Status of $|V_{cb}|$ determinations

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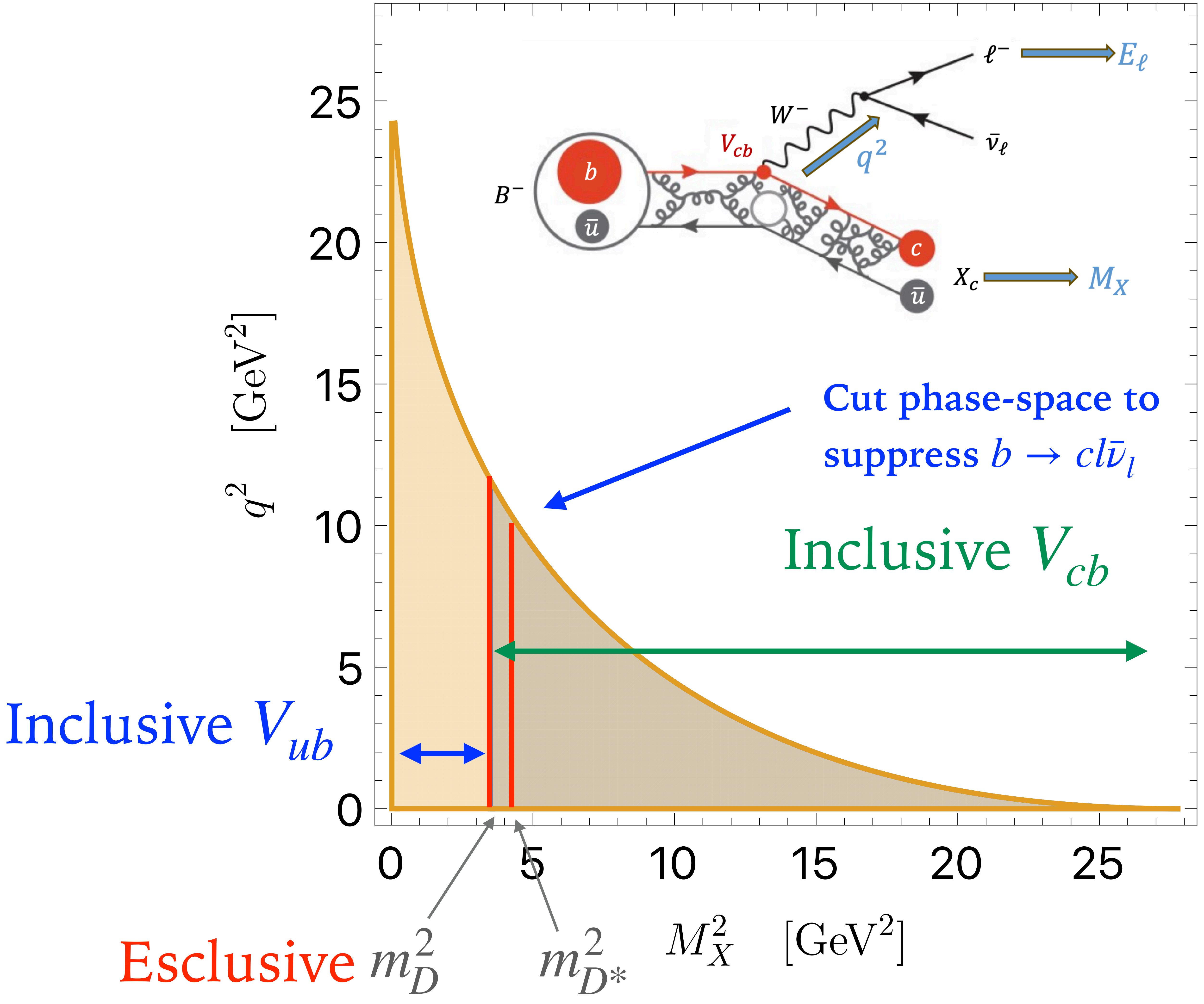
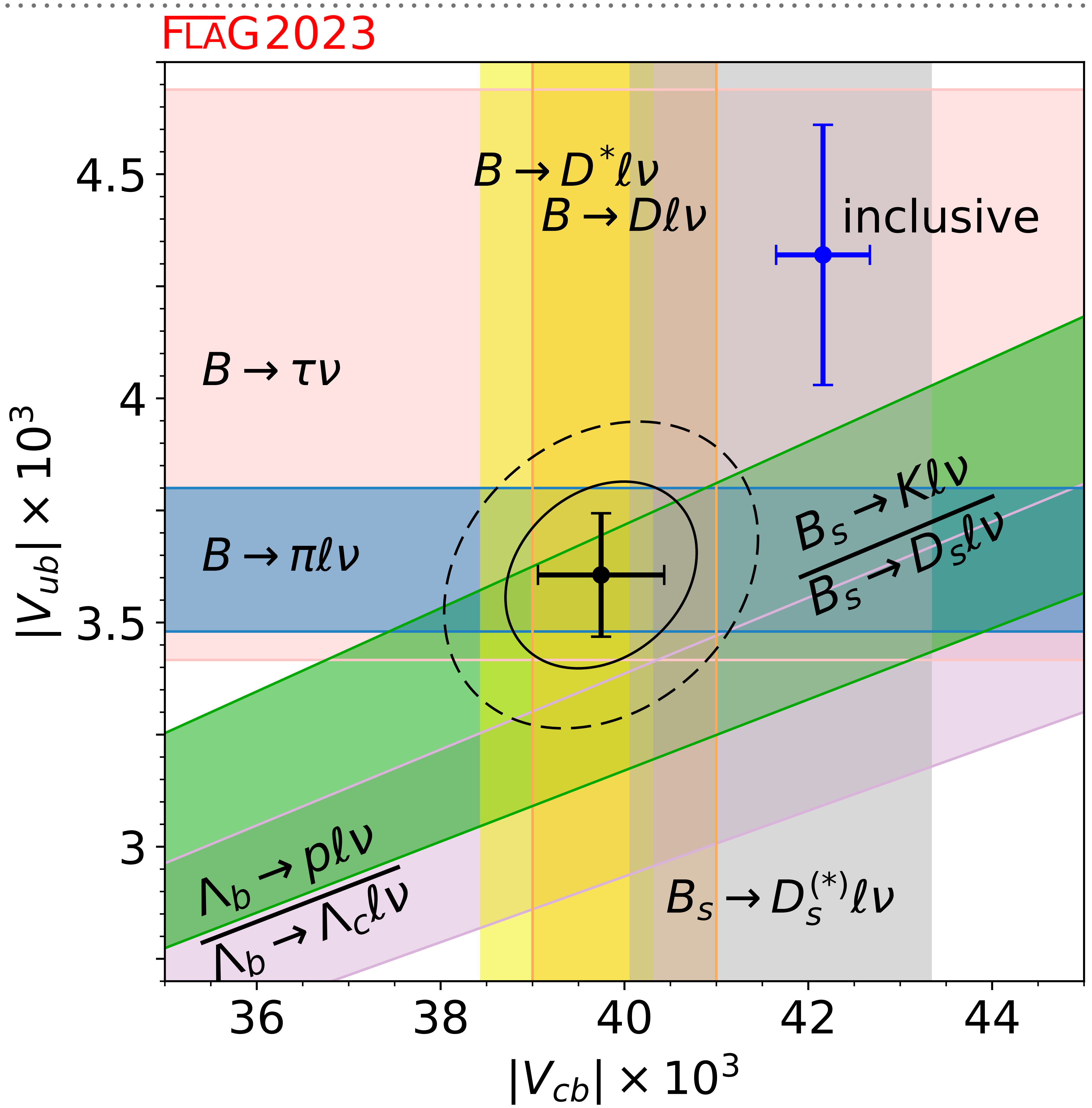
SM@LHC - Roma - 10 Maggio 2024

MARIE CURIE ACTIONS

Funded by
the European Union

V_{ub} AND V_{cb} EXTRACTION



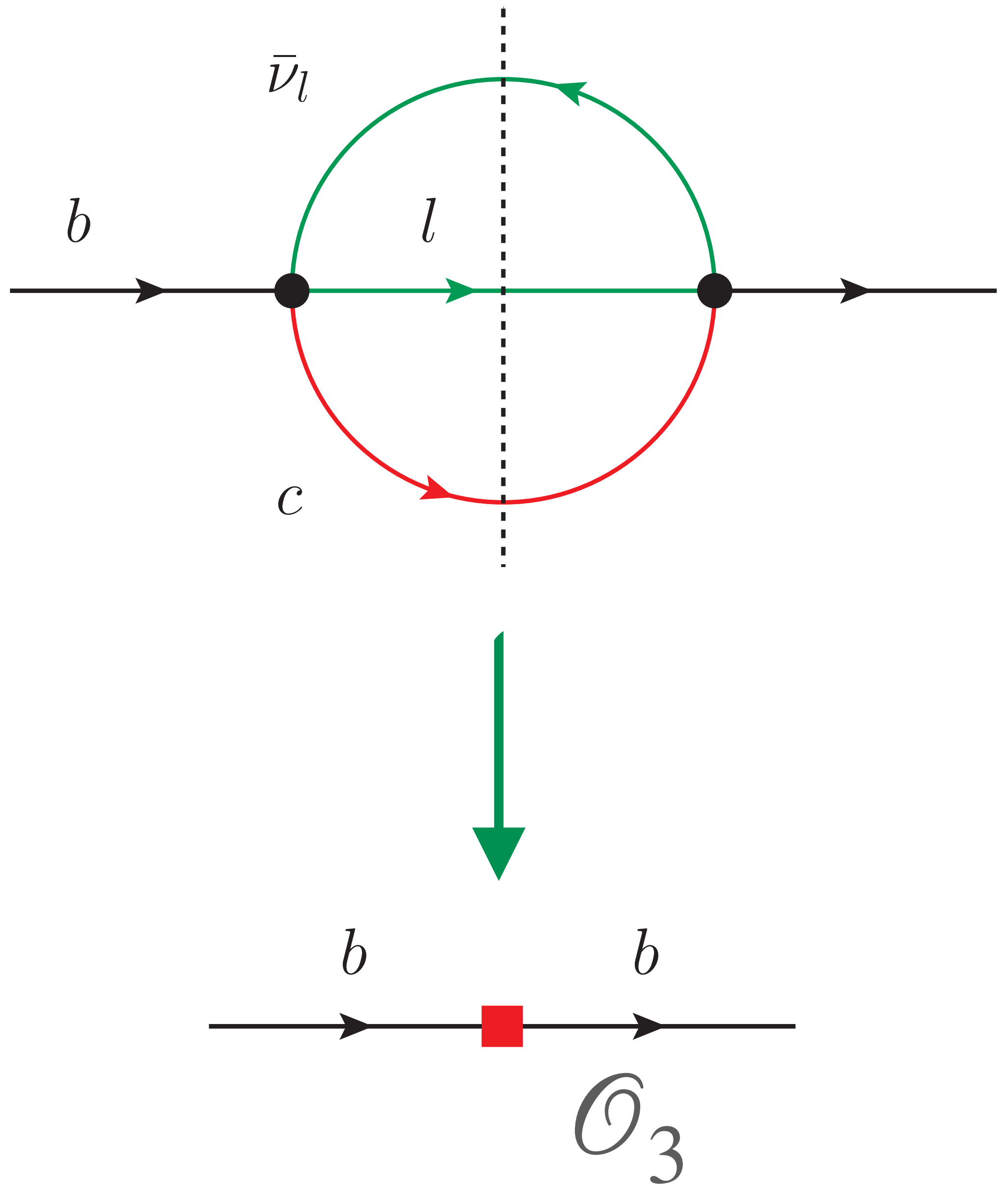


THE HEAVY QUARK EXPANSION

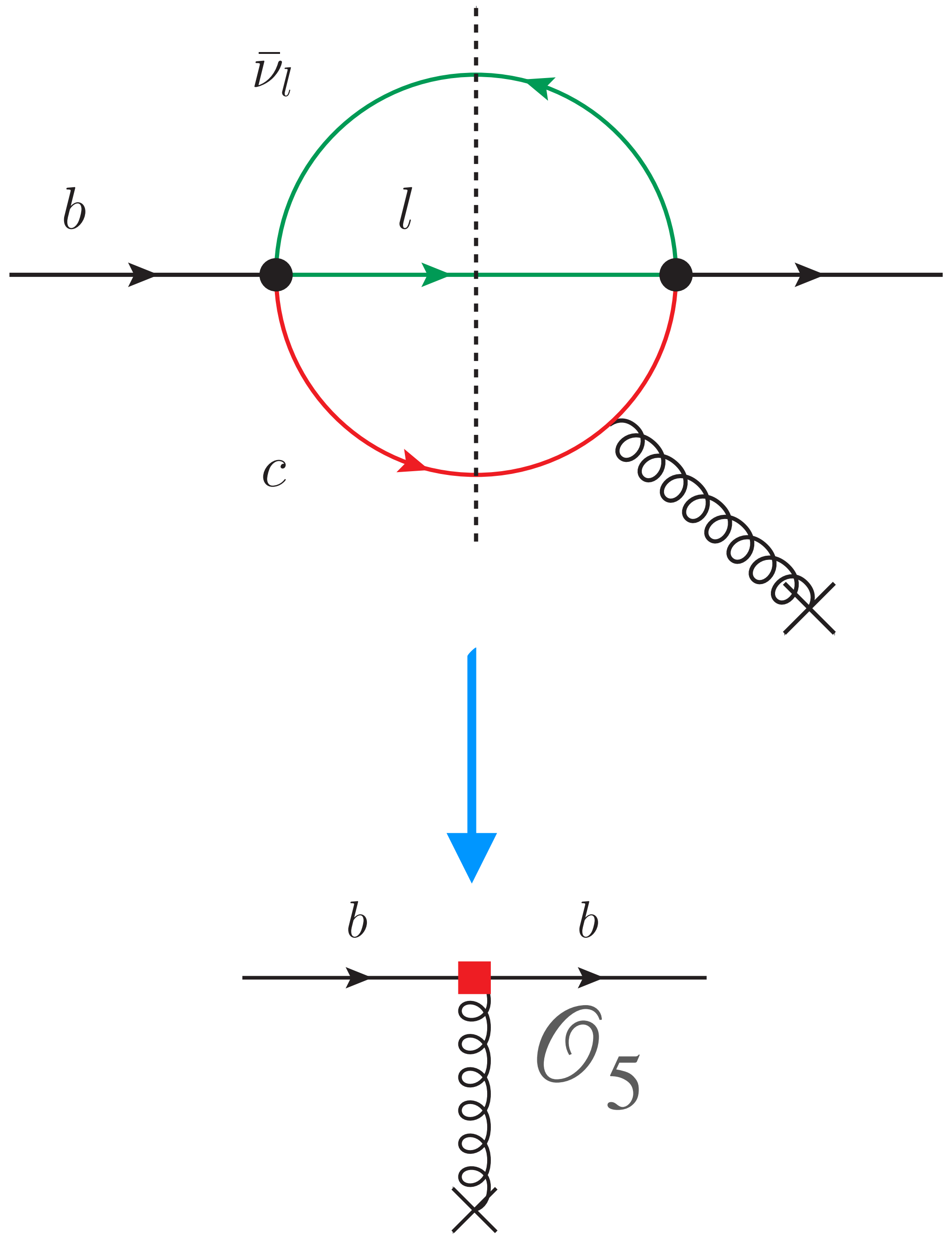
Perturbative QCD!

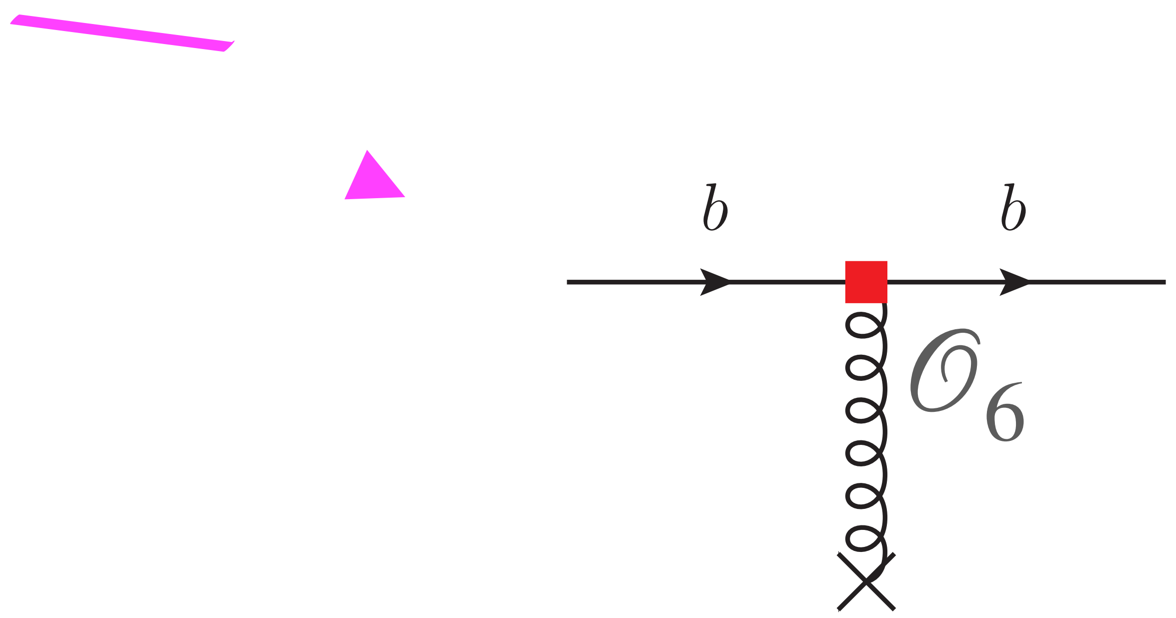
$$\Gamma_3 = \Gamma_3^{(0)} + \alpha_s \Gamma_3^{(1)} + \alpha_s^2 \Gamma_3^{(2)} + \alpha_s^3 \Gamma_3^{(3)}$$

$$\boxed{\Gamma = \Gamma_3 + \Gamma_5 \frac{\langle B | \mathcal{O}_5 | B \rangle}{m_b^2} + \Gamma_6 \frac{\langle B | \mathcal{O}_6 | B \rangle}{m_b^3} + \ldots}$$



Free quark decay



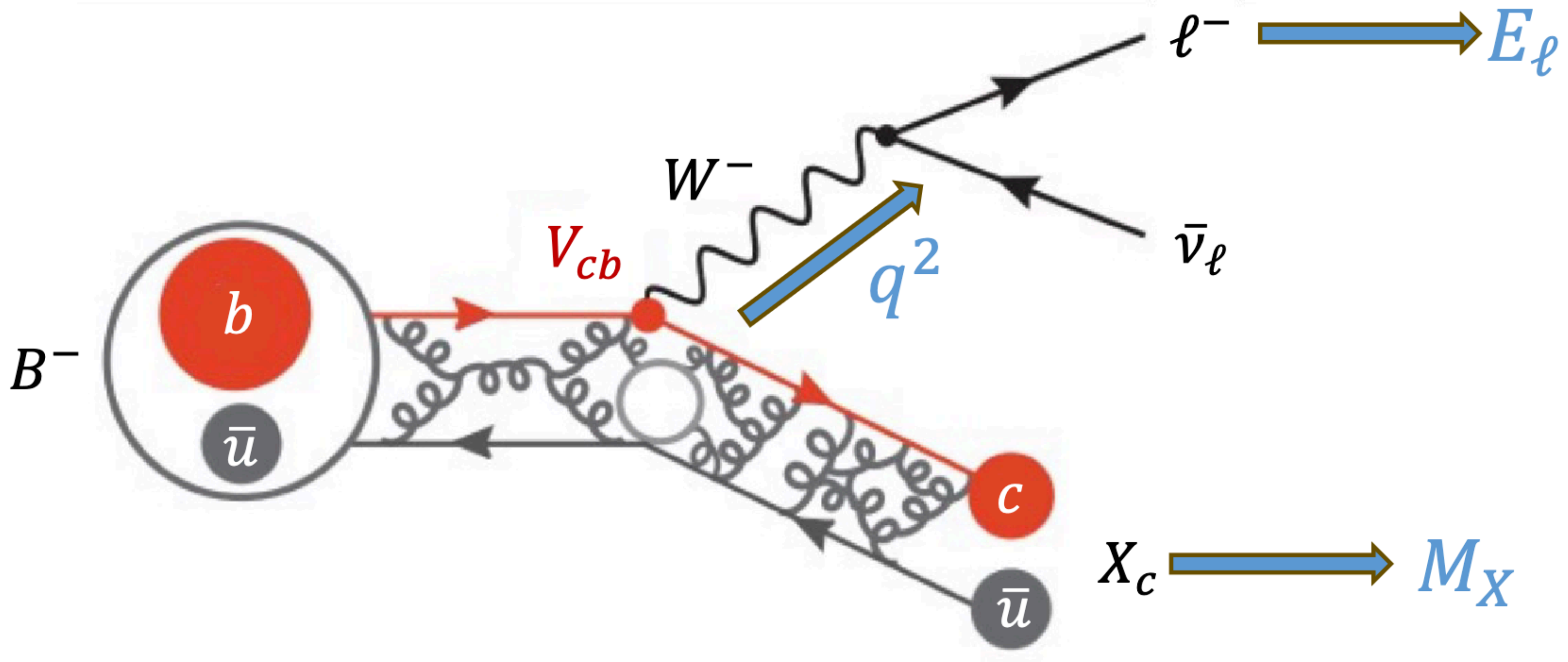


Darwin term ρ_D^3

Spin-Orbit term ρ_{LS}^3

Kinetic term μ_π^2, chromomagnetic term μ_G^2

SPECTRAL MOMENTS



$$\langle O^n \rangle_{\text{cut}} = \int_{\text{cut}} (O)^n \frac{d\Gamma}{d\Phi} d\Phi \Big/ \int_{\text{cut}} \frac{d\Gamma}{d\Phi} d\Phi$$

Cut: moments are measured with progressive cuts in E_l or q^2

$$O = (p_l + p_\nu)^2 = q^2$$ leptonic invariant mass

$$O = (p_B - q)^2 = M_X^2$$ hadronic invariant mass

$$O = v_B \cdot p_l = E_l$$ lepton energy

$\langle E_l \rangle, \langle M_X^2 \rangle, \langle q^2 \rangle$

↓

$\mu_\pi, \mu_G, \rho_D, \rho_{LS}, \ldots$

↓

$$\Gamma = \Gamma_3 + \Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \ldots \rightarrow |V_{cb}|$$

Q2 MOMENTS

$$\frac{\partial O}{\partial v_B} = 0$$

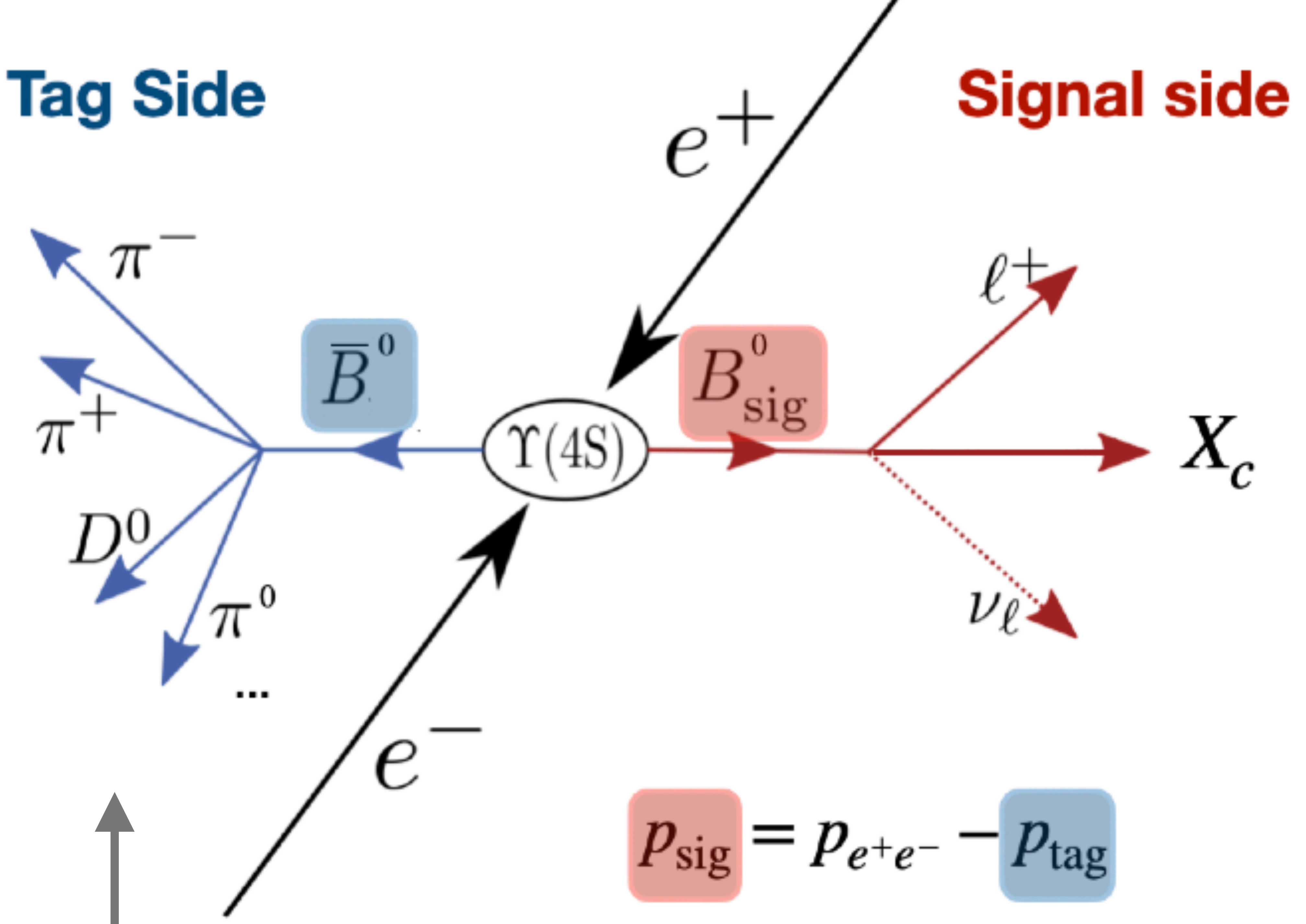
$$O = (p_l + p_\nu)^2 = q^2$$

$$O = (m_B v_B - q)^2 = M_X^2$$

$$O = v_B \cdot p_l = E_l$$

- $\Gamma_{\rm sl}$ and $\langle q^{2n} \rangle$ are invariant under reparametrization
- HQE parameters: 8 instead of 13 up to $1/m_b^4$
- NEW METHOD: extract $|V_{cb}|$ from q^2 moments

MF, Mannel, Vos, JHEP 02 (2019) 177



$$p_{\rm sig} = p_{e^+e^-} - p_{\rm tag}$$

Hadronic Tagging with Belle II algorithm

Keck et al, Comput.Softw. Big Sci. 3 (2019) 1, 6

First new data since 2010!

New: Inclusive semileptonic $b \to c l \bar{\nu}_l$ decays to order $1/m_b^5$

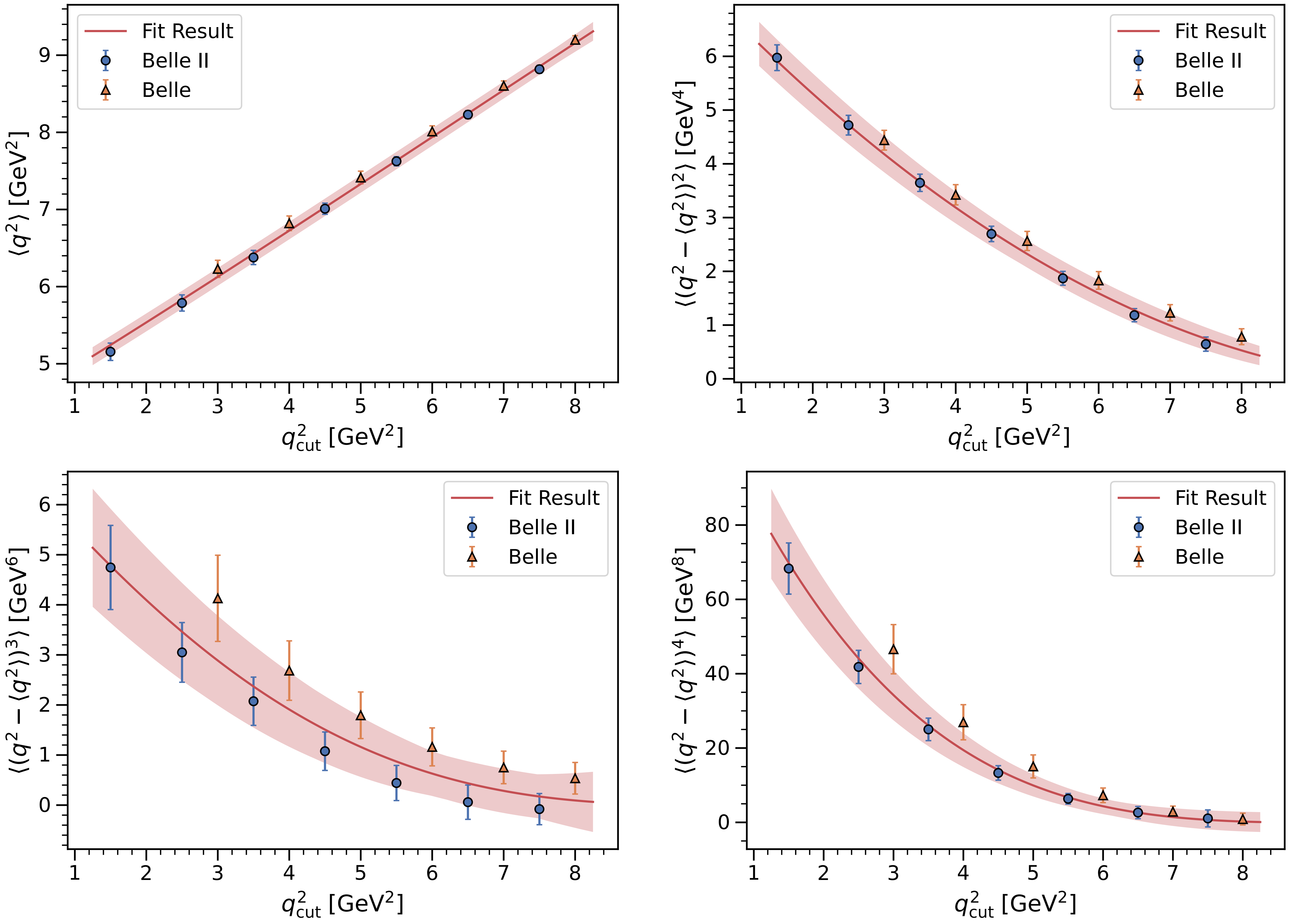
Mannel, Milutin, Vos, hep-ph/2311.12002

- 10 instead of 19 HQE parameters

Measurements of q^2 moments of inclusive $B \to X_c l^+ \nu_l$ decays with hadronic tagging

Belle, Phys. Rev. D 104, 112011 (2022)
Belle II, Phys. Rev. D 107, 072002 (2023)

$|V_{cb}|$ FROM q^2 MOMENTS



$$|V_{cb}| = (41.69 \pm 0.59_{\text{fit}} \pm 0.23_{\text{h.o.}}) \times 10^{-3}$$
$$= (41.69 \pm 0.63) \times 10^{-3}$$

Bernlochner, **MF**, Olschwesky, Person, van Tonder, Vos, Welsch, JHEP 10 (2022) 068

Γ	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓	✓	✓
μ_G^2	✓	✓		
ρ_D^3	✓	✓		
$1/m_b^4$	✓			
$m_b^{\text{kin}}/\overline{m}_c$		✓	✓	✓

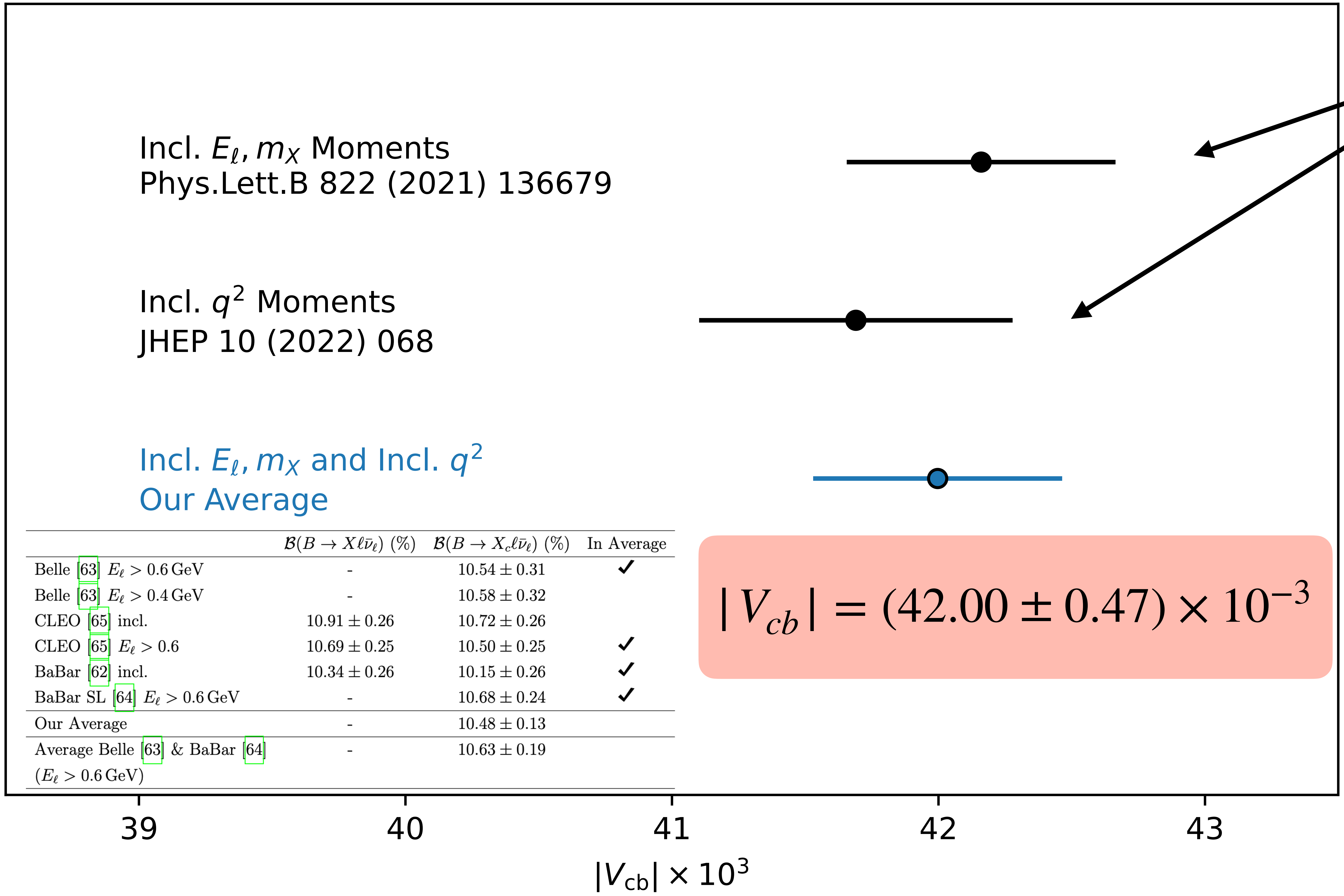
$\langle (q^2)^n \rangle$	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓		
μ_G^2	✓	✓		
ρ_D^3	✓	✓		
$1/m_b^4$	✓			

NNLO corrections missing!

N3LO corrections to the total rate!

MF, Schönwald, Steinhauser, Phys.Rev.Lett. 125 (2020) 5, 052003
Phys.Rev.D 103 (2021) 1, 014005, Phys.Rev.D 104 (2021) 1, 016003

Independent sets of data



	$\mathcal{B}(B \to X\ell\bar{\nu}_\ell)$ (%)	$\mathcal{B}(B \to X_c\ell\bar{\nu}_\ell)$ (%)	In Average
Belle [63] $E_\ell > 0.6\,\mathrm{GeV}$	-	10.54 ± 0.31	✓
Belle [63] $E_\ell > 0.4\,\mathrm{GeV}$	-	10.58 ± 0.32	
CLEO [65] incl.	10.91 ± 0.26	10.72 ± 0.26	
CLEO [65] $E_\ell > 0.6$	10.69 ± 0.25	10.50 ± 0.25	✓
BaBar [62] incl.	10.34 ± 0.26	10.15 ± 0.26	✓
BaBar SL [64] $E_\ell > 0.6\,\mathrm{GeV}$	-	10.68 ± 0.24	✓
Our Average	-	10.48 ± 0.13	
Average Belle [63] & BaBar [64] ($E_\ell > 0.6\,\mathrm{GeV}$)	-	10.63 ± 0.19	

$$|V_{cb}| = (42.00 \pm 0.47) \times 10^{-3}$$

MF, Prim, Vos, Eur. Phys. J. Spec. Top. (2024). https://doi.org/10.1140/epjs/s11734-024-01090-w

- Difference mainly driven by the $\mathrm{Br}(B \to X_c l \bar{\nu}_l)$ average
- We need new $\mathrm{Br}(B \to X_c l \bar{\nu}_l)$ measurements to improve.
- Challenging control sub-percent effects in the HQE

NNLO CORRECTIONS q^2 SPECTRUM

MF, Herren, hep-ph/2403.03976

$$\frac{d\Gamma}{d\hat{q}^2} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \left[F_0(\rho,\hat{q}^2) + \frac{\alpha_s}{\pi} F_1(\rho,\hat{q}^2) + \left(\frac{\alpha_s}{\pi}\right)^2 F_2(\rho,\hat{q}^2) \right] + O\left(\frac{1}{m_b^2}\right)$$

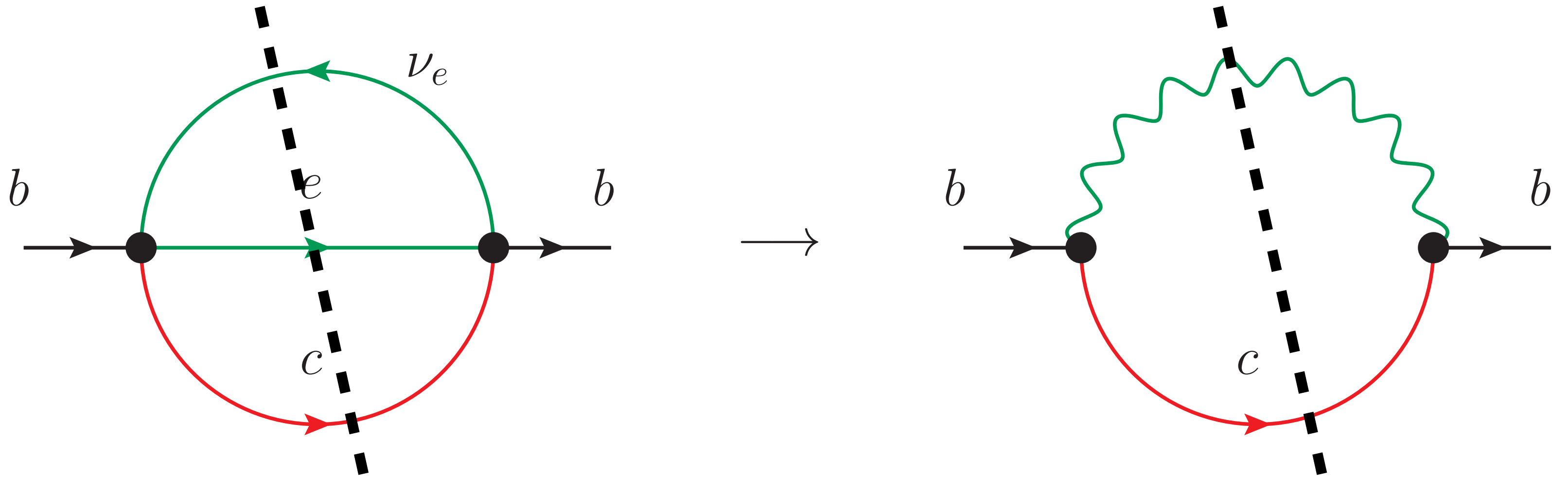
NEW: Analytic expressions at NNLO!

with $\rho = m_c/m_b$

Integration w.r.t. neutrino-electron phase space

$$\mathscr{L}^{\mu\nu}(p_L) = \int L^{\mu\nu} d\Phi_2(p_L; p_l, p_\nu) = \frac{1}{384\pi^5}\left(1 - \frac{m_\ell^2}{p_L^2}\right)^2 \left[\left(1 + \frac{2m_\ell^2}{p_L^2}\right) p_L^\mu p_L^\nu - g^{\mu\nu} p_L^2 \left(1 + \frac{m_\ell^2}{2p_L^2}\right)\right]$$

$\delta(p_L^2 - q^2) \times$

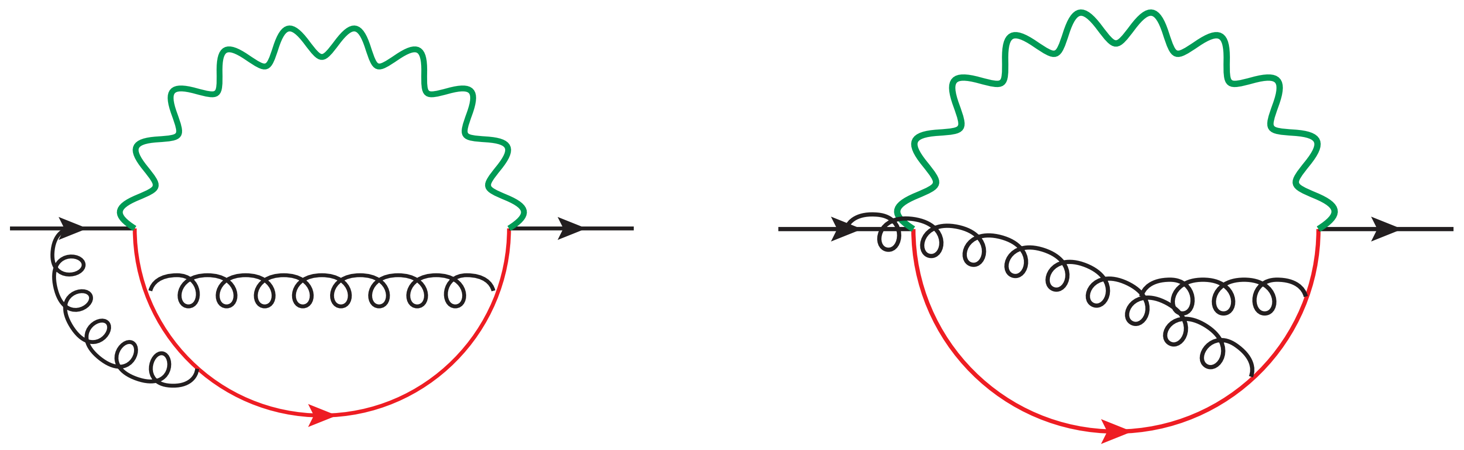


Inverse unitarity

$$\delta(p_L^2 - q^2) \to \frac{1}{2\pi i}\left[\frac{1}{p_L^2 - q^2 - i0} - \frac{1}{p_L^2 - q^2 + i0}\right]$$

NNLO calculation

- Three-loop diagrams
- Three different masses: m_b^2, m_c^2, q^2

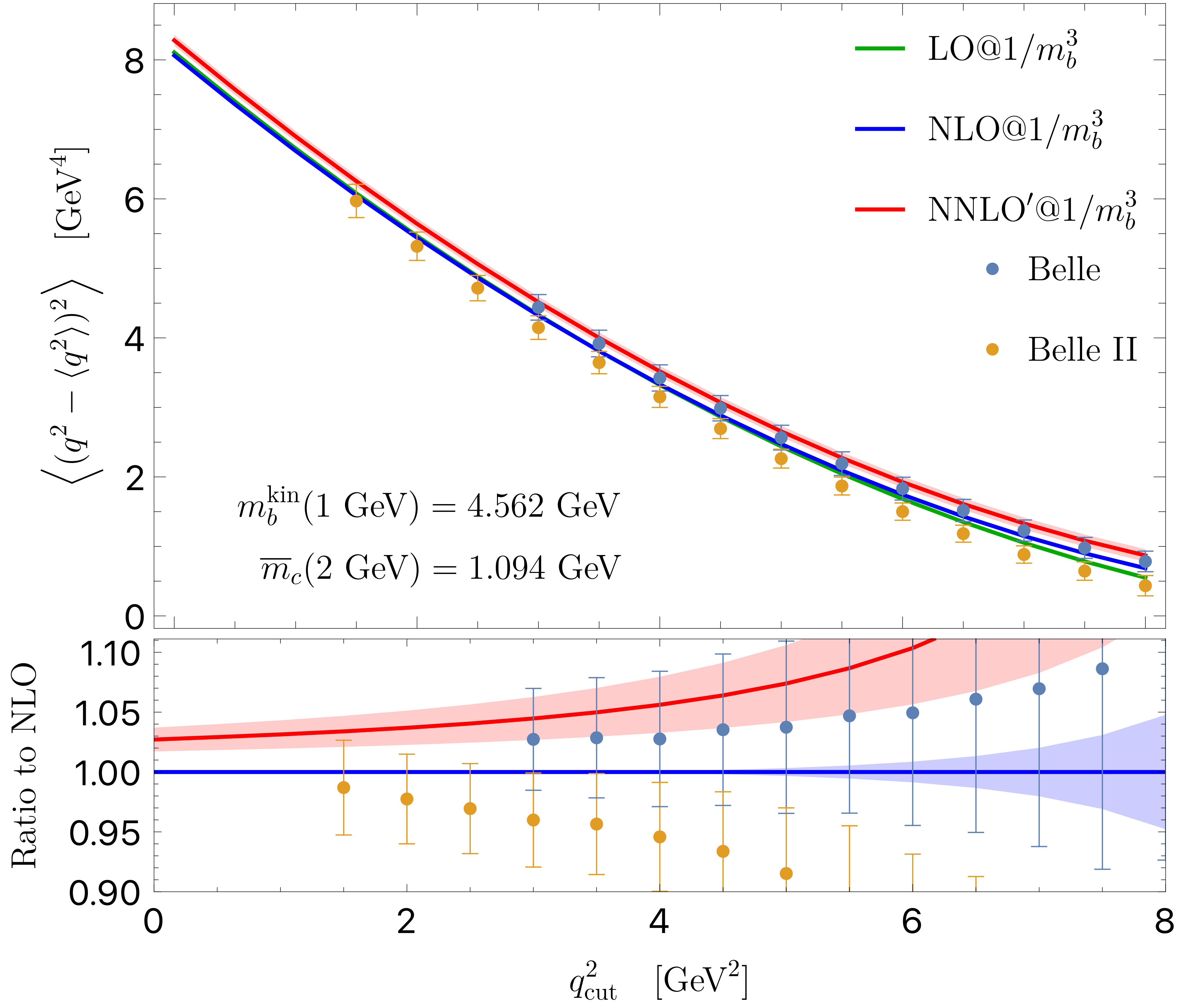


NEW: NNLO CORRECTIONS Q2 SPECTRUM

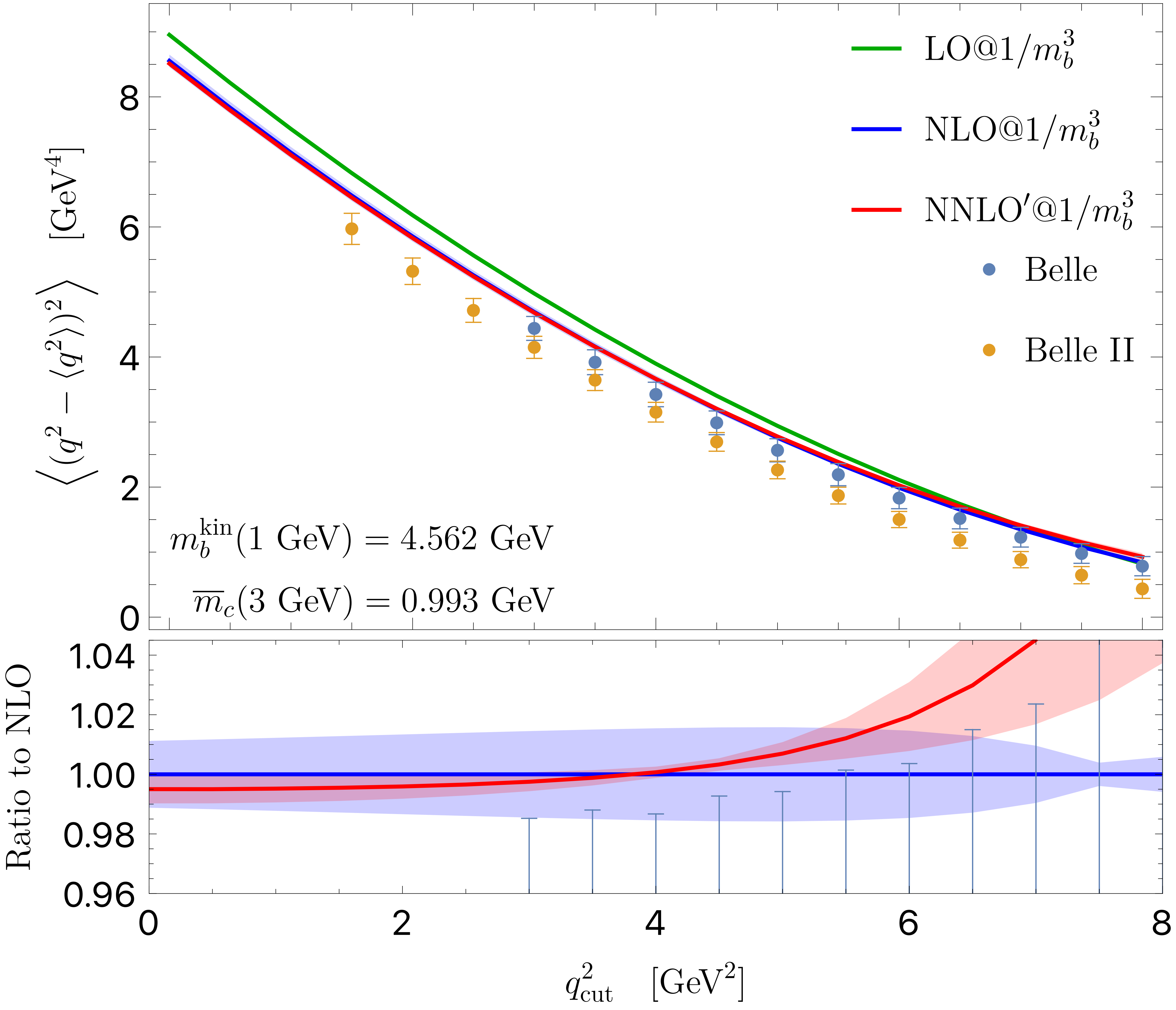
MF, Herren, hep-ph/2403.03976

NNLO effects mainly re-absorbed in the fit into a shift of ρ_D, r_E and r_G with reduced uncertainty. No major shift in $|V_{cb}|$.

setup from: Bernlochner, **MF,** et al, JHEP 10 (2022) 068



Unfortunate choice of $\overline{m}_c(2\,\mathrm{GeV})$



Much better $\overline{m}_c(3\,\mathrm{GeV})$

INCLUSIVE DECAYS: OPEN-SOURCE LIBRARY

MF, Milutin, Vos, to appear soon

Open-source python package: **KOLYA**

https://gitlab.com/vcb-inclusive/kolya

STAY TUNED!

- Interface to CRunDec for automatic α_s, m_b^{kin} and $\overline{m}_c$ RGE evolution
  Florian Herren, Matthias Steinhauser, arXiv:1703.03751
- SM h.o. effects and NP effects
  MF, Rahimi, Vos, JHEP 02 (2023) 086
- Observables
  - Γ_{sl}, $\Delta\text{Br}(E_{\text{cut}})$
  - Centralised moments $\langle E_\ell \rangle_{E_{\text{cut}}}$, $\langle M_X^2 \rangle_{E_{\text{cut}}}$
  - Centralised moments $\langle q^2 \rangle_{q^2_{\text{cut}}}$

Nikolai Uraltsev 1957 - 2013

Total Rate

The branching ratio is given by the function `BranchingRatio_KIN_MS(Vcb,par,hqe,wc)`

```
[10]: Vcb = 42.2e-2
      kolya.TotalRate.BranchingRatio_KIN_MS(Vcb,par,hqe,wc)

[10]: 10.64996041511315
```

Centralized Q2 moments

Q2 moments are evaluated with `Q2moments.moment_n_KIN_MS(q2cut, par, hqe, wc)`, for instance

```
[11]: q2cut = 8.0
      kolya.Q2moments.moment_1_KIN_MS(q2cut,par,hqe,wc)

[11]: 8.971188963587162
```

COMBINED FIT: q^2, E_l AND M_X^2 MOMENTS

Finauri, Gambino, JHEP 02 (2024) 206

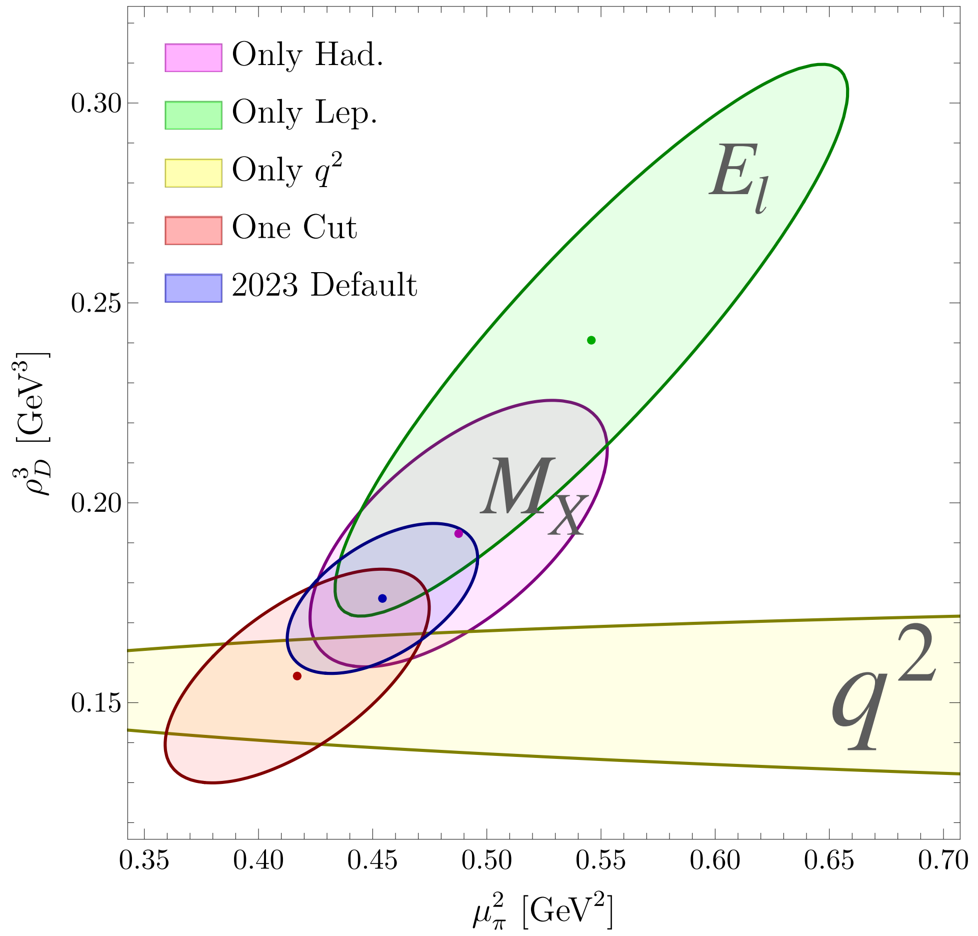
- Old DELPHI, CDF, BaBar, Belle data:
  $\langle E_l\rangle_{E_\mathrm{cut}}, \langle M_X^2\rangle_{E_\mathrm{cut}}, \Delta\mathrm{Br}_{E_\mathrm{cut}}$
- New Belle & Belle II: $\langle q^2\rangle_{q^2_\mathrm{cut}}$

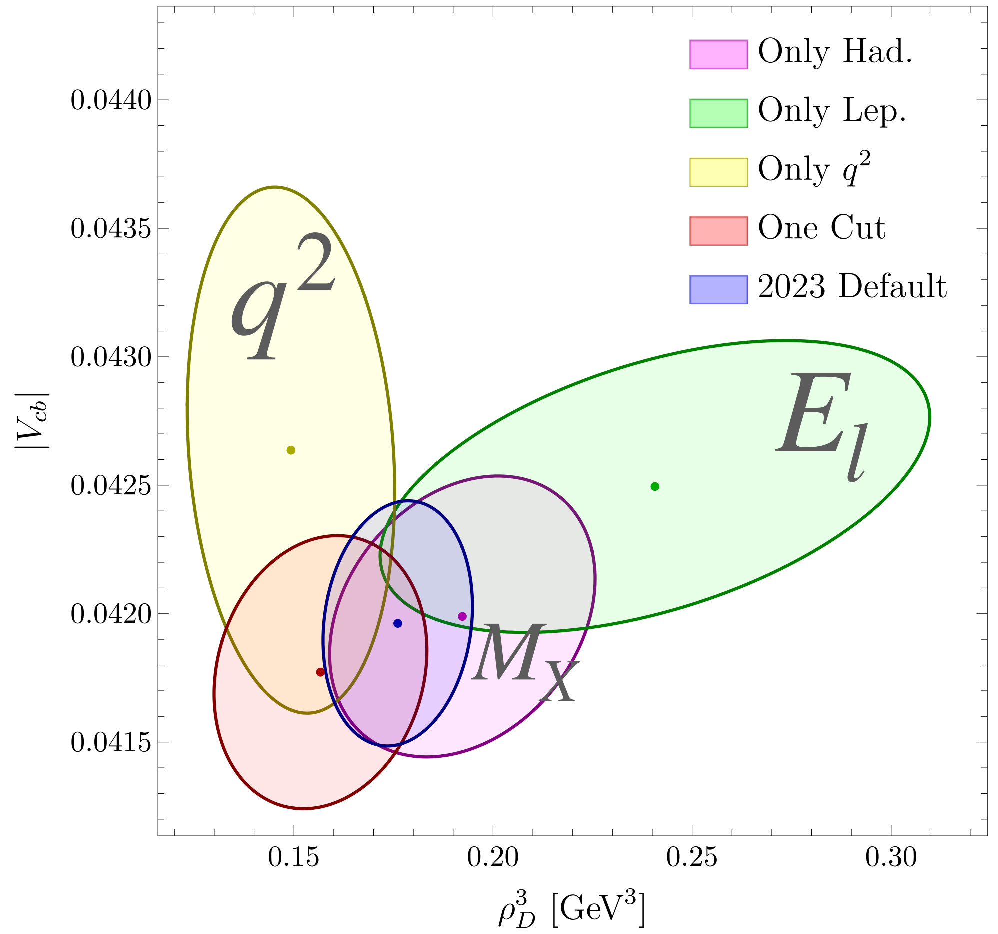
$$\begin{aligned} |V_{cb}| &= (41.97 \pm 0.27_\mathrm{exp} \pm 0.31_\mathrm{th} \pm 0.25_\Gamma) \times 10^{-3} \\ &= (41.97 \pm 0.48) \times 10^{-3} \end{aligned}$$

Compared with 2021 fit: 0.51 → 0.48 reduction

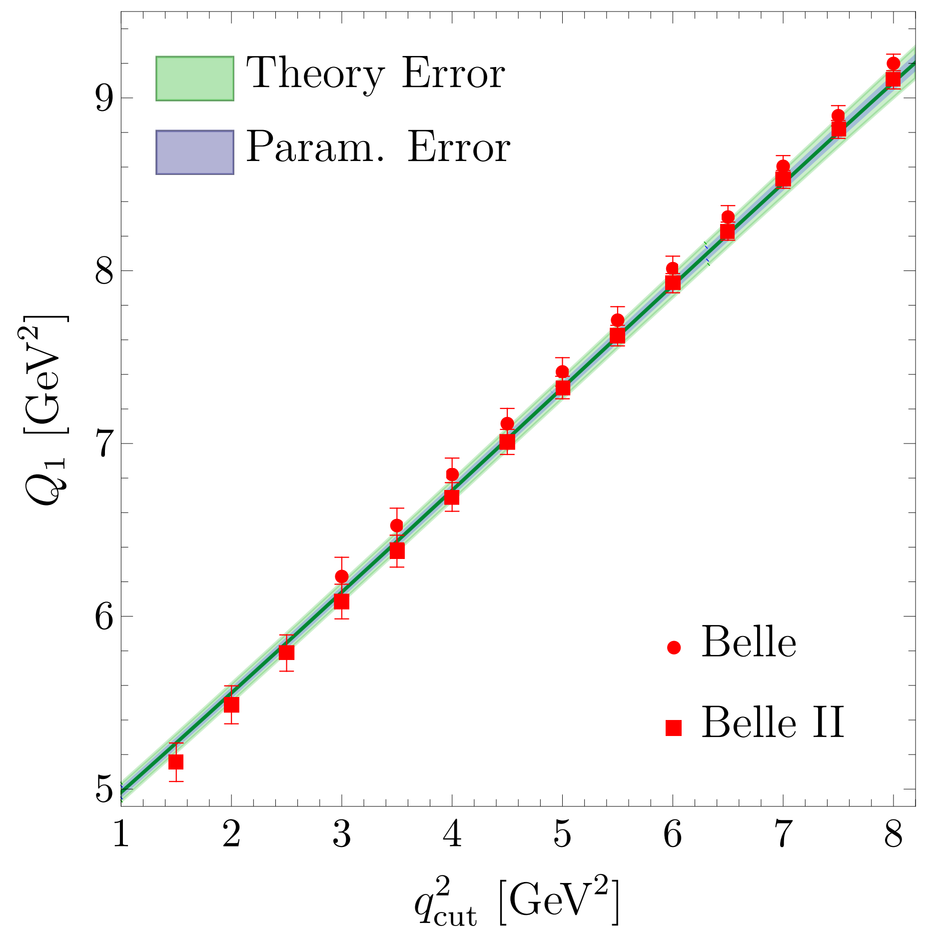
0.031 → 0.018 reduction

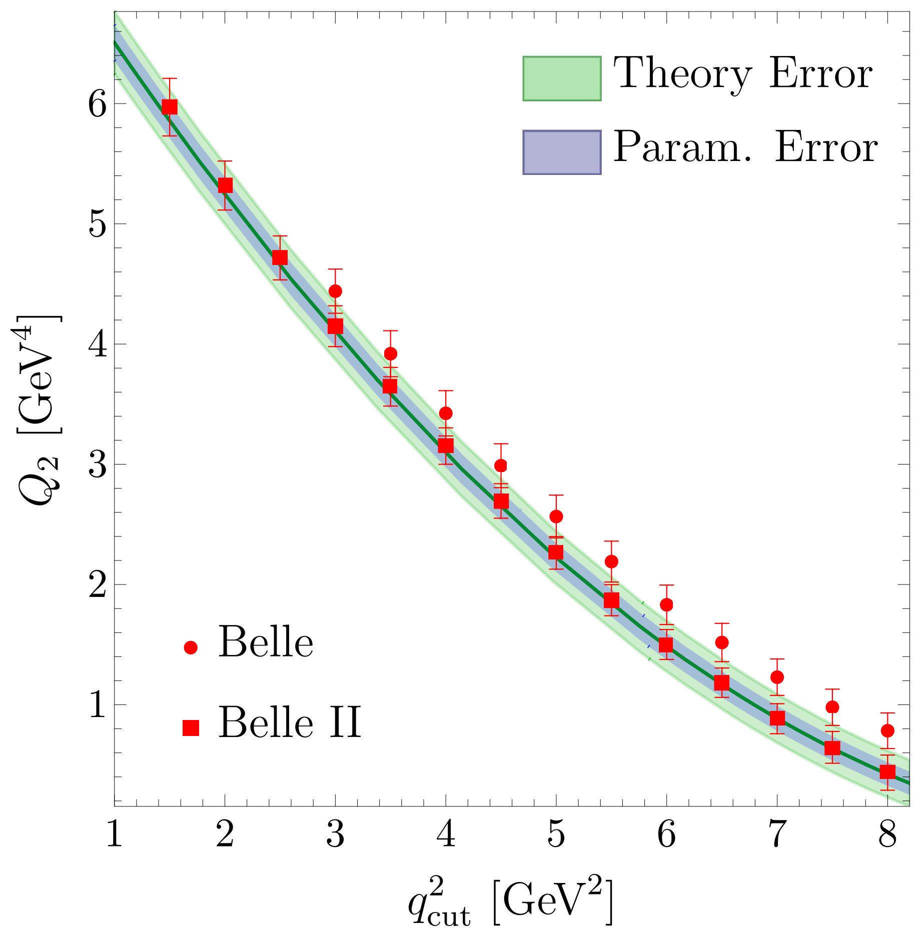
m_b^kin	$\overline{m}_c(2\,\mathrm{GeV})$	μ_π^2	$\mu_G^2(m_b)$	$\rho_D^3(m_b)$	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.573	1.090	0.454	0.288	0.176	−0.113	10.63	41.97
0.012	0.010	0.043	0.049	0.019	0.090	0.15	0.48
1	0.380	-0.219	0.557	-0.013	-0.172	-0.063	-0.428
	1	0.005	-0.235	-0.051	0.083	0.030	0.071
		1	-0.083	0.537	0.241	0.140	0.335
			1	-0.247	0.010	0.007	-0.253
				1	-0.023	0.023	0.140
					1	-0.011	0.060
						1	0.696
							1

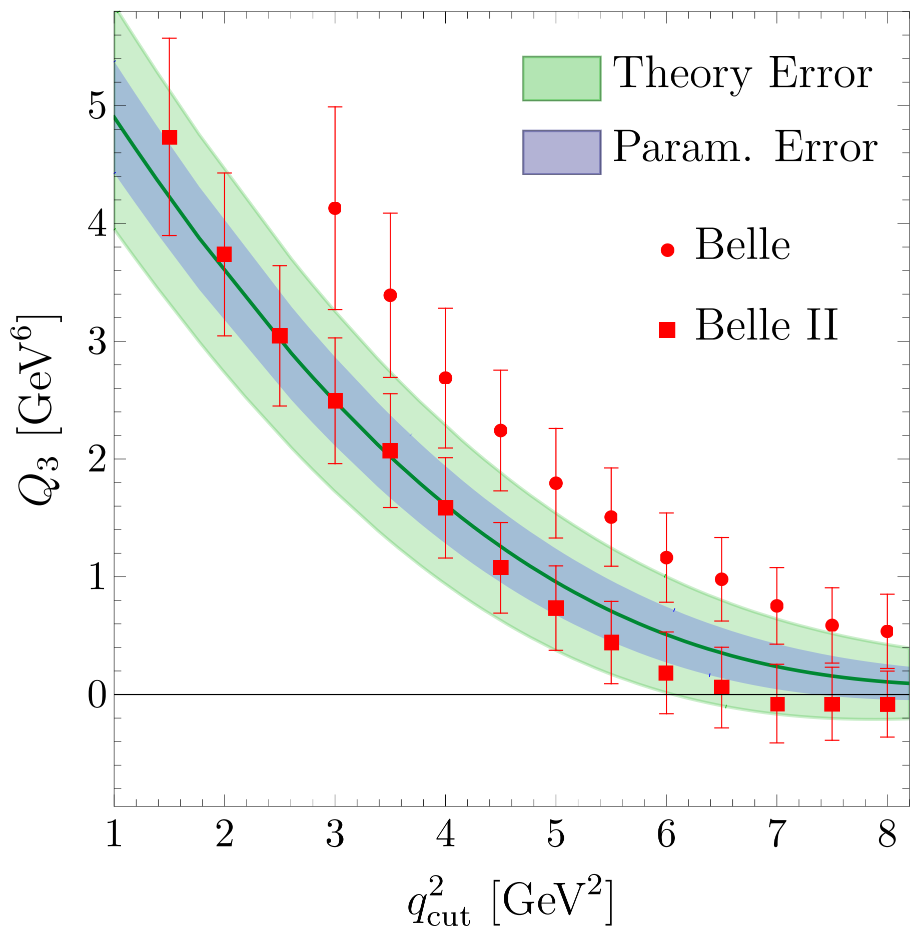


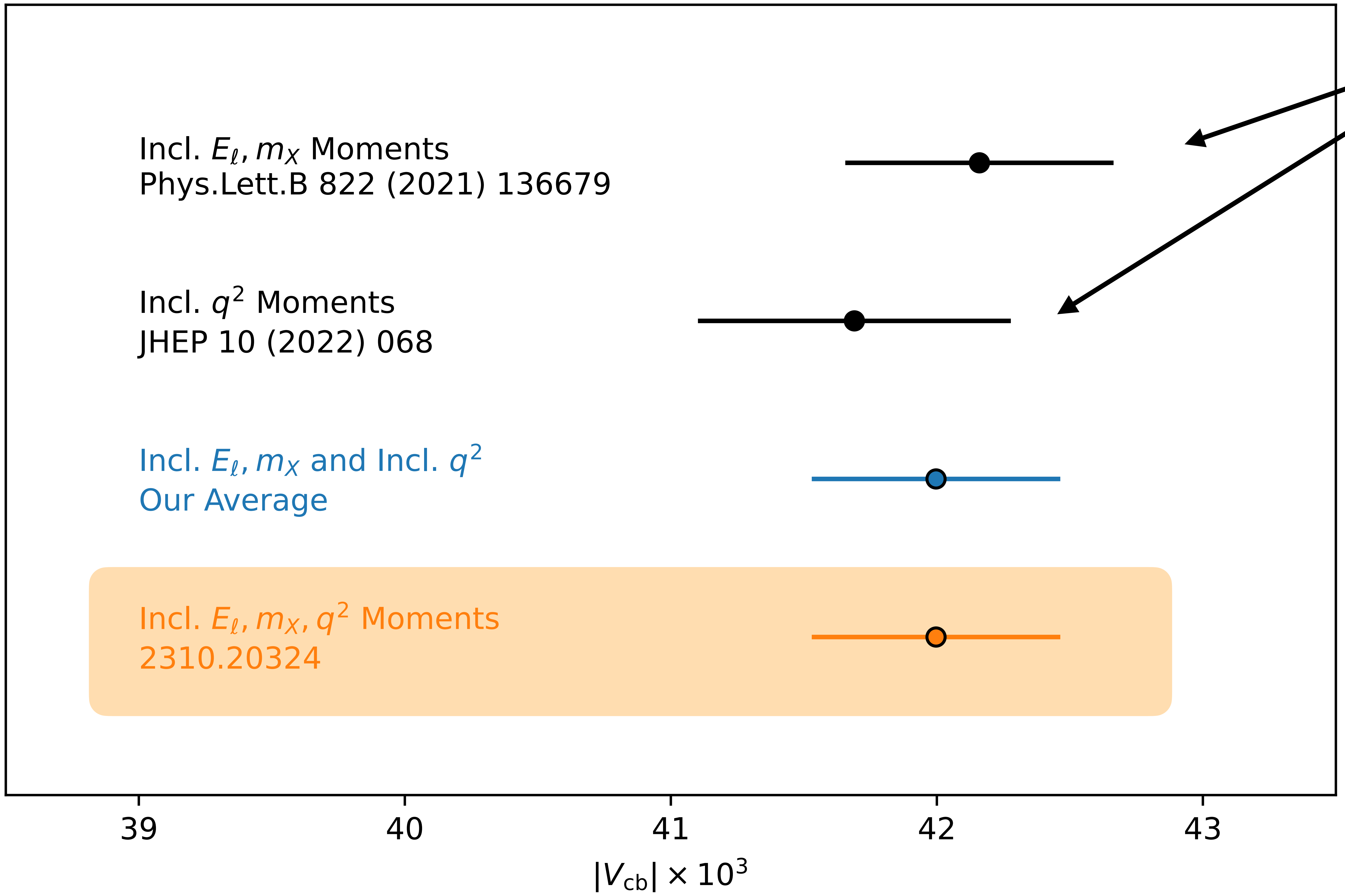


Only $\alpha_s^2\beta_0$ corrections included for $\langle q^2\rangle$









Independent sets of data

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- Challenging control sub-percent effects in the HQE

MF, Prim, Vos, Eur. Phys. J. Spec. Top. (2024). https://doi.org/10.1140/epjs/s11734-024-01090-w

NEW: BELLE II MEASUREMENT OF R(X)

$$R(X_{\ell_1/\ell_2}) = \frac{\Gamma_{B\to X\ell_1\bar\nu_1}}{\Gamma_{B\to X\ell_2\bar\nu_{l_2}}}$$

$$R^{\text{exp}}(X_{e/\mu}) = 1.007 \pm 0.009(\text{stat}) \pm 0.019(\text{syst})$$

Belle II, Phys.Rev.Lett. 131 (2023) 5, 051804

$$R^{\text{exp}}(X_{\tau/l}) = 0.228 \pm 0.016(\text{stat}) \pm 0.036(\text{syst})$$

Belle II, hep-ex/2311.07248

$$R^{\text{SM}}(X_{\tau/l}) = 0.225 \pm 0.005$$

Rahimi, Vos, JHEP 11 (2022) 007
Ligeti, Luke, Tackmann, Phys. Rev. D 105, 073009 (2022)

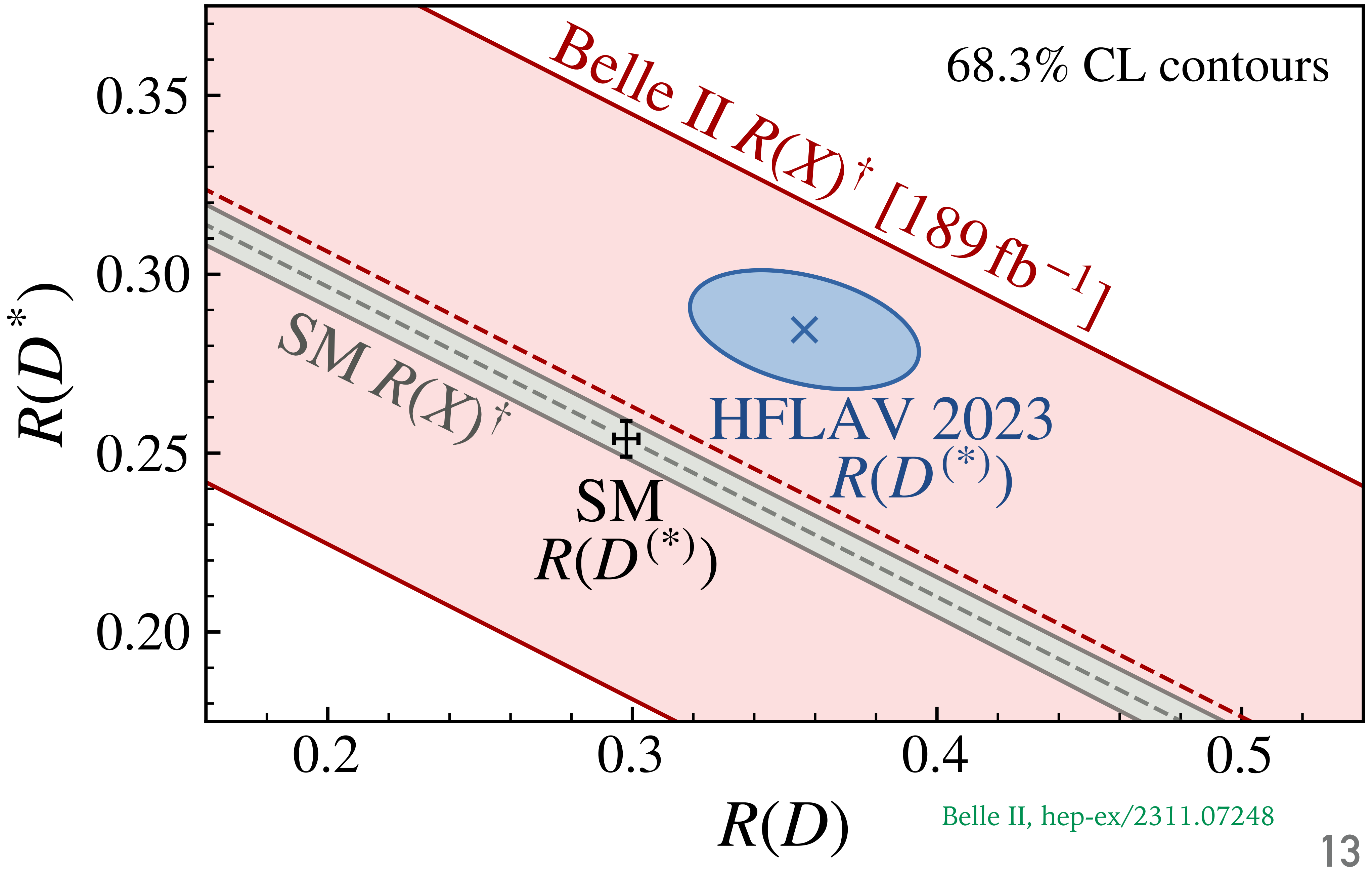
Enrichment with q^2 selection cut

$$R(X_c) = 0.241\left[1 - 0.156\frac{\alpha_s}{\pi} - 1.766\left(\frac{\alpha_s}{\pi}\right)^2\right]$$

$$R(X_c)\Big|_{q^2>6\,\text{GeV}^2} = 0.350\left[1 - 0.782\frac{\alpha_s}{\pi} - 8.355\left(\frac{\alpha_s}{\pi}\right)^2\right]$$

MF, Herren, hep-ph/2403.03976

$\dagger$ = with expected SM contributions of $D^{**}_{(\text{gap})}, X_u$ removed



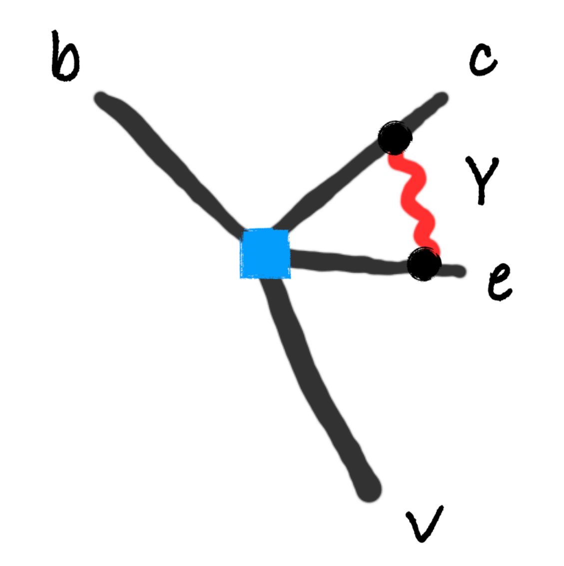
Belle II, hep-ex/2311.07248

QED EFFECTS

Bigi, Bordone, Gambino, Haisch, Piccione, JHEP 11 (2023) 163

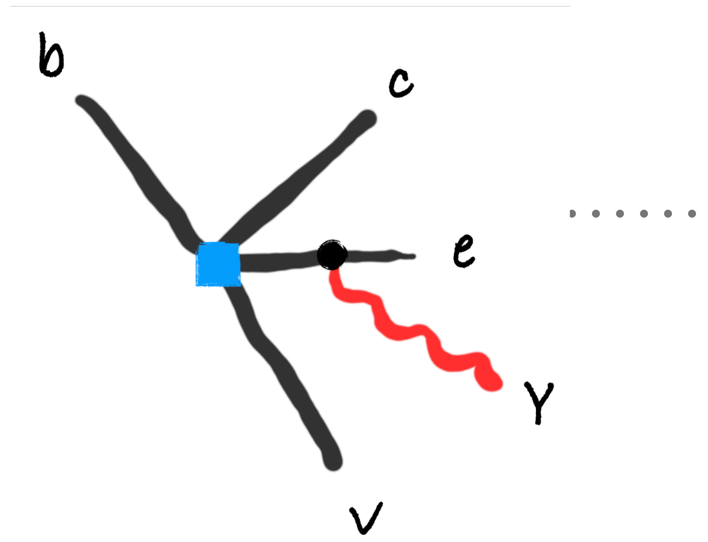
Exact $\mathcal{O}(\alpha_{\text{em}})$ calculation vs collinear and threshold enhancements

Threshold effects



$$\sim \frac{4\pi\alpha_{\text{em}}}{9}$$

Collinear enhancement



$$\sim \log\left(\frac{m_b^2}{m_e^2}\right) - 1$$

RGE evolution

Wilson coefficient

Exact $\mathcal{O}(\alpha_{\text{em}})$

$$\frac{\Gamma}{\Gamma_{\text{LO}}} = 1 + \frac{\alpha}{\pi}\left(\log\frac{M_Z^2}{m_b^2} - \frac{11}{6} + 5.516\,(14)\right)$$

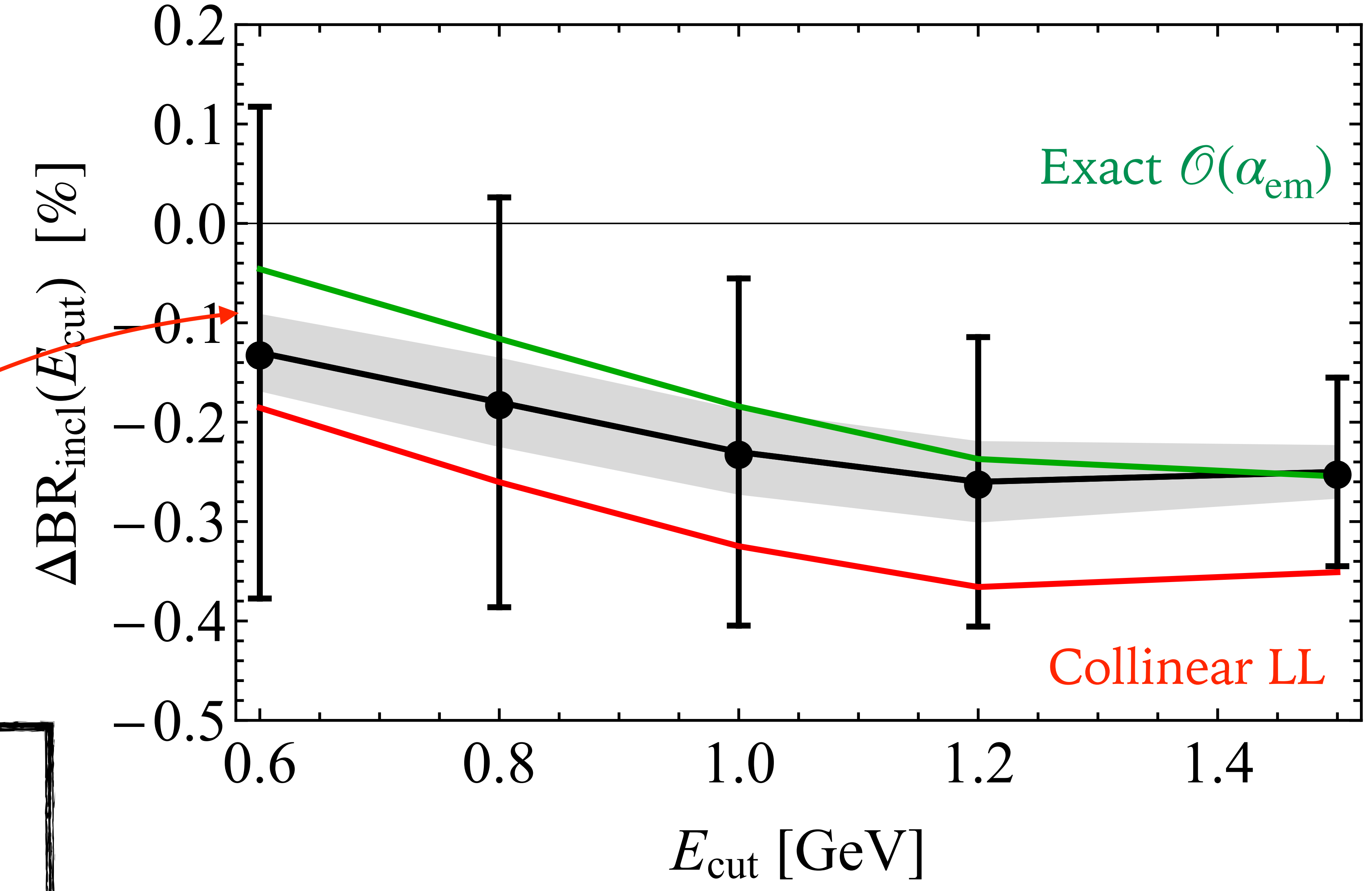
$$= 1 + 1.43\% - 0.44\% + 1.32\,\% = 1 + 2.31\,\%$$

Well approximated by EW logs and threshold effects

Effects not in PHOTOS: 0.8% up shift

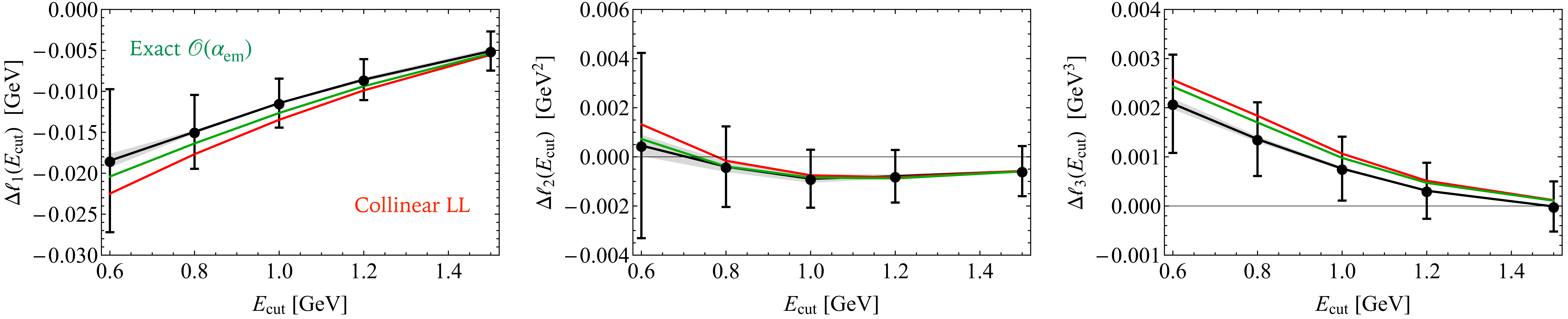
BaBar correction using PHOTOS

BaBar, Phys. Rev. D 69 (2004) 111104



ELECTRON ENERGY MOMENTS

BaBar correction using PHOTOS



Differences below 0.19% (ℓ_1, ℓ_2) up to 21 % for the third moment

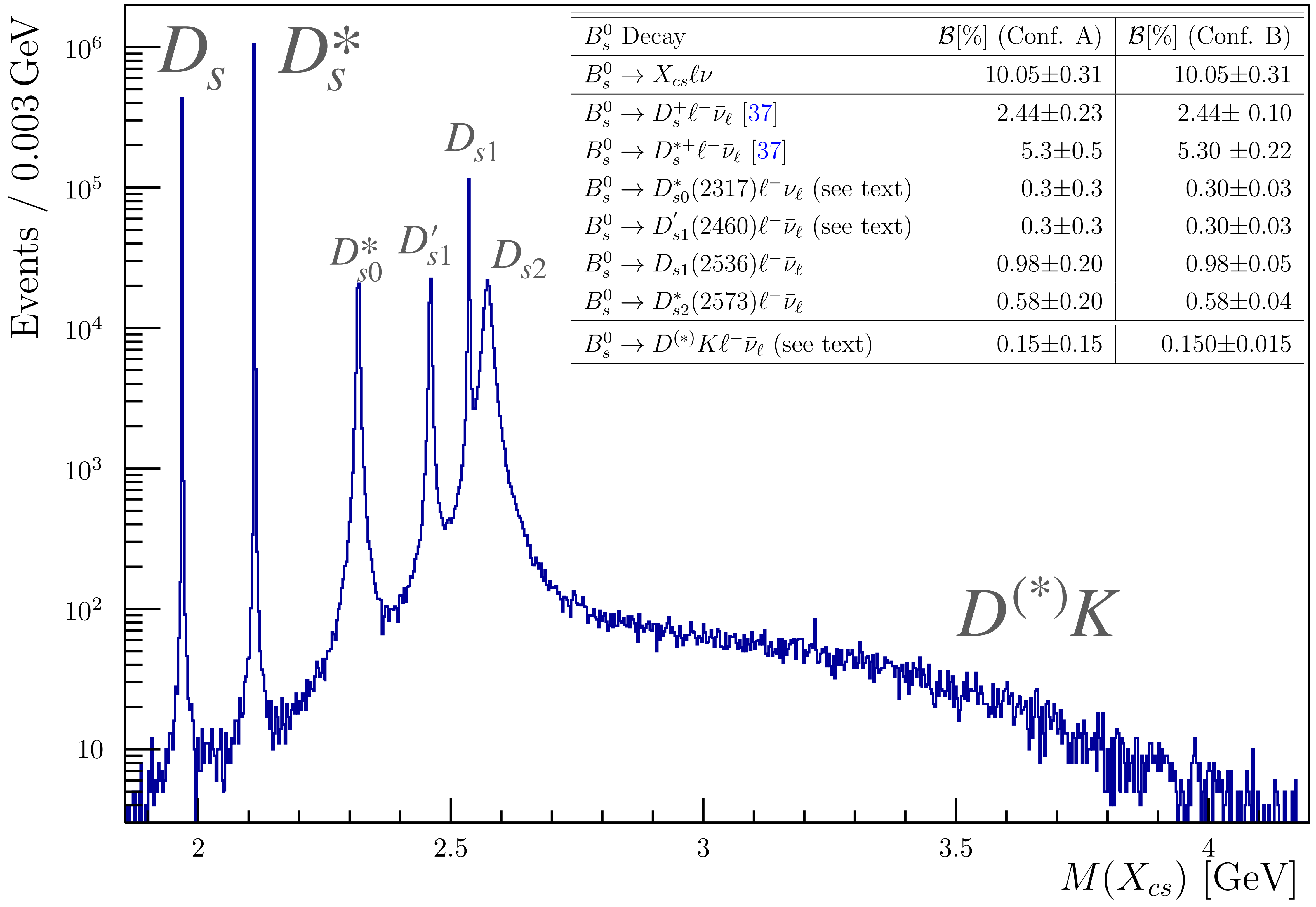
LHCb: Inclusive Semileptonic B_s^0 Decays

De Cian, Feliks, Rotondo, Vos, 2312.05147 [hep-ph]

- Semileptonic B_s: well-separated D_s spectrum
- Avoid amplitude analysis and interference effects
- Inclusive $\overline{B}_s \to X_{cs}\ell\bar{\nu}_\ell$:
  sum-of-exclusive technique
- Study SU(3) breaking in HQE

$$h_i = \left\langle \left(M_X^2 - \langle M_X^2 \rangle\right)^n \right\rangle$$

MOMENTS OF THE HADRONIC INVARIANT MASS



B_s^0 Decay	$\mathcal{B}$[%] (Conf. A)	$\mathcal{B}$[%] (Conf. B)
$B_s^0 \to X_{cs}\ell\nu$	10.05±0.31	10.05±0.31
$B_s^0 \to D_s^+\ell^-\bar{\nu}_\ell$ [37]	2.44±0.23	2.44± 0.10
$B_s^0 \to D_s^{*+}\ell^-\bar{\nu}_\ell$ [37]	5.3±0.5	5.30 ±0.22
$B_s^0 \to D_{s0}^*(2317)\ell^-\bar{\nu}_\ell$ (see text)	0.3±0.3	0.30±0.03
$B_s^0 \to D'_{s1}(2460)\ell^-\bar{\nu}_\ell$ (see text)	0.3±0.3	0.30±0.03
$B_s^0 \to D_{s1}(2536)\ell^-\bar{\nu}_\ell$	0.98±0.20	0.98±0.05
$B_s^0 \to D_{s2}^*(2573)\ell^-\bar{\nu}_\ell$	0.58±0.20	0.58±0.04
$B_s^0 \to D^{(*)}K\ell^-\bar{\nu}_\ell$ (see text)	0.15±0.15	0.150±0.015

- **Improve knowledge semileptonic $B_s^0 \to D_s^{**}$**
- **Understand non-resonant $B_s^0 \to D^{(*)}K\ell\bar{\nu}_\ell$**
- **For Br($B_s^0 \to X_{cs}\ell\bar{\nu}_\ell$) and $|V_{cb}|$: Belle II@$\Upsilon(5S)$**

INCLUSIVE DECAYS ON THE LATTICE

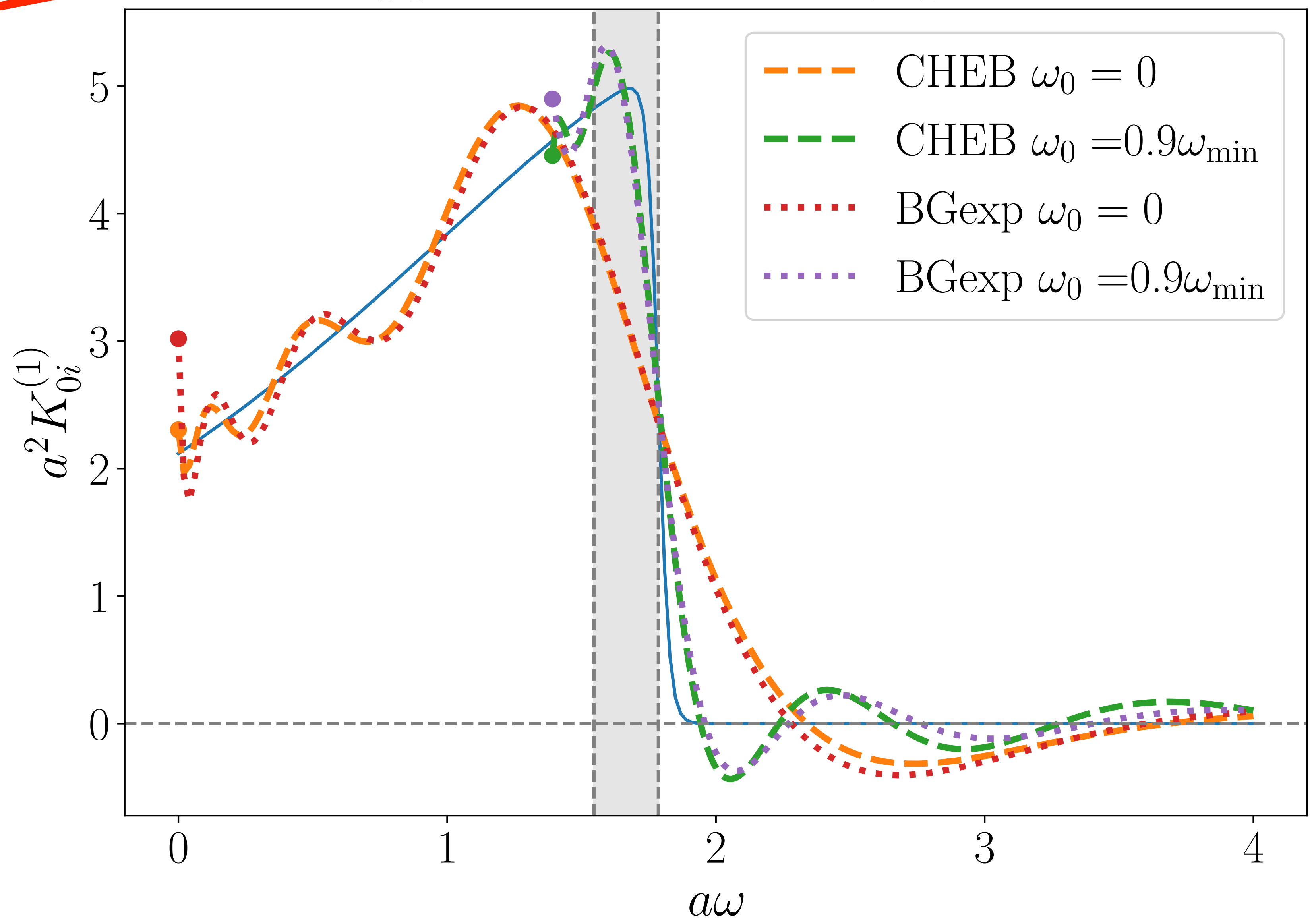
$$L_{\mu\nu}(\mathbf{q},\omega) \simeq c_{\mu\nu,0}(\mathbf{q}) + c_{\mu\nu,1}(\mathbf{q})e^{-\omega} + \ldots + c_{\mu\nu,N}(\mathbf{q})e^{-N\omega}$$

$$\Gamma = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \int_0^{\mathbf{q}^2} d\mathbf{q}^2 \sqrt{\mathbf{q}^2}\, \overline{X}(\mathbf{q}^2)$$

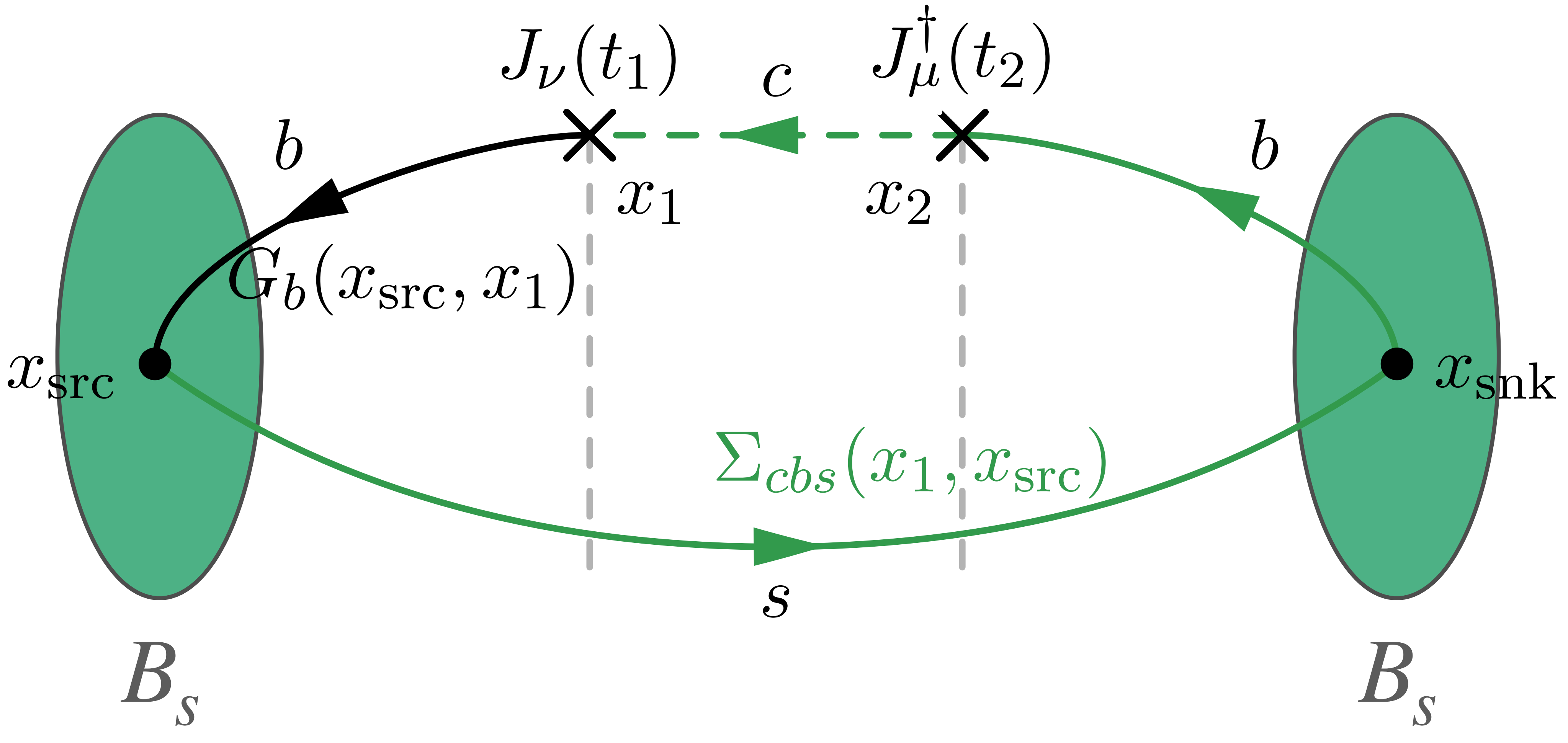
Smeared hadronic tensor

$$\overline{X}(\mathbf{q}^2) = \int_{\omega_{\min}}^{\omega_{\max}} W_{\mu\nu} L^{\mu\nu} \to \sum_{k=0}^{N} c_{\mu\nu,k} C^{\mu\nu}(\mathbf{q}, k + 2t_0)$$

Kernel approximation $N = 9$, $\mathbf{q}^2 = 4.77\ \mathrm{GeV}^2$



Barone, Hashimoto, Jüttner, Kaneko, Kellermann, JHEP 07 (2023) 145



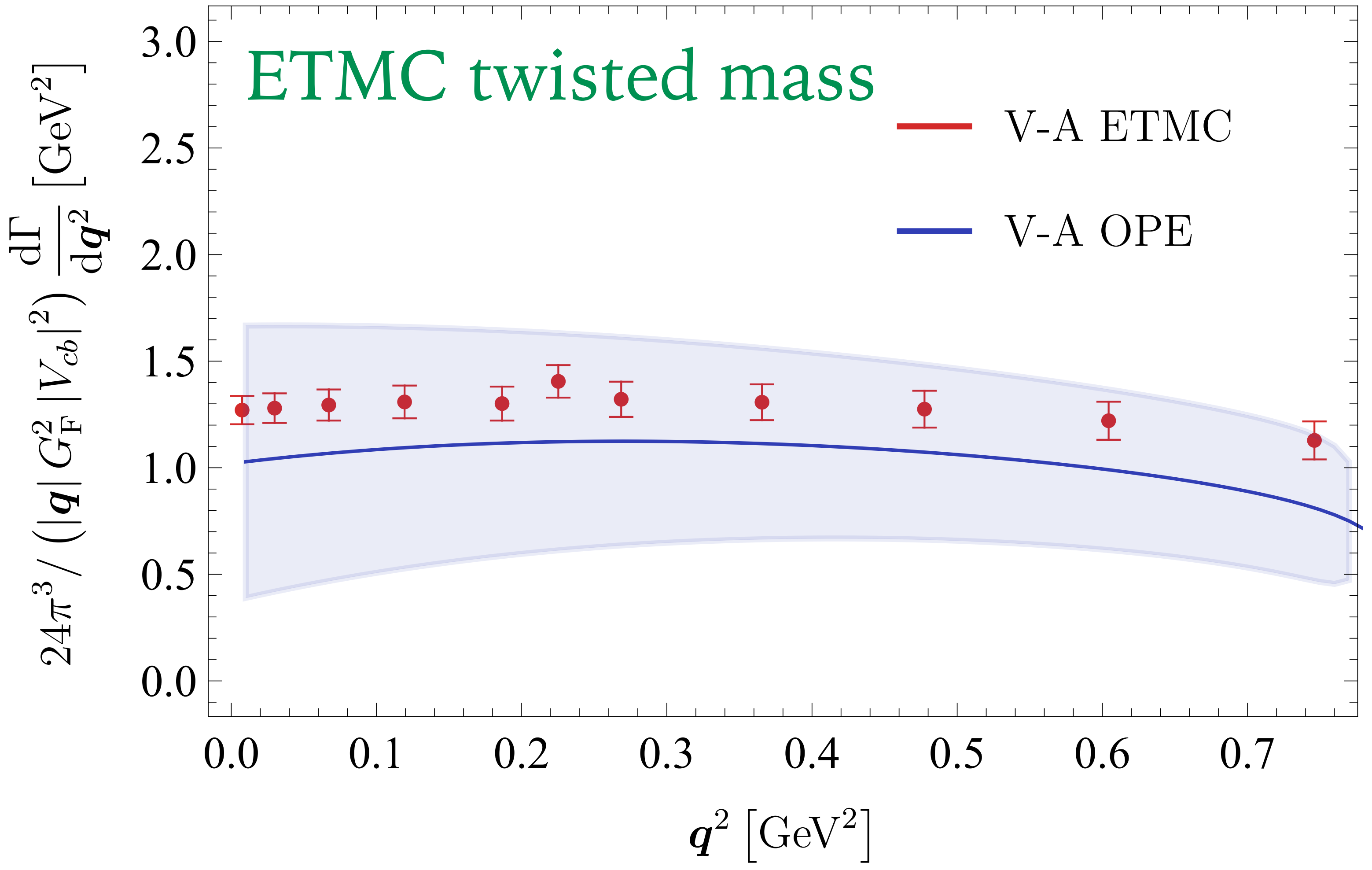
$$\simeq C_{\mu\nu}(\mathbf{q}, t) = \int_0^\infty \mathrm{d}\omega\, W_{\mu\nu}(\mathbf{q},\omega)\, e^{-\omega t}$$

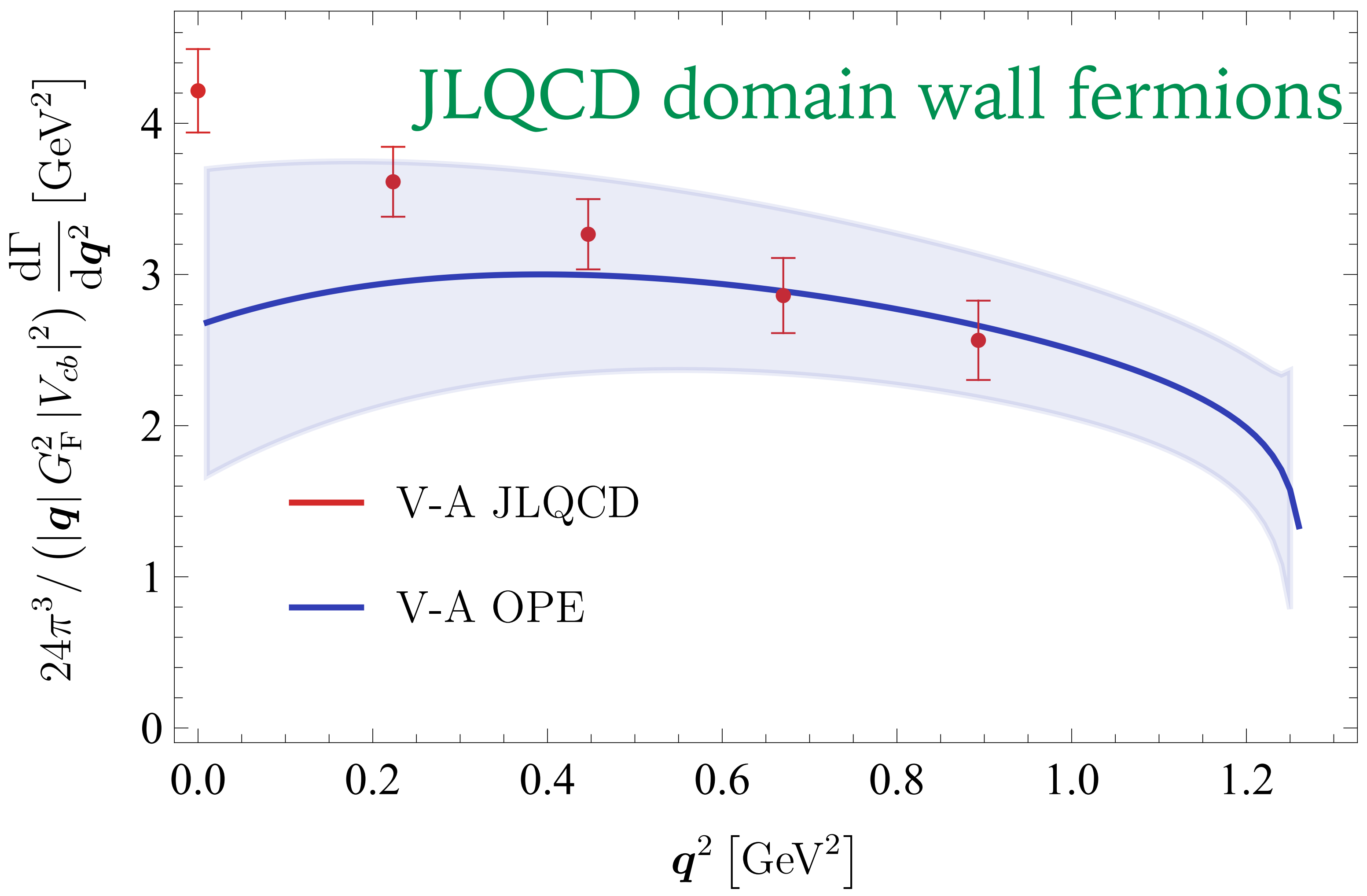
↑

Ill-posed inverse problem

Gambino, Hashimoto, Phys.Rev.Lett. 125 (2020) 3, 032001
Gambino, et al, JHEP 07 (2022) 083
Barone, Hashimoto, Jüttner, Kaneko, Kellermann, JHEP 07 (2023) 145

HQE VS LATTICE



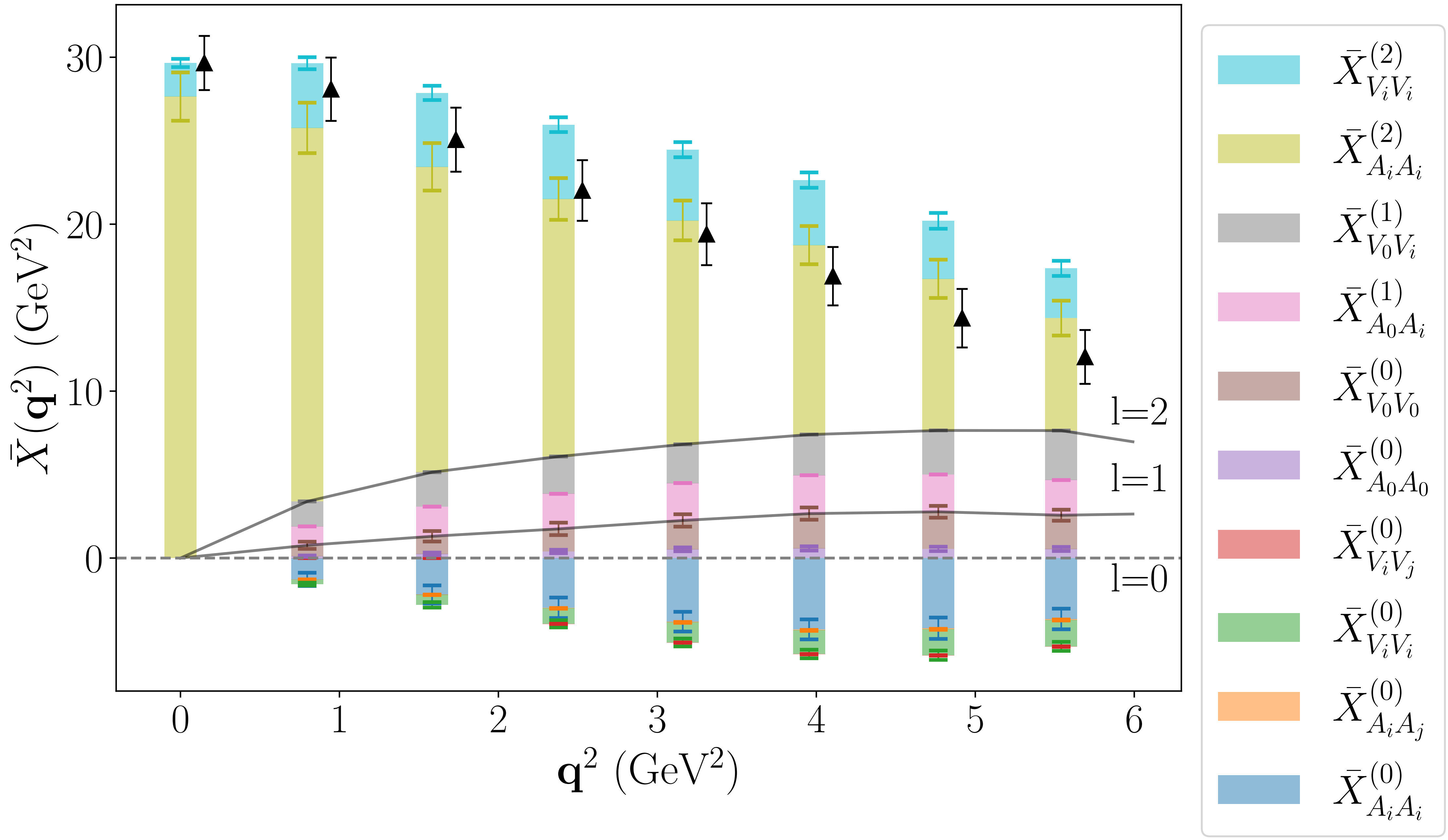


Gambino, Hashimoto, Mächler, Panero, Sanfilippo, Simula, Smecca, Tantalo, JHEP 07 (2022) 083

- Various moments of inclusive $B \to X_c \ell \bar{\nu}_\ell$
- Ensembles generated by the JLQCD and ETMC
- Unphysical light values of the b quark mass (2.7 GeV and 2.4 GeV)
- m_c close to its physical value.
- To Do: continuum and infinite-volume limits

INCLUSIVE DECAYS ON THE LATTICE

Contributions to $\overline{X}(\mathbf{q})$ with Chebyshev-polynomial approach, $N = 9$, $\omega_0 = 0.9\omega_{\min}$



Barone, Hashimoto, Jüttner, Kaneko, Kellermann, JHEP 07 (2023) 145

- Bottom quark: relativistic heavy quark formalism
- Valance quarks with DMF, approx. physical masses
- RBC/UKQCD ensambles
- Compatible Chebyshev and Backus-Gilbert approach for kernel expansion
- Final error on Γ_{sl} about 5%
- To Do: polynomial approximation, finite-volume effects, discretisation errors, continuum limit.

$B \to D\pi\ell\bar{\nu}_\ell$ DECAYS: MODEL INDEPENDENT DESCRIPTION

Gustafson, Herren, Van de Water, van Tonder, Wagman, hep-ph/2311.00864

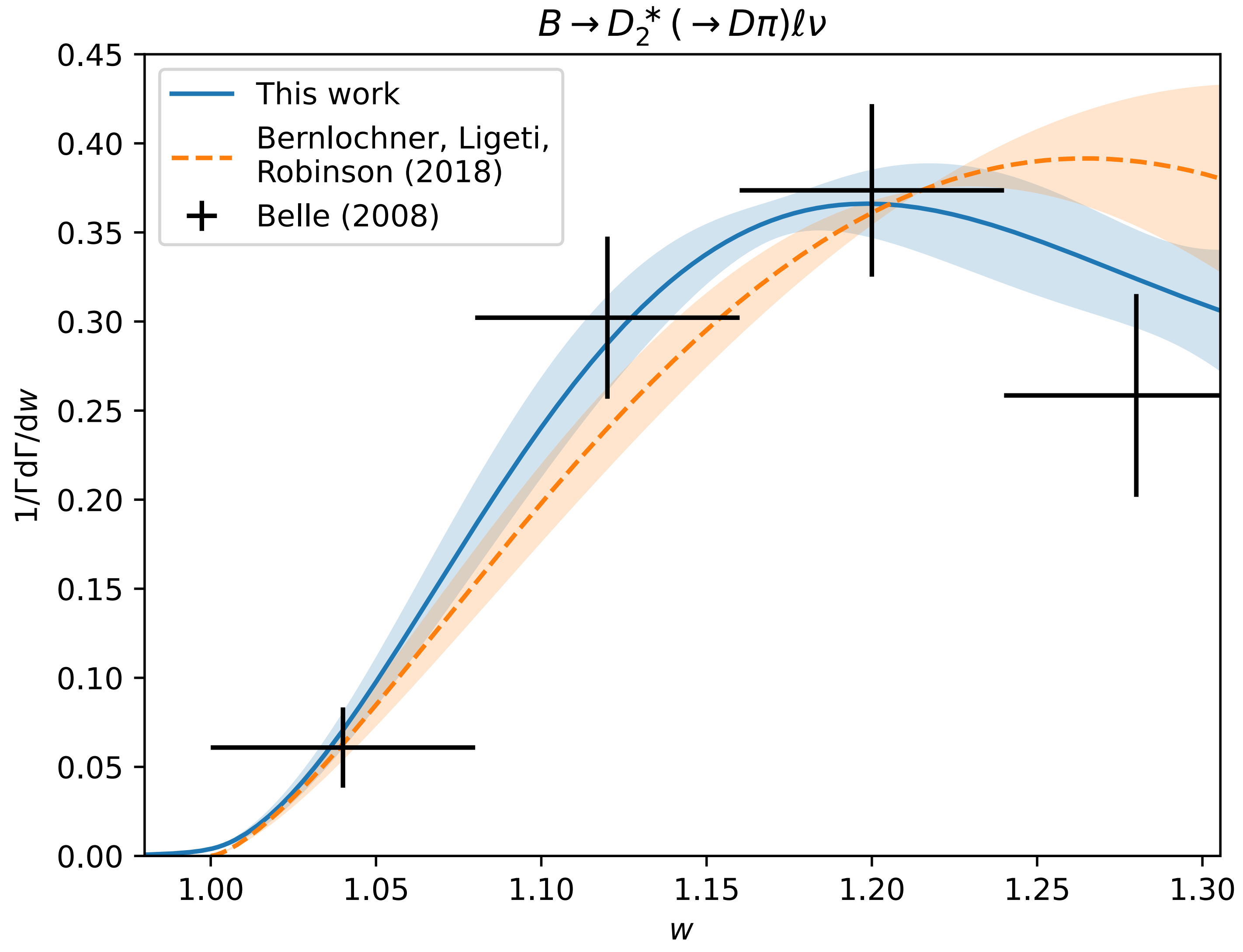
Semileptonic gap

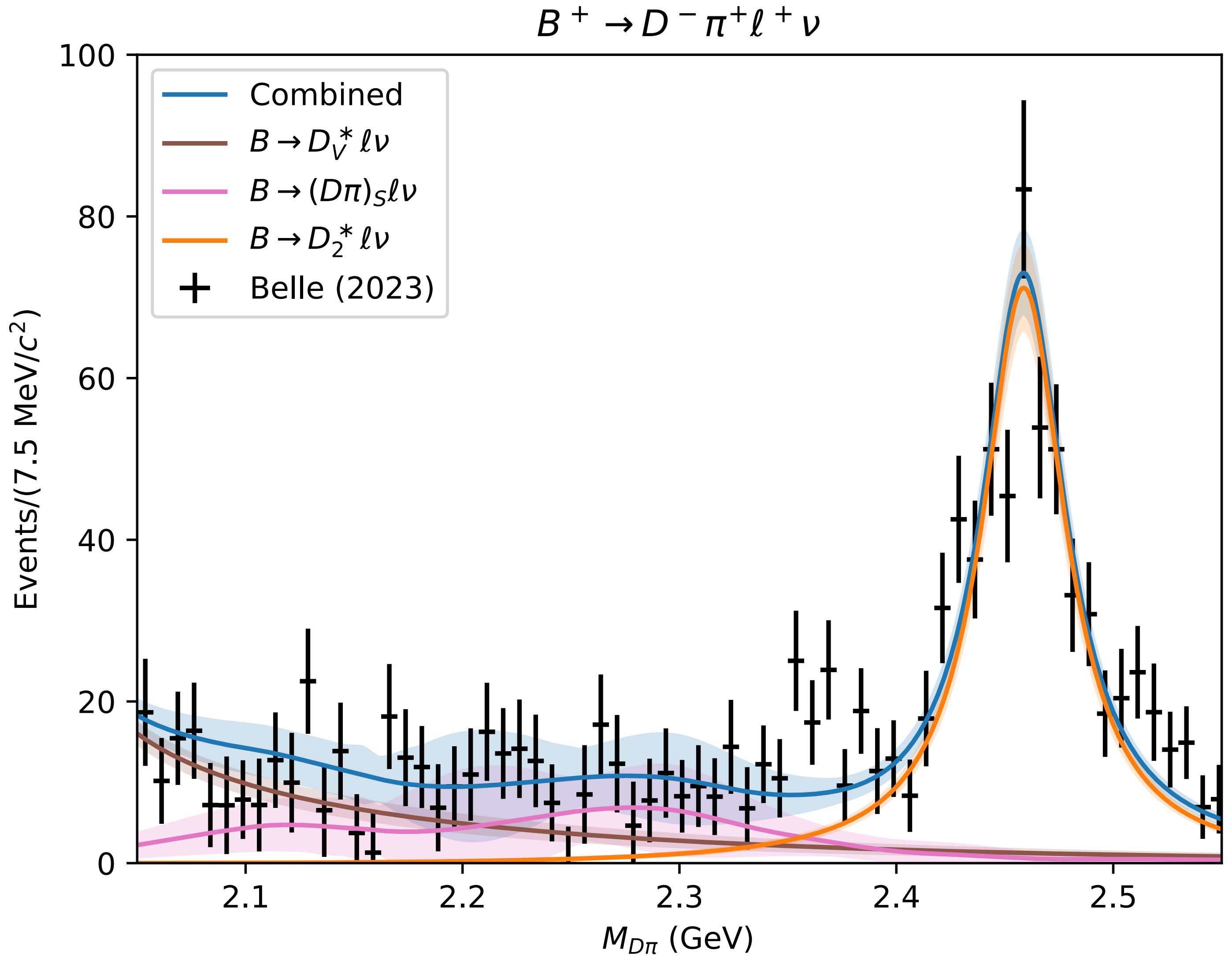
TABLE I. Branching fractions used in the simulation of $B \to X_c\,\ell\,\bar{\nu}_\ell$.

Decay	$\mathcal{B}(B^+)$	$\mathcal{B}(B^0)$
$B \to D\,\ell\,\nu_\ell$	$(2.4 \pm 0.1) \times 10^{-2}$	$(2.2 \pm 0.1) \times 10^{-2}$
$B \to D^*\,\ell\,\nu_\ell$	$(5.5 \pm 0.1) \times 10^{-2}$	$(5.1 \pm 0.1) \times 10^{-2}$
$B \to D_1\,\ell\,\nu_\ell$	$(6.6 \pm 1.1) \times 10^{-3}$	$(6.2 \pm 1.0) \times 10^{-3}$
$B \to D_2^*\,\ell\,\nu_\ell$	$(2.9 \pm 0.3) \times 10^{-3}$	$(2.7 \pm 0.3) \times 10^{-3}$
$B \to D_0^*\,\ell\,\nu_\ell$	$(4.2 \pm 0.8) \times 10^{-3}$	$(3.9 \pm 0.7) \times 10^{-3}$
$B \to D_1'\,\ell\,\nu_\ell$	$(4.2 \pm 0.9) \times 10^{-3}$	$(3.9 \pm 0.8) \times 10^{-3}$
$B \to D\pi\pi\,\ell\,\nu_\ell$	$(0.6 \pm 0.9) \times 10^{-3}$	$(0.6 \pm 0.9) \times 10^{-3}$
$B \to D^*\pi\pi\,\ell\,\nu_\ell$	$(2.2 \pm 1.0) \times 10^{-3}$	$(2.0 \pm 1.0) \times 10^{-3}$
$B \to D\eta\,\ell\,\nu_\ell$	$(4.0 \pm 4.0) \times 10^{-3}$	$(4.0 \pm 4.0) \times 10^{-3}$
$B \to D^*\eta\,\ell\,\nu_\ell$	$(4.0 \pm 4.0) \times 10^{-3}$	$(4.0 \pm 4.0) \times 10^{-3}$
$B \to X_c\,\ell\,\bar{\nu}_\ell$	$(10.8 \pm 0.4) \times 10^{-2}$	$(10.1 \pm 0.4) \times 10^{-2}$

Belle, Phys. Rev. D 104, 112011
Belle II, Phys. Rev. D 107, 072002 (2023)

100% uncertainty on the “gap”





- Model-independent description based on unitarity and analyticity
- Generalisation of the BGL formalism for $B \to D^{(*)}\ell\bar{\nu}_\ell$ to multi-hadron states
- Fit of the measured $M_{D\pi}$-spectrum. Coupled S-wave $D\pi$, $D\eta$ and D_sK channels
  Belle, Phys. Rev. D 107, 092003 (2023)
- Prediction: $\mathrm{Br}(B \to D\eta\ell\bar{\nu}_\ell) = (1.9 \pm 1.7) \times 10^{-5}$

EXCLUSIVE $|V_{cb}|$ FROM $B \to D^*\ell\bar{\nu}_\ell$

$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2|\eta_{\mathrm{EW}}|^2(w^2-1)^{1/2}P(w)|\mathscr{F}(w)|^2$$

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{V}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} = \frac{1}{2}\epsilon^{\nu *}\varepsilon^{\mu\nu}_{\ \ \rho\sigma}v_B^\rho v_{D^*}^\sigma \boldsymbol{h_V}(w)$$

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{A}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} =$$

$$\frac{i}{2}\epsilon^{\nu *}\left[g^{\mu\nu}(1+w)\boldsymbol{h_{A_1}}(w) - v_B^\nu\left(v_B^\mu \boldsymbol{h_{A_2}}(w) + v_{D^*}^\mu \boldsymbol{h_{A_3}}(w)\right)\right]$$

- Discard CLN and provide data in a parametrisation independent way.
- BGL is the appropriate framework for FFs fits.

Boyd, Grinstein, Lebed, Phys. Rev. Lett.74, 4603 (1995)

$$F(z) = \frac{1}{P_F(z)\phi_F(z)}\sum_{n=0}^{\infty} a_n z^n \text{ with } \sum|a_n|^2 = 1$$

- Truncation and the related uncertainties require careful consideration.

Grinstein, Kobach, Phys. Lett.B771, 359 (2017); Bordone, Jung, van Dyk (2019), Eur.Phys.J.C 80 (2020) 2, 74; Bernlochneret al., Phys. Rev.D95, 11, 115008 (2017); Gambino, Jung, Schacht, Phys. Lett.B795, 386 (2019); Bernlochner, Z. Ligeti and D. J. Robinson, Phys. Rev.D100, 1, 013005 (2019)

- Dispersive method approach

Di Carlo et al., Phys. Rev. D104, 5, 054502 (2021); Martinelli, Simula, Vittorio, Phys.Rev.D 104 (2021) 9, 094512

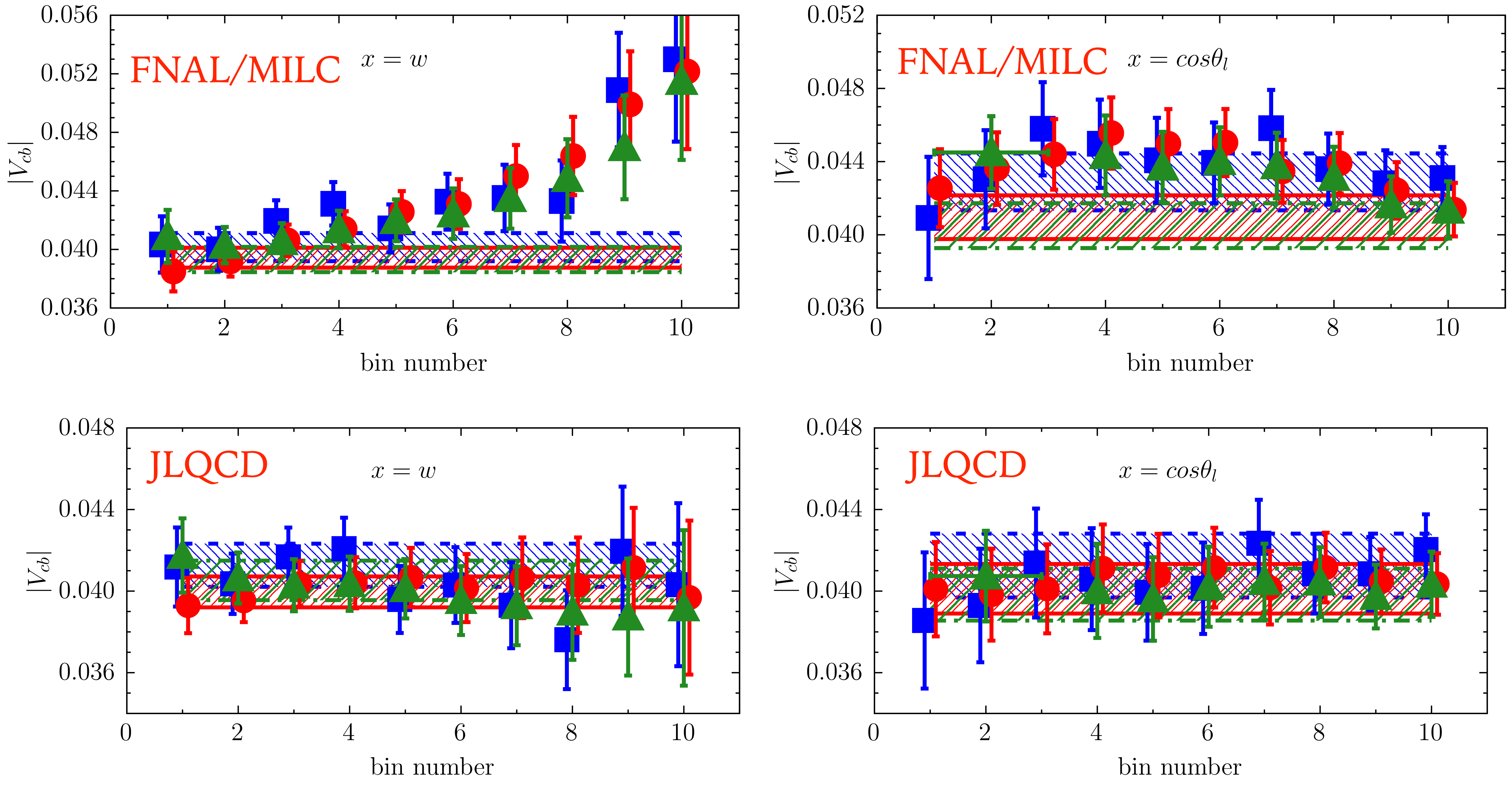
- Lattice form factors at non-zero recoils

EXCLUSIVE $|V_{cb}|$ FROM $B \to D^* \ell \bar{\nu}_\ell$

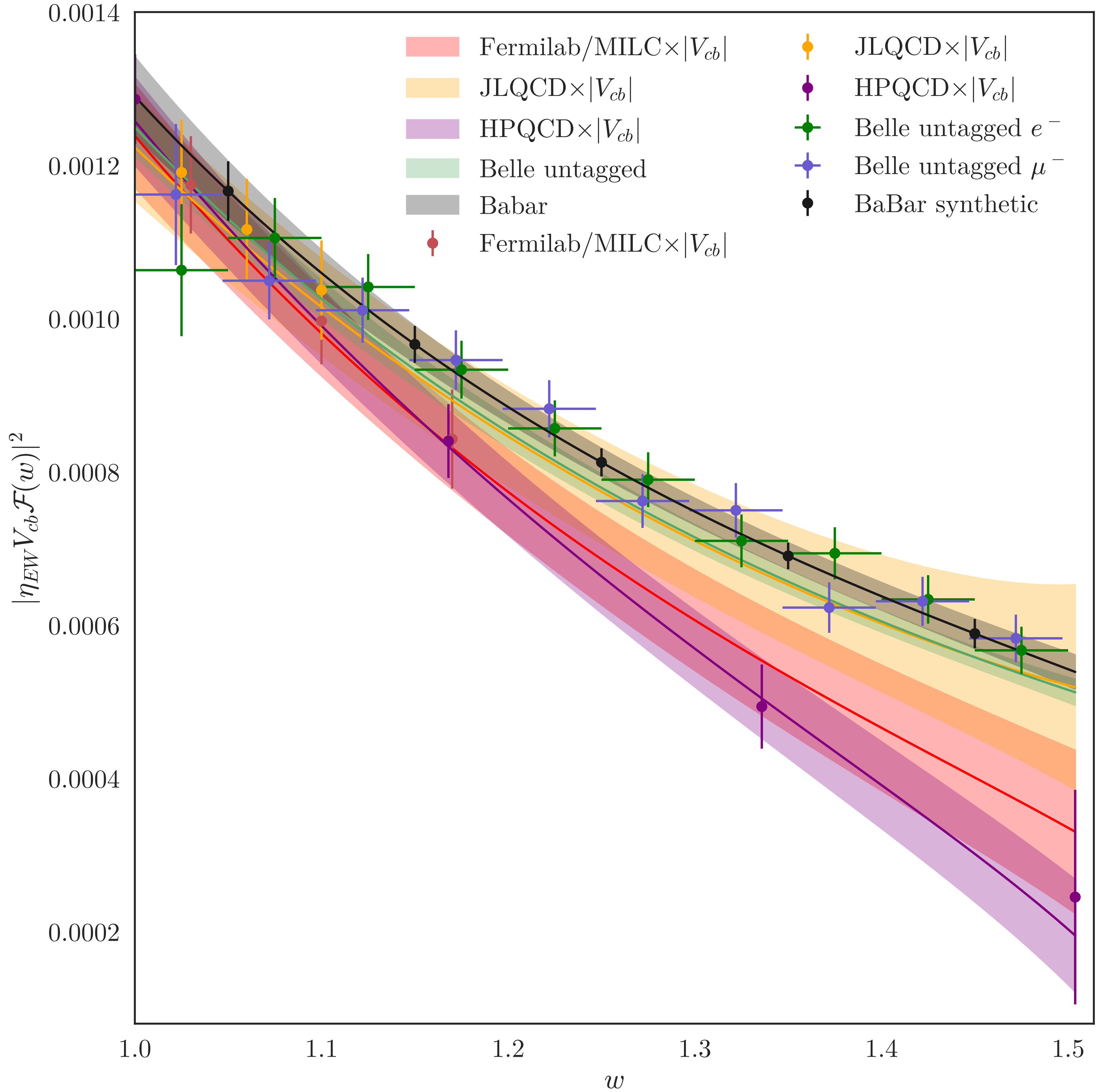
- Bin-per-bin exclusive $|V_{cb}|$ extraction (Dispersive Method & BGL)

  Martinelli, Simula, Vittorio, Eur.Phys.J. C 84 (2024) 4, 400
- Differential distributions from

Belle, Phys. Rev. D 108 (2023) 012002, Belle II, Phys. Rev. D 108 (2023) 092013, Belle, Phys. Rev. D 100 (2019) 052007



I do not show $x = \cos\theta_v$ and $x = \chi$



plot by A. Vaquero

$$|V_{cb}| = 38.17\,(85) \times 10^{-3}$$ FNAL/MILC hep-lat/2105.14019

$$|V_{cb}| = 39.19\,(90) \times 10^{-3}$$ JLQCD hep-lat/2306.05657

$$|V_{cb}| = 39.83\,(87) \times 10^{-3}$$ HPQCD hep-lat/2304.03137v2

CONCLUSIONS AND OUTLOOK

- Inclusive $|V_{cb}|$ extraction from q^2 moments is **robust and gives consistent results** with older data on $\langle E_l^2 \rangle$, $\langle M_X^{2n} \rangle$. Recent N3LO calculations leads to 1.2% uncertainty on $|V_{cb}|$.
- **What's next:** Measure all kin. moments as a function of q^2_{cut} and E_{cut} in a single analysis: capture full experimental correlations (also w/ and w/o FSR QED effects).
- New measurements of Br$(B \to X\ell\bar{\nu}_\ell)$ are also necessary!
- **LHCb can enter the inclusive business** with M_X moments in $B_s \to X_{cs}\ell\bar{\nu}_\ell$.
- First calculations of inclusive decays on the lattice but not mature yet.
  Validate, complement and improve the HQE.
- $B \to D^*$ FFs at non-zero recoil from three lattice group.
- New data published on differential distributions of $B \to D^*\ell\bar{\nu}_\ell$.
- JLQCD seems to give a more consistent picture but **the situation is still puzzling**.