+19.5\%}_{-16.3\%}$ fb	$13.37(2)^{+3.6\%}_{-11.4\%}$ fb	1.15



QCD only

↗ Reduction of the scale error by more than a factor of 2

↗ 15 % correction to the NLO (QCD+EW) prediction due to NLL' resummation

# TOP DECAY@NLO; 4 TOPS

[Dimitrakopoulos, Worek'24]

NLO QCD corrections to 4t production and decay in the 4 lepton channel in the NWA

$$pp \rightarrow t\bar{t}t\bar{t} + X \rightarrow W^+W^-W^+W^-b\bar{b}b\bar{b} + X \rightarrow \ell^+\nu_\ell \ell^-\bar{\nu}_\ell \ell^+\nu_\ell \ell^-\bar{\nu}_\ell b\bar{b}b\bar{b} + X$$

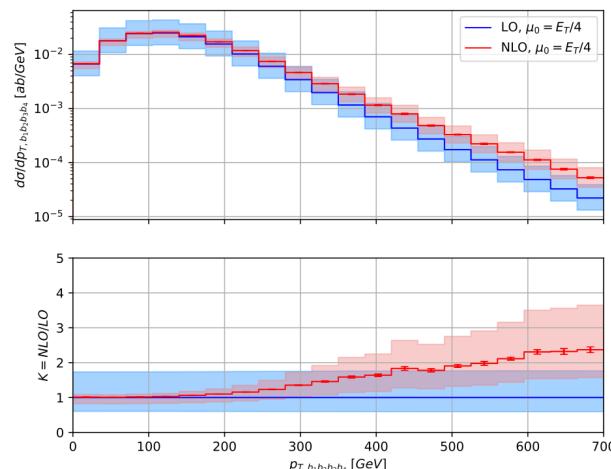
↗ Integrated fiducial cross sections

Decay treatment	$\sigma_i^{\text{NLO}}$ [ab]	$+\delta_{\text{scale}}$ [ab]	$-\delta_{\text{scale}}$ [ab]
$\mu_R = \mu_F = \mu_0 = 2m_t$			
full	5.462(3)	+0.156 (3%)	-0.853 (16%)
LO <sub>dec</sub>	5.295(3)	+1.123 (21%)	-1.224 (23%)
exp	4.895(2)	+0.624 (13%)	-1.002 (20%)

↗ NLO corrections to top decay impact both the size of the cross section and their scale uncertainties

↗ Differentially, for the majority of observables, stable and moderate corrections, similar to the integrated ones

↗ Apart from observables sensitive to additional jet radiation



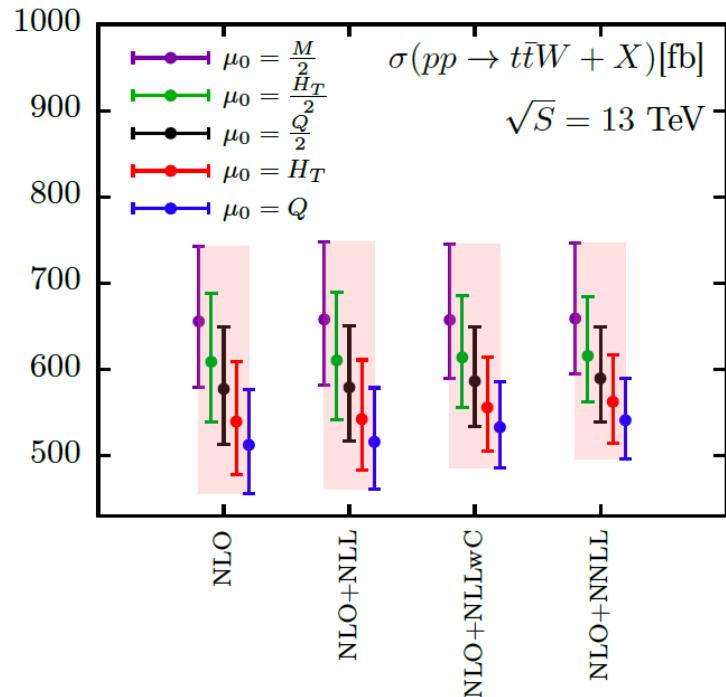
# SUMMARY

- ↗ Ongoing progress on increasing accuracy of the fixed-order predictions to NNLO -> two-loop virtual contributions; proof-of-concept calculations emerging
- ↗ ttH, ttW cross-sections (amplitudes) for on-shell production available at NNLO in the soft Higgs/soft W+massification approximations
- ↗ ttH: further improvement of theoretical precision achieved with NNLL resummation, very good agreement at NNLO+NNLL between two independent calculations; NNLL (+NLO) results for ttZ also available
- ↗ ttH, ttW, ttZ: off-shell effects calculated at NLO, including EW effects

# BACKUP

# NNLL FOR $t\bar{t}W$

[AK, Motyka, Schwartländer, Stebel, Theeuwes'18-20']



↗ NNLL resummation provides only a moderate correction: only  $q\bar{q}$  initial channel at LO

	$\mu_0$	NLO[fb]	NLO+NNLL[fb]	$K_{\text{NNLL}}$
$t\bar{t}W$	$Q$	$512^{+12.5\%}_{-11.1\%}$	$541^{+8.9\%}_{-8.4\%}$	1.06
	$H_T$	$539^{+13.0\%}_{-11.3\%}$	$562^{+9.6\%}_{-8.5\%}$	1.04
	$Q/2$	$577^{+12.5\%}_{-11.1\%}$	$590^{+10.0\%}_{-8.5\%}$	1.02
	$H_T/2$	$609^{+13.0\%}_{-11.5\%}$	$616^{+11.2\%}_{-8.8\%}$	1.01
	$M/2$	$656^{+13.2\%}_{-11.7\%}$	$659^{+13.3\%}_{-9.8\%}$	1.00