# Matching of EFT to UV complete models

**Javier Fuentes-Martín**
University of Granada

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# The Effective Field Theory approach

EFTs are essential to interpret experimental observations

UV physics

$$\mathscr{L}_{\mathrm{EFT}}(\eta_L) = \mathscr{L}_{d=4}(\eta_L) + \sum_{\ell=0}^{\infty}\sum_{n=5}^{\infty}\sum_{k} \frac{C_{n,k}^{(\ell)}}{(16\pi^2)^{\ell}\,\Lambda^{n-4}} O_{n,k}(\eta_L)$$

- Bottom → Up

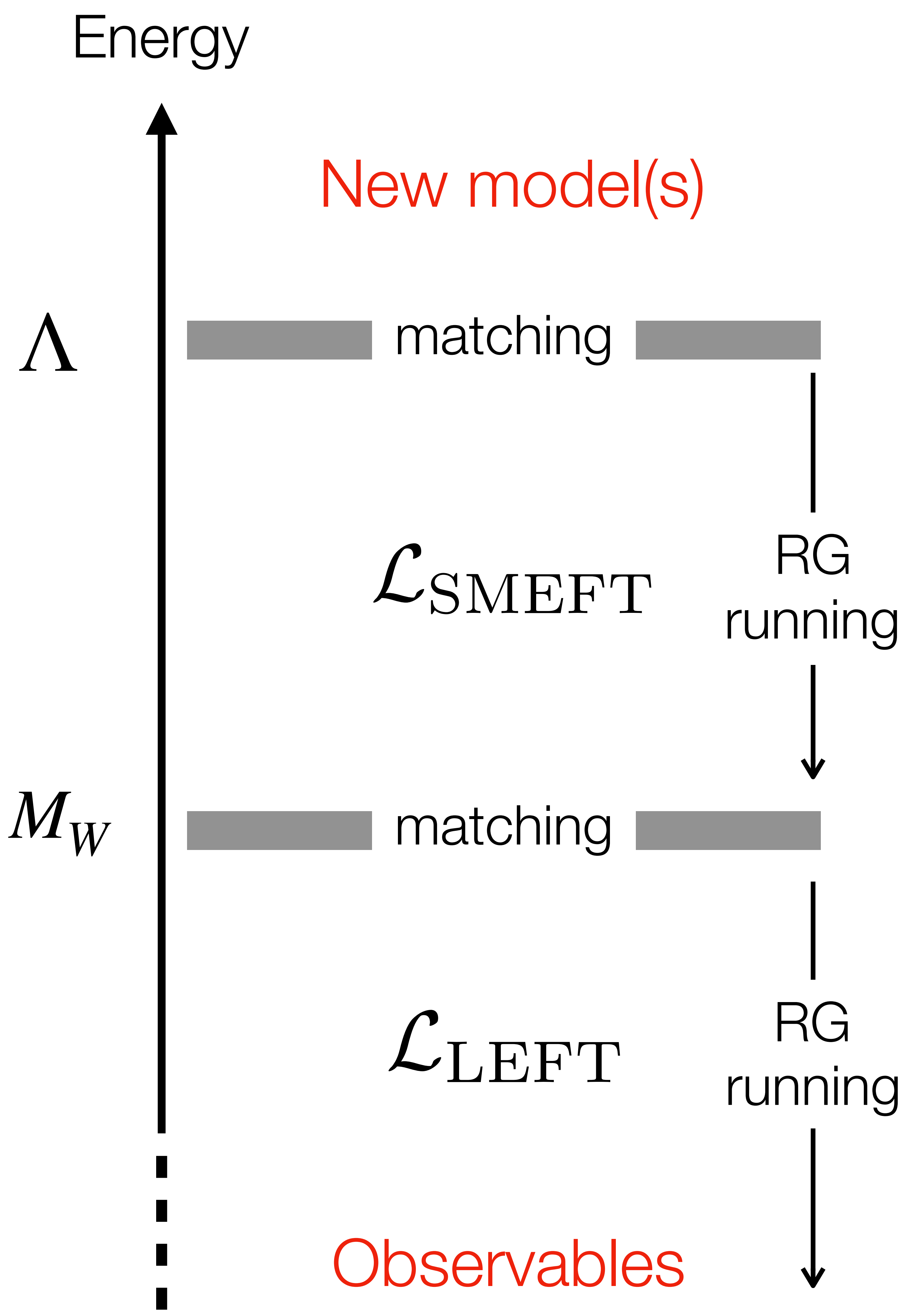
  EFTs offer a model comprehensive (“model independent”) approach to study deviations form the SM, organized in a double expansion in $E/\Lambda$ and loop orders.

- Top → Down

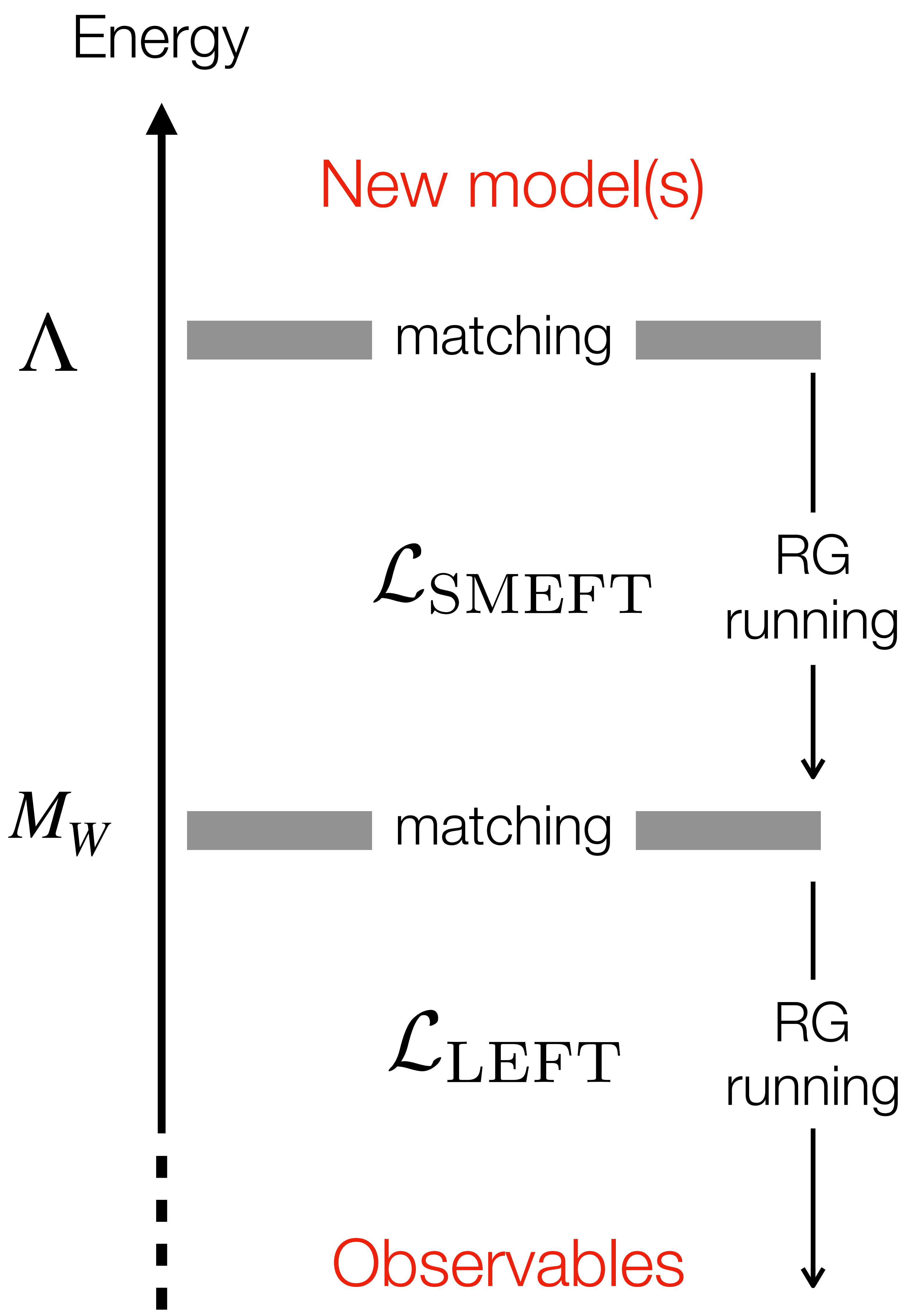
  (B)SM computations of experimental observables are multi-scale problems:
  Precision requires using EFTs ( Renormalization Group (RG) resummation of large logs )

  Multiple BSM models share the same EFT, so many computations are reusable ( “compute once for all” )

# The rise of automation



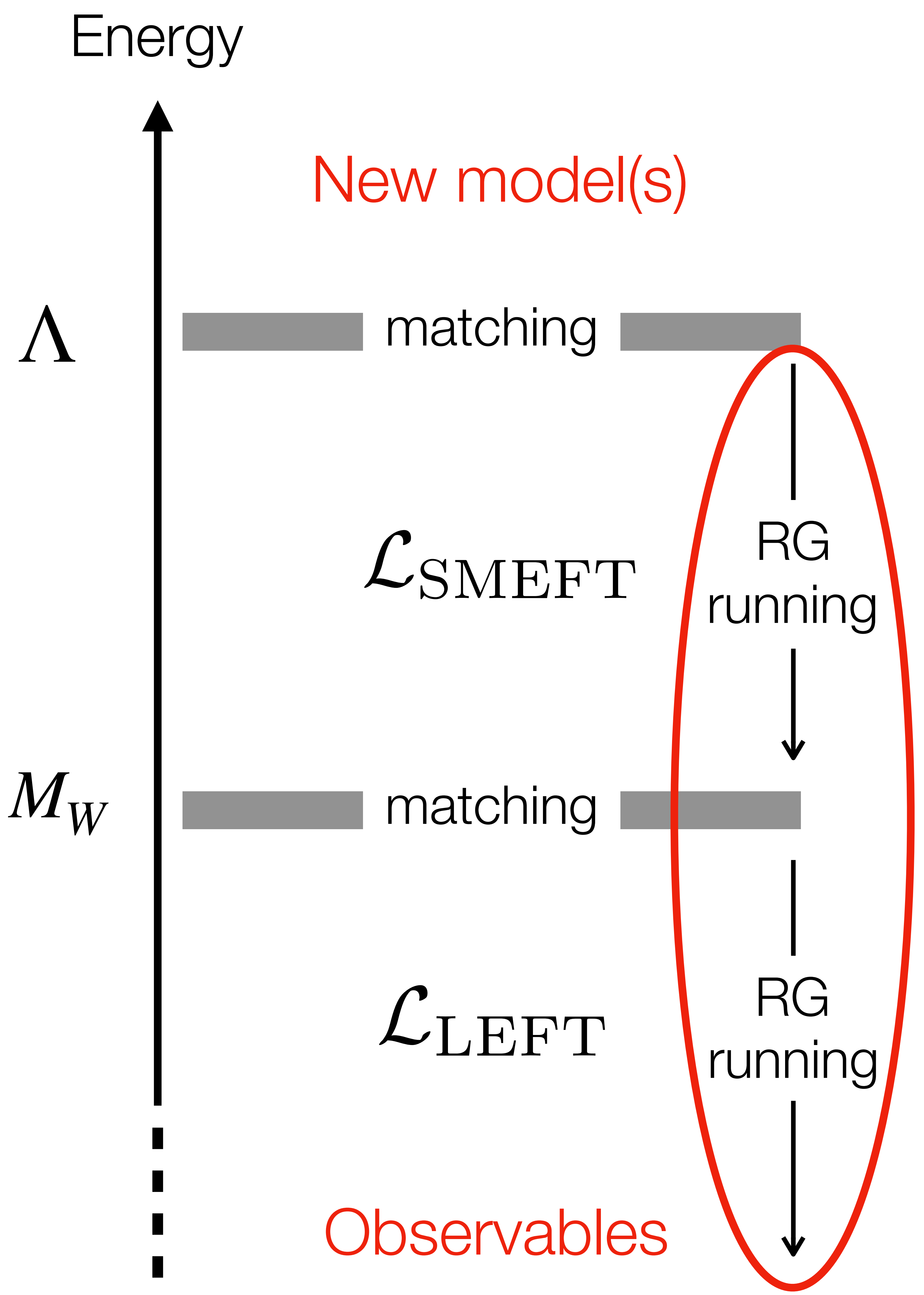
# The rise of automation




**Main motivation**

The vast landscape of BSM models and the repetitive nature of EFT computations call for automated solutions

# The rise of automation




DsixTools

JFM et al. '17 & '21

wilson

Aebischer et al. '18

"Hard-coded" one-loop results based on:

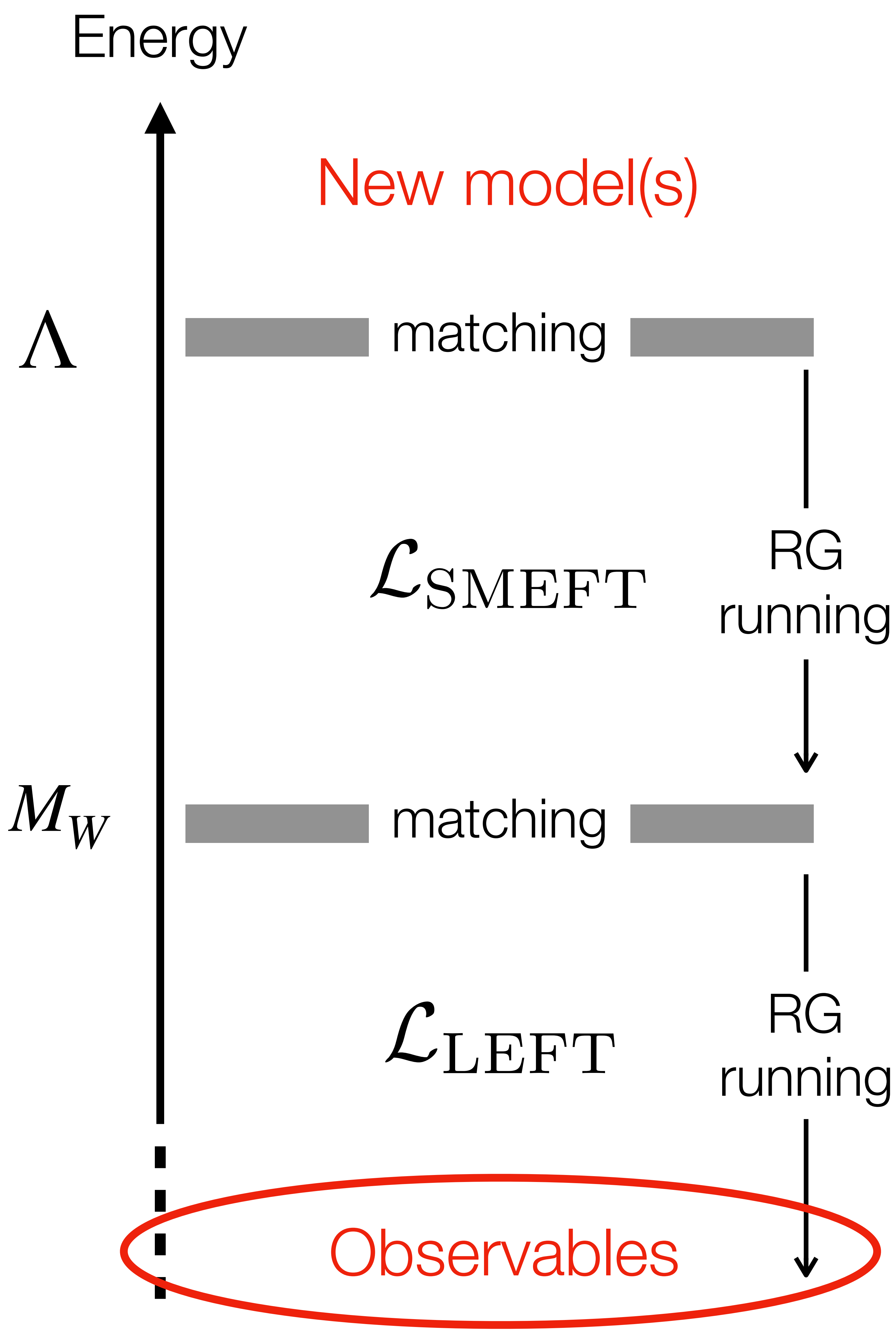
SMEFT running: Jenkins et al. '13, '14; Alonso et al. '14

LEFT basis: Jenkins et al. '18

SMEFT-LEFT matching: Dekens, Stoffer '19

LEFT running: Jenkins et al. '18

# The rise of automation




SMEFT likelihood ( smelli )

Aebischer et al. '18

flavio

Straub '16

HighPT

Allwicher et al. '22

HEPfit

De Blas et al. '19

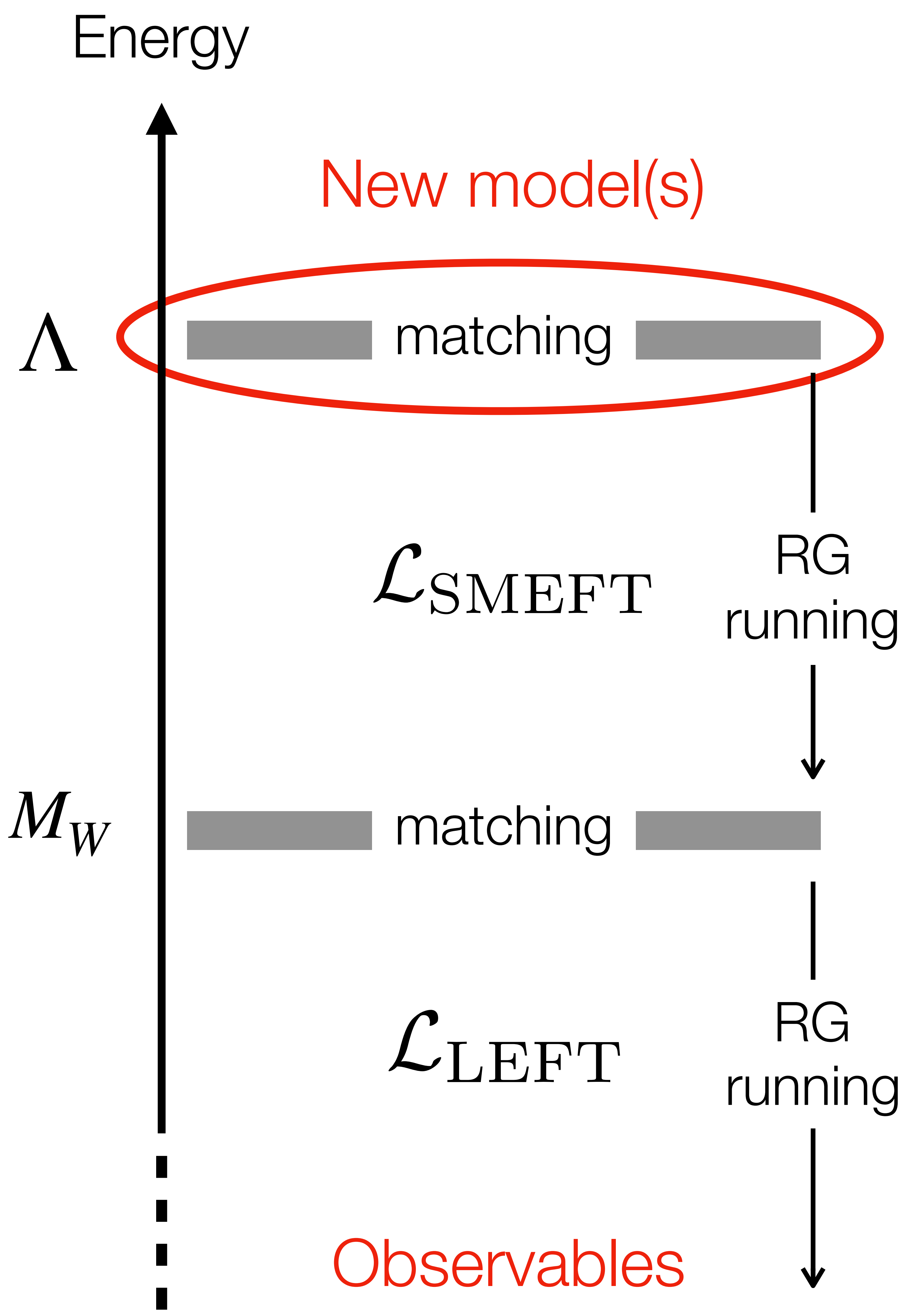
SMEFiT

Giani et al. '23

+ others

Involvement of experimental collaborations into this program is crucial

# The rise of automation



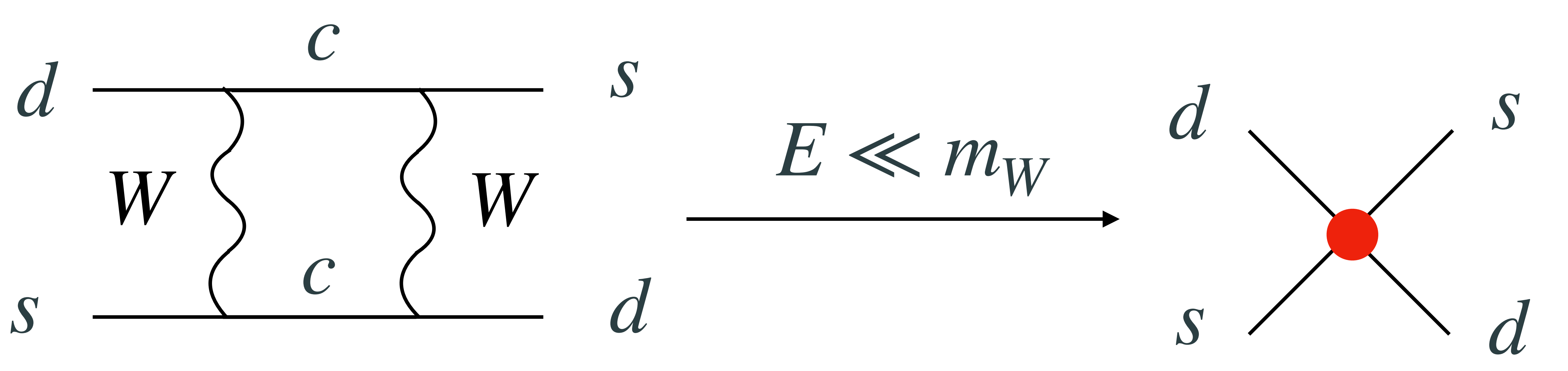

Much progress has been made:

- Tree-level matching to the SMEFT is a solved problem

  [ de Blas, Criado, Pérez-Victoria, Santiago, '17 ]

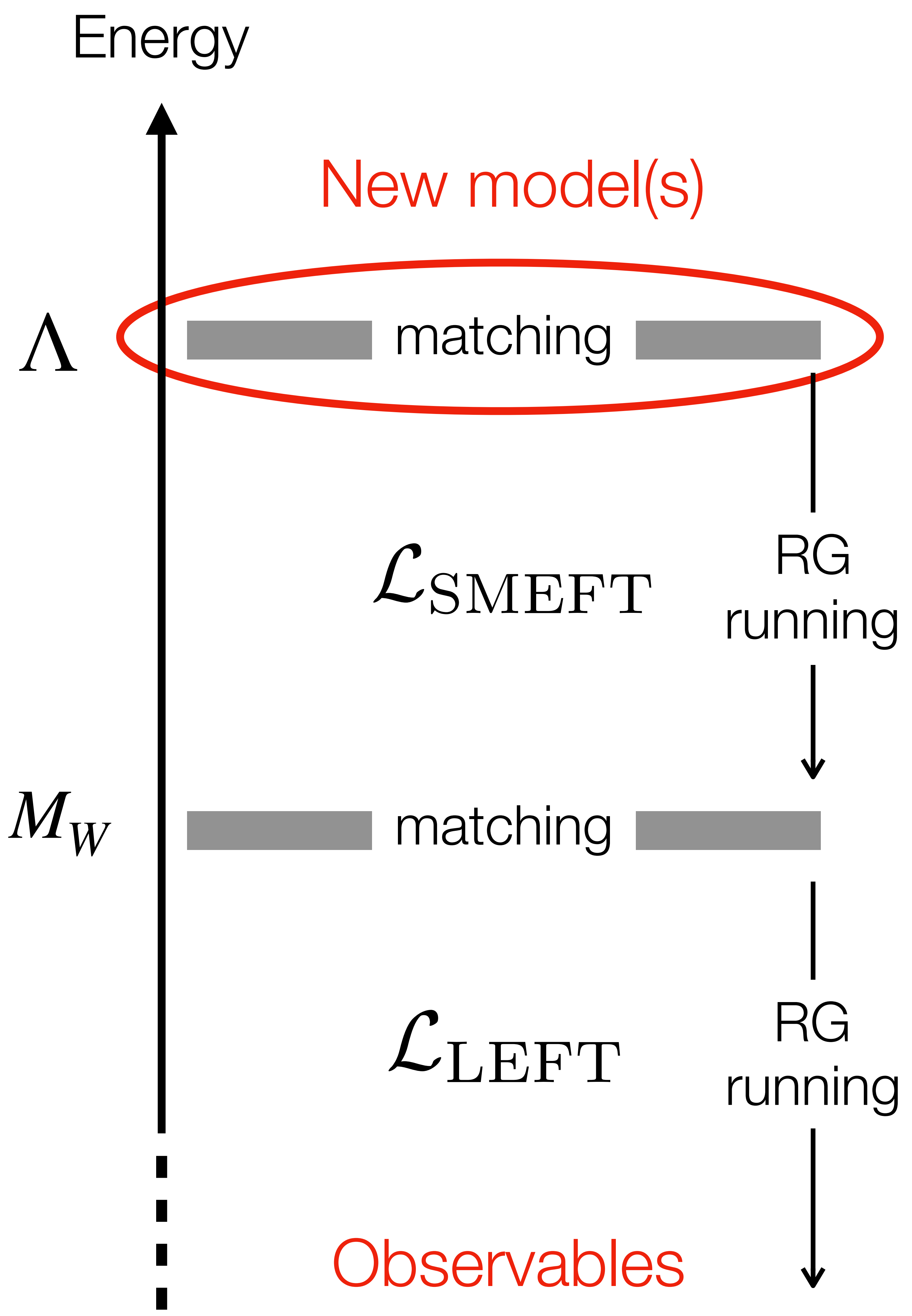
  MatchingTools: [ Criado '17 ]

- One-loop can be the leading effect in important processes. E.g., in the SM




Similarly, in BSM models: dipoles, FCNCs, EW precision…

# The rise of automation



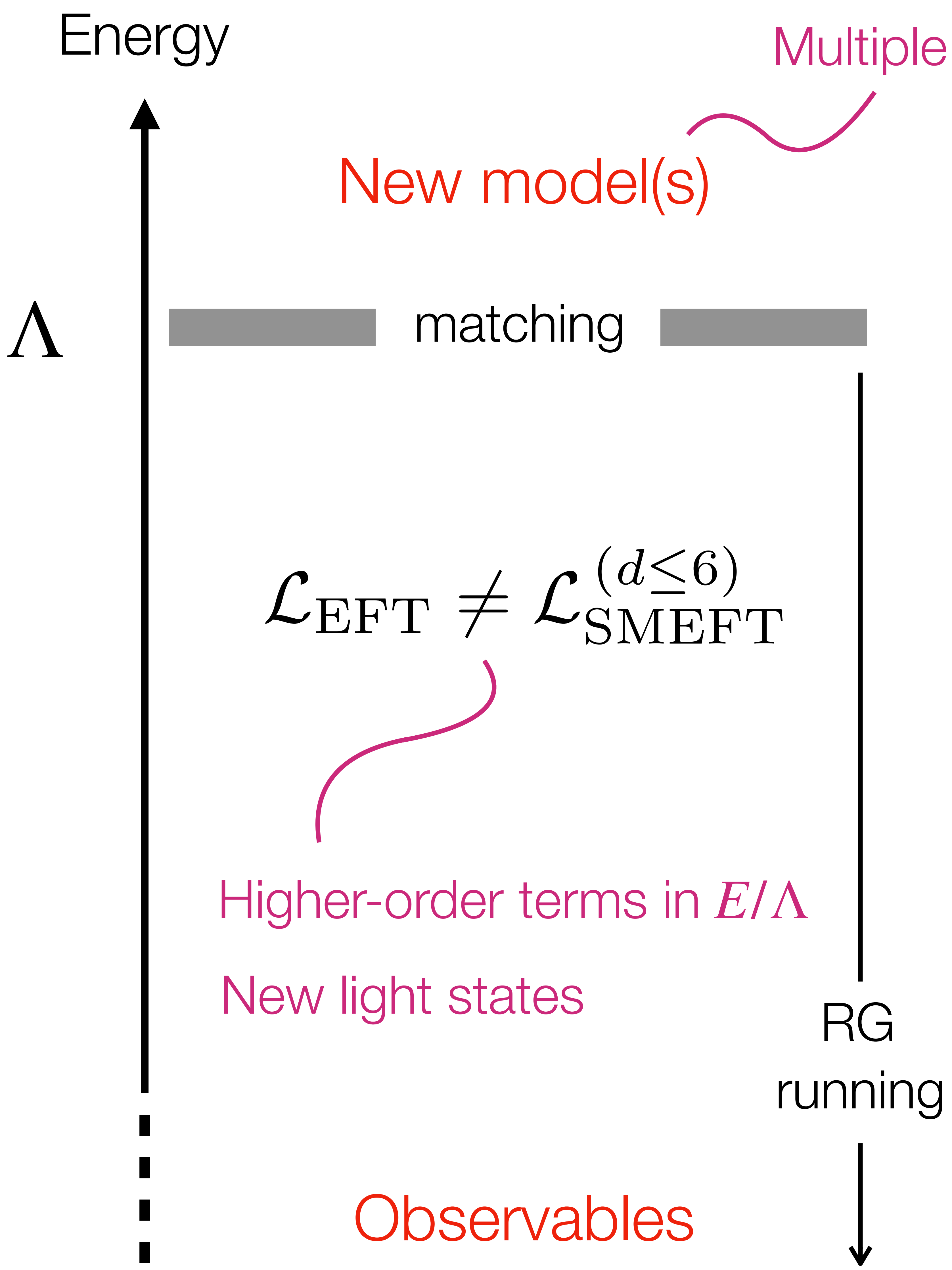

matchmakereft

Carmona et al. '22

JFM et al. '23

Automated one-loop matching of *many* models

# The rise of automation




## Present limitations

Full automation only for simpler scenarios (no heavy vectors!)

Some steps/approaches require prior knowledge of the target EFT

(Automated) inclusion of higher-loop orders is (so far) non-trivial

# EFT matching

The path-integral approach in a nutshell

# The traditional approach to matching

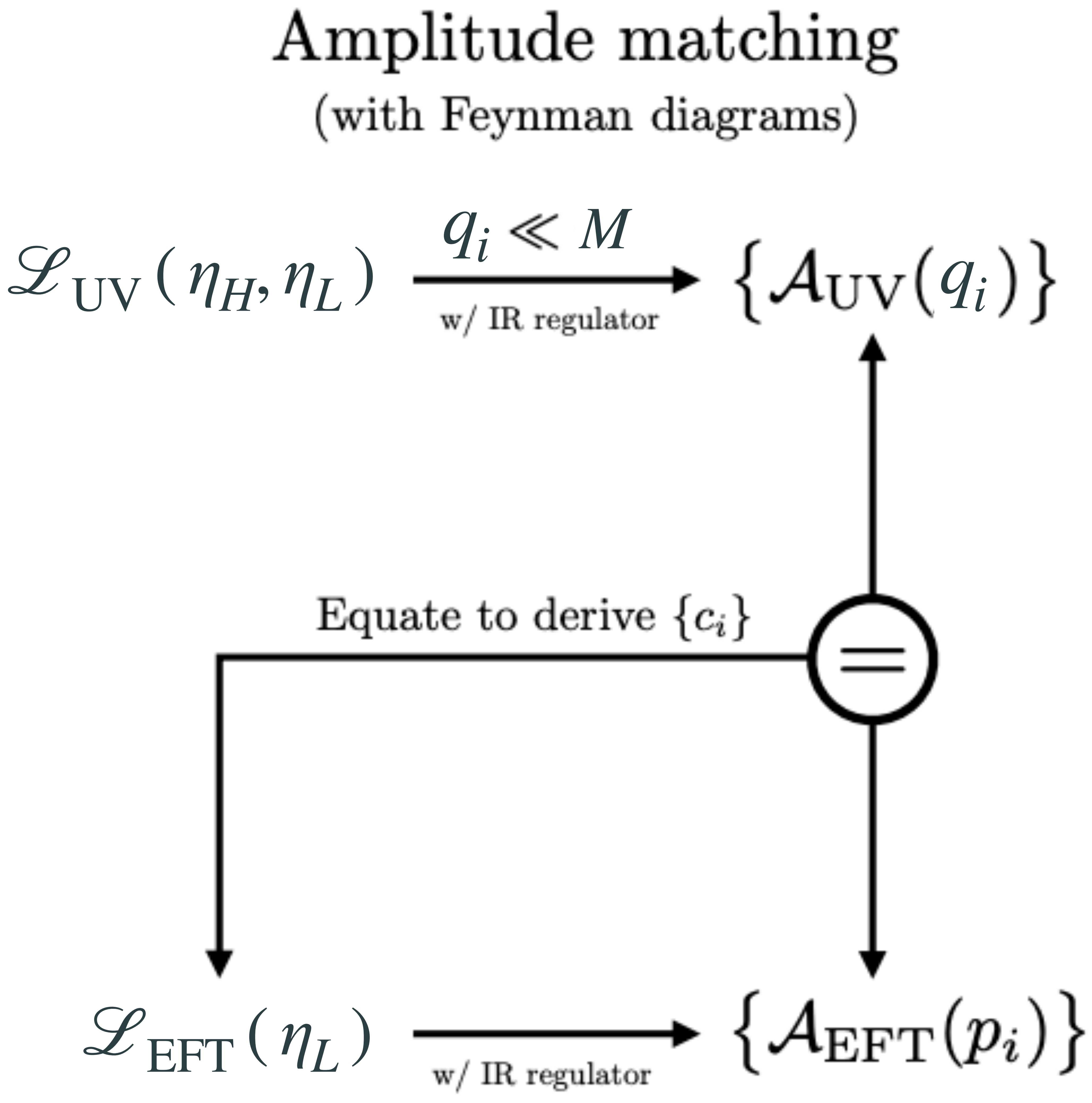



Figure from Cohen, Lu, Zhang, '21

- Well-established procedure to any loop order
- Matching usually done off-shell: Additional redundancies but need to consider 1LPI diagrams only
- Explicit breaking of gauge symmetry in intermediate steps
- Need a priori knowledge of the EFT Lagrangian in off-shell d-dimensional basis (e.g. with Fierz related ops.)

SMEFT basis in Gherardi, Marzocca, Venturini, '20; Carmona, Lazopoulos, Olgoso, Santiago, '21

# Functional matching

- **Lagrangian:** $\mathscr{L}_{\mathrm{UV}}$ with fields $\eta = (\eta_H \; \eta_L)^\intercal$ and hierarchy $m_H \gg m_L$

- **Background field method:** shift all fields $\eta \to \hat{\eta} + \eta$

  $\hat{\eta}$ : background fields (satisfy the quantum EOM) [ Tree lines in Feynman graphs ]

  $\hat{\eta}$ : quantum fluctuations [ Loop lines in Feynman graphs ]

- **Quantum effective action:**

$$e^{i\,\Gamma_{\mathrm{UV}}[\hat{\eta}]} = \int \mathscr{D}\eta \; \exp\left( i \int \mathrm{d}^d x \; \mathscr{L}_{\mathrm{UV}}(\eta + \hat{\eta}) \right)$$

**Goal:** Evaluate the path integral (“integrate out” the quantum configurations) and isolate the EFT contribution

# Functional matching

- **Expanding the Lagrangian in $\eta$:**

$$\mathscr{L}_{\mathrm{UV}}(\hat{\eta}+\eta) = \mathscr{L}_{\mathrm{UV}}(\hat{\eta}) + \left.\frac{\delta \mathscr{L}_{\mathrm{UV}}}{\delta \eta_i}\right|_{\eta=\hat{\eta}} \eta_i + \frac{1}{2}\bar{\eta}_i \left.\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta \eta_j\, \delta \bar{\eta}_i}\right|_{\eta=\hat{\eta}} \eta_j + \mathscr{O}(\eta^3)$$

# Functional matching

- **Expanding the Lagrangian in $\eta$:**

$$\mathscr{L}_{\text{UV}}(\hat{\eta}+\eta) = \boxed{\mathscr{L}_{\text{UV}}(\hat{\eta})} + \left.\frac{\delta \mathscr{L}_{\text{UV}}}{\delta \eta_i}\right|_{\eta=\hat{\eta}} \eta_i + \frac{1}{2}\bar{\eta}_i \left.\frac{\delta^2 \mathscr{L}_{\text{UV}}}{\delta \eta_j\, \delta \bar{\eta}_i}\right|_{\eta=\hat{\eta}} \eta_j + \mathscr{O}(\eta^3)$$

- **Tree-level:** $\mathscr{L}_{\text{EFT}}^{(0)} = \mathscr{L}_{\text{UV}}\big(\hat{\eta}_L, \hat{\eta}_H(\hat{\eta}_L)\big)$

  — Substitute $\hat{\eta}_H$ by its EOM expanded in $m_H^{-1}$

$$\left.\frac{\delta \mathscr{L}_{\text{UV}}}{\delta \eta_H}\right|_{\eta=\hat{\eta}} = 0$$

# Functional matching

- **Expanding the Lagrangian in $\eta$:**

$$\mathscr{L}_{\rm UV}(\hat\eta+\eta) = \boxed{\mathscr{L}_{\rm UV}(\hat\eta)} + \overset{0}{\cancel{\frac{\delta \mathscr{L}_{\rm UV}}{\delta \eta_i}\bigg|_{\eta=\hat\eta}}}\, \eta_i + \frac{1}{2}\bar\eta_i \frac{\delta^2 \mathscr{L}_{\rm UV}}{\delta \eta_j\, \delta\bar\eta_i}\bigg|_{\eta=\hat\eta} \eta_j + \mathcal{O}(\eta^3)$$

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  — Substitute $\hat{\eta}_H$ by its EOM expanded in $m_H^{-1}$

$$\left.\frac{\delta \mathscr{L}_{\mathrm{UV}}}{\delta \eta_H}\right|_{\eta=\hat{\eta}} = 0$$

- **1-loop:** $e^{i\Gamma^{(1)}_{\mathrm{UV}}[\hat{\eta}]} = \int \mathscr{D}\eta \, \exp\left(i \int \mathrm{d}^d x \, \bar{\eta}_i Q_{ij} \eta_j\right) \quad \Longrightarrow \quad \Gamma^{(1)}_{\mathrm{UV}}[\hat{\eta}] = -\, i \ln \mathrm{SDet}\, Q^{-1/2} = \frac{i}{2} \mathrm{STr} \ln Q$

Gaussian integration

# Functional matching

- **Expanding the Lagrangian in $\eta$:**

$$\mathscr{L}_{\rm UV}(\hat\eta+\eta) = \boxed{\mathscr{L}_{\rm UV}(\hat\eta)} + \underbrace{\cancel{\frac{\delta \mathscr{L}_{\rm UV}}{\delta \eta_i}\bigg|_{\eta=\hat\eta}}}_{0}\,\eta_i + \frac{1}{2}\bar\eta_i \underbrace{\boxed{\frac{\delta^2 \mathscr{L}_{\rm UV}}{\delta \eta_j\,\delta\bar\eta_i}\bigg|_{\eta=\hat\eta}}}_{\equiv\, Q_{ij}}\,\eta_j + \boxed{\mathcal{O}(\eta^3)}$$

Higher-loop orders (more later)

- **Tree-level:** $\mathscr{L}^{(0)}_{\rm EFT} = \mathscr{L}_{\rm UV}\big(\hat\eta_L, \hat\eta_H(\hat\eta_L)\big)$

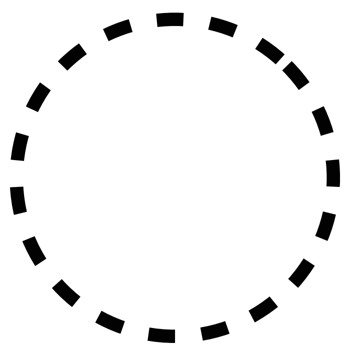
— Substitute $\hat\eta_H$ by its EOM expanded in $m_H^{-1}$

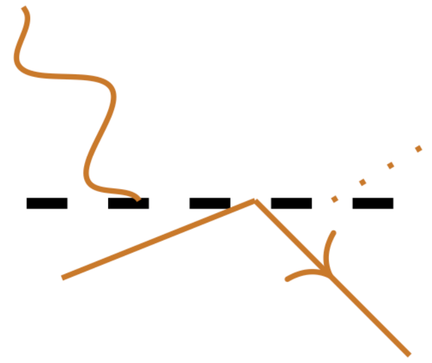
$$\frac{\delta \mathscr{L}_{\rm UV}}{\delta \eta_H}\bigg|_{\eta=\hat\eta} = 0$$

- **1-loop:** $e^{i\Gamma^{(1)}_{\rm UV}[\hat\eta]} = \int \mathscr{D}\eta\ \exp\left(i\int {\rm d}^d x\ \bar\eta_i Q_{ij}\eta_j\right) \Longrightarrow \Gamma^{(1)}_{\rm UV}[\hat\eta] = -\,i \ln {\rm SDet}\, Q^{-1/2} = \frac{i}{2}\,{\rm STr} \ln Q$

Gaussian integration

# Evaluating supertraces

- **Supertraces:** $\Gamma^{(1)}_{\mathrm{UV}}[\hat{\eta}] = \frac{i}{2}\,\mathrm{STr}\ln Q = \pm\frac{i}{2}\int\frac{d^dk}{(2\pi)^d}\,\langle k\,|\,\mathrm{tr}\ln Q[\hat{\eta}]\,|\,k\rangle \;=\pm\frac{i}{2}\ln$ 

$Q\,[\hat{\eta}] = \Big($  $\Big)^{-1}$

Dressed with bkg. fields

**Disclaimer:** (quantum) effective action $\neq$ EFT action!

Integrate out *all* quantum effects!
[ non-local ]

Integrate out *only* heavy particles
[ local ]

# The EFT Lagrangian comes from the hard part

We can separate $\Gamma^{(1)}_{\rm UV}$ in two regions ( for $q^2, m^2 \ll M^2$ ): hard ( $p^2 \sim M^2$ ) & soft ( $p^2 \sim m^2$ )

Method of regions: Beneke, Smirnov '97, Jantzen '11

$$\Gamma^{(1)}_{\rm UV} = \Gamma^{(1)}_{\rm UV}\Big|_{\rm hard} + \Gamma^{(1)}_{\rm UV}\Big|_{\rm soft}$$

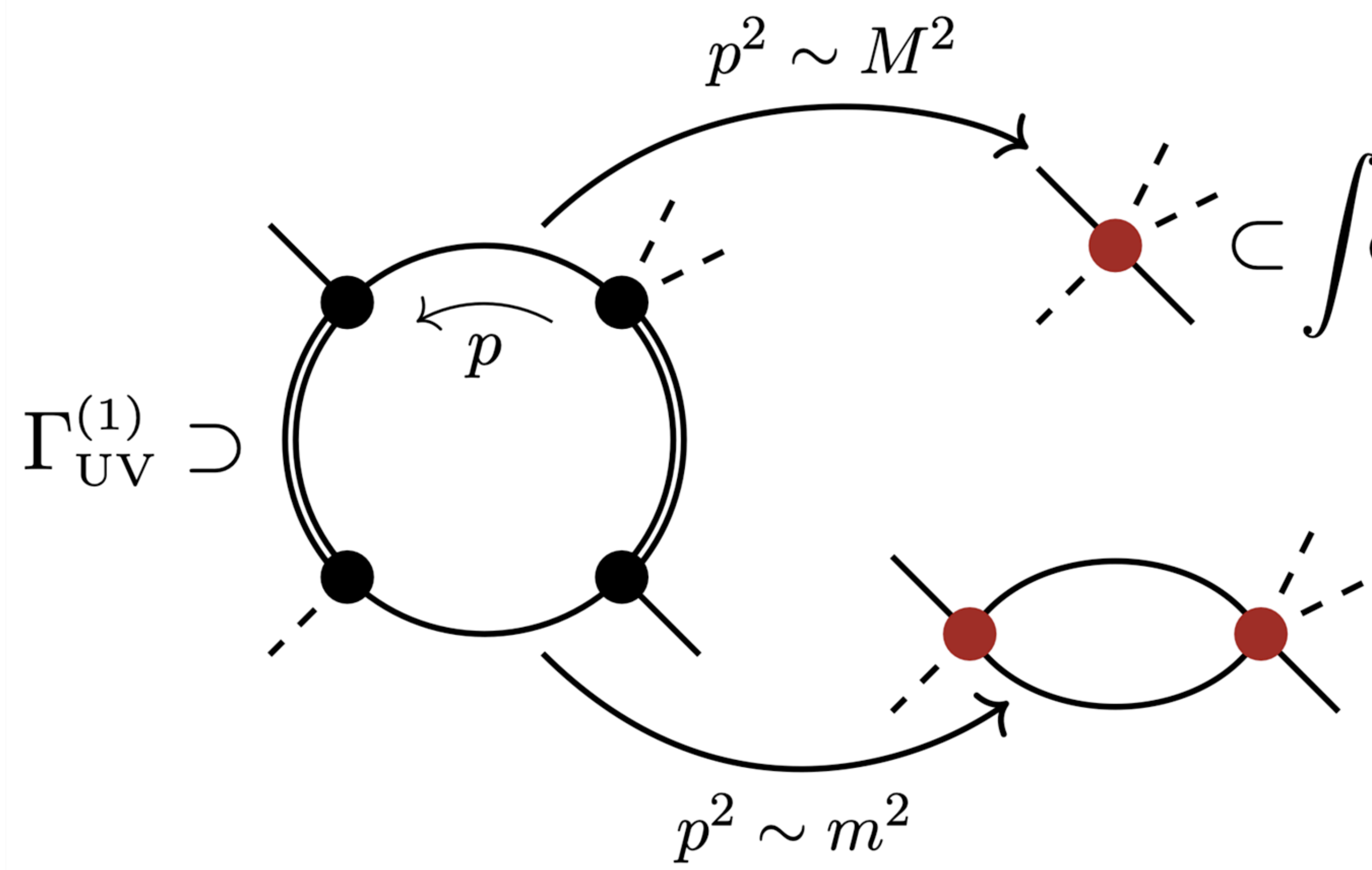
Heavy field loops

Mixed field loops

Light field loops

If only the hard part of the loop is considered, we get the EFT Lagrangian *directly*

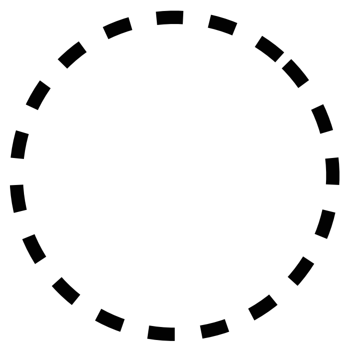
JFM, Portolés, Ruiz-Femenía, '16

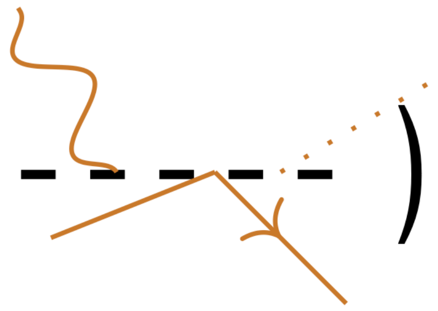



$$\subset \int d^4x\, \mathcal{L}^{(1)}_{\rm EFT} = \Gamma^{(1)}_{\rm UV}\Big|_{\rm hard}$$

$$\subset \int [\mathcal{D}\eta_{\rm L}]\, e^{i\int d^4x\, \mathcal{L}^{(0)}_{\rm EFT}} = \Gamma^{(1)}_{\rm EFT} = \Gamma^{(1)}_{\rm UV}\Big|_{\rm soft}$$

# Evaluating supertraces

- **Supertraces:** $\displaystyle\int d^dx\, \mathscr{L}_{\rm EFT} = \Gamma^{(1)}_{\rm UV}\Big|_{\rm hard} = \frac{i}{2}\,{\rm STr}\ln Q\Big|_{\rm hard} = \pm\frac{i}{2}\ln$  $\Big|_{\rm hard}$

$Q\,[\hat{\eta}] = \Big($  $\Big)^{-1}$

Dressed with bkg. fields

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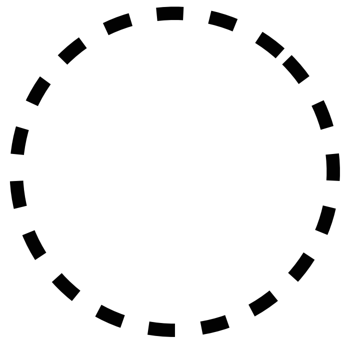
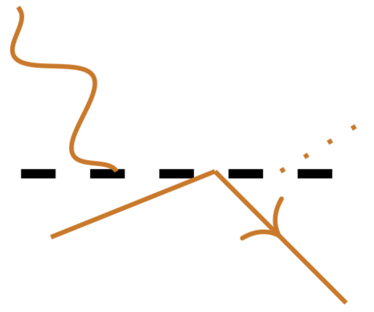
- **Supertraces:** $\displaystyle \int d^dx\, \mathscr{L}_{\rm EFT} = \Gamma^{(1)}_{\rm UV}\Big|_{\rm hard} = \frac{i}{2}\,{\rm STr}\ln Q\Big|_{\rm hard} = \pm\frac{i}{2}\ln$  $\Big|_{\rm hard}$

$Q\,[\hat{\eta}] = \Big($  $\Big)^{-1}$

Dressed with bkg. fields

- **Fluctuation operator:** $\displaystyle Q_{ij} \equiv \frac{\delta^2 \mathscr{L}_{\rm UV}}{\delta\eta_j\,\delta\bar{\eta}_i}\Bigg|_{\eta=\hat{\eta}} = \delta_{ij}\Delta_i^{-1} - X_{ij} = \Delta_i^{-1}(\delta_{ij} - \Delta_i X_{ij})$

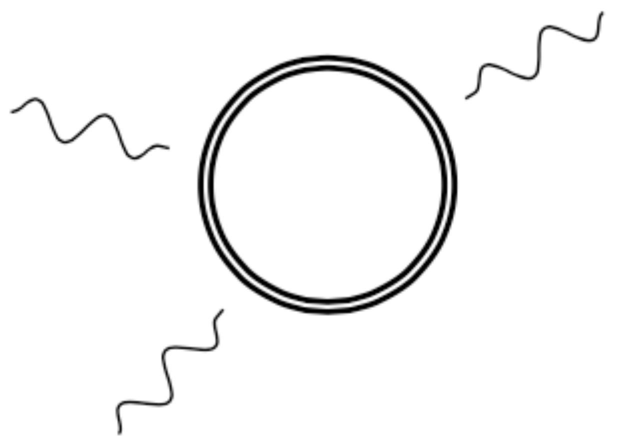
interaction terms

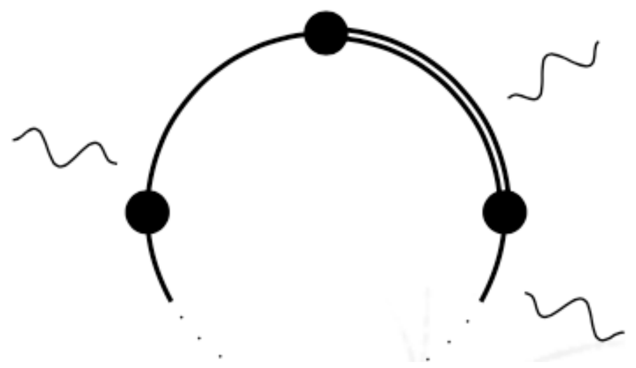
propagators

$$\Delta_i^{-1} = \begin{cases} -(D^2 + M_i^2) \\ i\gamma^\mu D_\mu - M_i \\ g^{\mu\nu}(D^2 + M_i^2) \end{cases}$$

Expanding the logarithm and taking $\Delta X$ at most $\mathcal{O}(m_H^{-1})$

$$\int d^dx\, \mathscr{L}_{\rm EFT} = \Gamma^{(1)}_{\rm UV}\Big|_{\rm hard} = \frac{i}{2}\,{\rm STr}\,\ln\,\Delta^{-1}\Big|_{\rm hard} - \frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\,{\rm STr}\left[(\Delta X)^n\right]\Big|_{\rm hard}$$





Covariant evaluation:

Chan '86; Cheyette '88; Gaillard '86

# Going beyond one loop

[ JFM, Thomsen, Palavrić, 2311.13630 ]

$$C_{ijk}[\hat{\eta}] \equiv \left.\frac{\delta^3 \mathscr{L}_{\mathrm{UV}}}{\delta\eta_i\,\delta\eta_j\,\delta\eta_k}\right|_{\eta=\hat{\eta}}$$

$$\Gamma_{\mathrm{UV}}[\hat{\eta}] = S_{\mathrm{UV}}[\hat{\eta}] - i\ln\int\mathscr{D}\eta\exp\left[i\left(\frac{\eta^2}{2}Q[\hat{\eta}] + \frac{\eta^3}{3!}C[\hat{\eta}] + \frac{\eta^4}{4!}D[\hat{\eta}] + \ldots\right)\right]$$

$$D_{ijkl}[\hat{\eta}] \equiv \left.\frac{\delta^4 \mathscr{L}_{\mathrm{UV}}}{\delta\eta_i\,\delta\eta_j\,\delta\eta_k\,\delta\eta_l}\right|_{\eta=\hat{\eta}}$$

$$= S_{\mathrm{UV}}[\hat{\eta}] - i\ln\int\mathscr{D}\eta e^{\frac{i}{2}(Q[\hat{\eta}]+Q[\hat{\eta}]^{(1)})\,\eta^2}\left[1 + \frac{i}{24}\eta^4 D[\hat{\eta}] - \frac{1}{72}\eta^6 C^2[\hat{\eta}] + \mathscr{O}(\hbar^3)\right]$$

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[ JFM, Thomsen, Palavrić, 2311.13630 ]

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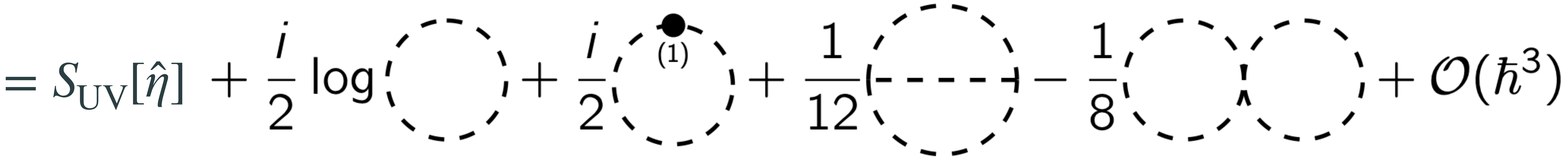
$$= S_{\mathrm{UV}}[\hat{\eta}] + \frac{1}{2}\mathrm{STr}\ln Q + \frac{i\hbar^2}{2}Q^{-1}_{ij}Q^{(1)}_{ij} - \frac{\hbar^2}{8}Q^{-1}_{ij}\,D_{ijkl}\,Q^{-1}_{kl} + \frac{\hbar^2}{12}C_{ijk}\,Q^{-1}_{il}Q^{-1}_{jm}Q^{-1}_{kn}\,C_{lmn} + \mathcal{O}(\hbar^3)$$



# General EFT matching formula

The EFT action is given by

$$S_{\mathrm{EFT}}[\phi] = \Gamma_{\mathrm{UV}}[\hat{\Phi}, \phi]\Big|_{\mathrm{hard}} \qquad \frac{\delta \Gamma_{\mathrm{UV}}\big|_{\mathrm{hard}}}{\delta \Phi}[\hat{\Phi}, \phi] = 0$$

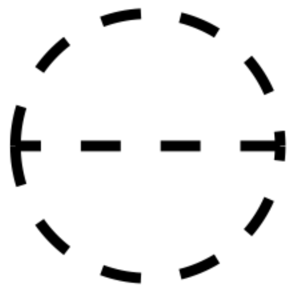
$\Phi$ : Heavy

$\phi$ : Light

"hard" denotes the part where all loop momenta are $p \sim \Lambda$ ( incl. tree-level contributions ) ( * )

- Already used at one loop order [ JFM, Portolés, Ruiz-Femenía, 1607.02142; Z. Zhang 1610.00710 ]
- Explicit proof to two-loop order [ JFM, Thomsen, Palavrić, 2311.13630 ]
- The hard region is by far the easiest to compute ( only vacuum diagrams at zero external momenta )
- Enables functional matching at any loop order

# Two-loop functional evaluation

How to evaluate  ?

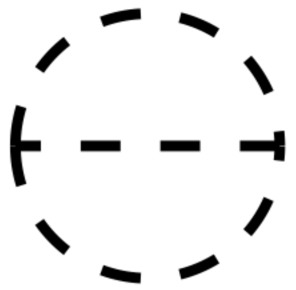
bkg. field dependent

diff. op. bkg. field dependent

$$G_{\mathrm{ss.}} = \sum_{m,n,m',n'} (-1)^{m+n} \sum_{r=0}^{\infty} \frac{(-i)^r}{r!} \int_x \int_{k\ell} C^{(\underline{m},\underline{n})}_{abc}(x)\, \partial_x^{\underline{r}} C^{(\underline{m}',\underline{n}')}_{def}(x) \left[\partial_{\underline{k}}^{r} Q^{-1}_{cf}(x,\, P_x - k - \ell)\right]$$
$$\times \left[(P_x + k)^{\underline{m}} Q^{-1}_{ad}(x,\, P_x + k)(P_x + k)^{\underline{m}'}\right]\left[(P_x + \ell)^{\underline{n}} Q^{-1}_{be}(x,\, P_x + \ell)(P_x + \ell)^{\underline{n}'}\right]$$

- Hard-region expansion (matching and UV counterterms) of dressed propagators, similar to 1-loop order
- The method has been applied to a toy-model matching and RG calculation @ 2-loop order

# Two-loop functional evaluation

How to evaluate  ?

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U(x, x')

$$\times \left[(P_x + k)^{\underline{m}} Q^{-1}_{ad}(x,\, P_x + k)(P_x + k)^{\underline{m}'}\right]\left[(P_x + \ell)^{\underline{n}} Q^{-1}_{be}(x,\, P_x + \ell)(P_x + \ell)^{\underline{n}'}\right]$$

U(x, x') U(x, x')

- Hard-region expansion (matching and UV counterterms) of dressed propagators, similar to 1-loop order
- The method has been applied to a toy-model matching and RG calculation @ 2-loop order
- Explicit gauge covariance with Wilson lines

polynomial

$$P_x^{\mu_1} \cdots P_x^{\mu_n} U(x, y)\Big|_{y=x} = p_n\big(P_x,\, G_{\mu\nu}\big)$$

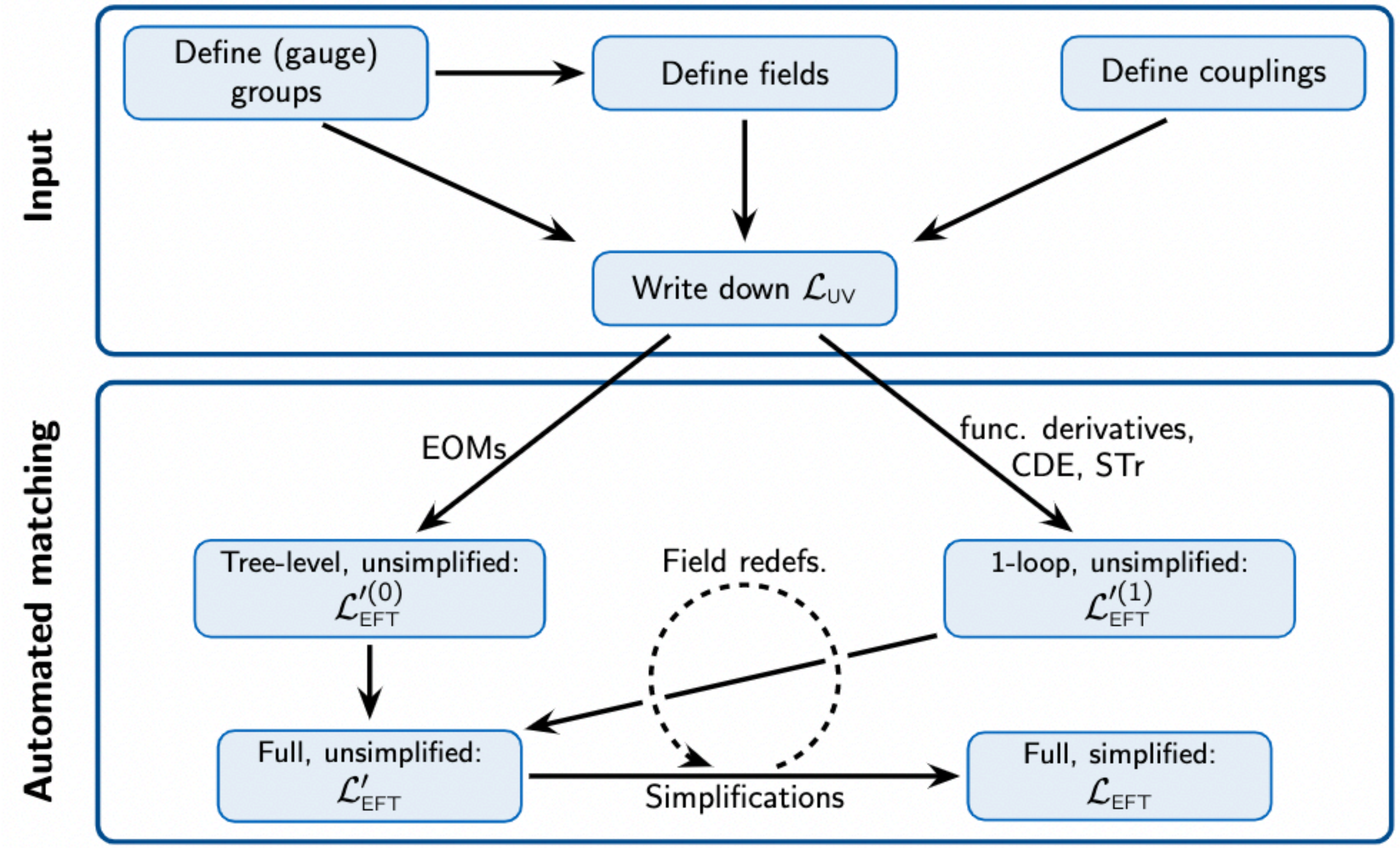
[ JFM, Palavric, Sánchez, Thomsen, WIP ]

MATCHETE

To make your way through the BSM jungle

# The Matchete package

MATCHETE is a Mathematica package aimed at fully automating EFT matching and RG evolution of arbitrary weakly-coupled UV theories using functional methods




[ JFM, König, Pagès, Thomsen, Wilsch, 2212.04510 ]

**Matchete v0.2** now publicly available:

- One-loop matching of *any* model with heavy scalars and/or fermions
- Simple and intuitive input/output
- Handles *all* group theory ( any group and reps )
- Fully automated simplifications to EFT basis ( IBP, field redefinitions/EOMs,… )

# An example

SM + Vector-like lepton

$$E \sim (\mathbf{1},\mathbf{1})_{-1}$$

## Setup

### SM Lagrangian

In[3]:=
```
LSM = LoadModel["SM"];
```

### Define new field

In[4]:=
```
DefineField[EE, Fermion, Charges → {U1Y[-1]}, Mass → {Heavy, ME}]
```

### Define new coupling

In[5]:=
```
DefineCoupling[yE, EFTOrder → 0, Indices → {Flavor}]
```

### Write interactions

In[6]:=
```
Lint = -yE[p] × Bar@l[i, p] ** PR ** EE[] × H[i] // PlusHc;
Lint // NiceForm
```

Out[7]//NiceForm=

$$-\overline{yE}^{p}\,\bar{H}_{i}\,\left(\overline{EE}\cdot P_{L}\cdot l^{ip}\right)-yE^{p}\,H^{i}\,\left(\bar{l}^{p}_{i}\cdot P_{R}\cdot EE\right)$$

### Define full UV Lagrangian

In[8]:=
```
LUV = LSM + FreeLag[EE] + Lint;
LUV // NiceForm
```

Out[9]//NiceForm=

$$-\frac{1}{4}B^{\mu\nu 2}-\frac{1}{4}G^{\mu\nu A 2}-\frac{1}{4}W^{\mu\nu I 2}+D_{\mu}\bar{H}_{i}\,D_{\mu}H^{i}+\mu^{2}\,\bar{H}_{i}\,H^{i}+i\left(\bar{d}^{p}_{a}\cdot\gamma_{\mu}P_{R}\cdot D_{\mu}d^{ap}\right)+i\left(\bar{e}^{p}\cdot\gamma_{\mu}P_{R}\cdot D_{\mu}e^{p}\right)+$$
$$i\left(\overline{EE}\cdot\gamma_{\mu}\cdot D_{\mu}EE\right)-ME\left(\overline{EE}\cdot EE\right)+i\left(\bar{l}^{p}_{i}\cdot\gamma_{\mu}P_{L}\cdot D_{\mu}l^{ip}\right)+i\left(\bar{q}^{p}_{ai}\cdot\gamma_{\mu}P_{L}\cdot D_{\mu}q^{aip}\right)+i\left(\bar{u}^{p}_{a}\cdot\gamma_{\mu}P_{R}\cdot D_{\mu}u^{ap}\right)-$$
$$\frac{1}{2}\lambda\,\bar{H}_{i}\,\bar{H}_{j}\,H^{i}\,H^{j}-\overline{Yd}^{pr}\,\bar{H}_{i}\left(\bar{d}^{r}_{a}\cdot P_{L}\cdot q^{aip}\right)-\overline{Ye}^{pr}\,\bar{H}_{i}\left(\bar{e}^{r}\cdot P_{L}\cdot l^{ip}\right)-Ye^{pr}\,H^{i}\left(\bar{l}^{p}_{i}\cdot P_{R}\cdot e^{r}\right)-Yd^{pr}\,H^{i}\left(\bar{q}^{p}_{ai}\cdot P_{R}\cdot d^{ar}\right)-$$
$$Yu^{pr}\,\bar{H}_{i}\left(\bar{q}^{p}_{aj}\cdot P_{R}\cdot u^{ar}\right)\varepsilon^{ji}-\overline{Yu}^{pr}\,H^{j}\left(\bar{u}^{r}_{a}\cdot P_{L}\cdot q^{aip}\right)\bar{\varepsilon}_{ij}-\overline{yE}^{p}\,\bar{H}_{i}\left(\overline{EE}\cdot P_{L}\cdot l^{ip}\right)-yE^{p}\,H^{i}\left(\bar{l}^{p}_{i}\cdot P_{R}\cdot EE\right)$$

# An example: SM + Vector-like lepton

Main matching routine

In[9]:=
```
LEFT = Match[LUV, LoopOrder → 1, EFTOrder → 6] /. ϵ^-1 → 0;
```

Simplification to on-shell basis

In[10]:=
```
LEFTOnShell = LEFT // EOMSimplify;
Length@%
```

```
» The Lagrangian contains terms of lower power than dimension 4. Defining effective couplings and assuming these terms to
  be dimension 4. Use 'PrintEffectiveCouplings' and 'ReplaceEffectiveCouplings' to recover explicit expressions.
» Added new CG cg1 with indices {Bar[SU2L[fund]], SU2L[adj], Bar[SU2L[fund]]}
```

Out[11]= 66

Select Higgs-lepton current operator

In[12]:=
```
SelectOperatorClass[LEFTOnShell, {e, Bar@e, H, Bar@H}, 1] // GreensSimplify // NiceForm
```

Out[12]//NiceForm=

$$\frac{i}{360}\,\hbar\,\frac{1}{ME^2}\left(48\,gY^4\,\delta^{pr}+5\,\overline{yE}^{s}\left(3\,yE^{t}\,\overline{Ye}^{tr}\,Ye^{sp}\left(1+6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\right)-2\,yE^{s}\,gY^2\left(13+6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\right)\delta^{pr}\right)\right)$$
$$\left(-D_\mu\bar{H}_i\,H^i\left(\bar{e}^r\cdot\gamma_\mu P_R\cdot e^p\right)+\bar{H}_i\,D_\mu H^i\left(\bar{e}^r\cdot\gamma_\mu P_R\cdot e^p\right)\right)$$

$$Q_{He}^{pr}=(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$$

# What's new since December 2022?

`v0.1.0` $\longrightarrow$ `v0.2.0`

- More robust simplification routines: flavor, symmetry-vanishing operators…
- Changed evaluation of supertraces: from CDE to Wilson lines [ Theory: JFM, Palavric, Sánchez, Thomsen, WIP ]
- Significant performance improvements!

| Version | `Match` [s] | `EOMSimplify` [s] |
|---|---|---|
| `v0.1.0` | 74 | 281 |
| `v0.2.0` | 12 | 81 |

Dimension-six one-loop matching

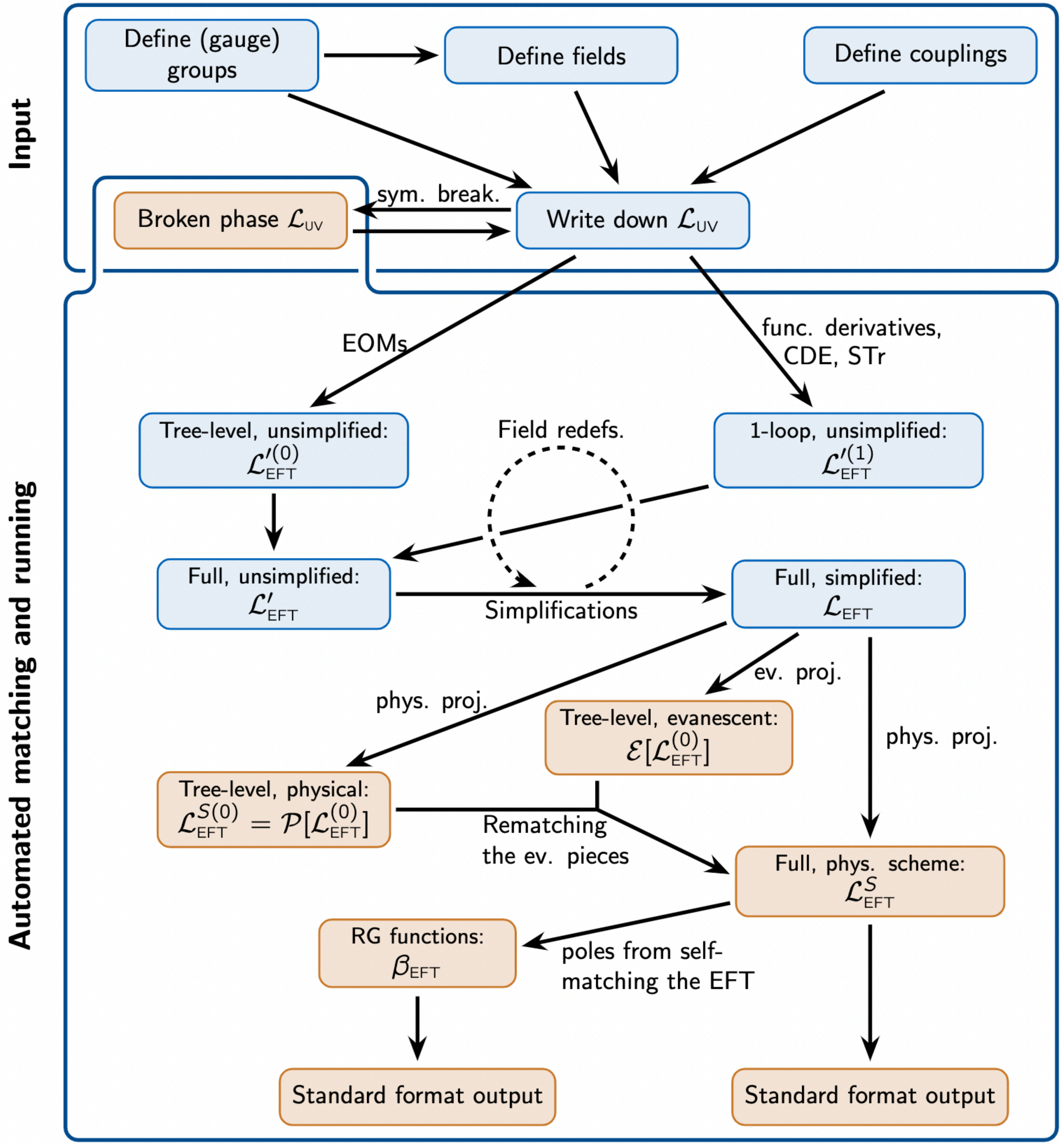
Model: SM + S1 + S3 (LQs)

CPU: Apple M3 (single core)

- Bug fixing: matching, group theory, simplifications…

**The community has been a tremendous help bringing issues to our attention!**

# Work in progress and future plans




Upcoming!

- One-loop RG computations
- Handling of evanescent contributions
- Interface with other EFT tools

Longer term:

- Heavy vectors and symmetry breaking
- Matching and running beyond one loop

# Work in progress: Counterterm evaluation and RG equations

- The functional approach can be easily adapted to extract UV divergencies
- A taste of what it will look like using **Matchete** for the SM

```
LSM // NiceForm
```

eForm=

$$-\frac{1}{4}B^{\mu\nu 2}-\frac{1}{4}G^{\mu\nu A2}-\frac{1}{4}W^{\mu\nu I2}+D_\mu\bar{H}_i D_\mu H^i+\mu^2\bar{H}_i H^i+i\left(\bar{d}^p_a\cdot\gamma_\mu P_R\cdot D_\mu d^{ap}\right)+i\left(\bar{e}^p\cdot\gamma_\mu P_R\cdot D_\mu e^p\right)+i\left(\bar{l}^p_i\cdot\gamma_\mu P_L\cdot D_\mu l^{ip}\right)+i\left(\bar{q}^p_{ai}\cdot\gamma_\mu P_L\cdot D_\mu q^{aip}\right)+i\left(\bar{u}^p_a\cdot\gamma_\mu P_R\cdot D_\mu u^{ap}\right)-$$
$$\frac{1}{2}\lambda\bar{H}_i\bar{H}_j H^i H^j-\overline{Yd}^{pr}\bar{H}_i\left(\bar{d}^r_a\cdot P_L\cdot q^{aip}\right)-\overline{Ye}^{pr}\bar{H}_i\left(\bar{e}^r\cdot P_L\cdot l^{ip}\right)-Ye^{pr}H^i\left(\bar{l}^p_i\cdot P_R\cdot e^r\right)-Yd^{pr}H^i\left(\bar{q}^p_{ai}\cdot P_R\cdot d^{ar}\right)-Yu^{pr}\bar{H}_i\left(\bar{q}^p_{aj}\cdot P_R\cdot u^{ar}\right)\varepsilon^{ji}-\overline{Yu}^{pr}H^j\left(\bar{u}^r_a\cdot P_L\cdot q^{aip}\right)\bar{\varepsilon}_{ij}$$

```
UVDivergentAction[LSM, EFTOrder → 4] // NiceForm
```

eForm=

$$\frac{1}{\epsilon}\Bigg(-\frac{41}{24}\hbar\, gY^2 B^{\mu\nu 2}+\frac{5}{2}\hbar\, gs^2 G^{\mu\nu A2}+\frac{31}{24}\hbar\, gL^2 W^{\mu\nu I2}+\frac{1}{8}\hbar\,\mu^2\left(15\,gL^2+5\,gY^2-24\,\overline{Yd}^{pr}Yd^{pr}-8\,\overline{Ye}^{pr}Ye^{pr}-24\,\overline{Yu}^{pr}Yu^{pr}-24\,\lambda\right)\bar{H}_i H^i+$$
$$\hbar\left(\frac{9}{16}gL^4-\frac{1}{16}gY^4-3\,\overline{Yd}^{pr}\overline{Yd}^{st}Yd^{pt}Yd^{sr}-\overline{Ye}^{pr}\overline{Ye}^{st}Ye^{pt}Ye^{sr}-3\,\overline{Yu}^{pr}\overline{Yu}^{st}Yu^{pt}Yu^{sr}-\frac{5}{8}\lambda\, gY^2+3\,\overline{Yd}^{pr}Yd^{pr}\lambda+\overline{Ye}^{pr}Ye^{pr}\lambda+3\,\overline{Yu}^{pr}Yu^{pr}\lambda+3\,\lambda^2-\frac{1}{8}gL^2\left(gY^2+15\,\lambda\right)\right)$$
$$\bar{H}_i\bar{H}_j H^i H^j+\hbar\left(\frac{3}{4}\left(\overline{Yd}^{ps}\overline{Yd}^{tr}Yd^{ts}-\overline{Yd}^{sr}\overline{Yu}^{pt}Yu^{st}\right)+\frac{1}{144}\overline{Yd}^{pr}\left(-27\,gL^2+192\,gs^2-17\,gY^2+216\,\overline{Yd}^{st}Yd^{st}+72\,\overline{Ye}^{st}Ye^{st}+216\,\overline{Yu}^{st}Yu^{st}\right)\right)\bar{H}_i\left(\bar{d}^r_a\cdot P_L\cdot q^{aip}\right)+$$
$$\frac{1}{16}\hbar\left(12\,\overline{Ye}^{ps}\overline{Ye}^{tr}Ye^{ts}+\overline{Ye}^{pr}\left(-3\,gL^2+7\,gY^2+24\,\overline{Yd}^{st}Yd^{st}+8\,\overline{Ye}^{st}Ye^{st}+24\,\overline{Yu}^{st}Yu^{st}\right)\right)\bar{H}_i\left(\bar{e}^r\cdot P_L\cdot l^{ip}\right)-$$
$$\frac{1}{16}\hbar\left(3\,Ye^{rp}gL^2-7\,Ye^{rp}gY^2-24\,\overline{Yd}^{st}Yd^{st}Ye^{rp}-12\,\overline{Ye}^{st}Ye^{rt}Ye^{sp}-8\,\overline{Ye}^{st}Ye^{rp}Ye^{st}-24\,Ye^{rp}\overline{Yu}^{st}Yu^{st}\right)H^i\left(\bar{l}^r_i\cdot P_R\cdot e^p\right)+$$
$$\frac{1}{144}\hbar\left(-27\,Yd^{rp}gL^2+192\,Yd^{rp}gs^2-17\,Yd^{rp}gY^2+108\,\overline{Yd}^{st}Yd^{rt}Yd^{sp}+216\,\overline{Yd}^{st}Yd^{rp}Yd^{st}+72\,Yd^{rp}\overline{Ye}^{st}Ye^{st}-108\,Yd^{sp}\overline{Yu}^{st}Yu^{rt}+216\,Yd^{rp}\overline{Yu}^{st}Yu^{st}\right)H^i\left(\bar{q}^r_{ai}\cdot P_R\cdot d^{ap}\right)+$$
$$\frac{1}{144}\hbar\left(-27\,Yu^{rp}gL^2+192\,Yu^{rp}gs^2+7\,Yu^{rp}gY^2+216\,\overline{Yd}^{st}Yd^{st}Yu^{rp}+72\,\overline{Ye}^{st}Ye^{st}Yu^{rp}-108\,\overline{Yd}^{st}Yd^{rt}Yu^{sp}+108\,\overline{Yu}^{st}Yu^{rt}Yu^{sp}+216\,\overline{Yu}^{st}Yu^{rp}Yu^{st}\right)\bar{H}_i\left(\bar{q}^r_{aj}\cdot P_R\cdot u^{ap}\right)\varepsilon^{ji}+$$
$$\hbar\left(\frac{1}{144}\overline{Yu}^{pr}\left(-27\,gL^2+192\,gs^2+7\,gY^2+216\,\overline{Yd}^{st}Yd^{st}+72\,\overline{Ye}^{st}Ye^{st}+216\,\overline{Yu}^{st}Yu^{st}\right)+\frac{3}{4}\overline{Yu}^{tr}\left(-\overline{Yd}^{ps}Yd^{ts}+\overline{Yu}^{ps}Yu^{ts}\right)\right)H^j\left(\bar{u}^r_a\cdot P_L\cdot q^{aip}\right)\bar{\varepsilon}_{ij}\Bigg)$$

*Ghost loops are not yet included

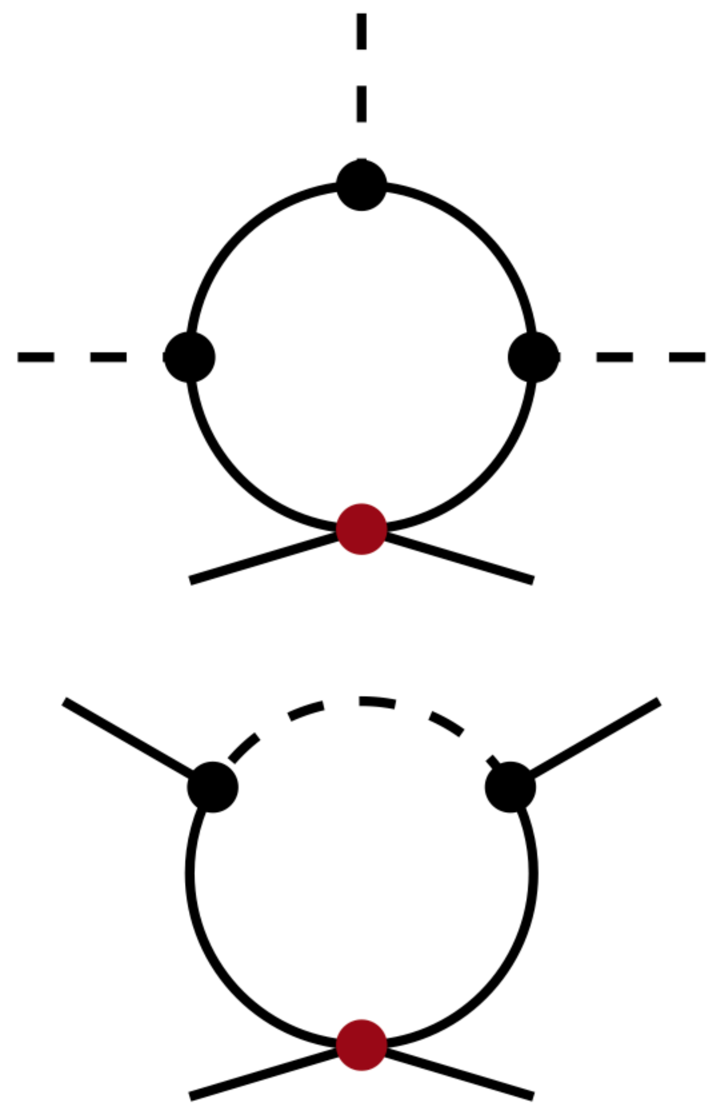
# Work in progress: Fierzing (and others) and evanescent contributions

Some operator identities (like Fierz) are only valid in strictly $d = 4$ dimensions

*Application to the SMEFT:* JFM, König, Pagès, Thomsen, Wilsch, 2211.09144

$$O_d = \underbrace{\mathcal{P}\, O_d}_{\text{phys. part}} + \underbrace{\overset{\text{id} - \mathcal{P}}{\mathcal{E}_{\mathcal{P}}}\, O_d}_{\text{ev. part}}$$

$$\mathcal{L}_{\text{EFT}} + \text{ev.} \longrightarrow \mathcal{L}_{\text{EFT}} + \Delta\mathcal{L}$$



*Sample diagrams

- Initial step: automatic identification of evanescent operators!

```
redOp = CRqe[p, r, s, t] (Bar@q[c, i, p] ** e[r]) (Bar@e[s] ** q[c, i, t]);
% // NiceForm
```

:eForm=

$\mathtt{CRqe}^{prst} \left(\bar{e}^{s} \cdot P_L \cdot q^{cit}\right) \left(\bar{q}^{p}_{ci} \cdot P_R \cdot e^{r}\right)$

```
GreensSimplify[redOp, Basis4D → Evanescent] // NiceForm
```

:eForm=

$\mathtt{CRqe}^{prst}\, E_1^{stpr} - \frac{1}{2}\, \mathtt{CRqe}^{tpsr} \left(\bar{e}^{s} \cdot \gamma_\mu P_R \cdot e^{p}\right) \left(\bar{q}^{t}_{ai} \cdot \gamma_\mu P_L \cdot q^{air}\right)$

# Work in progress: Matching to a particular SMEFT basis

Compute on-shell EFT Lagrangian from UV model

```
LEFTOnShell = Match[LUV, LoopOrder → 1, EFTOrder → 6] // EOMSimplify // AdjustWIP;
```

```
The Lagrangian contains terms of lower power than dimension 4. Defining effective couplings and assuming these
  terms to be dimension 4. Use 'PrintEffectiveCouplings' and 'ReplaceEffectiveCouplings' to recover explicit expressions.
```

Load generic SMEFT Lagrangian

```
LSMEFT = LoadModel["SMEFT"] // EOMSimplify // ShiftRenCouplings;
```

Equate the two, to solve for the SMEFT coefficients

```
MatchLagrangians[LEFTOnShell, LSMEFT] // CleanUpFlavor // TableForm // NiceForm
```

iceForm=

$\mu \to \sqrt{-2\,\hbar\,\overline{yE}^{p}\,yE^{p}\,ME^{2} - 2\,\hbar\,\frac{1}{\epsilon}\,\overline{yE}^{p}\,yE^{p}\,ME^{2} + \mu^{2} - 2\,\hbar\,\overline{yE}^{p}\,yE^{p}\,ME^{2}\,\mathrm{Log}\left[\frac{\bar{\mu}^{2}}{ME^{2}}\right]}$

$\lambda \to \lambda$

$cHBox \to -\frac{1}{30}\,\hbar\,gY^{4}\,\frac{1}{ME^{2}} - \frac{5}{24}\,\hbar\,\overline{yE}^{p}\,yE^{p}\,gL^{2}\,\frac{1}{ME^{2}} + \frac{13}{72}\,\hbar\,\overline{yE}^{p}\,yE^{p}\,gY^{2}\,\frac{1}{ME^{2}} - \frac{1}{3}\,\hbar\,\overline{yE}^{p}\,\overline{yE}^{r}\,yE^{p}\,yE^{r}\,\frac{1}{ME^{2}} + \frac{3}{2}\,\hbar\,\overline{yE}^{p}\,yE^{r}\,\overline{Ye}^{rs}\,Ye^{ps}\,\frac{1}{ME^{2}} - \frac{1}{4}\,\hbar\,\overline{yE}^{p}\,yE^{p}\,gL^{2}\,\frac{1}{ME^{2}}\,\mathrm{Log}\left[\frac{\bar{\mu}^{2}}{ME^{2}}\right] + \frac{1}{12}\,\hbar\,\overline{yE}^{p}\,yE^{p}$

$cHD \to -\frac{2}{15}\,\hbar\,gY^{4}\,\frac{1}{ME^{2}} + \frac{13}{18}\,\hbar\,\overline{yE}^{p}\,yE^{p}\,gY^{2}\,\frac{1}{ME^{2}} - \frac{1}{2}\,\hbar\,\overline{yE}^{p}\,\overline{yE}^{r}\,yE^{p}\,yE^{r}\,\frac{1}{ME^{2}} + \frac{1}{2}\,\hbar\,\overline{yE}^{p}\,yE^{r}\,\overline{Ye}^{rs}\,Ye^{ps}\,\frac{1}{ME^{2}} + \frac{1}{3}\,\hbar\,\overline{yE}^{p}\,yE^{p}\,gY^{2}\,\frac{1}{ME^{2}}\,\mathrm{Log}\left[\frac{\bar{\mu}^{2}}{ME^{2}}\right] + \hbar\,\overline{yE}^{p}\,yE^{r}\,\overline{Ye}^{rs}\,Ye^{ps}\,\frac{1}{ME^{2}}\,\mathrm{Log}\left[\frac{\bar{\mu}^{2}}{ME^{2}}\right]$

$cH \to \frac{1}{3}\,\hbar\,\overline{yE}^{p}\,\overline{yE}^{r}\,\overline{yE}^{s}\,yE^{p}\,yE^{r}\,yE^{s}\,\frac{1}{ME^{2}} + 2\,\hbar\,\overline{yE}^{p}\,\overline{yE}^{r}\,yE^{p}\,yE^{s}\,\overline{Ye}^{st}\,Ye^{rt}\,\frac{1}{ME^{2}} - 2\,\hbar\,\overline{yE}^{p}\,yE^{r}\,\overline{Ye}^{rs}\,\overline{Ye}^{tu}\,Ye^{pu}\,Ye^{ts}\,\frac{1}{ME^{2}} - \frac{5}{18}\,\hbar\,\overline{yE}^{p}\,yE^{p}\,\lambda\,gL^{2}\,\frac{1}{ME^{2}} - \hbar\,\overline{yE}^{p}\,\overline{yE}^{r}\,yE^{p}\,yE^{r}\,\lambda\,\frac{1}{ME^{2}} + \hbar\,\overline{yE}^{p}$

$cHB \to \frac{1}{8}\,\hbar\,\overline{yE}^{p}\,yE^{p}\,gY^{2}\,\frac{1}{ME^{2}}$

$cHG \to 0$

$cG \to 0$

$cHWB \to -\frac{1}{6}\,\hbar\,gL\,gY\,\overline{yE}^{p}\,yE^{p}\,\frac{1}{ME^{2}}$

$cHW \to \frac{1}{24}\,\hbar\,\overline{yE}^{p}\,yE^{p}\,gL^{2}\,\frac{1}{ME^{2}}$

$cW \to 0$

$cHBt \to 0$ SMEFT coefficients

$cHGt \to 0$

$cHWtB \to 0$

$cHWt \to 0$

$cGt \to 0$

The aim is to use the SMEFT Warsaw basis to interface with smelli and HighPT!

Aebischer et al., 1810.07698

Allwicher et al., 2207.10756

*Example model: SM + vector-like lepton $E \sim (\mathbf{1}, \mathbf{1})_{-1}$

# Summary and conclusions

- (Automated) EFT matching and RG evolution is crucial to BSM phenomenology

- Functional matching is ideal for automation ( also useful for pen-and-paper computations! )

- Huge progress towards complete (one-loop) automation: Lagrangian in, fully simplified EFT Lagrangian out

- The ultimate goal is a tool ( or chain of tools ) that fully automates
  - Matching
  - RG evolution

  **Multi-step matching**

  - Connection to observables / fit to data **Interface with other EFT pheno codes**

  streamlining future BSM analyses

https://gitlab.com/matchete/matchete

# Thank you

Matching models is about to become easy!

# Backup

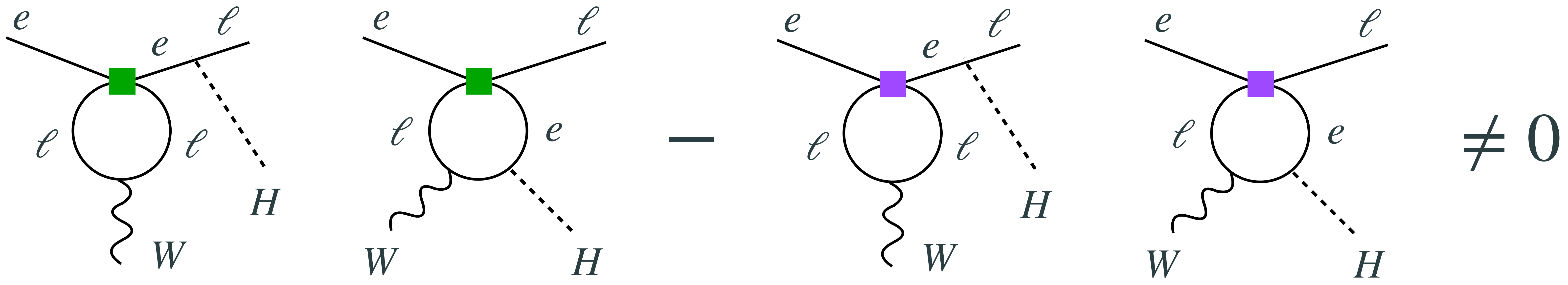
# Evanescent operators

In $d = 4$, we can use the Fierz identity $R_{\ell e} = -\frac{1}{2} Q_{\ell e}$

$$\mathscr{L}_{\text{EFT}} \supset C_{\ell e}^{prst} R_{\ell e}^{prst} \qquad R_{\ell e}^{prst} = (\bar{\ell}_p e_r)(\bar{e}_s \ell_t)$$

$$\mathscr{L}'_{\text{EFT}} \supset -\frac{1}{2} C_{\ell e}^{prst} Q_{\ell e}^{prst} \qquad Q_{\ell e}^{prst} = (\bar{\ell}_p \gamma_\mu \ell_t)(\bar{e}_s \gamma^\mu e_r)$$

so that $\mathscr{L}_{\text{EFT}} = \mathscr{L}'_{\text{EFT}}$ at tree level. However,



# Evanescent operators

In $d = 4$, we can use the Fierz identity $R_{\ell e} = -\frac{1}{2} Q_{\ell e}$

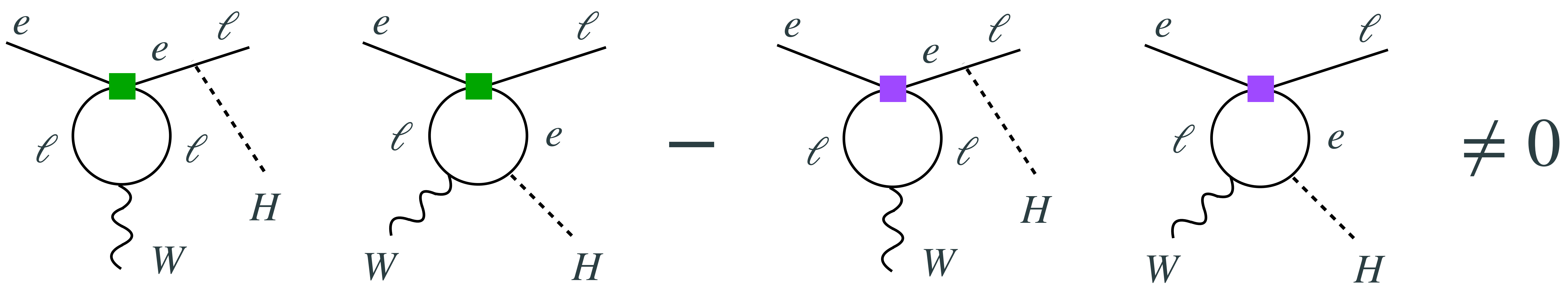
$$\mathscr{L}_{\text{EFT}} \supset C^{prst}_{\ell e} R^{prst}_{\ell e}$$

$$\mathscr{L}'_{\text{EFT}} \supset -\frac{1}{2} C^{prst}_{\ell e} Q^{prst}_{\ell e}$$

$$R^{prst}_{\ell e} = (\bar{\ell}_p e_r)(\bar{e}_s \ell_t)$$

$$Q^{prst}_{\ell e} = (\bar{\ell}_p \gamma_\mu \ell_t)(\bar{e}_s \gamma^\mu e_r)$$

so that $\mathscr{L}_{\text{EFT}} = \mathscr{L}'_{\text{EFT}}$ at <u>tree level</u>. However,




In $d = 4 - 2\epsilon$, there is an evanescent operator that also contributes to the amplitude

$$R^{prst}_{\ell e} = -\frac{1}{2} Q^{prst}_{\ell e} + E^{prst}_{\ell e} \qquad E^{prst}_{\ell e} \xrightarrow{\epsilon \to 0} 0 \qquad E^{prst}_{\ell e} \to -\frac{g_L\, y_e^{ts}}{128\pi^2} Q^{pr}_{eW} + \text{[ many other contributions ]}$$

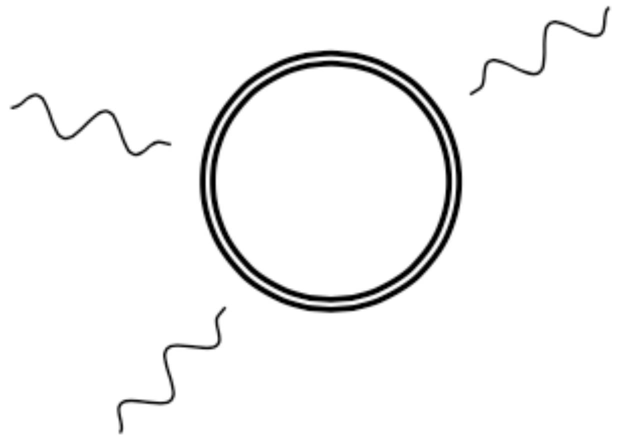
# Linear simplifications

Example: Integrating out a heavy fermion in the fundamental representation of SU(3)

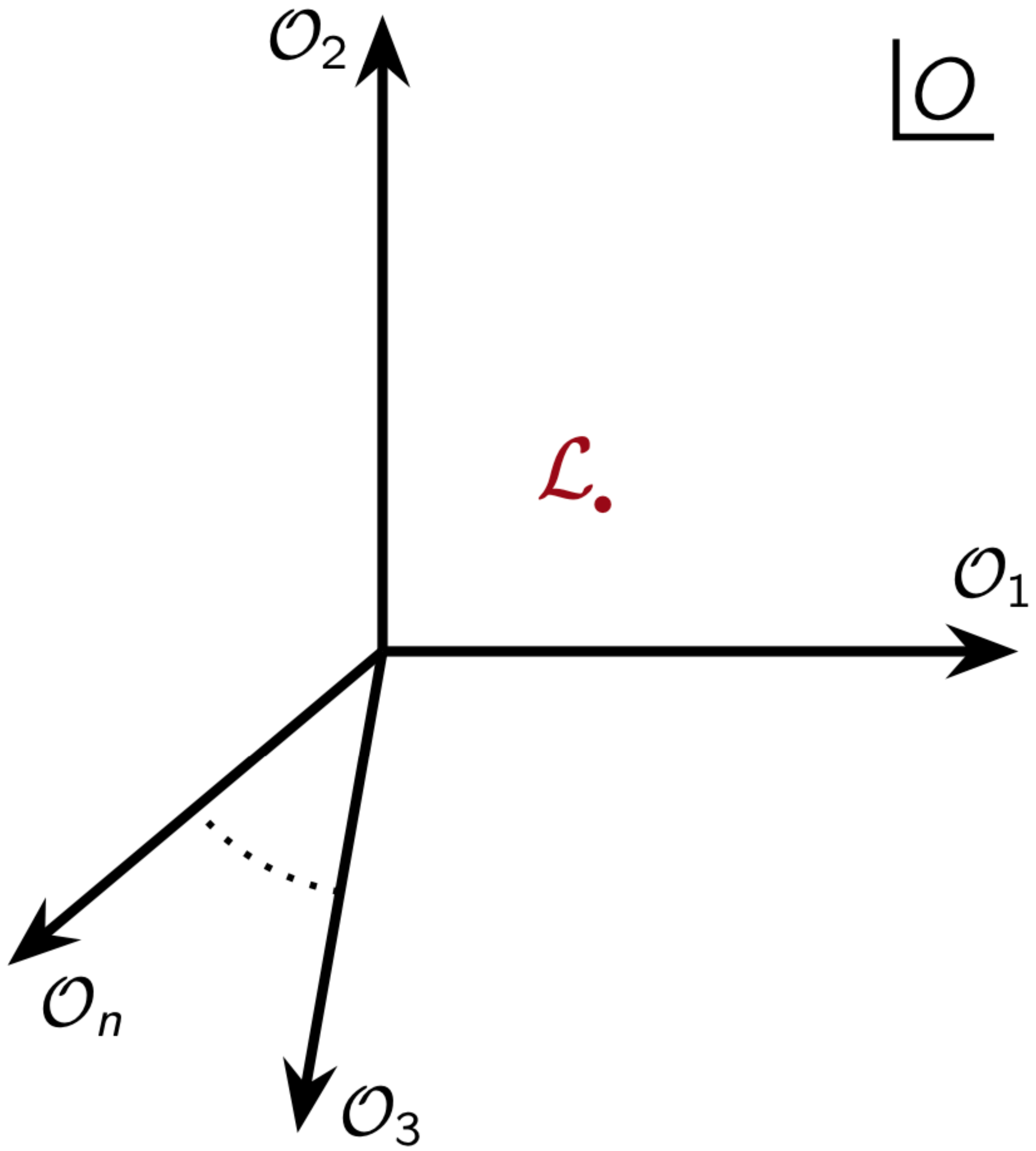
```
In[12]:= LEFT // NiceForm
Out[12]//NiceForm=
```

$$
\begin{aligned}
&\frac{7}{540}\,\hbar\,g^2\,\frac{1}{M\Psi^2}\left(D_\rho G^{\mu\nu A}\right)^2 + \\
&\frac{1}{40}\,\hbar\,g^2\,\frac{1}{M\Psi^2}\,G^{\mu\nu A}\,D^2 G^{\mu\nu A} + \frac{7}{540}\,\hbar\,g^2\,\frac{1}{M\Psi^2}\,D_\rho G^{\mu\nu A}\,D_\nu G^{\mu\rho A} - \\
&\frac{1}{180}\,\hbar\,g^2\,\frac{1}{M\Psi^2}\,D_\nu G^{\mu\nu A}\,D_\rho G^{\mu\rho A} + \frac{1}{40}\,\hbar\,g^2\,\frac{1}{M\Psi^2}\,G^{\mu\nu A}\,D_\nu D_\rho G^{\mu\rho A} + \\
&\frac{1}{40}\,\hbar\,g^2\,\frac{1}{M\Psi^2}\,G^{\mu\nu A}\,D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24}\,\hbar\,g^3\,\frac{1}{M\Psi^2}\,G^{\mu\nu A}\,G^{\mu\rho B}\,G^{\nu\rho C}\,f^{ABC}
\end{aligned}
$$

$$\mathscr{L}_{\mathrm{EFT}} = \sum_i C_i \mathcal{O}_i \in O$$



( log supertrace )



# Linear simplifications

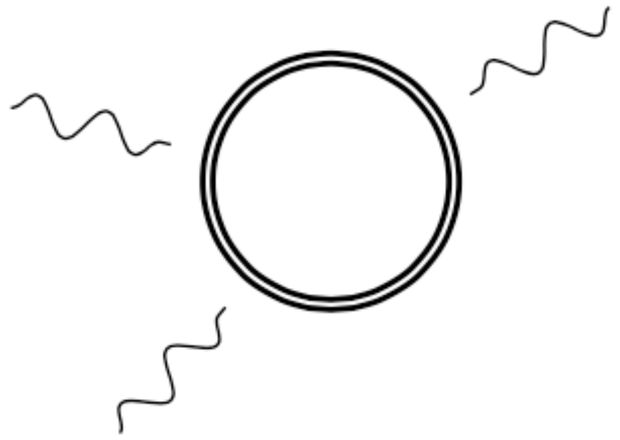
Example: Integrating out a heavy fermion in the fundamental representation of SU(3)

In[12]:= `LEFT // NiceForm`

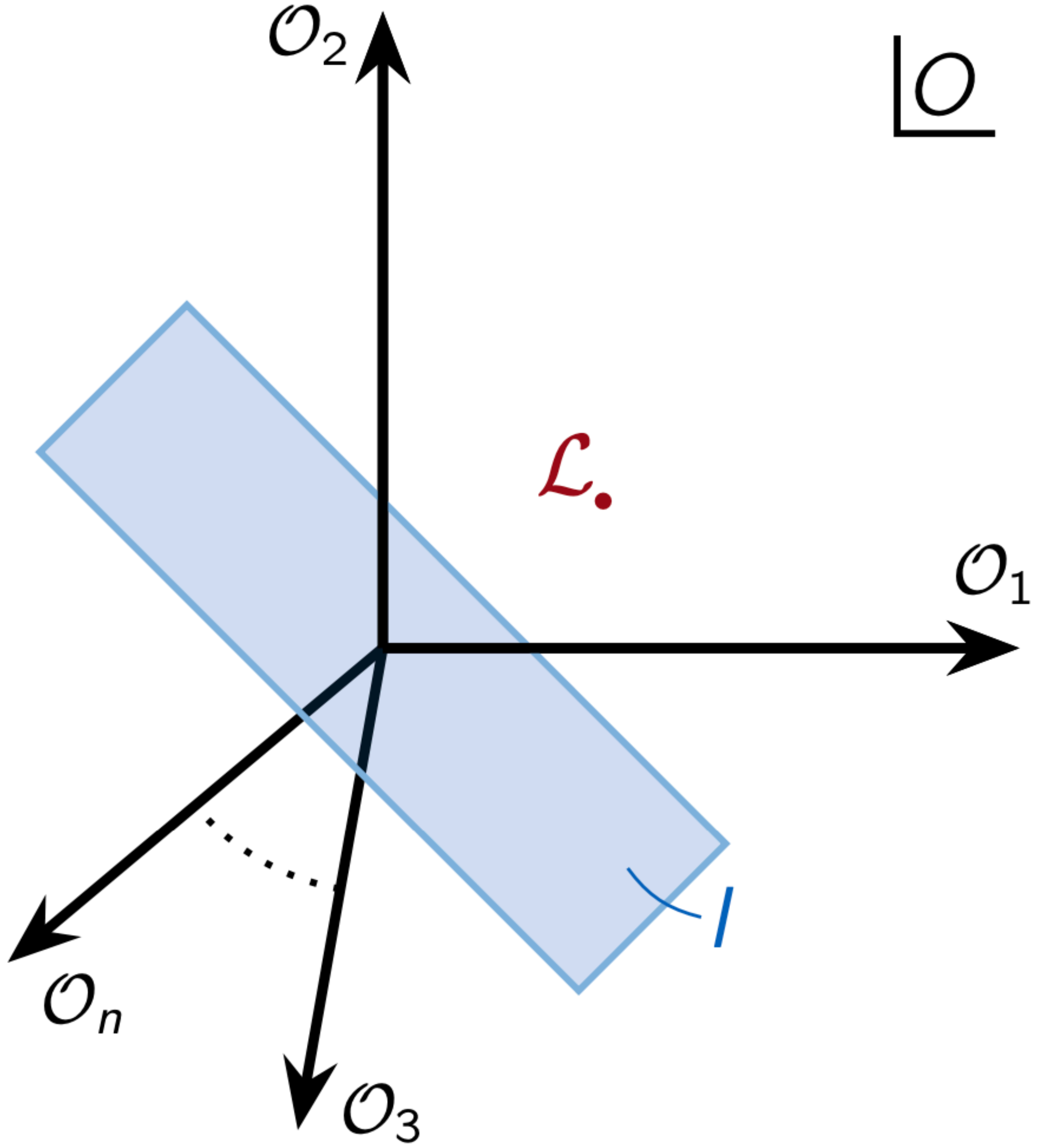
Out[12]//NiceForm=

$$\frac{7}{540}\,\hbar\, g^2\,\frac{1}{M\Psi^2}\left(D_\rho G^{\mu\nu A}\right)^2 + \frac{1}{40}\,\hbar\, g^2\,\frac{1}{M\Psi^2}\,G^{\mu\nu A}\,D^2 G^{\mu\nu A} + \frac{7}{540}\,\hbar\, g^2\,\frac{1}{M\Psi^2}\,D_\rho G^{\mu\nu A}\,D_\nu G^{\mu\rho A} - \frac{1}{180}\,\hbar\, g^2\,\frac{1}{M\Psi^2}\,D_\nu G^{\mu\nu A}\,D_\rho G^{\mu\rho A} + \frac{1}{40}\,\hbar\, g^2\,\frac{1}{M\Psi^2}\,G^{\mu\nu A}\,D_\nu D_\rho G^{\mu\rho A} + \frac{1}{40}\,\hbar\, g^2\,\frac{1}{M\Psi^2}\,G^{\mu\nu A}\,D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24}\,\hbar\, g^3\,\frac{1}{M\Psi^2}\,G^{\mu\nu A}\,G^{\mu\rho B}\,G^{\nu\rho C}\,f^{ABC}$$

$$\mathscr{L}_{\text{EFT}} = \sum_i C_i \mathcal{O}_i \in O$$



( log supertrace )




$I \subseteq O$ is the space of all operators identities, such as IBP relations, yielding e.g.

$$\mathcal{O}_1 + 2\,\mathcal{O}_3 = 0$$

interpreted as

$$\mathcal{O}_1 + 2\,\mathcal{O}_3 \in I$$

# Linear simplifications

Example: Integrating out a heavy fermion in the fundamental representation of SU(3)

In[12]:=
```
LEFT // NiceForm
```
Out[12]//NiceForm=

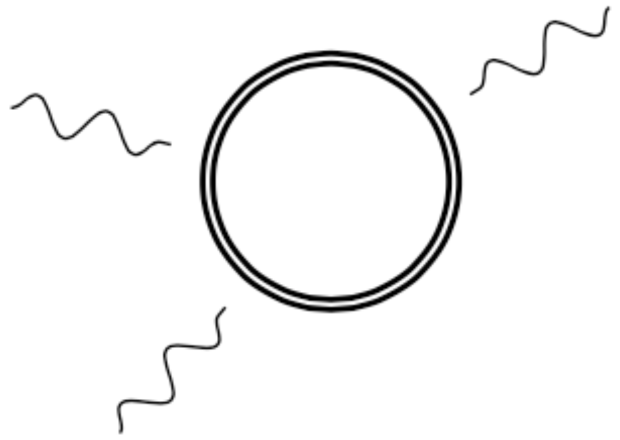
$$\frac{7}{540}\hbar g^2 \frac{1}{M\Psi^2}\left(D_\rho G^{\mu\nu A}\right)^2 + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D^2 G^{\mu\nu A} + \frac{7}{540}\hbar g^2 \frac{1}{M\Psi^2} D_\rho G^{\mu\nu A} D_\nu G^{\mu\rho A} - \frac{1}{180}\hbar g^2 \frac{1}{M\Psi^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\nu D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24}\hbar g^3 \frac{1}{M\Psi^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC}$$

$$\mathscr{L}_{\mathrm{EFT}} = \sum_i C_i \mathscr{O}_i \in O$$

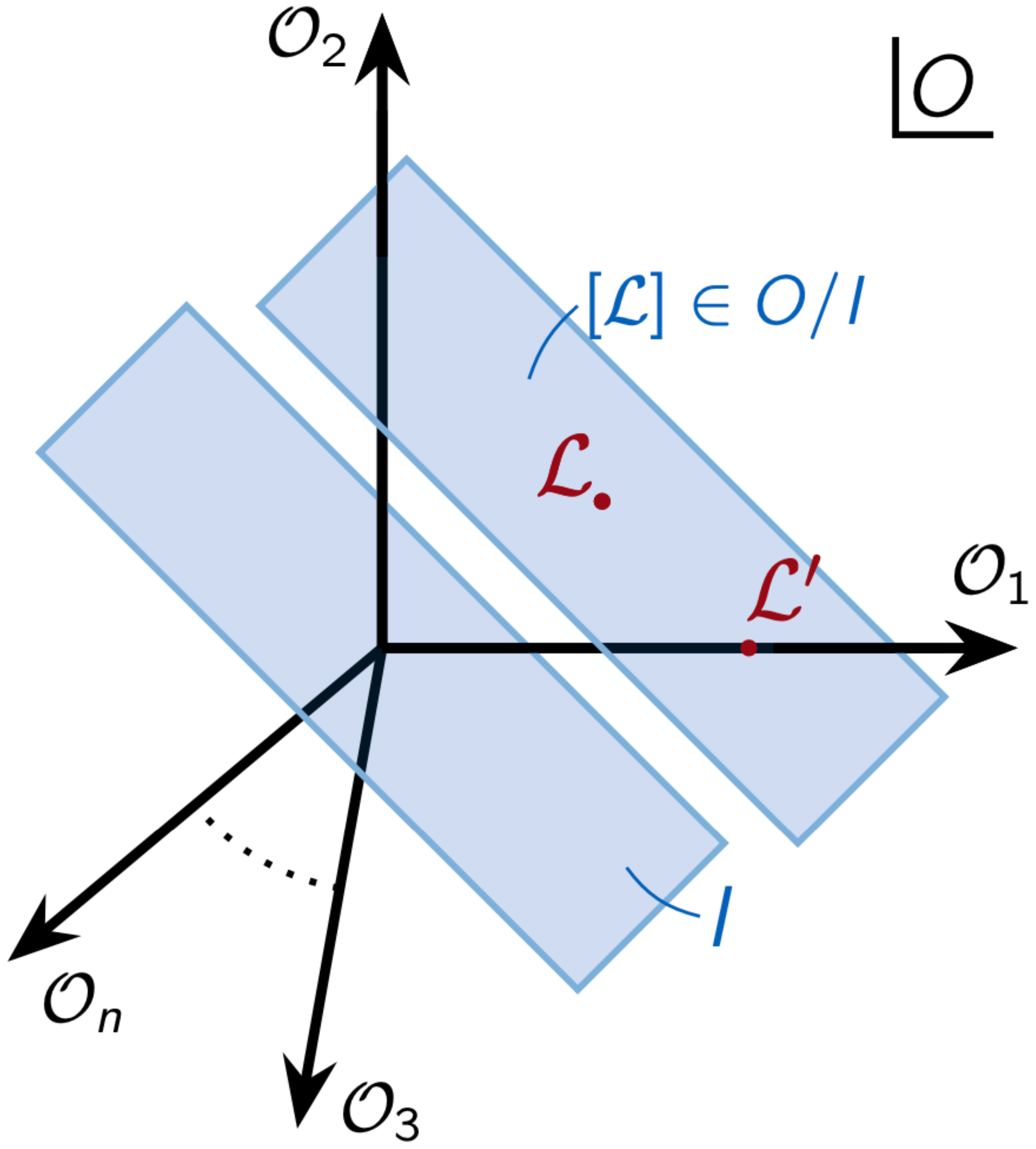
By gaussian elimination, we can *choose* a representative element for $[\mathscr{L}_{\mathrm{EFT}}] \in O/I$ to get an EFT basis

In[13]:=
```
LEFT // GreensSimplify // NiceForm
```
Out[13]//NiceForm=

$$-\frac{1}{15}\hbar g^2 \frac{1}{M\Psi^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} - \frac{1}{180}\hbar g^3 \frac{1}{M\Psi^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC}$$



( log supertrace )




$I \subseteq O$ is the space of all operators identities, such as IBP relations, yielding e.g.

$$\mathscr{O}_1 + 2\,\mathscr{O}_3 = 0$$

interpreted as

$$\mathscr{O}_1 + 2\,\mathscr{O}_3 \in I$$

# Linear simplifications with evanescent operators

Evanescent operators appear from a special type of linear simplification ( valid only for $d = 4$ )

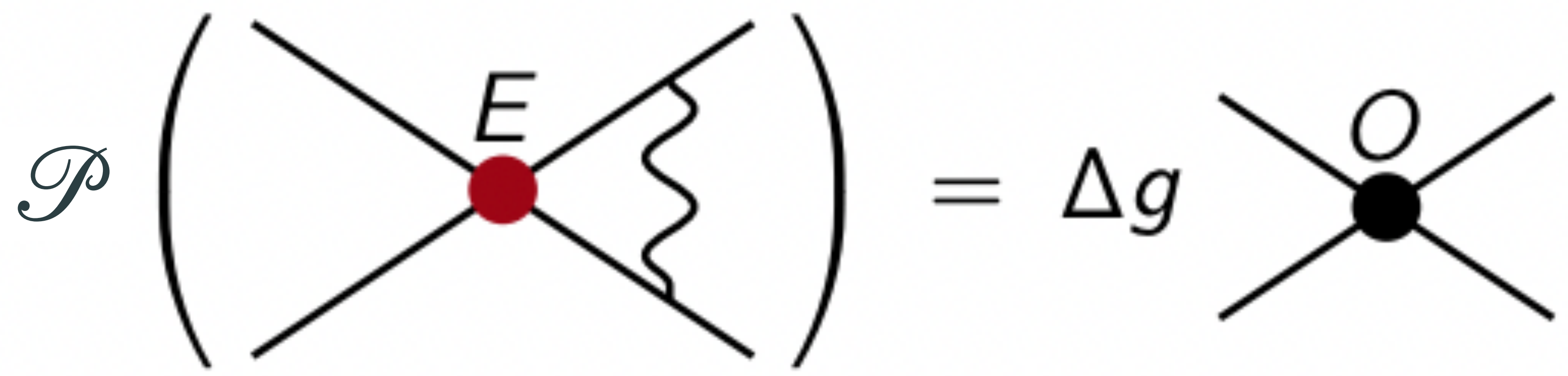
$$O_d = \underbrace{\mathscr{P}\, O_d}_{\text{Physical part}} + \underbrace{\overbrace{\mathscr{E}}^{Id - \mathscr{P}}\, O_d}_{\text{Evanescent part}}$$

$\mathscr{P} \equiv$ Projection to the physical ( $d = 4$ ) basis

E.g. Fierz identities

$$(\bar{\ell}_p e_r)(\bar{e}_s \ell_t) = -\frac{1}{2}(\bar{\ell}_p \gamma_\mu \ell_t)(\bar{e}_s \gamma^\mu e_r) + \overbrace{E_{\ell e}^{prst}}^{\text{rank}\,(d-4)} \longrightarrow (\bar{\ell}_p e_r)(\bar{e}_s \ell_t) + \frac{1}{2}(\bar{\ell}_p \gamma_\mu \ell_t)(\bar{e}_s \gamma^\mu e_r) - E_{\ell e}^{prst} \in I$$

Representative elements are chosen so evanescent operators are retained. Afterwards, these are removed by shifting the coefficients of physical operators




e.g. $E_{\ell e}^{prst} \to -\dfrac{g_L\, y_e^{ts}}{128\pi^2}\, Q_{eW}^{pr} +$ [ other contributions ]