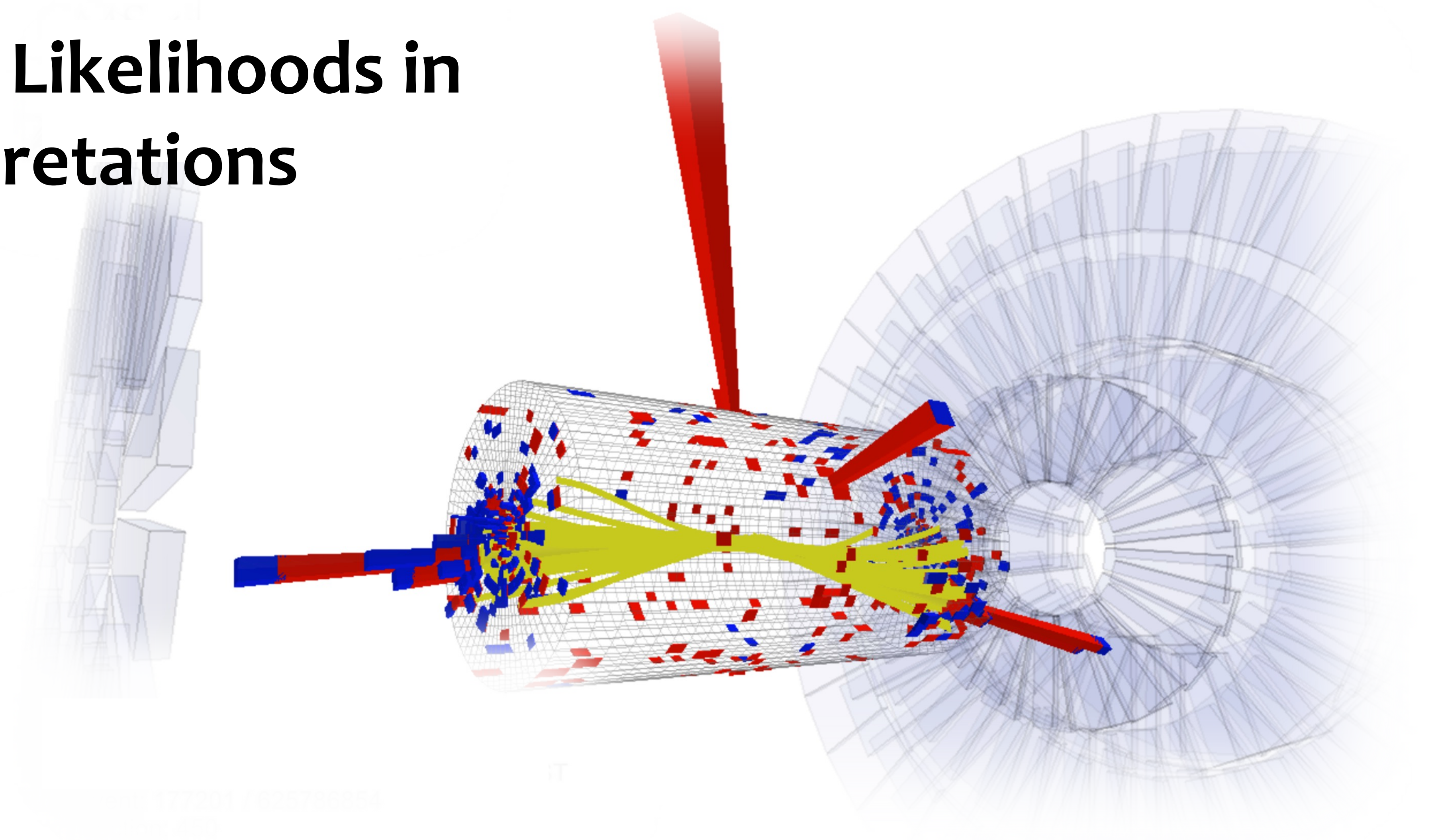
Imperial College London

Full vs Simplified Likelihoods in Higgs (Re-)interpretations

Nicholas Wardle



SM@LHC 2024

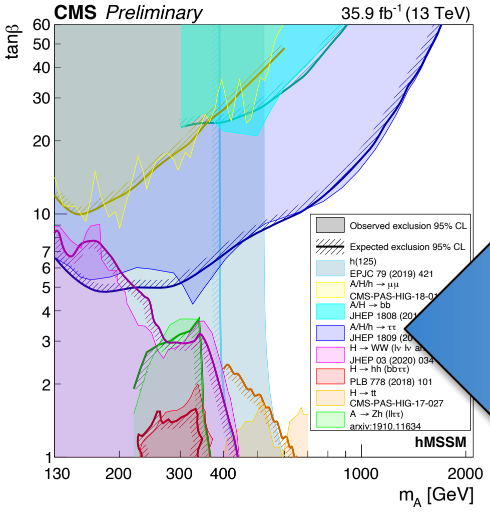
CNR, Rome

7-10 May 2024

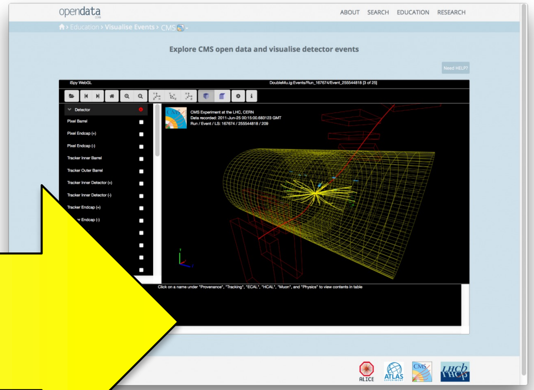
LHC “interpretation” spectrum

Highest level of interpretation “baked in”

Minimal interpretation



There is a *wide* spectrum of public information from LHC experiments (including that pertinent for Higgs physics)



Inspired by P. Owen @ Reinterp2021

Exclusion limits/contours
In UV-complete models

Raw data

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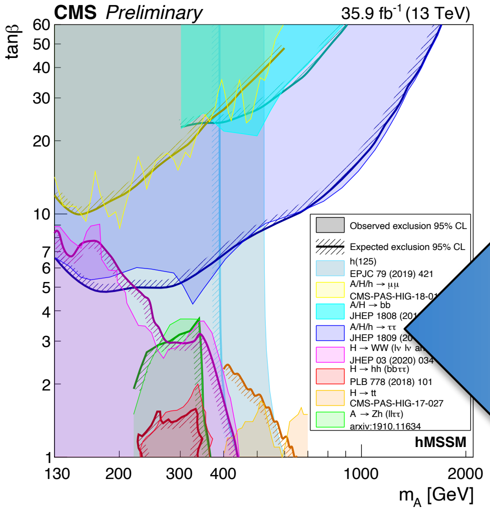

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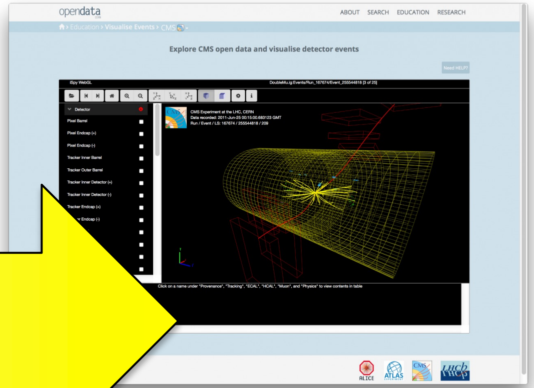

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Easy to communicate/use

Requires expertise to distill information

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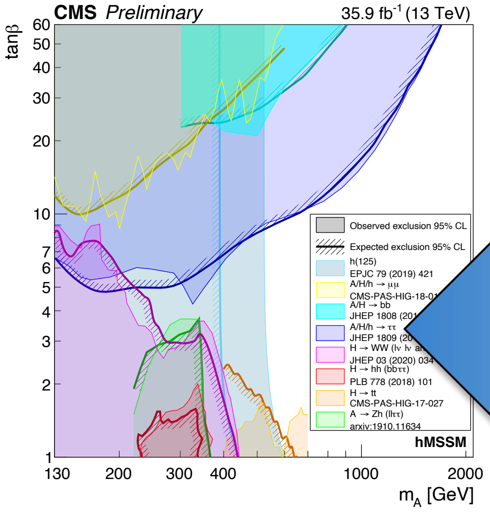

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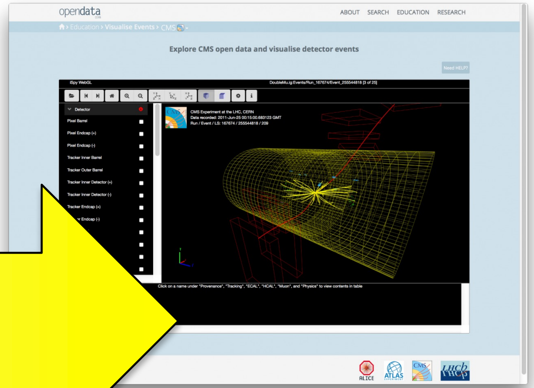

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Exclusion limits/contours
In UV-complete models

Raw data

I will ***not cover*** the latest Higgs interpretations in this talk.
Instead see :

- BSM interpretations of H-measurements (R. Santos)
- STXS & differential H-measurements (S. Heim)
- EFT interpretations of H/HH (A. Calandri)

Easy to communicate/use

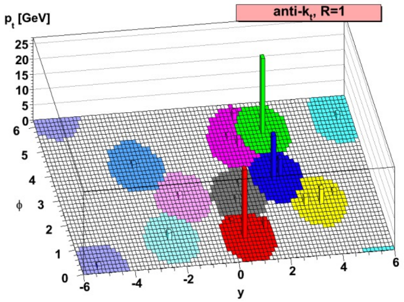
Requires expertise to distill information

“Full” Likelihood

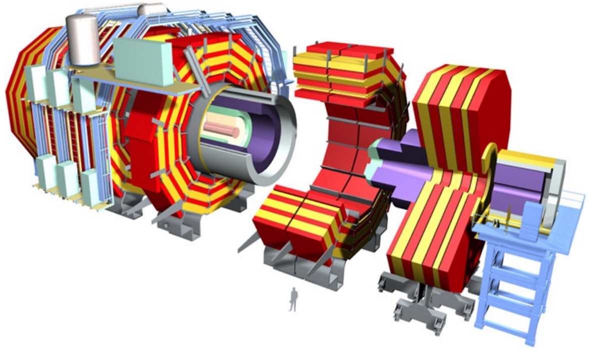
The LHC is a random number generator…

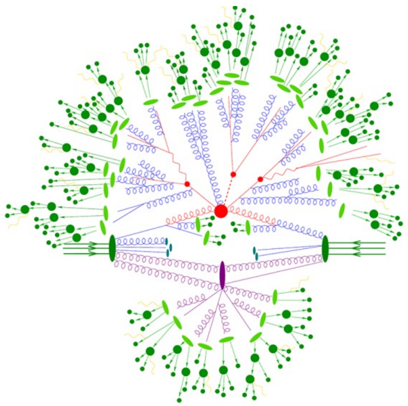
Observables	Detector interactions	Shower splittings	Parton-level momenta	Theory parameters

$$x \longleftarrow z_d \longleftarrow z_s \longleftarrow z_p \longleftarrow \theta$$

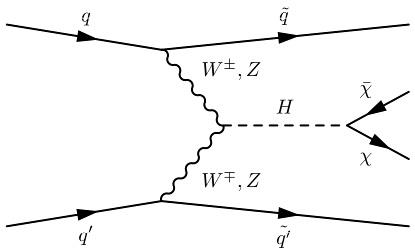


[Source: M. Cacciari, G. Salam, G. Soyez 0802.1189]





[Source: F. Krauss]



$$p(x|\theta) = \int \mathrm{d}z_d \int \mathrm{d}z_s \int \mathrm{d}z_p \, p(x|z_d) \quad p(z_d|z_s) \quad p(z_s|z_p) \quad p(z_p|\theta)$$

This is THE **full** likelihood if we substitute our observed data for *x*

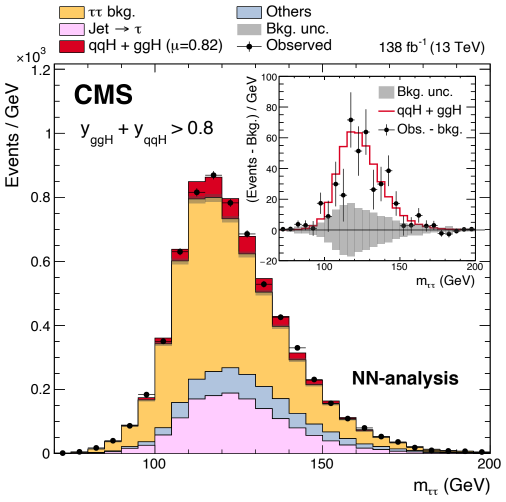
(Example from K. Cranmer)

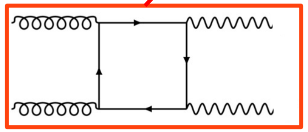
~~“Full”~~ “Experimental” Likelihood

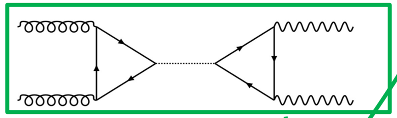
We never use the *full* likelihood for Higgs measurements/interpretations, so I call the most complete thing we use the *experimental* likelihood. For Higgs measurements, typically

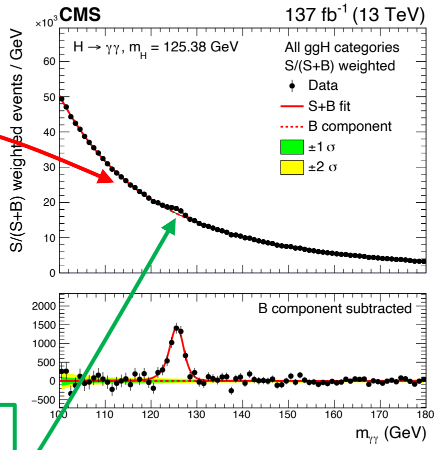
$$L(\vec{\mu},\vec{\nu}) = \prod_{n} p\left(x_n ; \sum_{i,f} \mu_i \mu^f S^f_{i,n}(\vec{\nu}) + \sum_{k} B_k(\vec{\nu}) \right) \cdot \prod_{i} p(y_i;\nu_i)$$

The “**data**” in each channel









Decompose expectation into ***signal*** and ***background*** contributions

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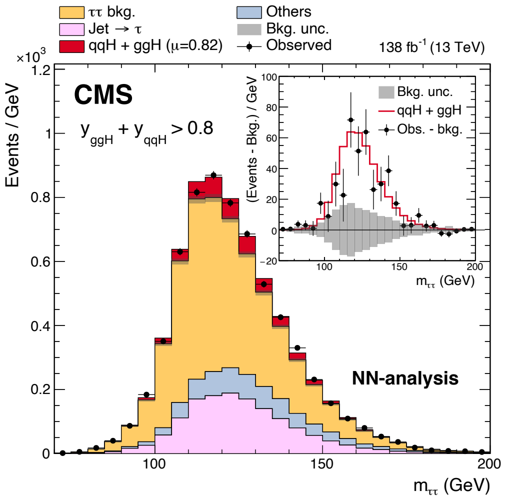
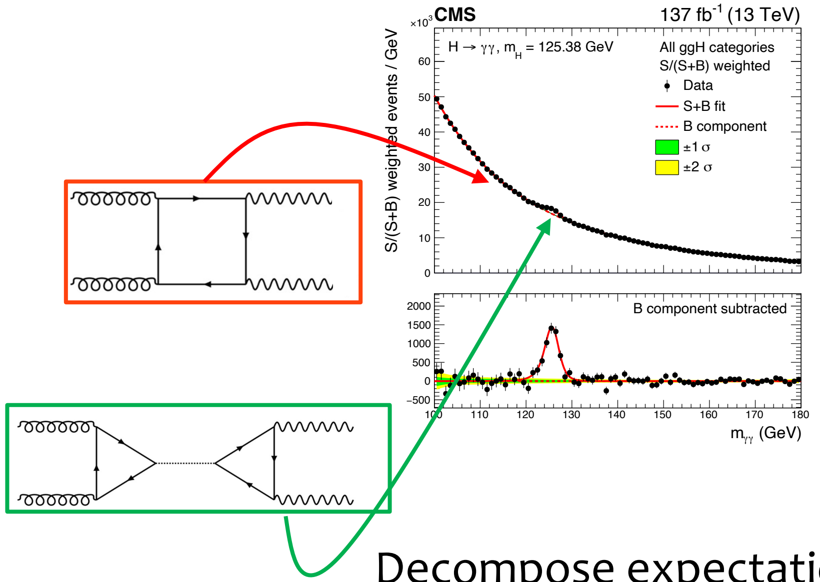
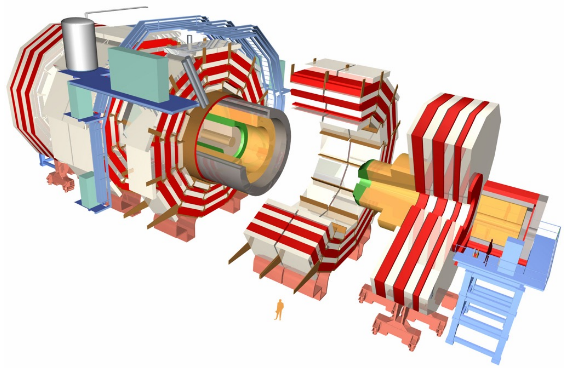


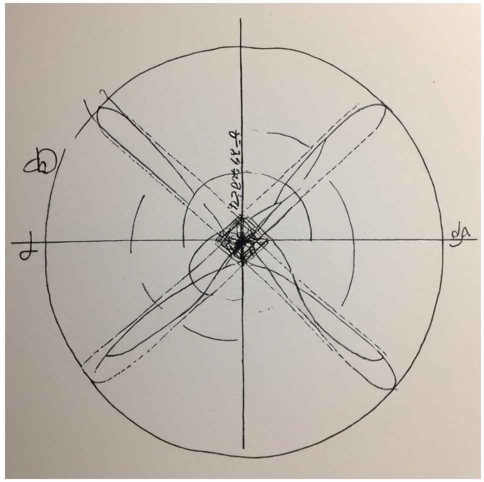


Decompose expectation into ***signal*** and ***background*** contributions

Parameters of interest and ***nuisance parameters*** parameterize physics model and experimental/theoretical systematic uncertainties



Experimental Likelihood interpretations

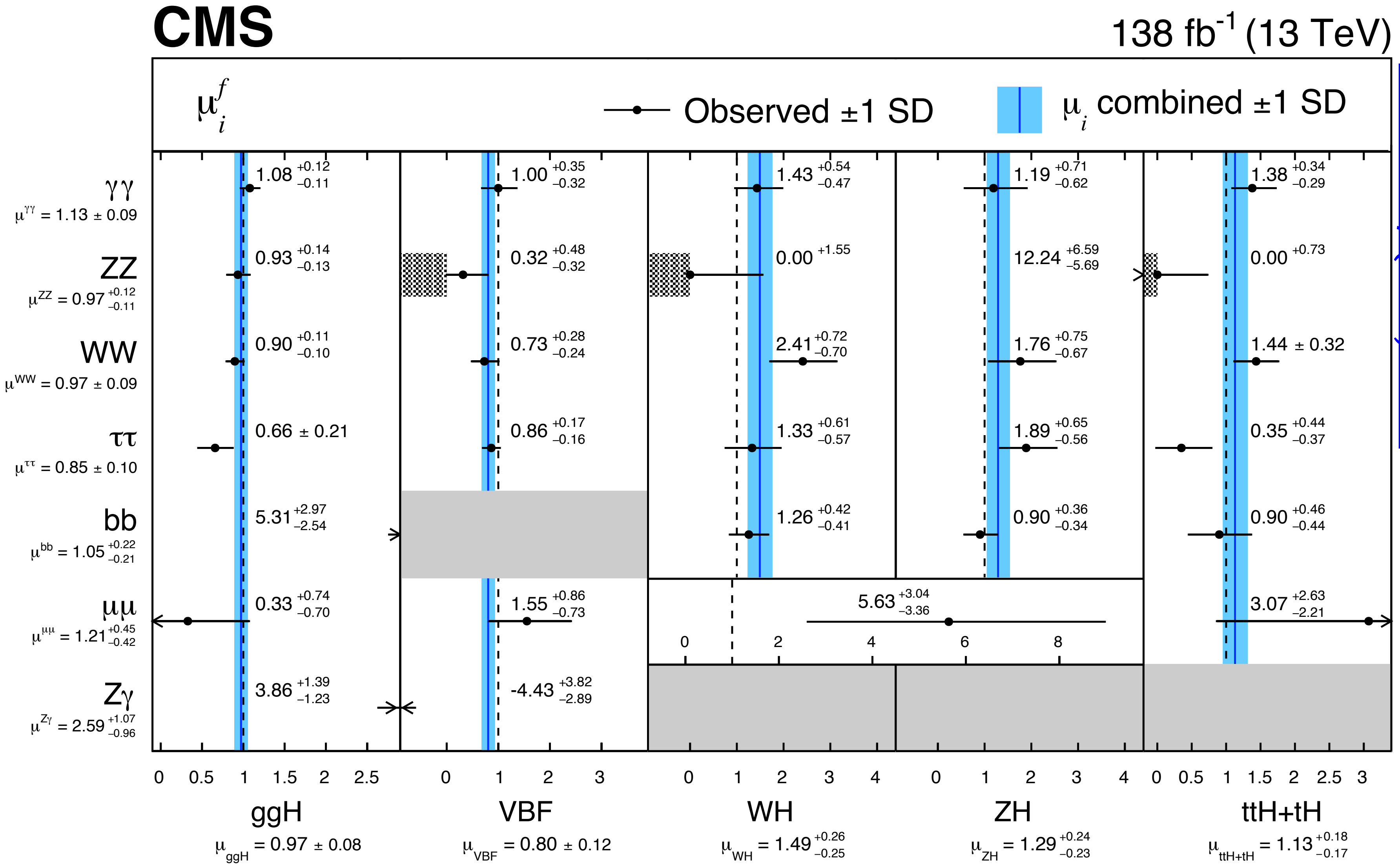


OpenArt's "interpretation of experimental likelihood"

Profiled Experimental Likelihood

Profiling out the nuisance parameters yields individual measurements of POIs

$$L(\vec{\mu},\vec{\nu}) \rightarrow L(\mu_i, \hat{\vec{\mu}}_{j\neq i}, \hat{\vec{\nu}})$$



Nature 607 (2022) 60-68

$$\mu_i = \frac{\sigma_i}{(\sigma_i)_{\rm SM}} \quad \text{and} \quad \mu^f = \frac{{\rm BR}^f}{({\rm BR}^f)_{\rm SM}}.$$

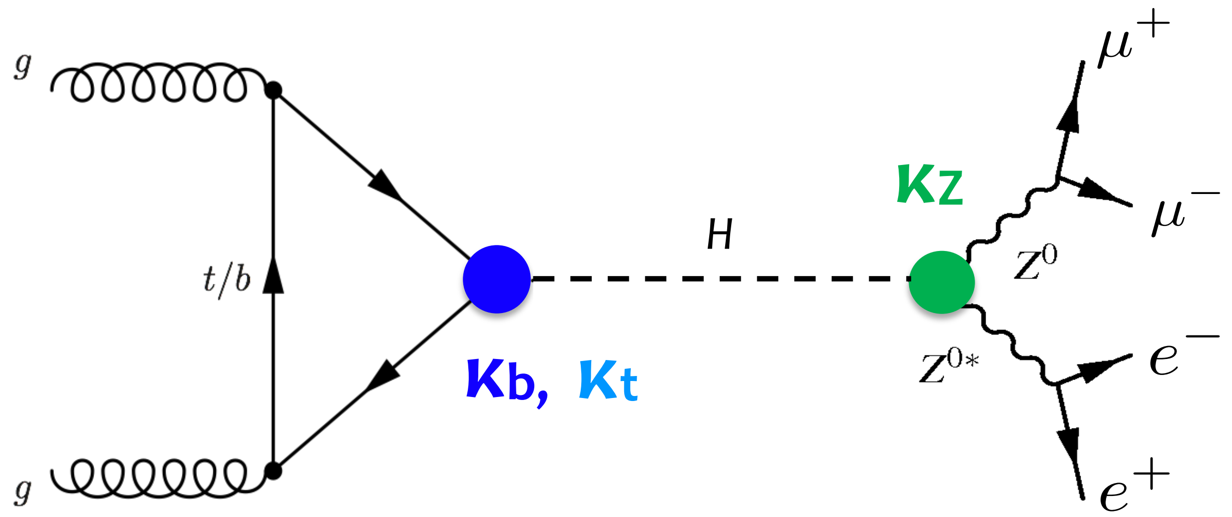
Interpretations - couplings

Higgs interpretations performed by *re-parameterizing* $L(\vec{\mu},\vec{\nu}) \rightarrow L(\vec{\kappa},\vec{\nu})$

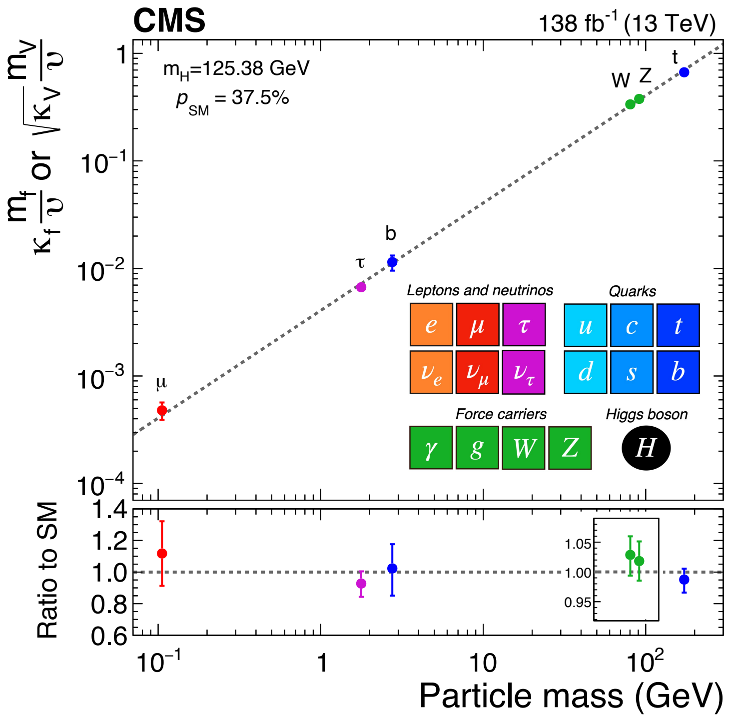
"Signal strengths" parameterized in terms of model parameters

Example : κappas $\vec{\mu} \rightarrow \vec{\mu}(\vec{\kappa})$

Standard model defined by $\vec{\kappa} = 1$



$$\mu_{ggH} \cdot \mu^{ZZ} \sim \frac{(1.06\kappa_t^2 + 0.01\kappa_b^2 - 0.07\kappa_b\kappa_t)\kappa_Z^2}{\kappa_H^2}$$



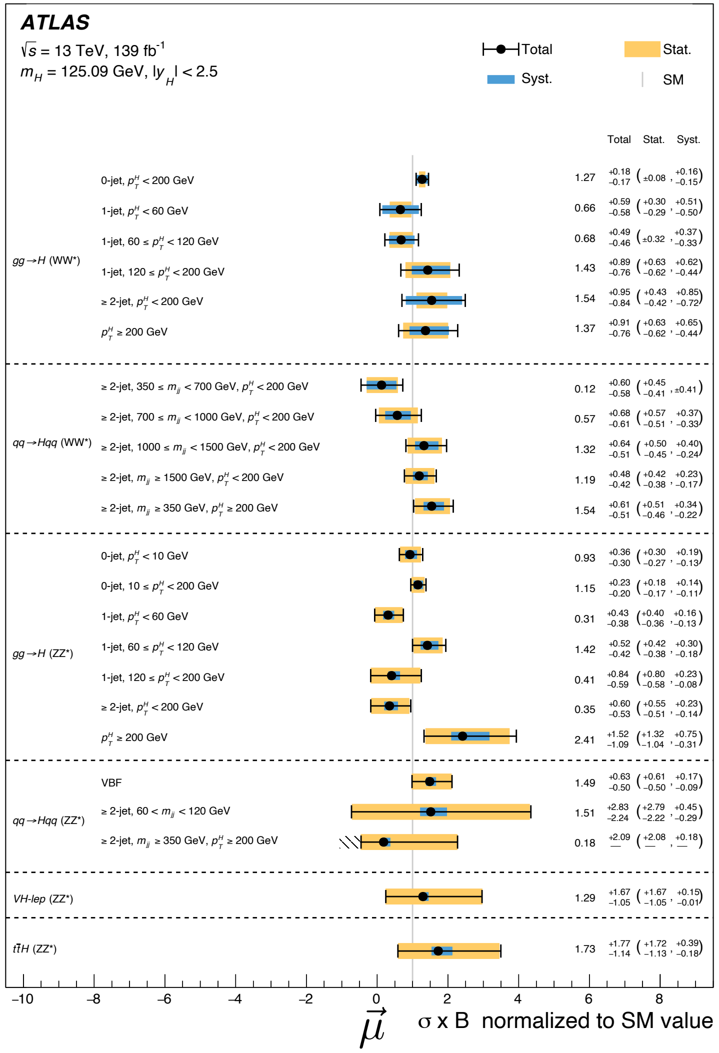
Re-profile likelihood

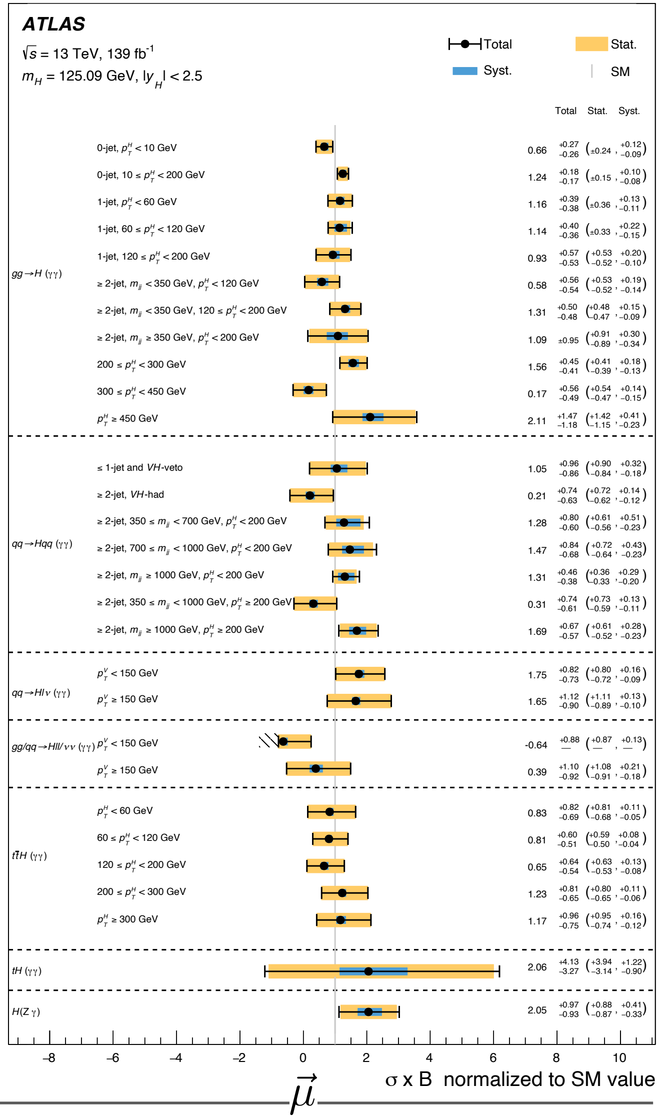
Interpretations – STXS

These days (with more data since LHC Run-2), we can measure more than global signal strengths and couplings

→ Differential measurements – eg STXS “$\vec{\mu}$” *

arXiv:2402.05742 (sub to JHEP)



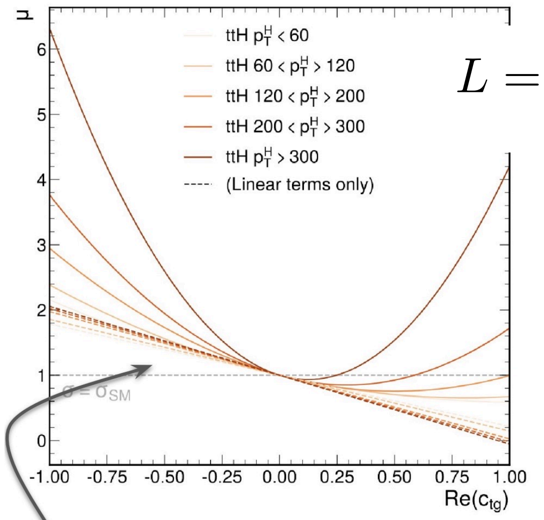


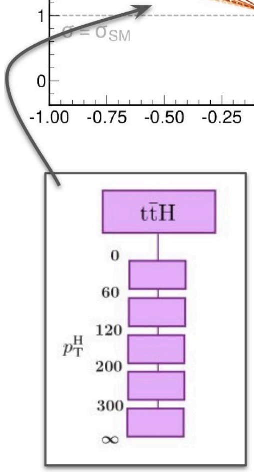
*ignores dominant theory uncertainties

Interpretations – STXS

These days (with more data since LHC Run-2), we can measure more than global signal strengths and couplings
→ Differential measurements – eg STXS "$\vec{\mu}$" *

Exploit sensitivity to kinematic dependence of BSM contributions → Effective field theory interpretations





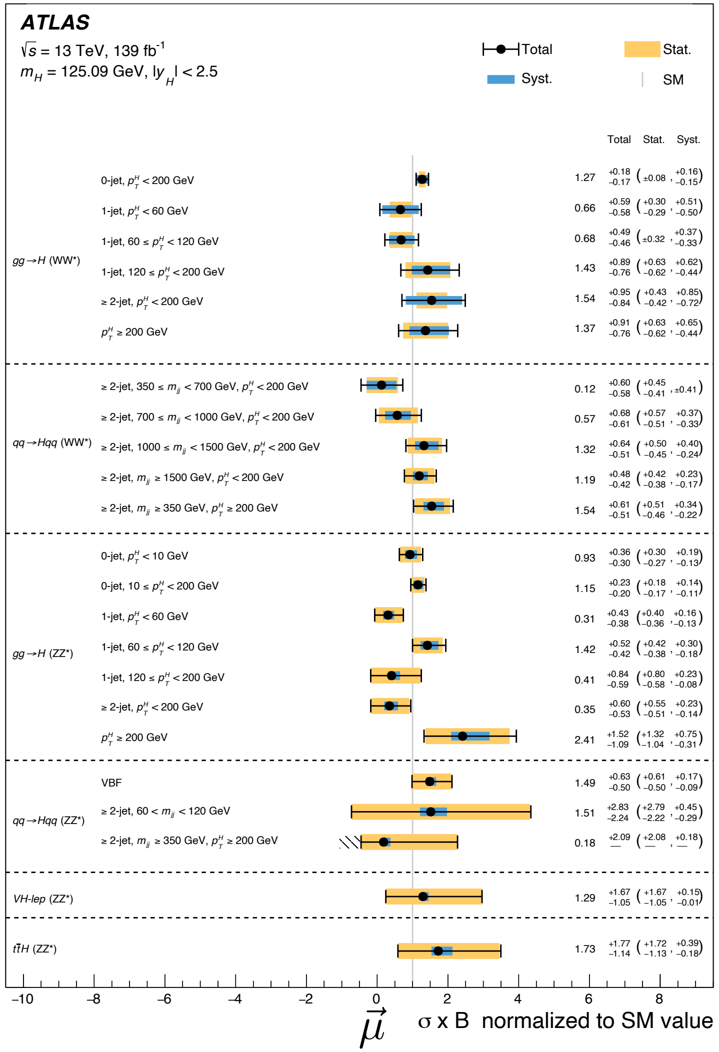
$$L = L_{SM} + \frac{1}{\Lambda}\sum_k \mathcal{O}_k + \ldots$$

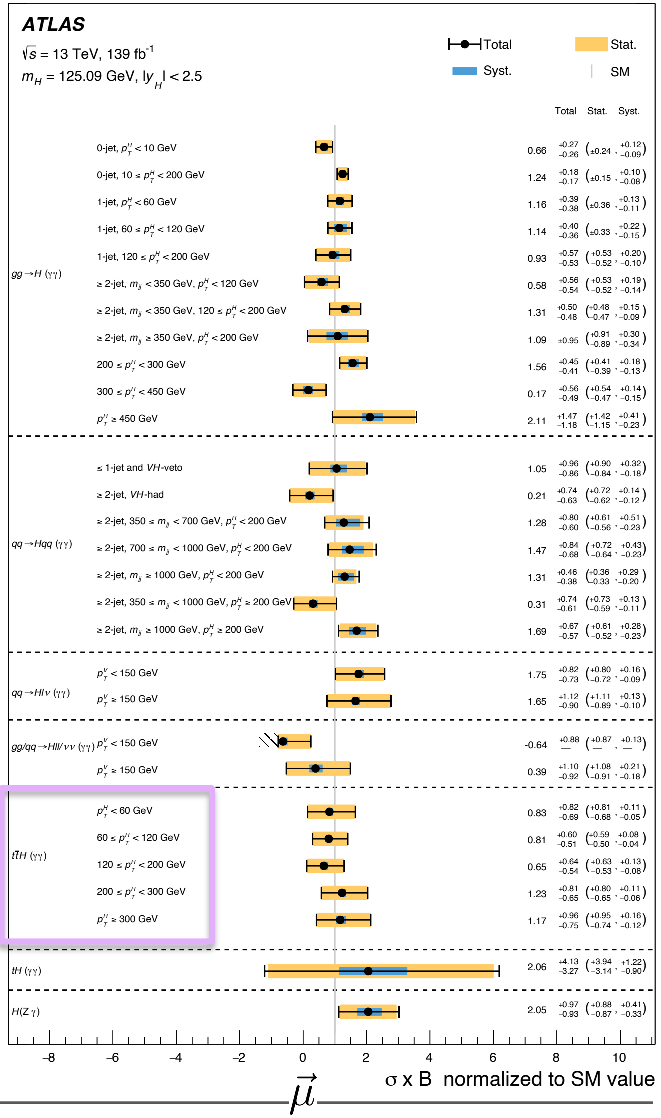
Parameterize BSM effects in terms of Wilson coefficients c_i

$$\vec{\mu} \rightarrow \vec{\mu}(\vec{c})$$

$$\mu = O^{\mathrm{EFT}}/O^{\mathrm{SM}} = 1 + \sum_j A_j c_j + \sum_{jk} B_{jk} c_j c_k$$

[arXiv:2402.05742](https://arxiv.org/abs/2402.05742) (sub to JHEP)





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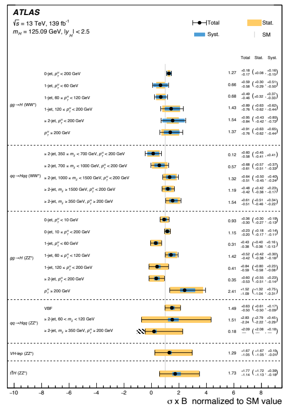
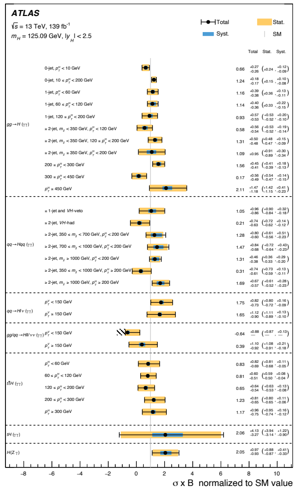
2HDM interpretations of STXS measurements

In 2HDM models, SM-like Higgs boson couplings are modified with respect to the SM predictions → rates of Higgs production/decay ($\vec{\mu}$) are modified



$$L(\vec{\mu}) \rightarrow L\Big(\vec{\mu}(\vec{\kappa}(\cos(\beta-\alpha),\tan\beta))\Big)$$



$$L(\vec{\mu}) \rightarrow L\Big(\vec{\mu}(\vec{c}(\cos(\beta-\alpha),\tan\beta))\Big)$$

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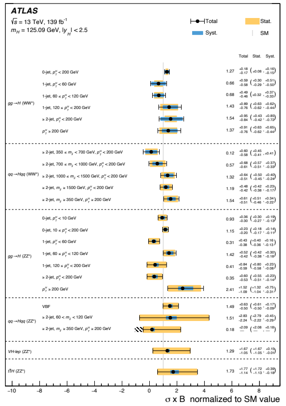
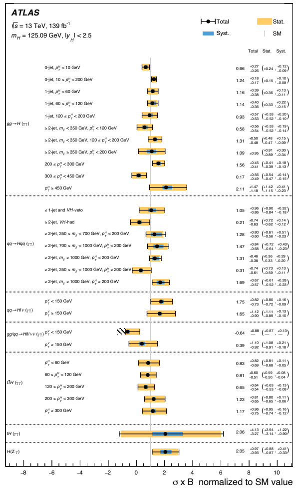


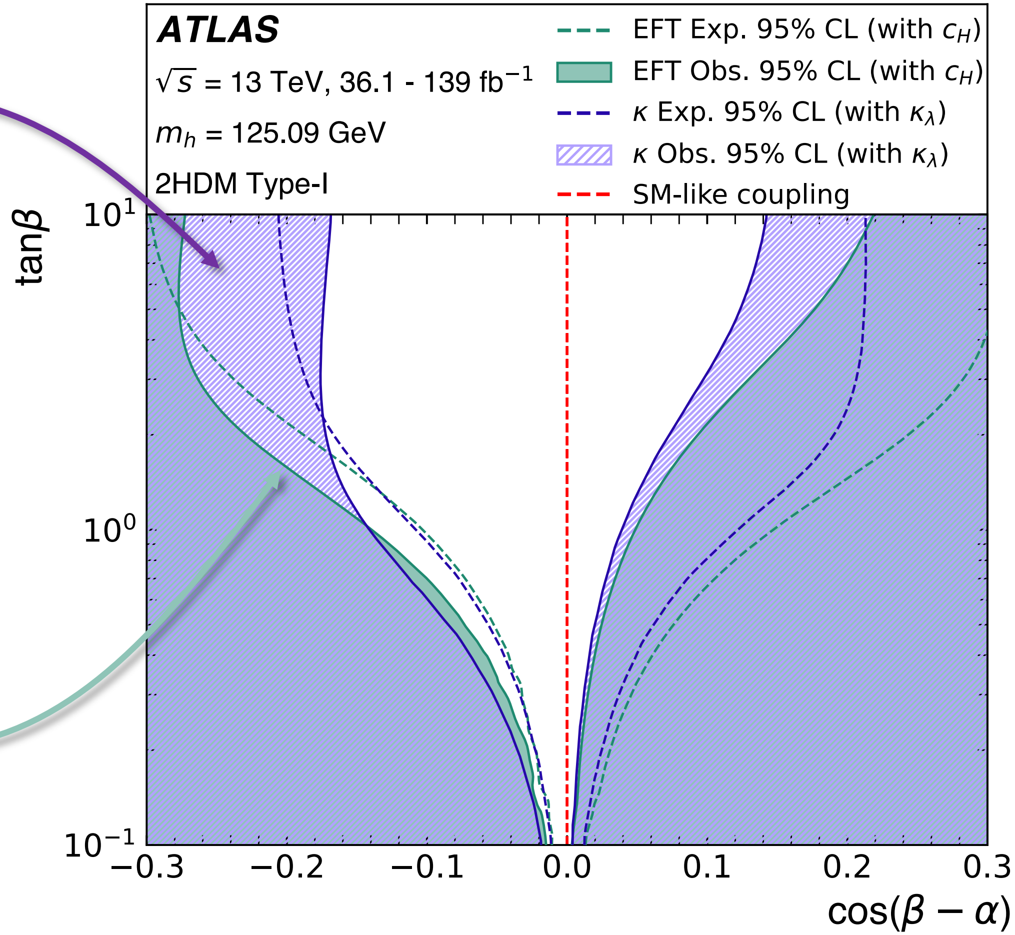


$\kappa_V, \kappa_F, \kappa_\lambda$

$$L(\vec{\mu}) \rightarrow L\left(\vec{\mu}(\vec{\kappa}(\cos(\beta-\alpha), \tan\beta))\right)$$

$$L(\vec{\mu}) \rightarrow L\left(\vec{\mu}(\vec{c}(\cos(\beta-\alpha), \tan\beta))\right)$$

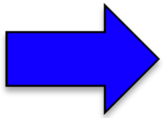
$C_{bH}, C_{tH}, C_{\tau H}, C_H$

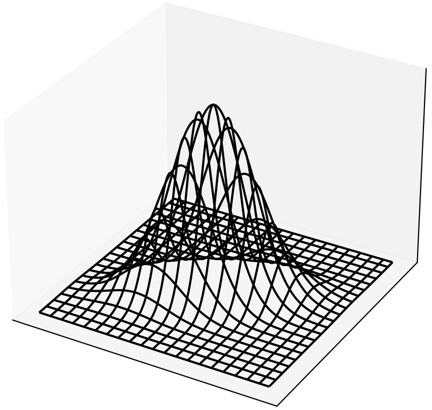


Parameterization of BSM scenario can impact resulting constraints on new physics!

Simplified Likelihoods

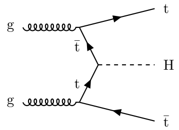
$$L(\vec{\mu},\vec{\nu}) = \prod_{n} p\left(x_n; \sum_{i,f} \mu_i \mu^f S^f_{i,n}(\vec{\nu}) + \sum_{k} B_k(\vec{\nu})\right) \cdot \prod_{i} p(y_i;\nu_i)$$



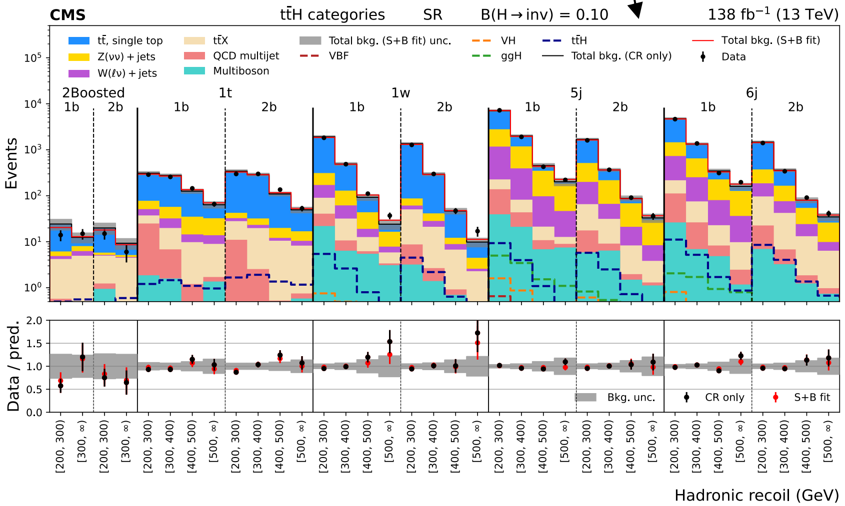


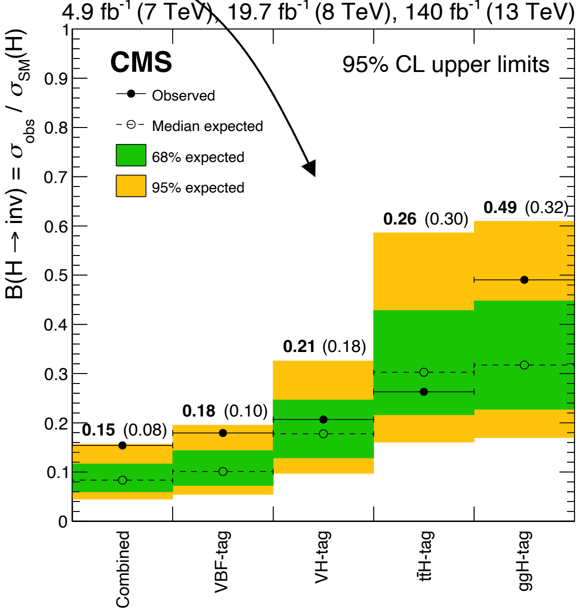
Interpretation of H→invisible

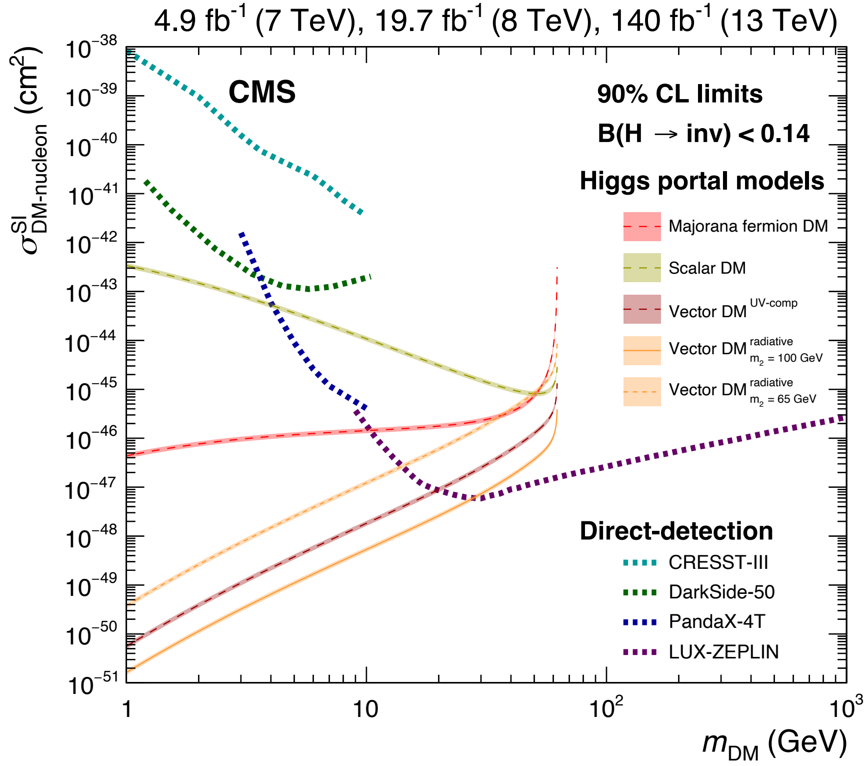
When the result is presented as single number, interpretation is "straightforward*"



[Eur. Phys. J. C 83 (2023) 933]







Event counts/expectations → Upper limits on B(H→inv) → Interpretations in H-portal models for DM

$$L(B_{H\to\text{inv}})$$

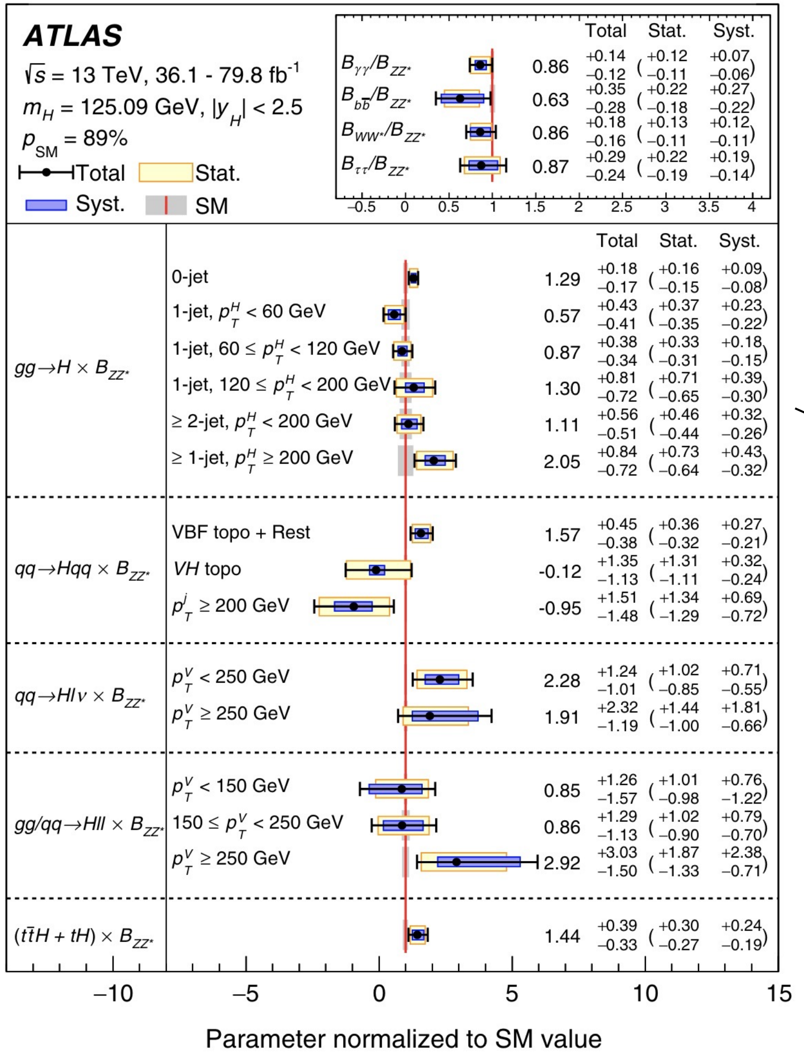
$$L(B_{H\to\text{inv}}(\sigma_{DM}^{SI})|m_{DM})$$

a nice feature of LH methods

* Nothing straightforward about the work needed to get there : PLB 709 (2012) 65, HEP 05 (2013) 036, EPJC 73 (2013) 2455

Gaussian approximations

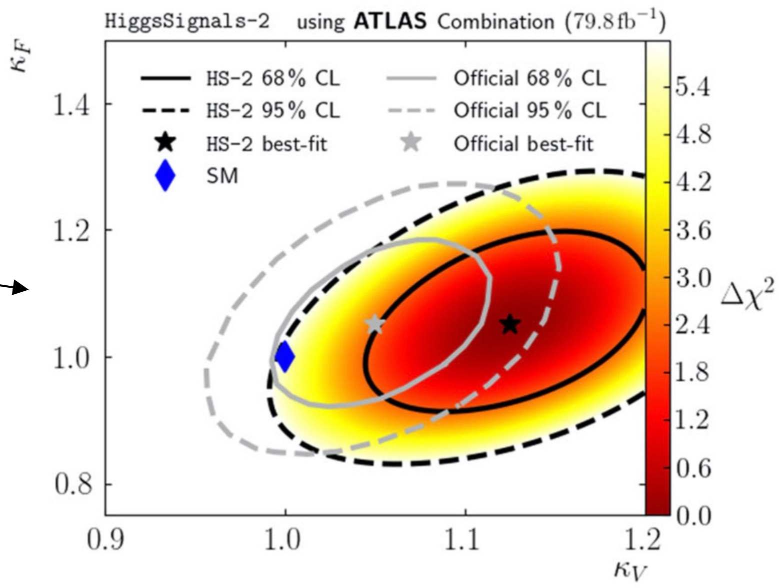
Phys. Rev. D 101(1), 012002 (2020)



Simple approximation of likelihood allows for fast/easy interpretation of Higgs boson measurements

$$-2\ln\left(\frac{L(\vec{\mu},\hat{\hat{\vec{\nu}}})}{L(\hat{\vec{\mu}},\hat{\vec{\nu}})}\right) \approx \left(\hat{\vec{\mu}}-\vec{\mu}\right)^{T}\left(\hat{\vec{\mu}}-\vec{\mu}\right)$$

$$\vec{\mu} \rightarrow \vec{\mu}(\kappa_V, \kappa_F)$$



Eur. Phys. J. C 81(2), 145 (2021)

Gaussian approximations

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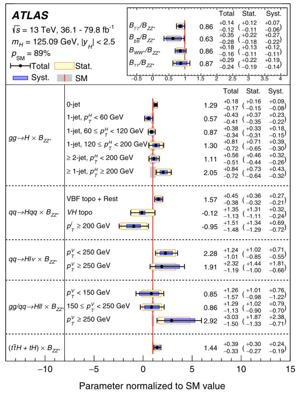

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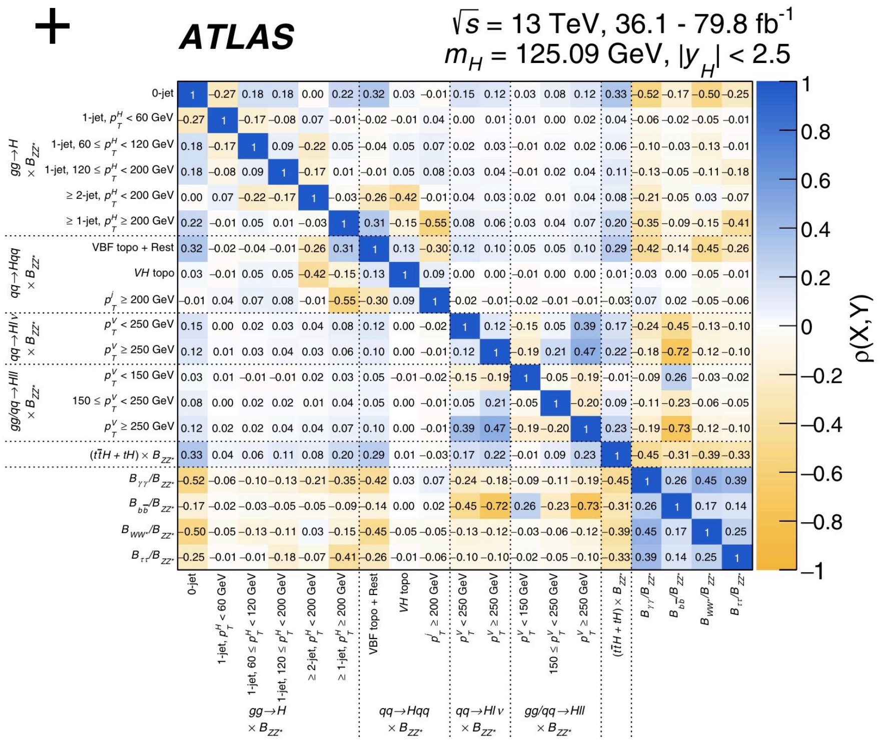
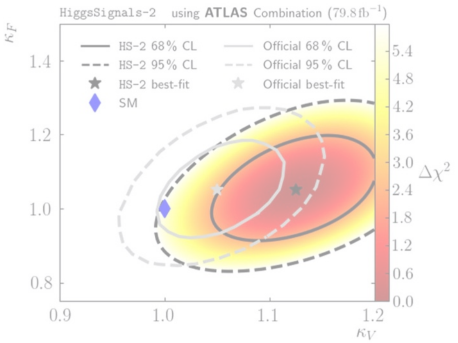


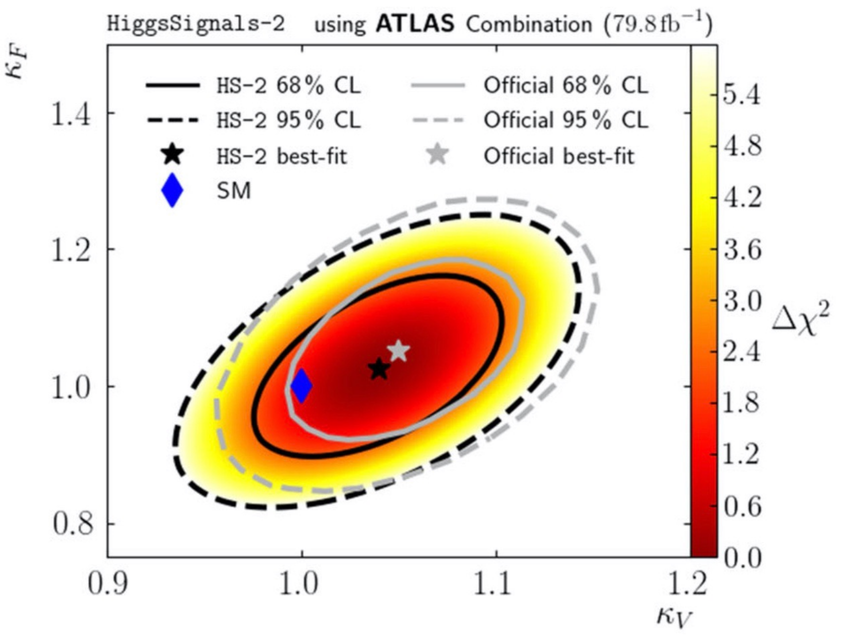
Correlations between measurements dramatically improve simplification

$$-2\ln\left(\frac{L(\vec{\mu},\hat{\hat{\vec{\nu}}})}{L(\hat{\vec{\mu}},\hat{\vec{\nu}})}\right) \approx \left(\hat{\vec{\mu}}-\vec{\mu}\right)^{T} \boldsymbol{C}_{\vec{\mu}}^{-1}\left(\hat{\vec{\mu}}-\vec{\mu}\right)$$

$$\vec{\mu} \rightarrow \vec{\mu}(\kappa_V, \kappa_F)$$



Eur. Phys. J. C 81(2), 145 (2021)



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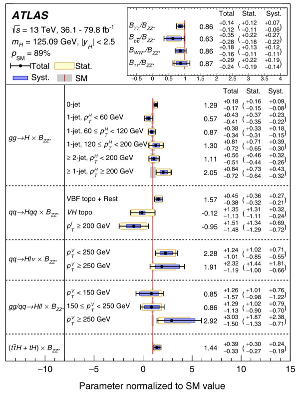

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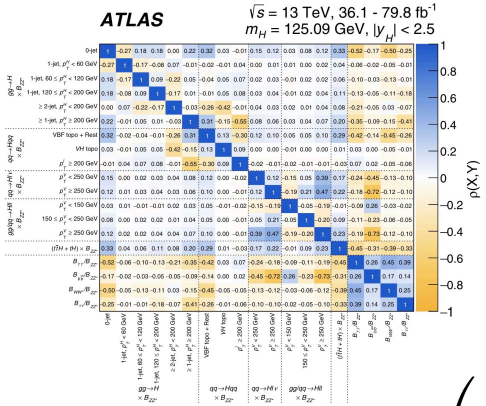

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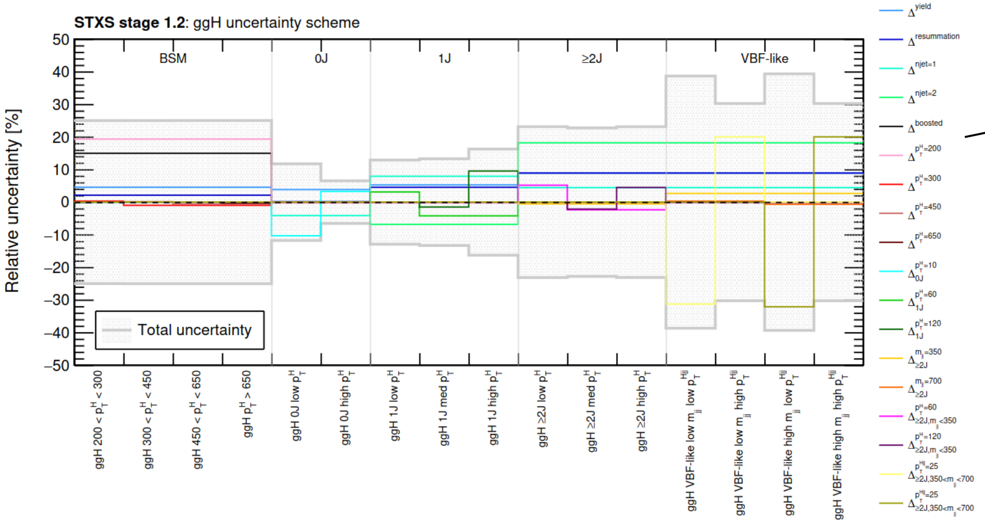
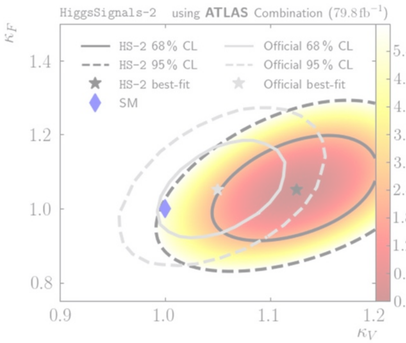
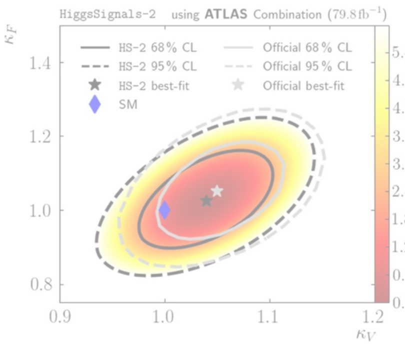


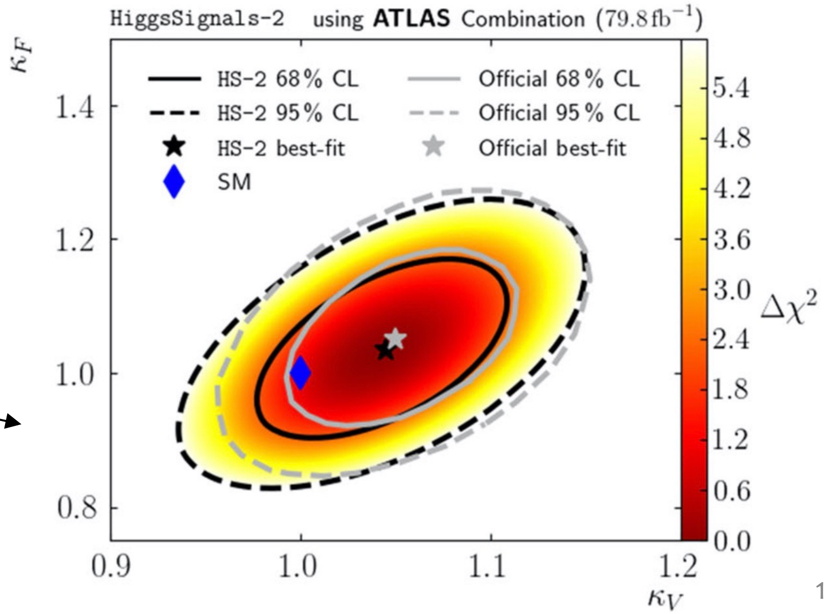
Accounting for correlations between STXS bins in **theory uncertainties** yields further improvements

$$-2\ln\left(\frac{L(\vec{\mu},\hat{\hat{\vec{\nu}}})}{L(\hat{\vec{\mu}},\hat{\vec{\nu}})}\right) \approx \left(\hat{\vec{\mu}}-\vec{\mu}\right)^T \left(\boldsymbol{C}_{\vec{\mu},\mathrm{Exp.}}+\boldsymbol{C}_{\vec{\mu},\mathrm{Th.}}\right)^{-1}\left(\hat{\vec{\mu}}-\vec{\mu}\right)$$

$$\vec{\mu} \rightarrow \vec{\mu}(\kappa_V,\kappa_F)$$



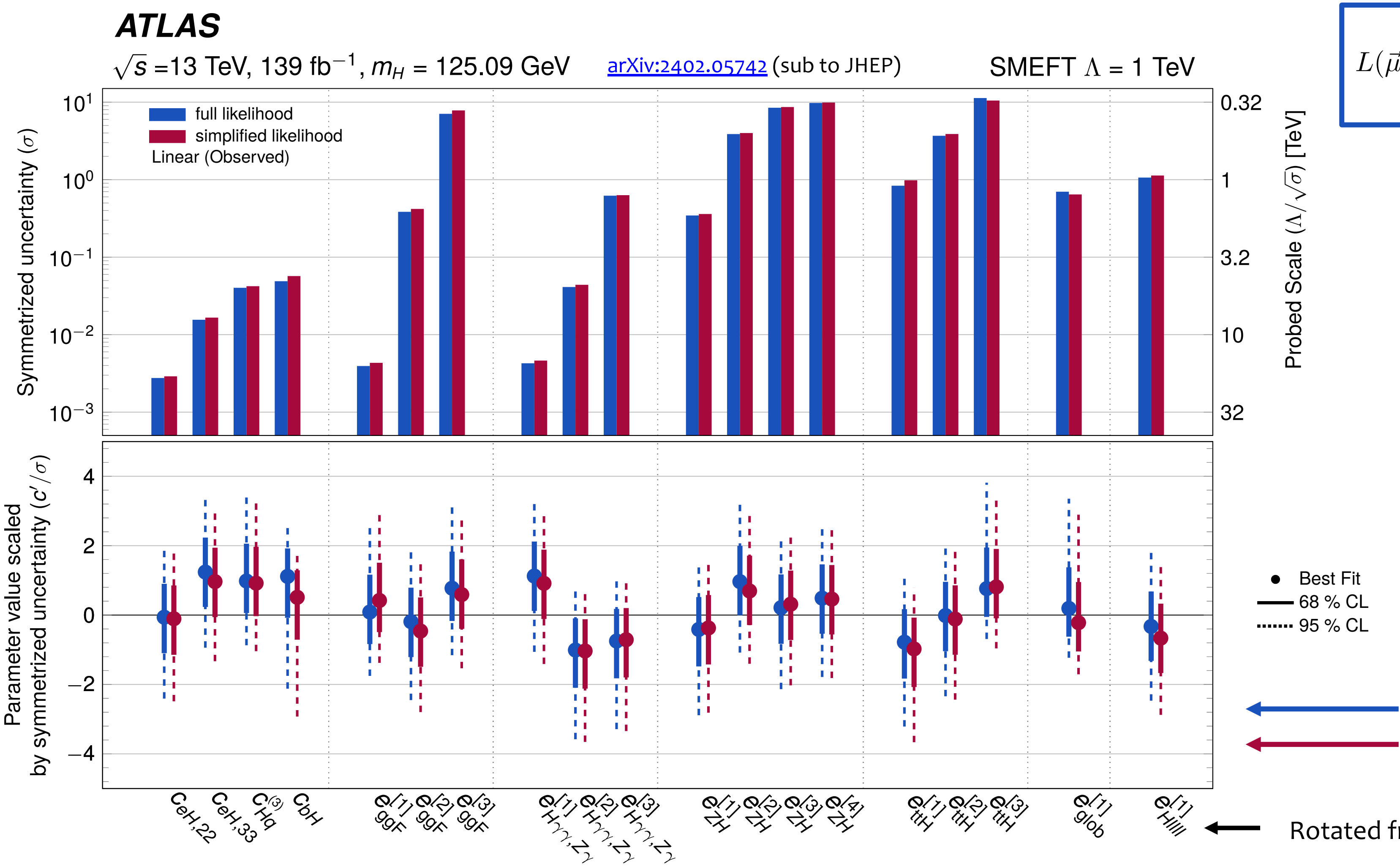




Eur. Phys. J. C 81(2), 145 (2021)

EFT interpretations with Simplified Likelihoods

Comparison of constraints on Wilson Coefficients from **simplified likelihood** approach with **experimental likelihood**



Rotated from Warsaw basis based on sensitive directions in fit

$$L(\vec{\mu},\vec{\nu}) = \prod_n p\left(x_n; \sum_{i,f} \mu_i \mu^f S^f_{i,n}(\vec{\nu}) + \sum_k B_k(\vec{\nu})\right) \cdot \prod_i p(y_i;\nu_i)$$

$$-2\ln\left(\frac{L(\vec{\mu},\hat{\hat{\vec{\nu}}})}{L(\hat{\vec{\mu}},\hat{\vec{\nu}})}\right)$$

$$\left(\hat{\vec{\mu}}-\vec{\mu}\right)^T \left(\boldsymbol{C}_{\vec{\mu}}\right)^{-1} \left(\hat{\vec{\mu}}-\vec{\mu}\right)$$

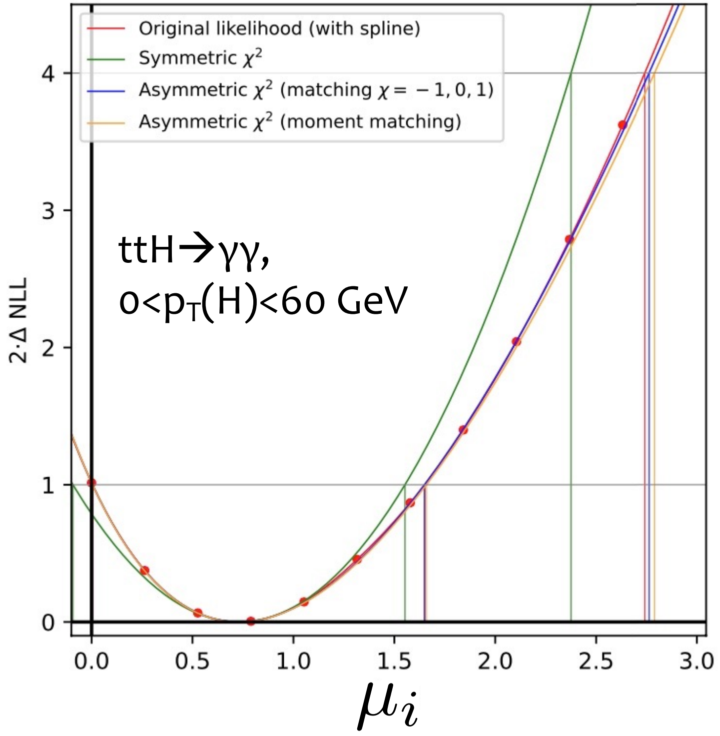
$$\vec{\mu} \rightarrow \vec{\mu}(\vec{c})$$

EFT interpretations with Simplified Likelihoods++

Inclusion of asymmetric uncertainties possible with slight extension to Gaussian approximation*

$$-2\ln\left(\frac{L(\vec{\mu},\hat{\hat{\vec{\nu}}})}{L(\hat{\vec{\mu}},\hat{\vec{\nu}})}\right) \approx \rho_{ij}\chi_i\chi_j\,,\ \mu_i = \alpha_i + \beta_i\chi_i + \gamma_i\chi_i^2$$

Coefficients α, β, γ, and matrix ρ determined from published best-fit, asymmetric uncertainties and correlation matrix [1,2]



*Can also use "*variable Gaussian*" as in **S. Kraml, T.Q. Loc, D.T. Nhung, L.D. Ninh** $C = \mathbf{\Sigma}(\boldsymbol{\mu}).\rho.\mathbf{\Sigma}(\boldsymbol{\mu})$

[1] J. Araz arxXiv:2307.06996 [2] JHEP04 (2019) 064

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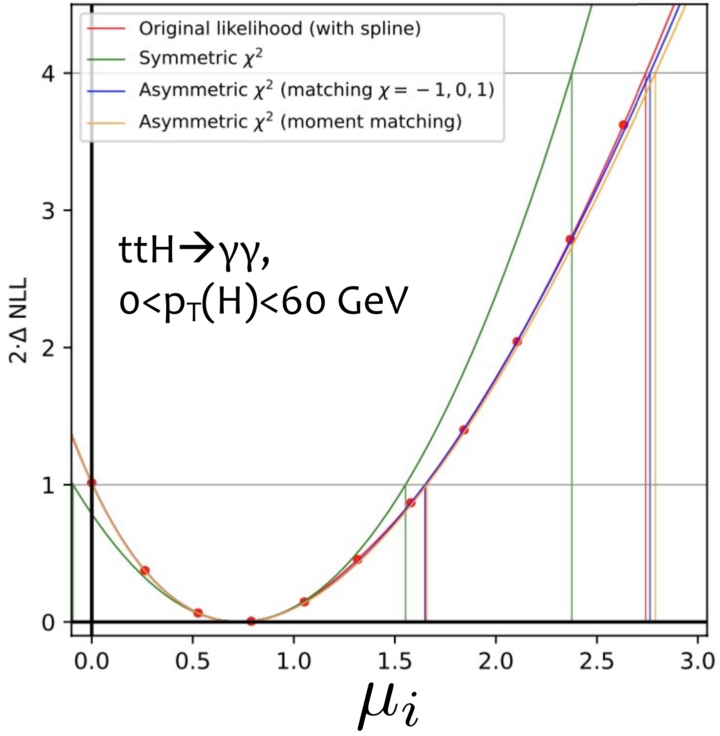

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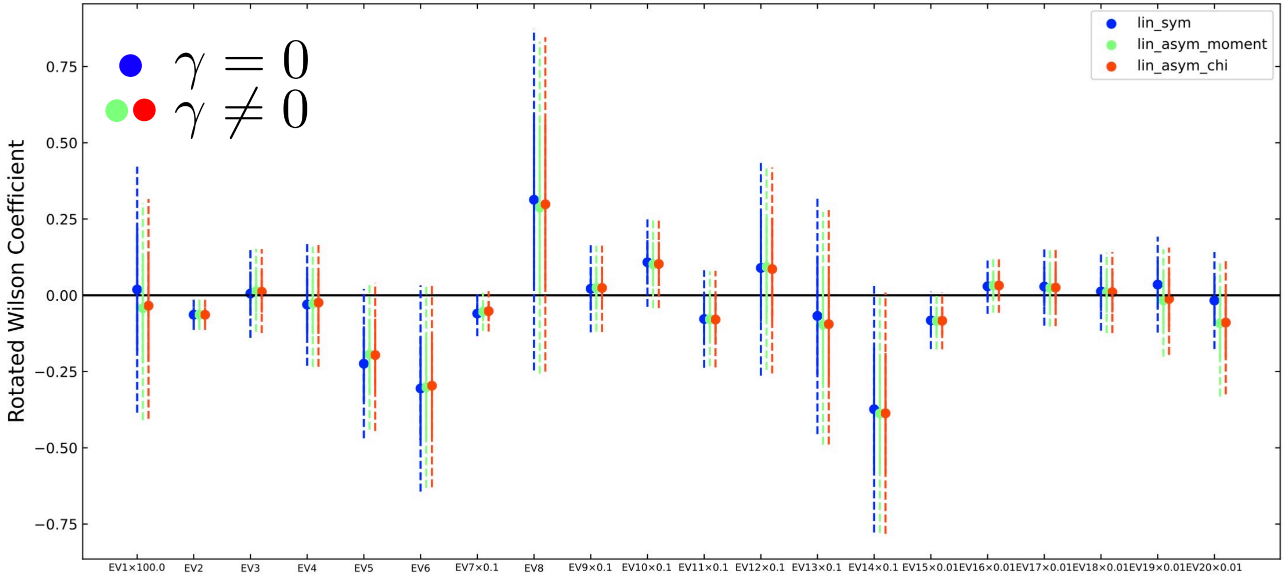

Small impact in terms of EFT constraints from Higgs (+other) measurements

*Can also use "*variable Gaussian*" as in **S. Kraml, T.Q. Loc, D.T. Nhung, L.D. Ninh** $C = \mathbf{\Sigma}(\boldsymbol{\mu}).\rho.\mathbf{\Sigma}(\boldsymbol{\mu})$

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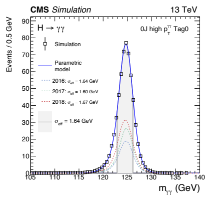
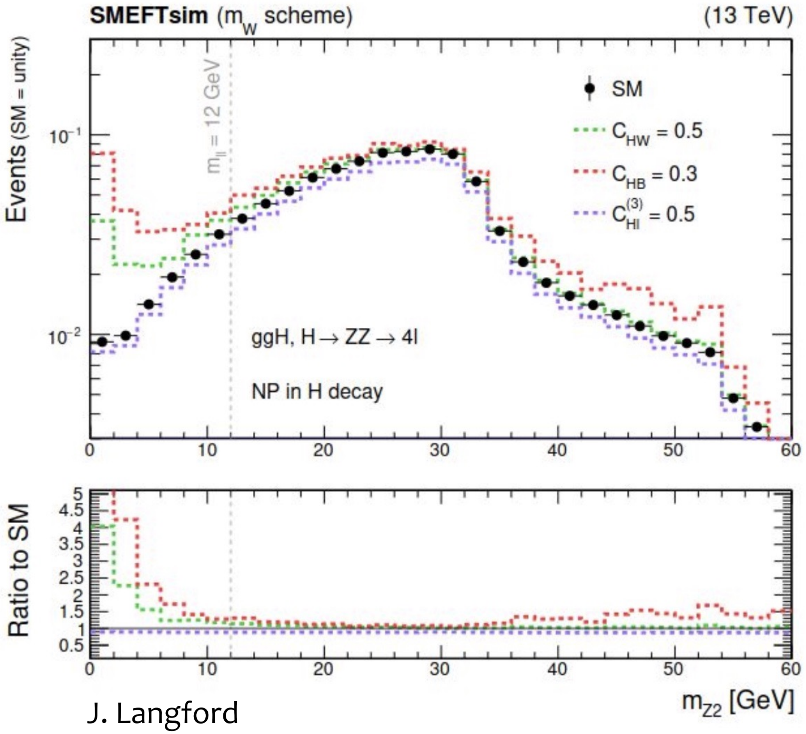
Caveats for Re-interpreting Higgs measurements

EPJC **80** 957 (2020)

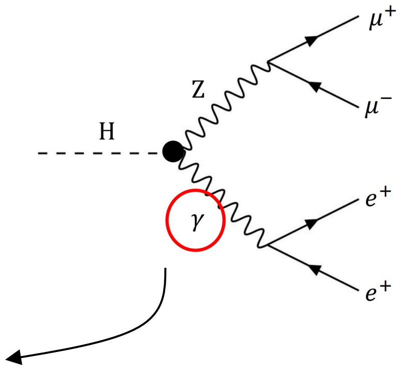
$$L(\vec{\mu},\vec{\nu}) = \prod_{n} p\left(x_n; \sum_{i,f} \mu_i \mu^f S^f_{i,n}(\vec{\nu}) + \sum_k B_k(\vec{\nu})\right) \cdot \prod_i p(y_i;\nu_i)$$

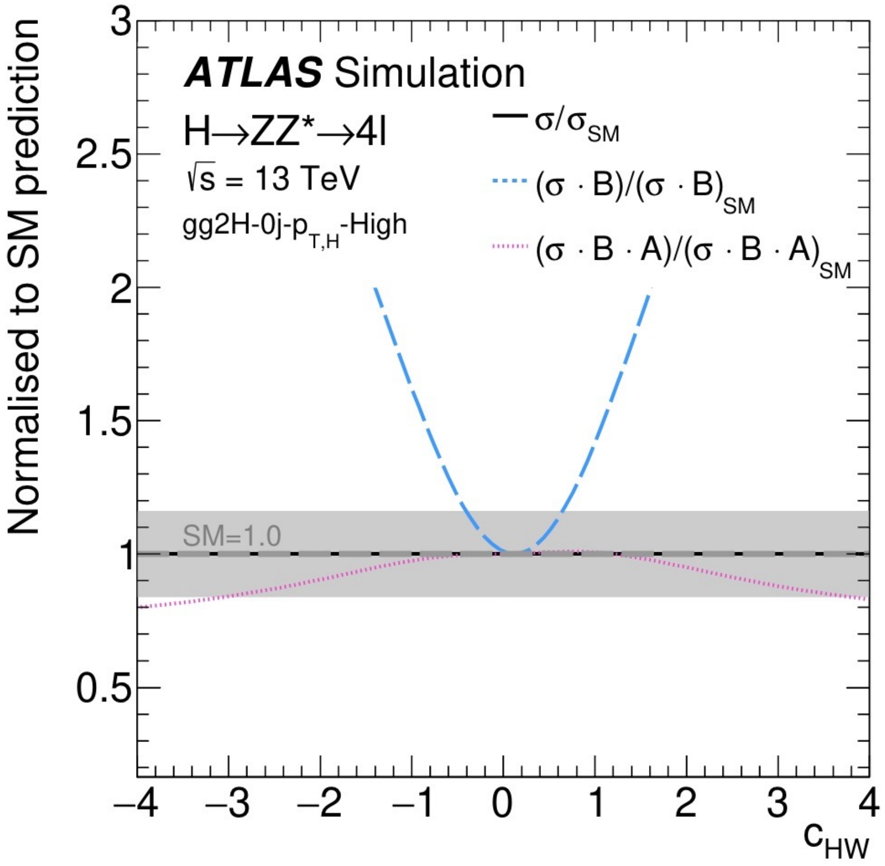
$$= \quad \text{[plot]} \quad \times \mathcal{L} \times \varepsilon \times A$$





J. Langford





No guarantee that assumptions made for SM measurements will hold for BSM

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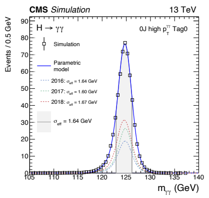
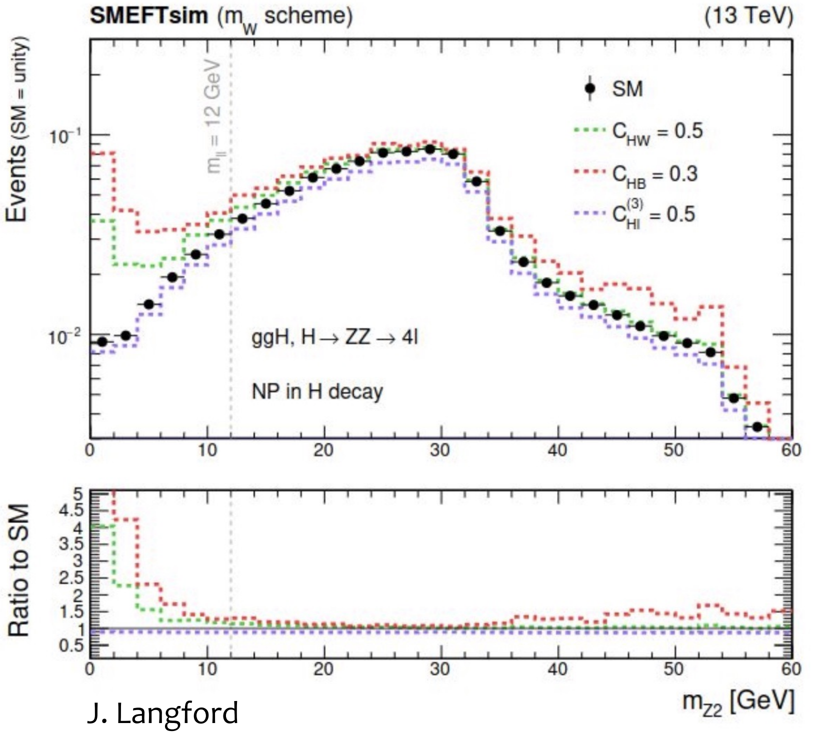
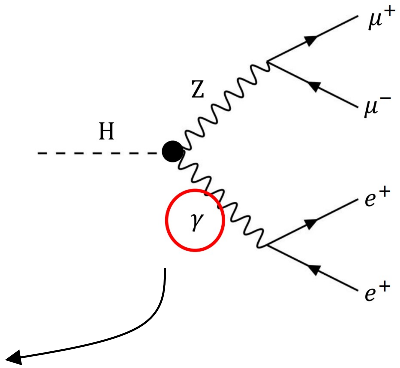
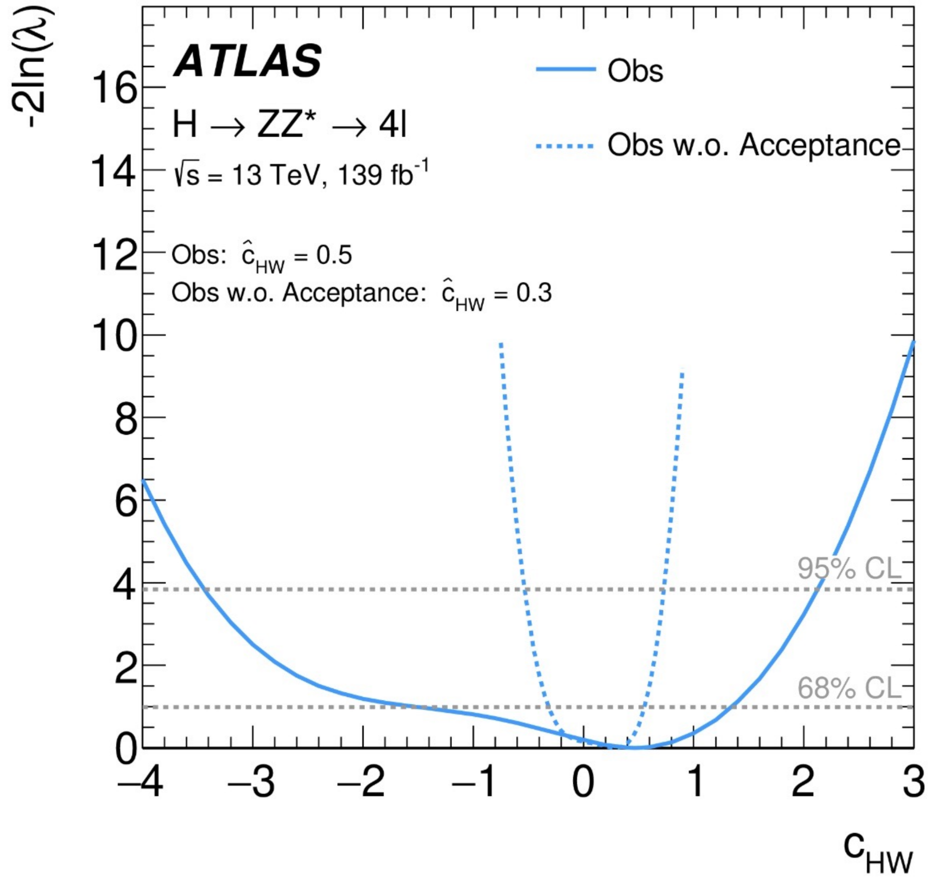
$= $ 

$\times \mathcal{L} \times \varepsilon \times A$



J. Langford





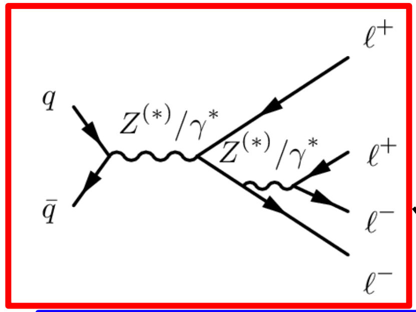
No guarantee that assumptions made for SM measurements will hold for BSM
→ Need to account for non-SM acceptance (A) when interpreting measurements under BSM scenarios!

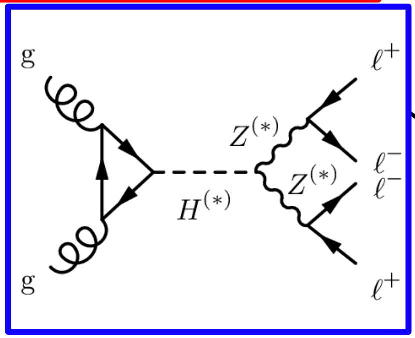
Caveats for Re-interpreting Higgs measurements

$$L(\vec{\mu},\vec{\nu}) = \prod_n p\left(x_n; \sum_{i,f} \mu_i \mu^f S^f_{i,n}(\vec{\nu}) + \sum_k B_k(\vec{\nu})\right) \cdot \prod_i p(y_i;\nu_i)$$

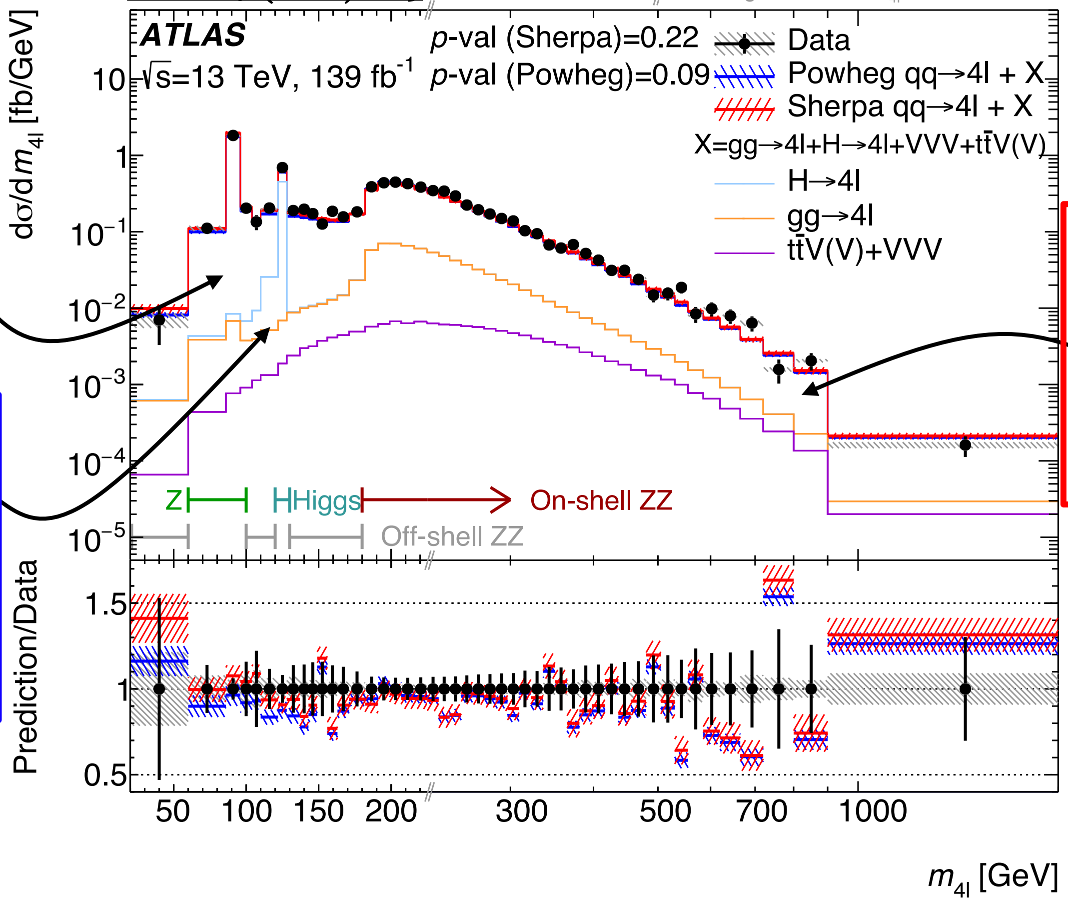
BSM interpretations of Higgs measurements should consider effects on backgrounds* too

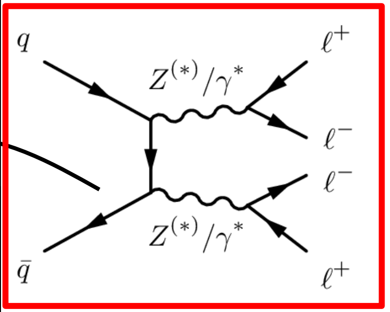
One measurement's **background** is another measurement's **signal**





JHEP 07 (2021) 005



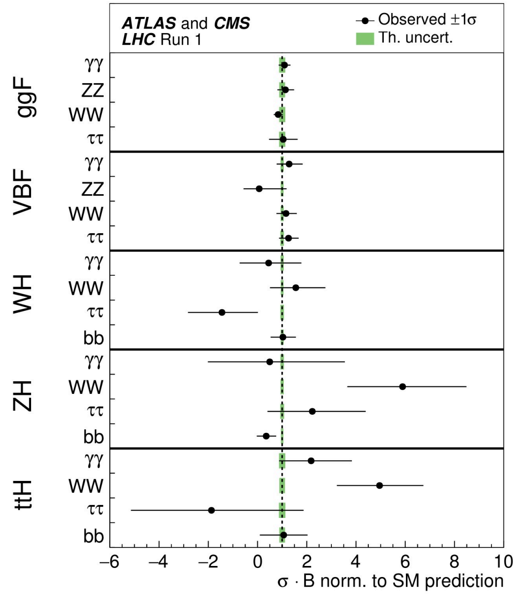


Also highlighted by [L. Nollen](#)

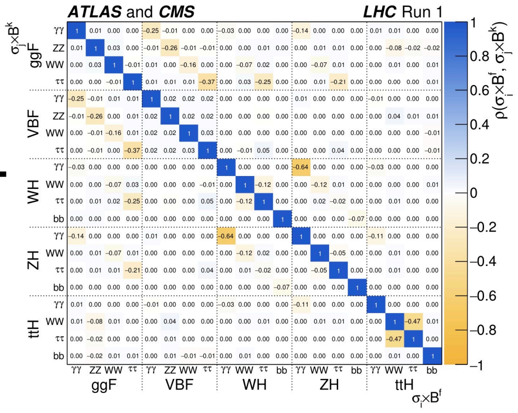
*different story if background is "data-driven"

A note on combinations of simplified likelihoods

Correlation information between measurements (eg due to theory uncertainties) often lost when using simplified likelihoods constructed from ***profiled likelihood*** …



+



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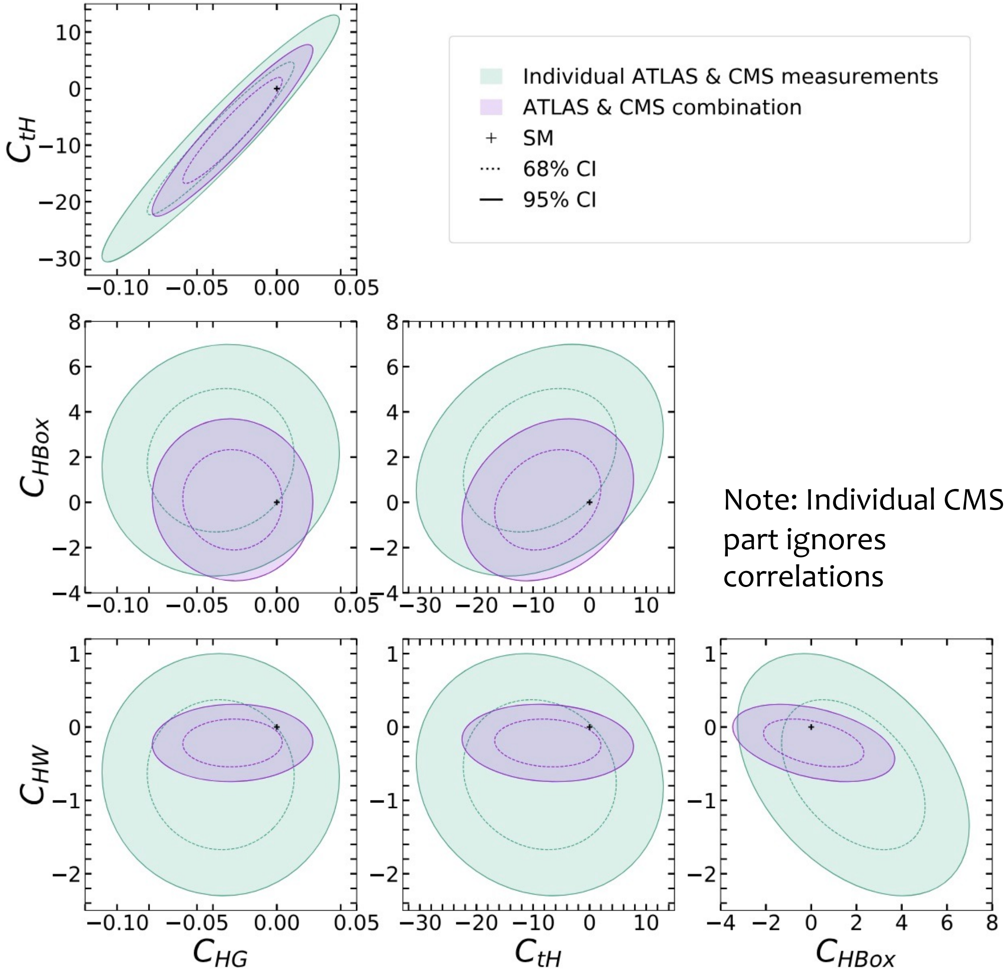

Note: Individual CMS part ignores correlations

arXiv:2109.04981

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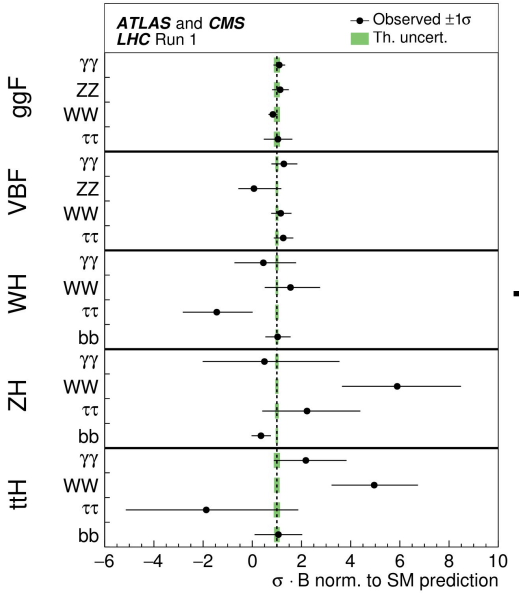

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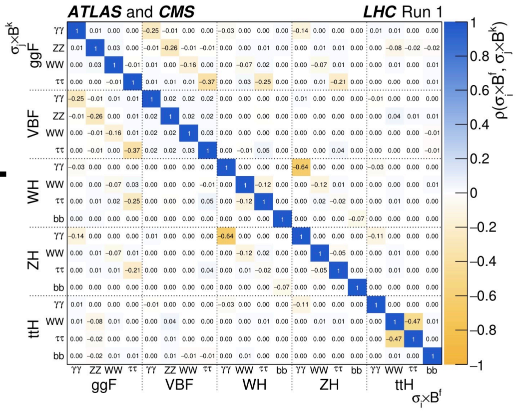

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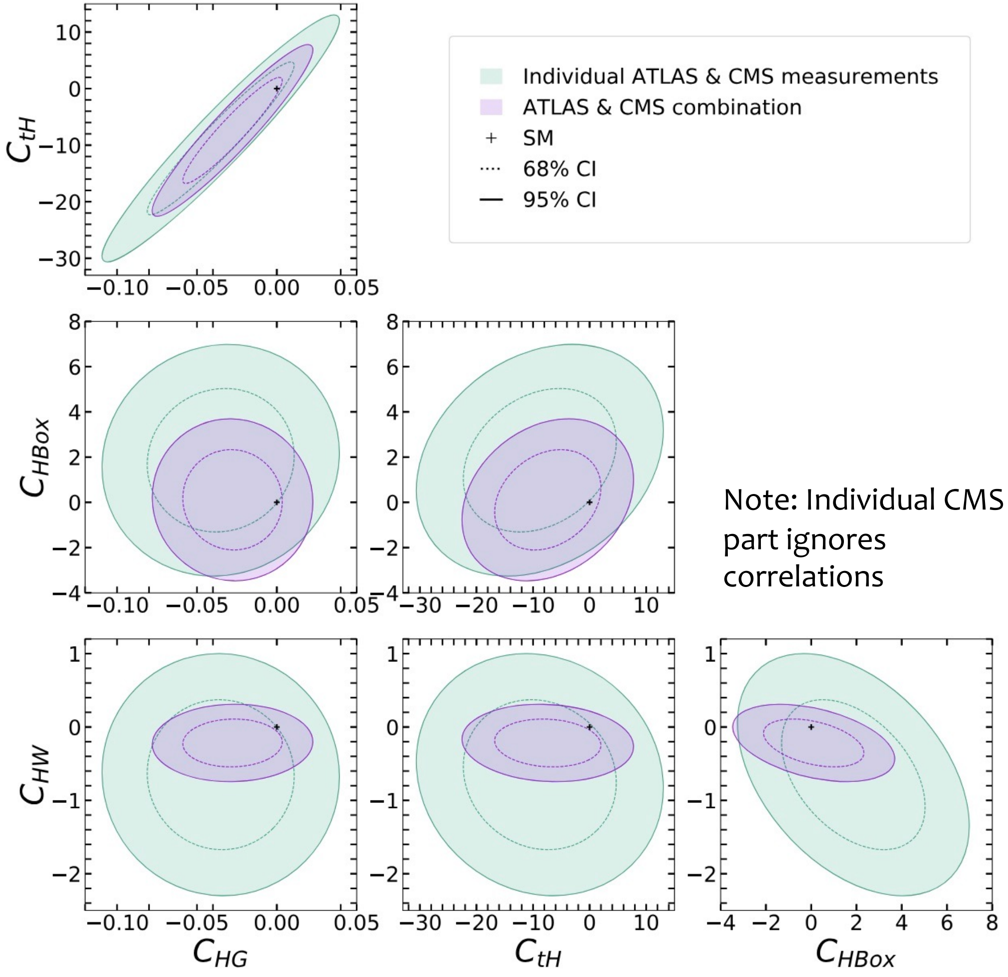


Note: Individual CMS part ignores correlations

$$\hat{L}_{\text{ATLAS}} + \hat{L}_{\text{CMS}} \neq \hat{L}_{\text{ATLAS+CMS}}$$

arXiv:2109.04981

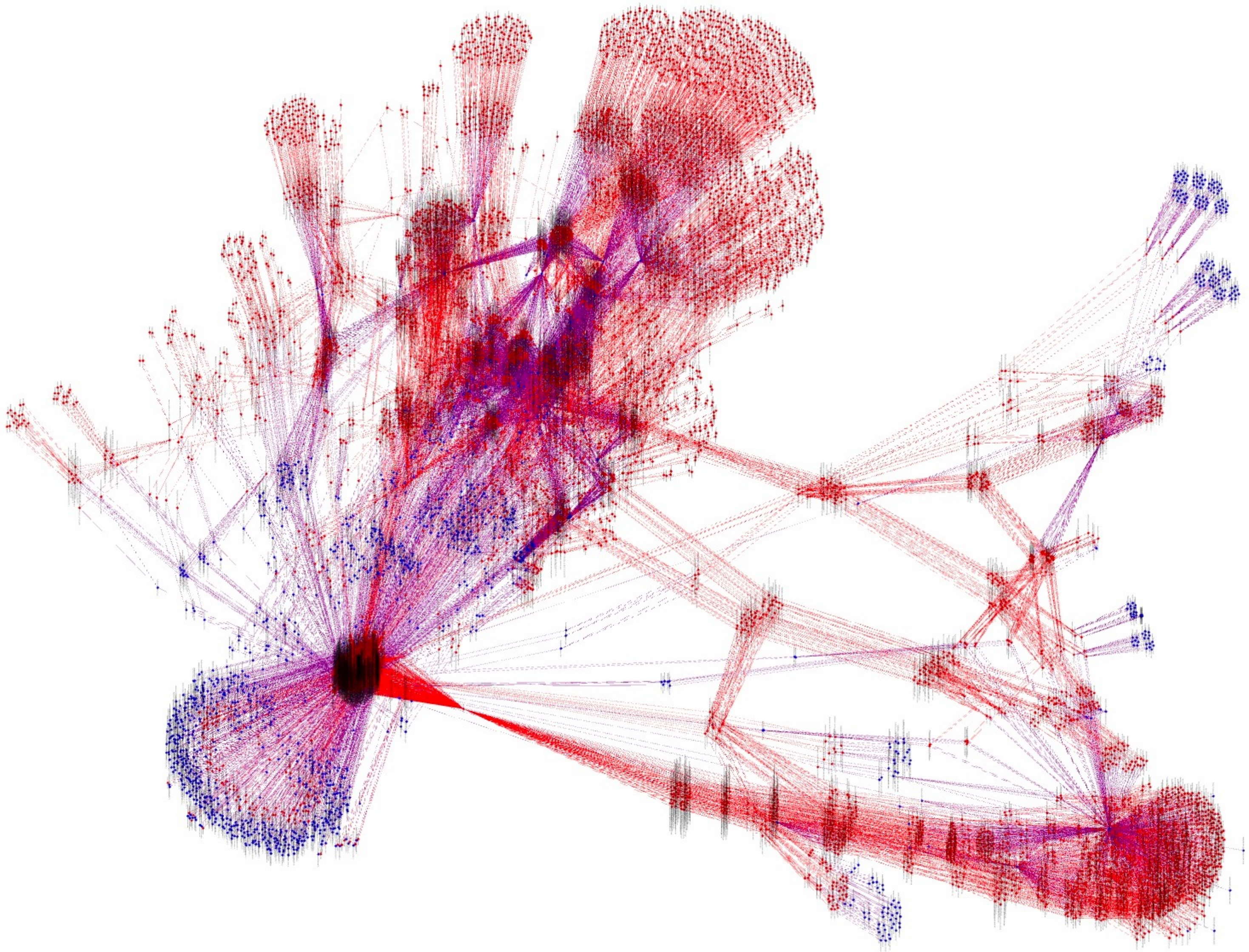
Why can’t we (you) just use the experimental likelihood then?

CMS Higgs observation LH

Even ~12 years ago our Higgs boson experimental likelihood composed of

- 5 decay channels (analyses)
- Mix of template/parametric analyses
- ~700 parameters

Extremely complicated statistical model



How can we communicate likelihood function for (re-)interpretations?

Like this …

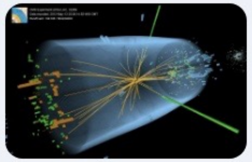
CERN
416,893 followers
2h •

CMS releases #HiggsBoson discovery data to the public

The CMS Collaboration has recently released, in electronic format, the combination of the measurements that contributed to establishing the discovery of the Higgs boson in 2012.

This release coincides with the publication of the Combine software – the statistical analysis tool that CMS developed during the first run of #LHC, to search for the unique particle, which has since been adopted throughout the collaboration.

Find out more: https://lnkd.in/gq_Tb5UB



CMS releases Higgs boson discovery data to the public
home.cern • 3 min read

CDS | Search records... | Communities | My dashboard | Log in

CMS statistical models

Published April 15, 2024 | Version v1.0 — Model — Open

CMS Higgs boson observation statistical model

CMS Collaboration

Introduction

This resource contains the full statistical model from the Higgs Run-1 combination, which led to the Higgs boson discovery, in the format of `Combine` datacards. The instructions below include a few basic examples on how to extract the significance and signal strength measurements, for more details please consult the `Combine` documentation.

Datacards

Datacards for the combination (and per-decay channel sub-combinations) leading to the Higgs-boson discovery at CMS are in the `125.5` folder. The nuisance parameters corresponding to different sources of systematic uncertainties are described in the `*.html` and `*.yml` files located in that folder.

For the full combination of decay channels, the relevant datacard is `125.5/comb.txt`. The individual datacards for each of the analyses in CMS targeting the main Higgs boson decay modes are also in the `125.5` folder.

Software instructions

General installation instructions for `Combine` can be found in the `Combine` documentation.

A container image is provided to ensure reproducible results. The results in this README are obtained using `v9.2.1`:

```
docker run --name combine -it gitlab-registry.cern.ch/cms-cloud/combine-standalone:v9.2.1
```

A slim version of the container image is also available at `gitlab-registry.cern.ch/cms-cloud/combine-standalone:v9.2.1-slim`. Versions of packages in the slim container image do not match exactly with the ones in the default container, so small differences in the output of commands with respect to the ones shown below are to be expected.

You can copy files (such as the datacards and other inputs for `combine`) using `docker cp` as documented here.

For the commands below, you may require running `ulimit -s unlimited; ulimit -u unlimited` to avoid memory issues.

1K VIEWS — 118 DOWNLOADS
Show more details

Versions
Version v1.0 — Apr 15, 2024
10.17181/c2948-e8875

Cite all versions? You can cite all versions by using the DOI 10.17181/2cp5k-ggn24. This DOI represents all versions, and will always resolve to the latest one. Read more.

Communities
CMS statistical models

Details
DOI (Cite this version - v1.0) DOI 10.17181/c2948-e8875
DOI (Cite all versions) DOI 10.17181/2cp5k-ggn24
Resource type: Model
Publisher: CERN

https://new-cds.cern.ch/records/c2948-e8875

Full **statistical model** + **data** = **experimental likelihood!**

Note : ATLAS pyHF full experimental statistical models (+data) also available since 2019 but none for **SM Higgs measurements** yet…

(Re-)using the likelihood

CMS statistical software (`Combine`) published to allow (re-) interpretation of published CMS statistical models (see `Combine` paper and online documentation)

PhysicsModels

	model	--PO	POIs	Description
Couplings with resolved loops	K1	--PO dohmm , --PO dohzg --PO dohcchgluglu --PO BRU --PO higgsMassRange=x,y	kappa_W , kappa_Z , kap pa_b , kappa_t , kappa_tau , kappa_mu	Higgs boson couplir $H \to \gamma\gamma$ loops are options doX=1 , the by the appropriate c are tied to other pro branching ratio unce of just using the unc with range $x < m_H$
Couplings with effective loops	K2	--PO dohmm , --PO dohzg --PO dohcchgluglu --PO BRU --PO higgsMassRange=x,y	kappa_g , kapppa_gam , kappa_Zgam , kappa_W , kappa_Z , kappa_b , kappa_t , kappa_tau , kappa_mu , kappa_Zg	Higgs boson couplir $H \to Z\gamma$ loops are the options doX=1 , scaled by the appro tied to other proces branching ratio unce of just using the unc with range $x < m_H$

Python based model implementation

```python
def getHiggsSignalYieldScale(self, production, decay, energy):
    name = "c7_XSBRscal_%s_%s_%s" % (production, decay, energy)
    if self.modelBuilder.out.function(name) == None:
        if production in ["ggH", "qqH", "ggZH", "tHq", "tHW"]:
            XSscal = ("@0", "Scaling_%s_%s" % (production, energy))
        elif production == "WH":
            XSscal = ("@0*@0", self.kappa_W)
        elif production == "ZH":
            XSscal = ("@0*@0", self.kappa_Z)
        elif production == "ttH":
            XSscal = ("@0*@0", "kappa_t")
        elif production == "bbH":
            XSscal = ("@0*@0", "kappa_b")
        else:
            raise RuntimeError("Production %s not supported" % production)
        BRscal = decay
        if not self.modelBuilder.out.function("c7_BRscal_" + BRscal):
            raise RuntimeError("Decay mode %s not supported" % decay)
        if decay == "hss":
            BRscal = "hbb"
        if production == "ggH" and (decay in self.add_bbH) and energy in ["7TeV", "8TeV", "13TeV", "14TeV"]:
            b2g = "CMS_R_bbH_ggH_%s_%s[%g]" % (decay, energy, 0.01)
            b2gs = "CMS_bbH_scaler_%s" % energy
            self.modelBuilder.factory_(
                'expr::%s("(%s + @1*@1*@2*@3)*@4", %s, kappa_b, %s, %s, c7_BRscal_%s)' % (name, XSscal[0], XSscal[1], b2g, b2gs, BRscal)
            )
        else:
            self.modelBuilder.factory_('expr::%s("%s*@1", %s, c7_BRscal_%s)' % (name, XSscal[0], XSscal[1], BRscal))
        print("[LHC-HCG Kappas]", name, production, decay, energy, ": ", end=" ")
        self.modelBuilder.out.function(name).Print("")
    return name
```

Command line interface to **construct + (re-)parameterize likelihood** eg : $\vec{\mu} \to \vec{\mu}(\kappa_V, \kappa_F)$

```
> text2workspace.py -P HiggsAnalysis.CombinedLimit.HiggsCouplings_ICHEP12:cVcF 125.5/comb.txt -m 125.5 -o comb_kVkF.root
> combine comb_kVkF.root -m 125.5 -M MultiDimFit --algo singles
```

Calculate result

```
 --- MultiDimFit ---
best fit parameter values and profile-likelihood uncertainties:
   CV :    +0.946   -0.120/+0.113 (68%)
   CF :    +0.497   -0.170/+0.203 (68%)
Done in 3.09 min (cpu), 3.09 min (real)
```

Summary

Interpretations of Higgs measurements of great interest to community

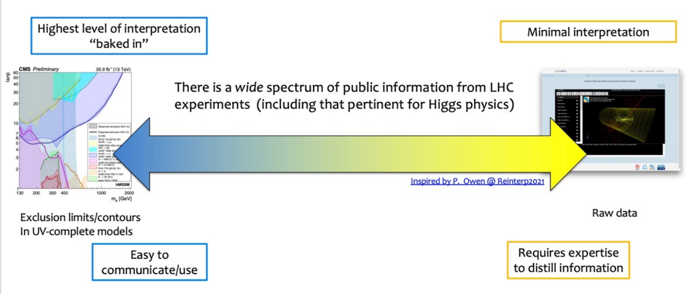
- Evolution with time from inclusive (→kappas) to differential (→EFT) focused measurements
- Ultimately aim is to place constraints on (or better yet discovery) new physics and the way we get to that matters

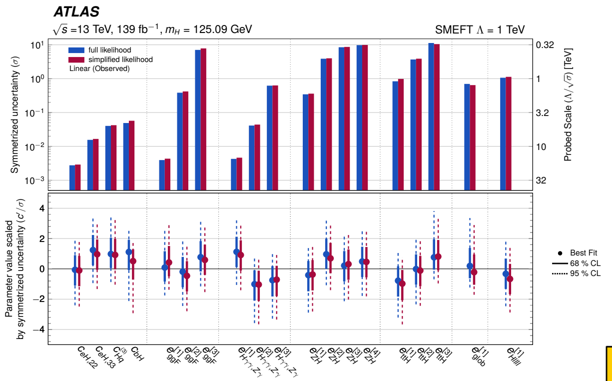
Simplifications facilitate communication + re-use of experimental results

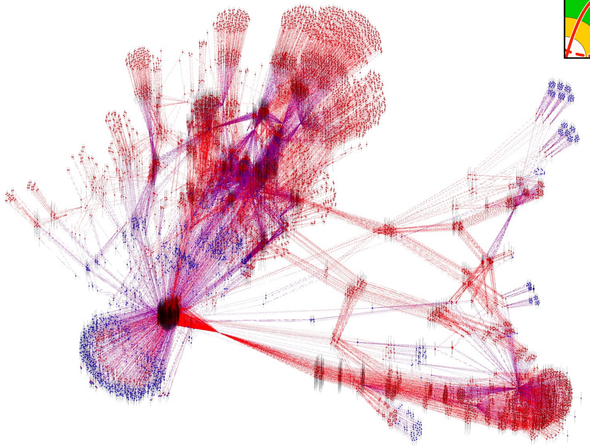
- Gaussian approximation extremely useful but need to be careful of
    - correlations (both experimental and theoretical)
    - Non-gaussian behavior (we never truly reach asymptotes so precision matters)
    - Generalizations of assumptions made from SM→BSM not always valid
- Truly full likelihoods not realized experimentally (yet) though several ideas emerging (see backup) on doing that

CMS has published first full experimental likelihood – the CMS Higgs discovery statistical model (+ the data used)

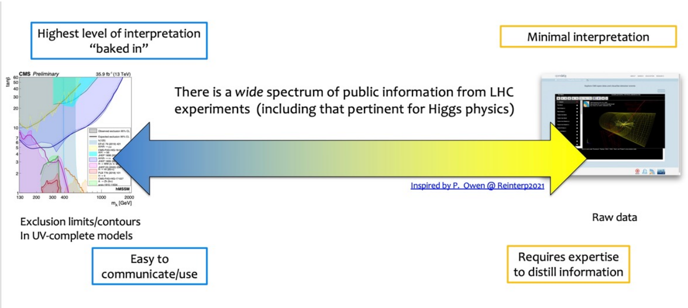
- Code available to (re-)use the model for interpretations of Higgs results
- Many more planned for the future







Summary



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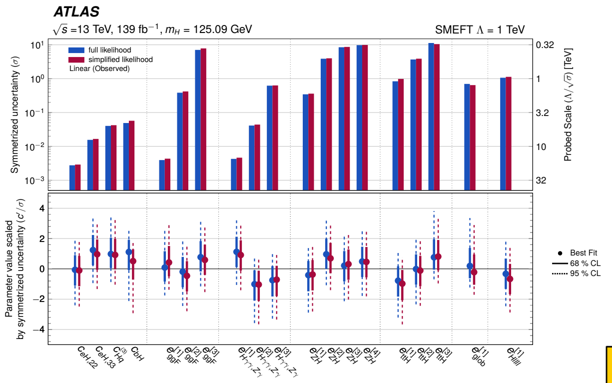

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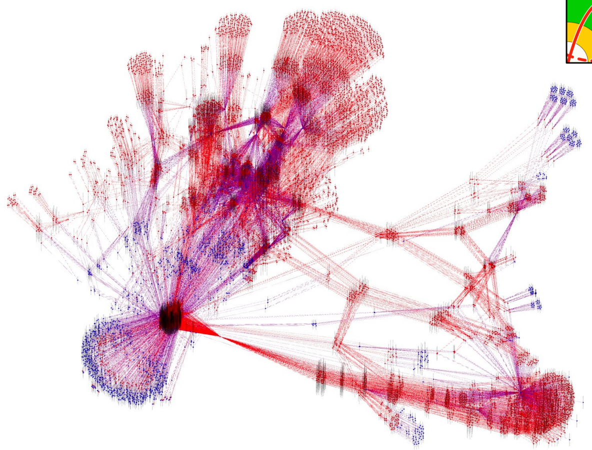

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THANKS!

<u>Backup Slides</u>

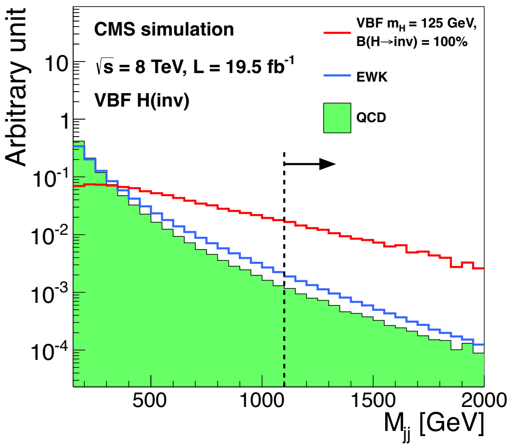
~~"Full"~~ "Experimental" Likelihood

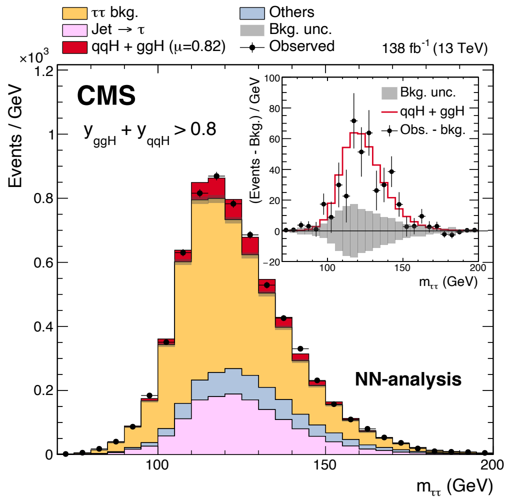
We never use the *full* likelihood for Higgs measurements/interpretations, so I call the most complete thing we use the *experimental* likelihood. For Higgs measurements, typically

$$L(\vec{\mu},\vec{\nu}) = \prod_n p\left(\boxed{x_n}; \sum_{i,f} \mu_i \mu^f S^f_{i,n}(\vec{\nu}) + \sum_k B_k(\vec{\nu}) \right) \cdot \prod_i p(y_i;\nu_i)$$

The "**data**" in each channel can be ...

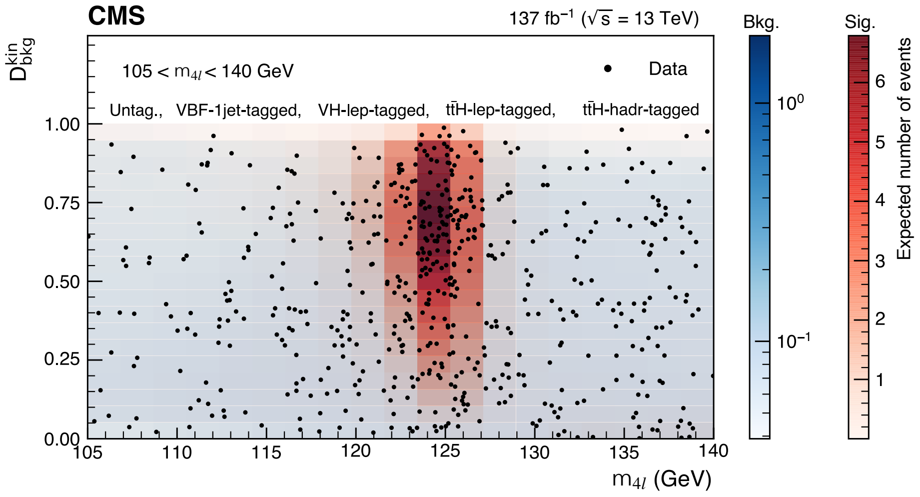
Event count(s) after some selection





Number of events in a given bin of some distribution

Multidimensional observable used to separate signal and background



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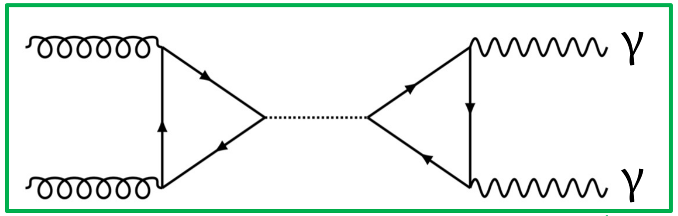
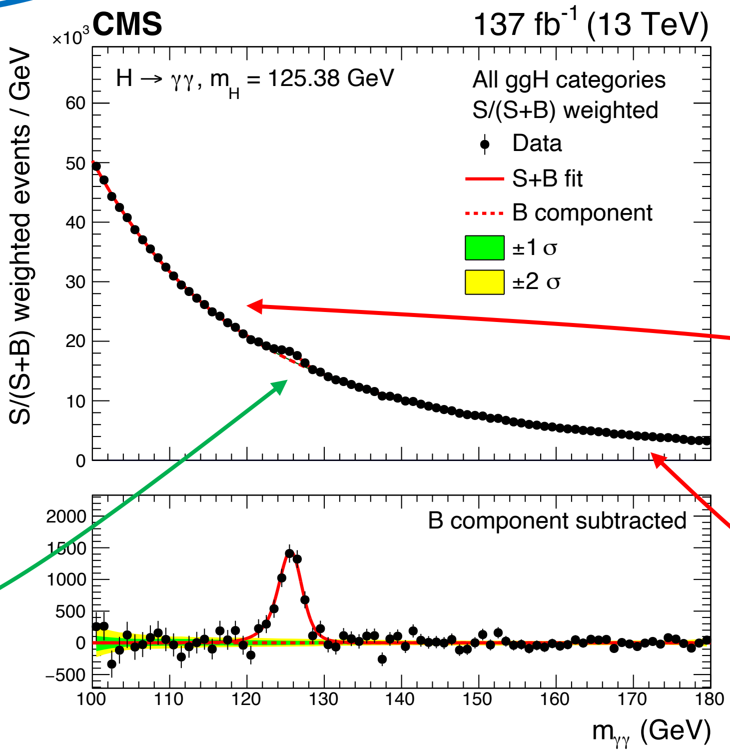
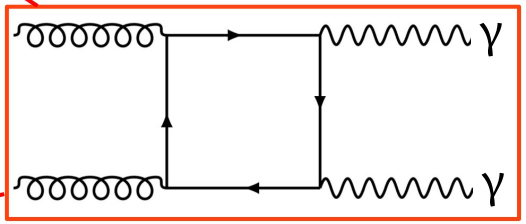
Parameters of interest*

$$\mu_i = \frac{\sigma_i}{(\sigma_i)_{\mathrm{SM}}} \quad \text{and} \quad \mu^f = \frac{\mathrm{BR}^f}{(\mathrm{BR}^f)_{\mathrm{SM}}}.$$





Decompose into ***signal*** and ***background*** contributions

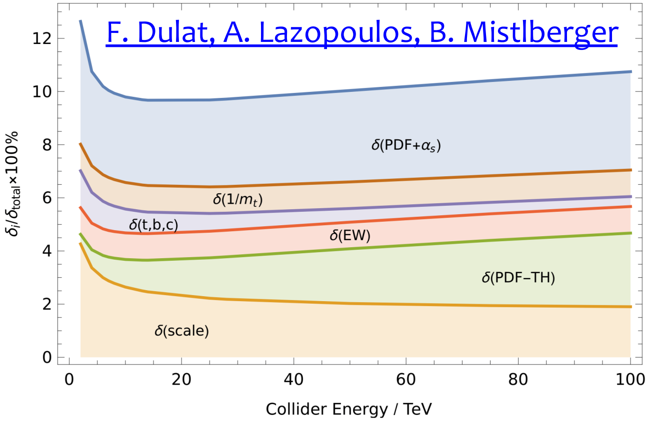


* Note several Higgs analyses directly parameterize in BSM terms eg AC H→VV

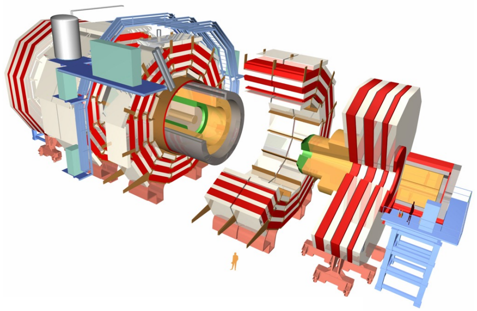
~~"Full"~~ "Experimental" Likelihood

We never use the *full* likelihood for Higgs measurements/interpretations, so I call the most complete thing we use the *experimental* likelihood. For Higgs measurements, typically

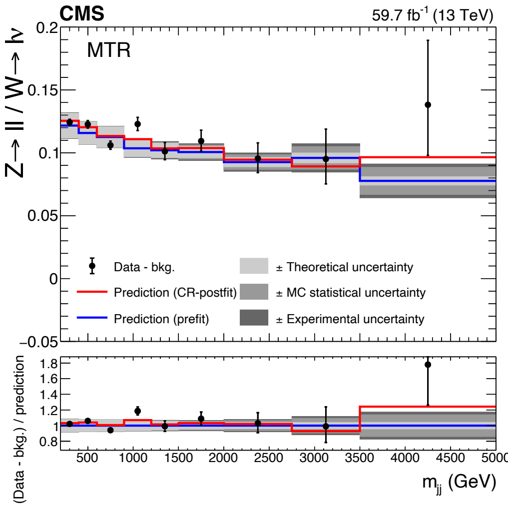
$$L(\vec{\mu},\vec{\nu}) = \prod_{n} p\left(x_n; \sum_{i,f} \mu_i \mu^f S^f_{i,n}(\vec{\nu}) + \sum_k B_k(\vec{\nu}) \right) \cdot \prod_i p(y_i;\nu_i)$$



Experimental/Detector systematics:
- Object efficiencies, energy scales, luminosity



Signal theory uncertainties:
- Inclusive x-section uncertainties, QCD scale, pdf, UEPS, Branching ratios, jet counting

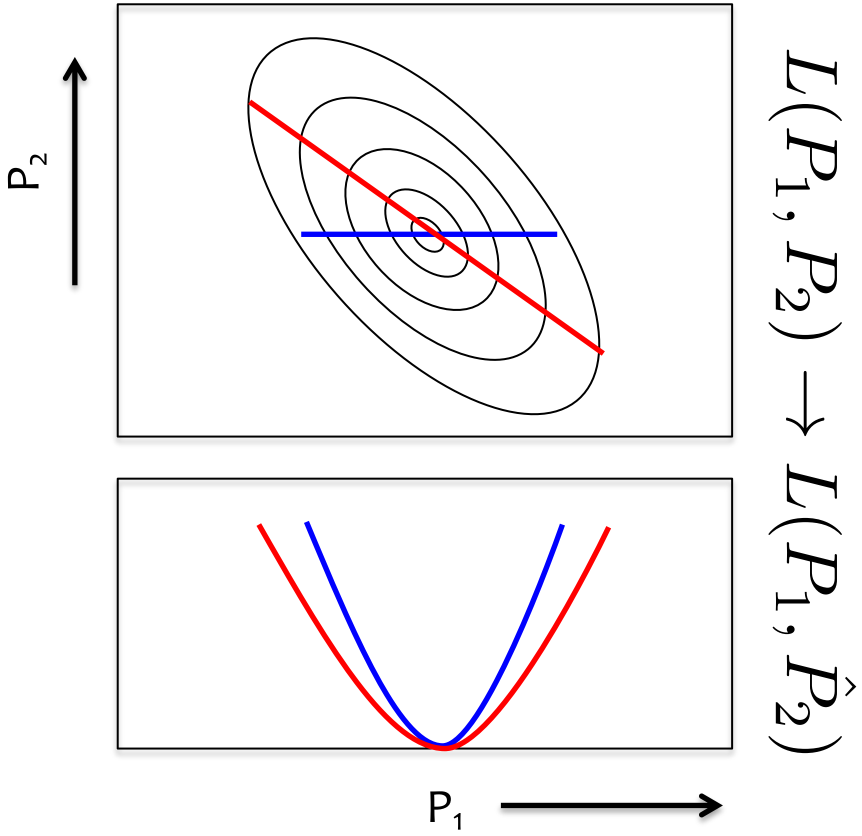


Background theory uncertainties:
- Often rather different phase-spaces considered for extrapolating from control regions for data-driven estimates

Higgs Combinations have **O(1000)'s nuisance parameters**

Profiling

To estimate parameters of the model (and intervals on the parameters of interest), (one or two at a time…), we eliminate parameters of likelihood via ***profiled likelihood***



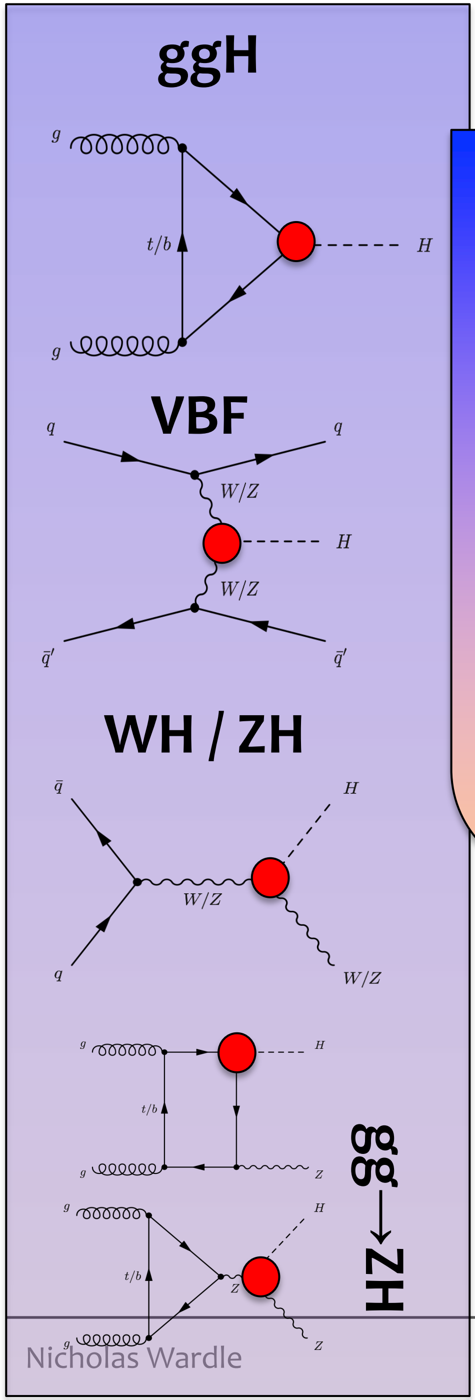
Example, say we have just 2 parameters

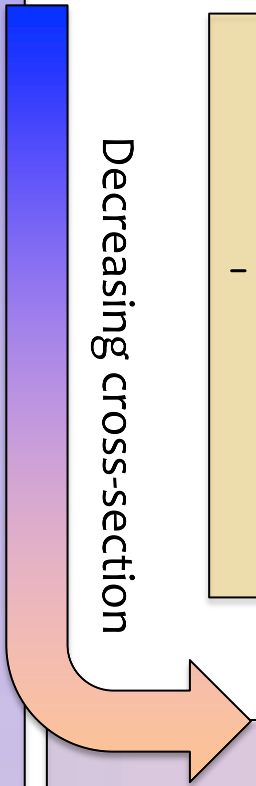
- $L(P_1,P_2)$ describes full likelihood
- Profiling out one of the parameters gives is a profiled likelihood
- We use Wilks' theorem to determine intervals from ratios of profiled log-likelihood (q)

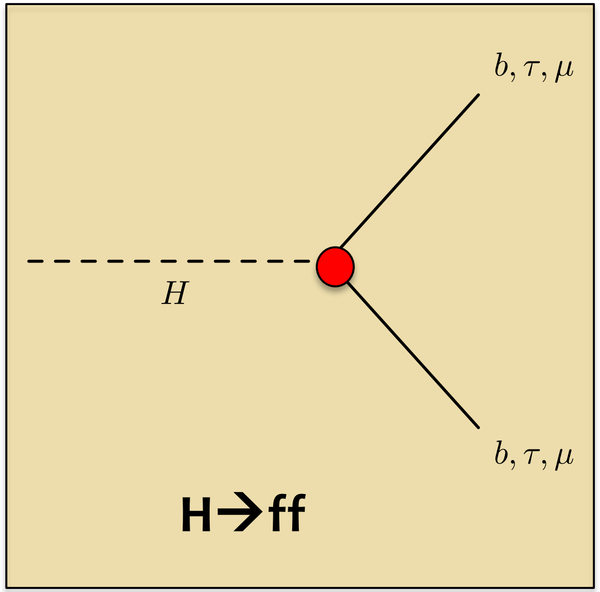
$$q(P_1) = -2\ln\left(\frac{L(P_1, \hat{\hat{P}}_2)}{L(\hat{P}_1, \hat{P}_2)}\right)$$

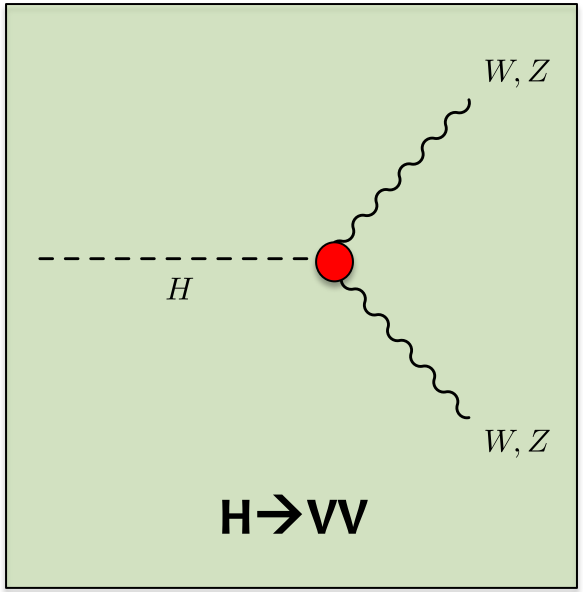
- q=0 → "best-fit" for P_1
- q≤1 → 1σ interval for P_1

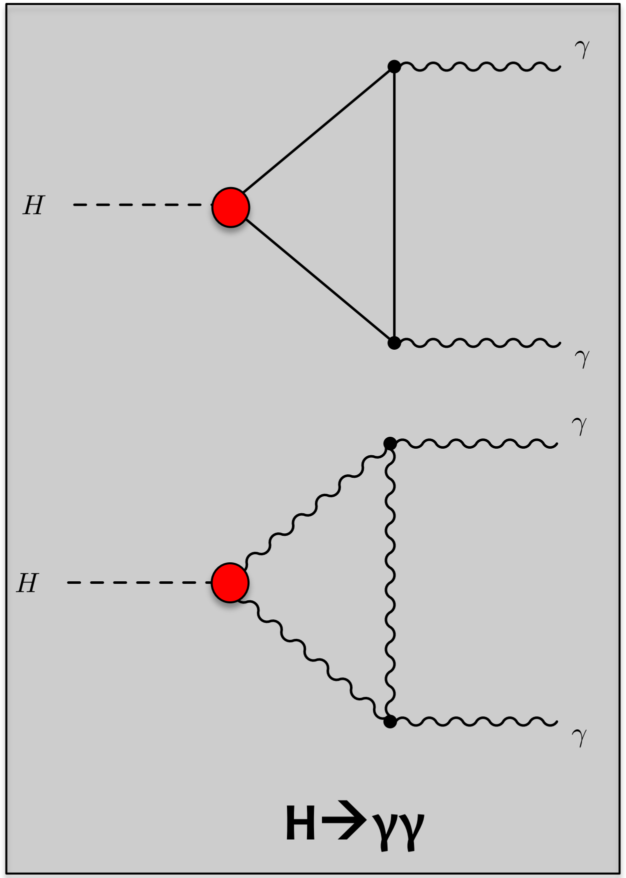
Higgs Production and Decay

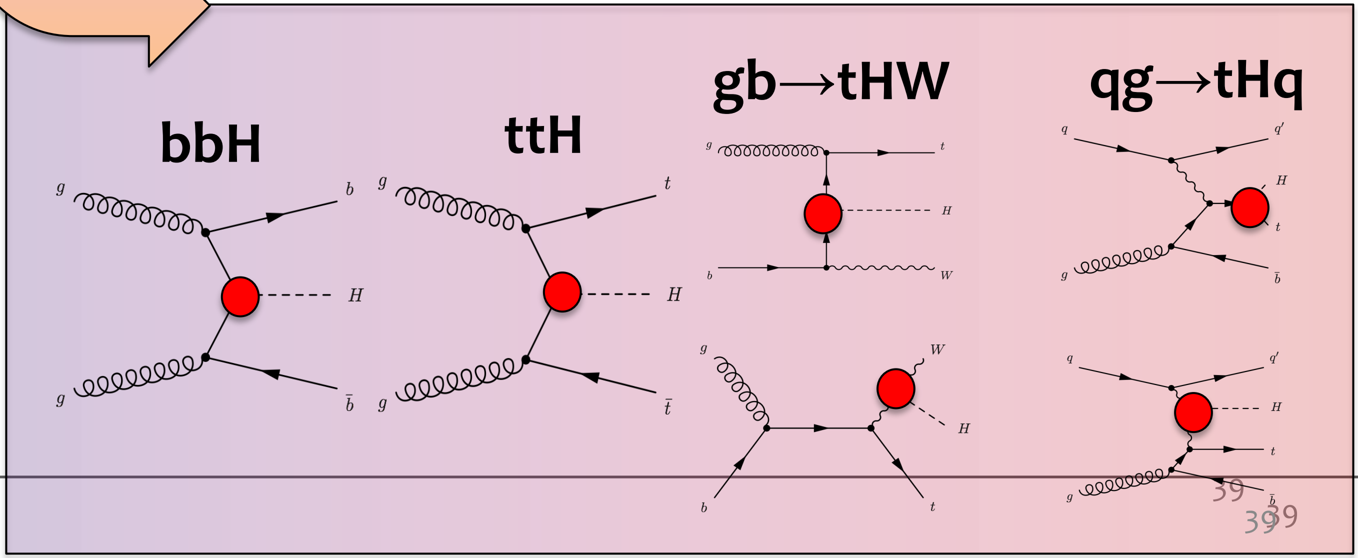












Many production and decay modes to study Higgs for m_H~125 GeV … access to many **Higgs-SM couplings**

Latest Higgs Combinations (inputs)

ATLAS: arXiv:2402.05742(sub to JHEP)

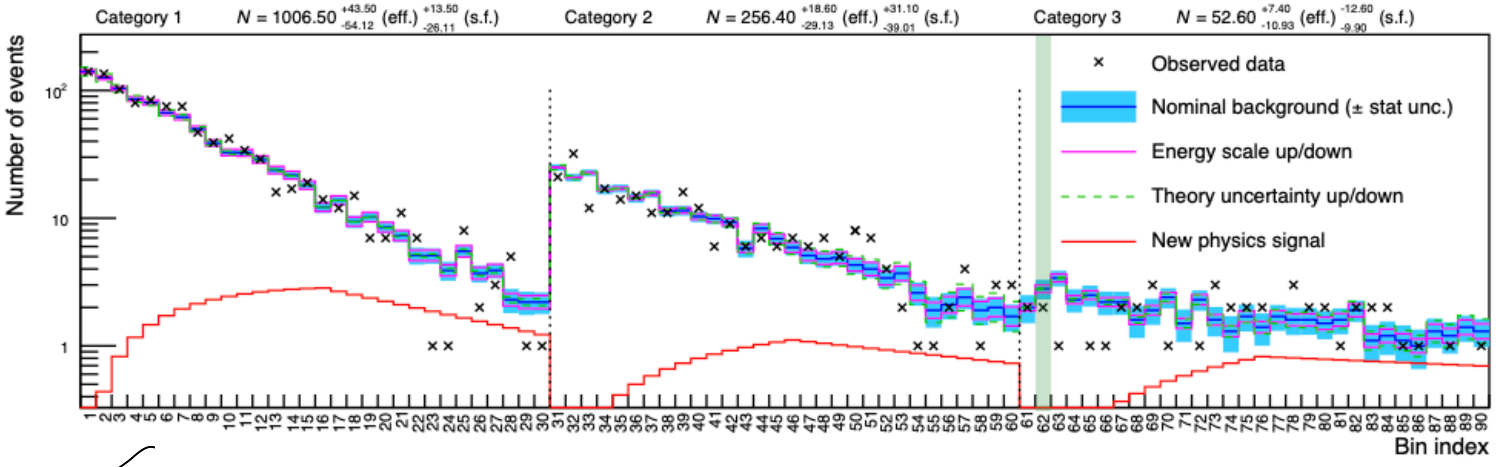
Decay channel	Analysis Production mode	$\mathcal{L}$ [fb^{-1}]	Reference	Binning	SMEFT	2HDM and (h)MSSM
$H \to \gamma\gamma$	(ggF, VBF, WH, ZH, $t\bar{t}H$, tH)	139	[38] [19]	STXS-1.2 differential	✓ ✓(subset)	✓
$H \to ZZ^*$	($ZZ^* \to 4\ell$: ggF, VBF, $WH+ZH$, $t\bar{t}H+tH$)	139	[22] [18]	STXS-1.2 differential	✓ ✓(subset)	✓
	($ZZ^* \to \ell\ell\nu\bar{\nu}/\ell\ell q\bar{q}$: $t\bar{t}H$ multileptons)	36.1	[27]	STXS-0*		✓
$H \to \tau\tau$	(ggF, VBF, $WH+ZH$, $t\bar{t}H+tH$)	139	[39]	STXS-1.2	✓	✓
	($t\bar{t}H$ multileptons)	36.1	[27]	STXS-0*		✓
$H \to WW^*$	(ggF, VBF)	139	[40]	STXS-1.2	✓	✓
	(WH, ZH)	36.1	[41]	STXS-0*		✓
	($t\bar{t}H$ multileptons)	36.1	[27]	STXS-0*		✓
$H \to bb$	(WH, ZH)	139	[42,25]	STXS-1.2	✓	✓
	(VBF)	126	[43]	STXS-1.2	✓	✓
	($t\bar{t}H+tH$)	139	[44]	STXS-1.2	✓	✓
	(boosted Higgs bosons: inclusive production)	139	[45]	STXS-1.2	✓	✓
$H \to Z\gamma$	(inclusive production)	139	[46]	STXS-0*	✓	✓
$H \to \mu\mu$	(ggF + $t\bar{t}H+tH$, VBF + $WH+ZH$)	139	[47]	STXS-0*	✓	✓

CMS: Nature 607 (2022) 60-68

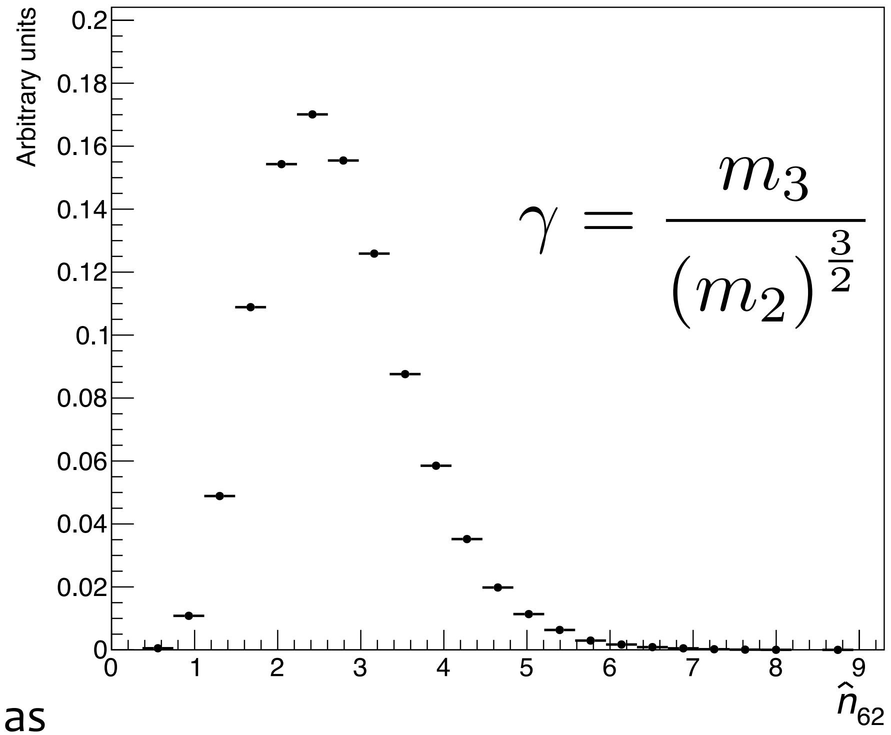
Analysis	Decay tags	Production tags
Single Higgs boson production		
$\mathrm{H} \to \gamma\gamma$ [42]	$\gamma\gamma$	ggH, $p_{\mathrm{T}}(\mathrm{H}) \times N_{\mathrm{j}}$ bins VBF/VH hadronic, $p_{\mathrm{T}}(\mathrm{Hjj})$ bins WH leptonic, $p_{\mathrm{T}}(\mathrm{V})$ bins ZH leptonic ttH $p_{\mathrm{T}}(\mathrm{H})$ bins, tH
$\mathrm{H} \to \mathrm{ZZ} \to 4\ell$ [43]	4μ, $2e2\mu$, $4e$	ggH, $p_{\mathrm{T}}(\mathrm{H}) \times N_{\mathrm{j}}$ bins VBF, m_{jj} bins VH hadronic VH leptonic, $p_{\mathrm{T}}(\mathrm{V})$ bins ttH
$\mathrm{H} \to \mathrm{WW} \to \ell\nu\ell\nu$ [44]	$e\mu/ee/\mu\mu$ $\mu\mu$+jj/ee+jj/$e\mu$+jj 3ℓ 4ℓ	ggH $\leq$ 2-jets VBF VH hadronic WH leptonic ZH leptonic
$\mathrm{H} \to \mathrm{Z}\gamma$ [45]	$\mathrm{Z}\gamma$	ggH VBF
$\mathrm{H} \to \tau\tau$ [46]	$e\mu$, $e\tau_{\mathrm{h}}$, $\mu\tau_{\mathrm{h}}$, $\tau_{\mathrm{h}}\tau_{\mathrm{h}}$	ggH, $p_{\mathrm{T}}(\mathrm{H}) \times N_{\mathrm{j}}$ bins VH hadronic VBF
$\mathrm{H} \to \mathrm{bb}$ [47–51]	$\mathrm{W}(\ell\nu)\mathrm{H(bb)}$ $\mathrm{Z}(\nu\nu)\mathrm{H(bb)}$, $\mathrm{Z}(\ell\ell)\mathrm{H(bb)}$ bb	VH, high-$p_{\mathrm{T}}(\mathrm{V})$ WH leptonic ZH leptonic ttH, $\to 0, 1, 2\ell$ + jets ggH, high-$p_{\mathrm{T}}(\mathrm{H})$ bins
$\mathrm{H} \to \mu\mu$ [52]	$\mu\mu$	ggH VBF
ttH production with $\mathrm{H} \to$ leptons [53]	2ℓ SS, 3ℓ, 4ℓ, $1\ell + \tau_{\mathrm{h}}$, 2ℓ SS+$1\tau_{\mathrm{h}}$, $3\ell + 1\tau_{\mathrm{h}}$	ttH
$\mathrm{H} \to$ Inv. [71, 72]	$p_{\mathrm{T}}^{\mathrm{miss}}$	ggH VBF VH hadronic ZH leptonic

Simplified likelihoods for searches

$$L(\boldsymbol{\alpha},\boldsymbol{\delta})\pi(\boldsymbol{\delta}) = \prod_{I=1}^{P} \Pr\Big(n_I^{\mathrm{obs}} \Big| n_I(\boldsymbol{\alpha},\boldsymbol{\delta})\Big)\pi(\boldsymbol{\delta})$$



Non-Gaussian effects can be accounted for



simplify

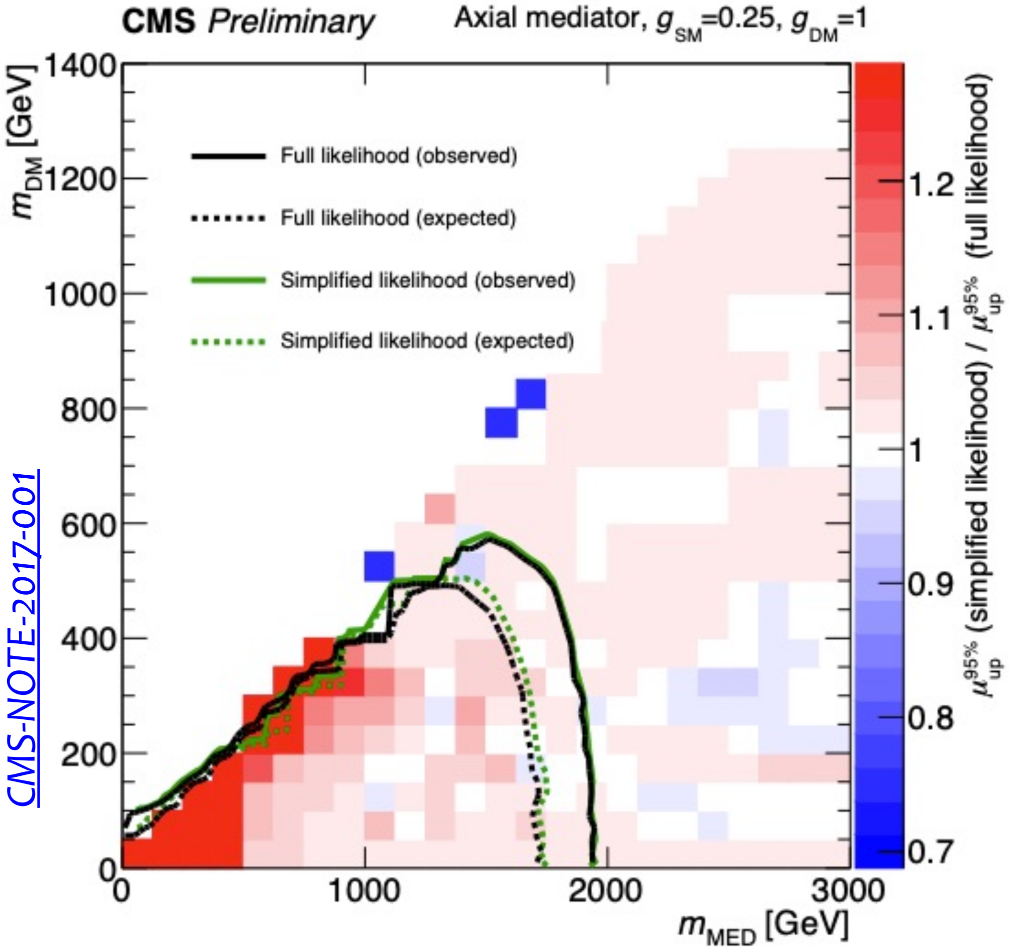
These are the same as the *full* likelihood

$$L(\mu,\boldsymbol{\delta})\pi(\boldsymbol{\delta}) \rightarrow L(\mu,\boldsymbol{\theta})\pi(\boldsymbol{\theta}) = \prod_{I=1}^{P=90} P(n_I^{\mathrm{obs}}|\mu\cdot n_{s,I} + a_I + b_I\theta_I + c_I\theta_I^2)\cdot\frac{1}{\sqrt{(2\pi)^P}}e^{-\frac{1}{2}\boldsymbol{\theta}^T\boldsymbol{\rho}^{-1}\boldsymbol{\theta}}$$

A. Buckley, M. Citron, S. Fichet, S. Kraml, W. Waltenberger, **NW** *J. High Energ. Phys.* **2019,** 64 (2019)

Simplified likelihoods in the wild!

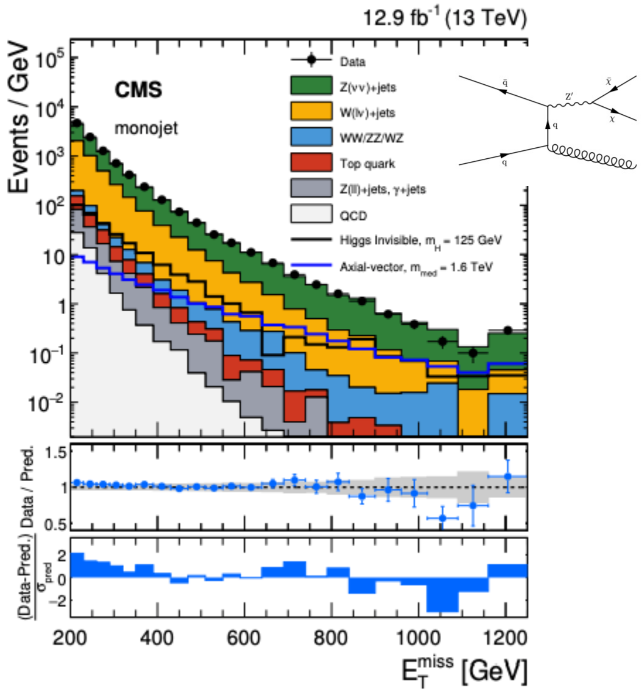
Real experimental likelihoods converted into simplified likelihoods*…

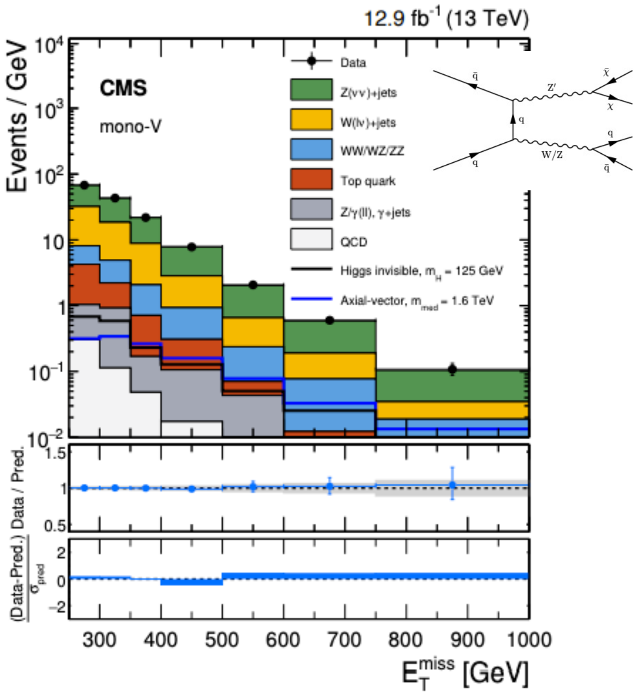


[CMS-NOTE-2017-001](#)

"Search for dark matter produced with an energetic jet or a hadronically decaying W or Z boson at √s= 13 TeV" [JHEP 07 (2017) 014](#)

- Data separated into 1 or 2 jet topologies
- Binned missing transverse momentum distribution used to separate signal from background → 29 bins





Workflows

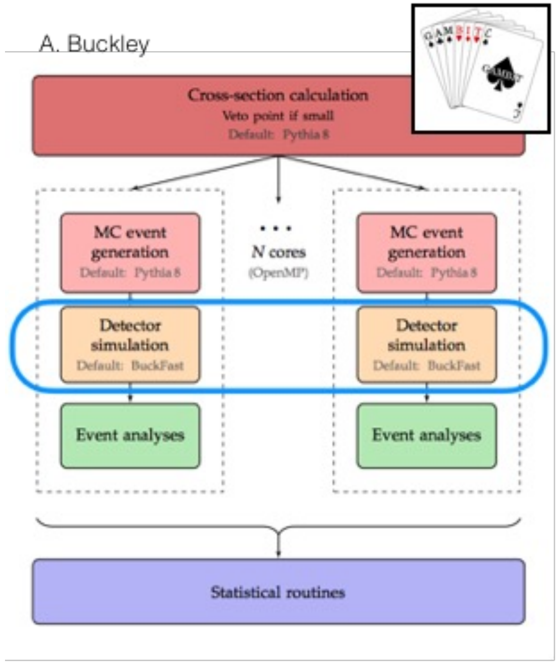
Standard workflow for predictions

New signal model
- UFO+param/proc/run card
- …

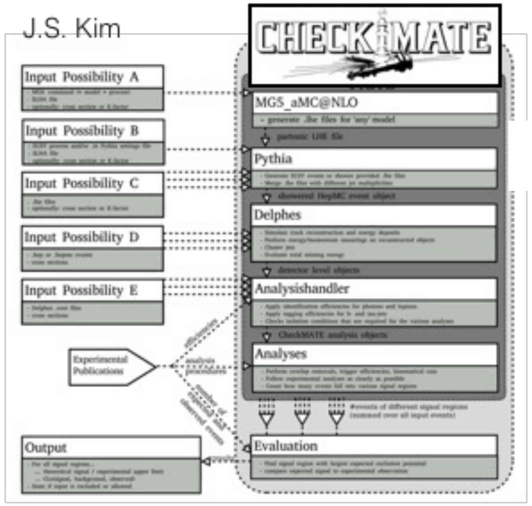
MC generation
- MG5+Pythia
- Herwig
- Sherpa
- …

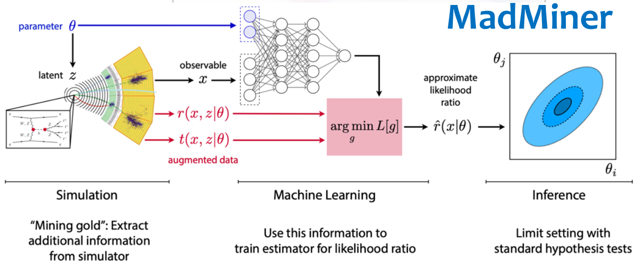
Detector Simulation
- Full/Fast Geant4
- Delphes
- BuckFast
- Transfer functions
- …

Event Selection/categorisation
- Rivet
- MadAnalysis
- …

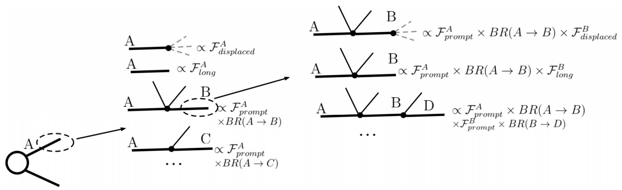


- Counts/observables for signal process
- Statistical inference



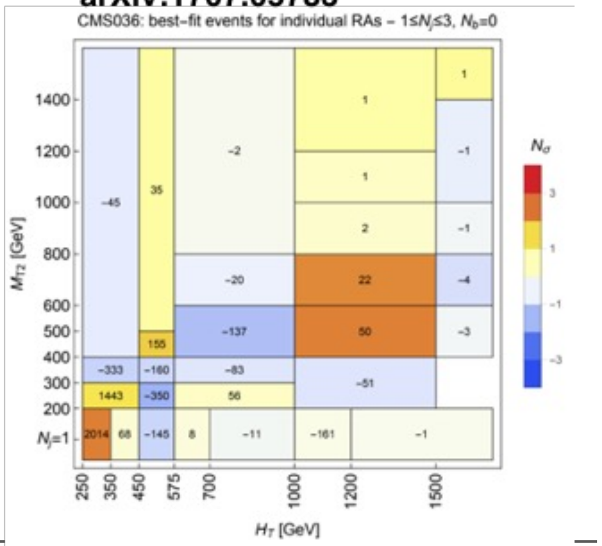


Reproducible research data analysis platform



SModelS

arXiv:1707.05783

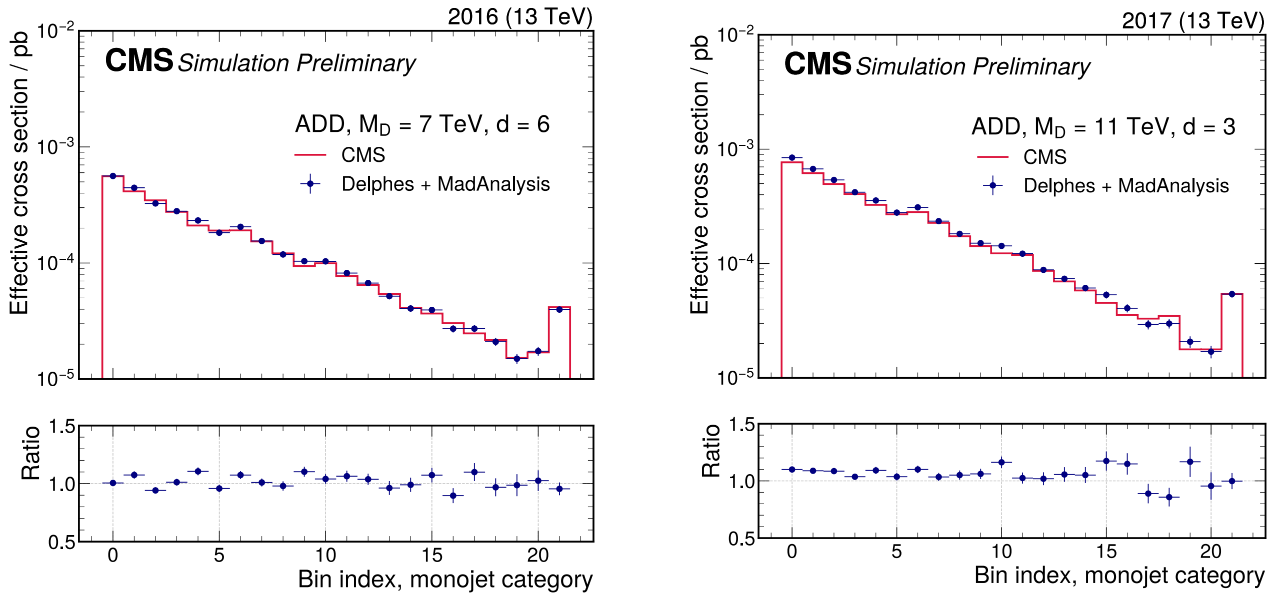


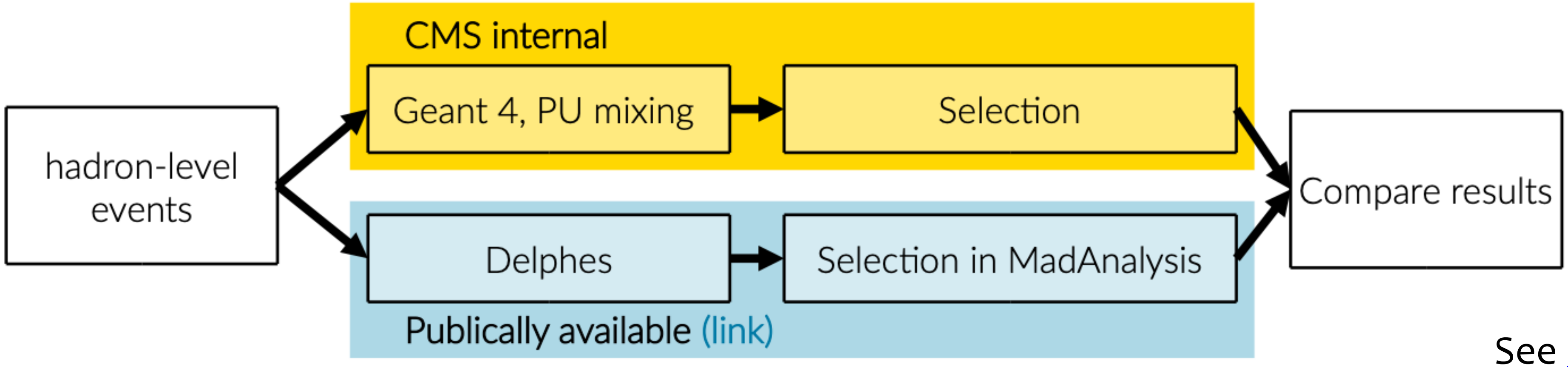
Example from CMS (EXO-20-004)

"Search for new particles in events with energetic jets and large missing transverse momentum in proton-proton collisions at 13 TeV" – Full Run-2 data update

HepData entry

- Signal templates & cutflows
- Simplified likelihood inputs
- MC Generator configs for various signals + MadAnalysis implementation

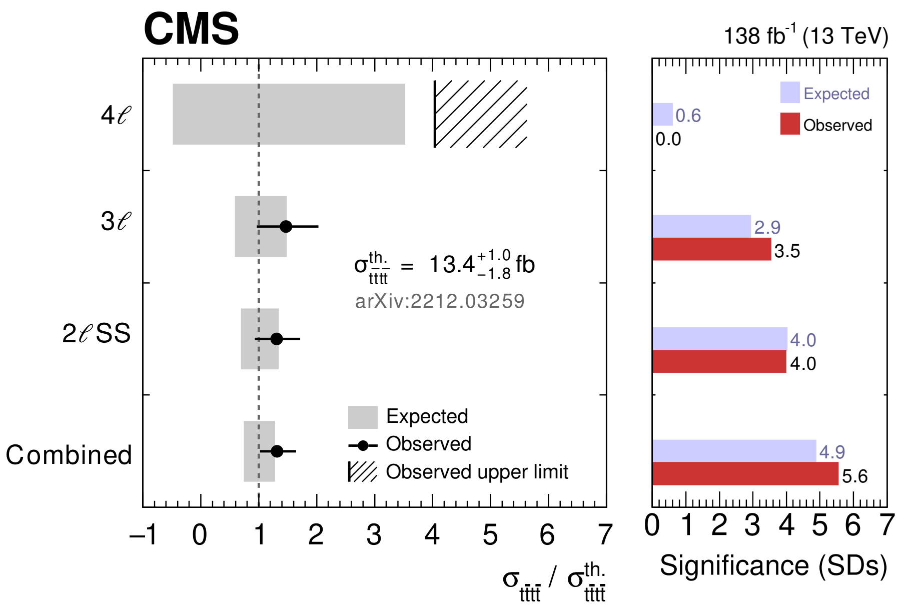




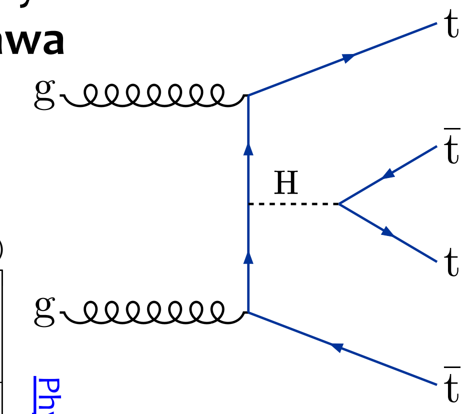
See RAMP talk by A. Albert

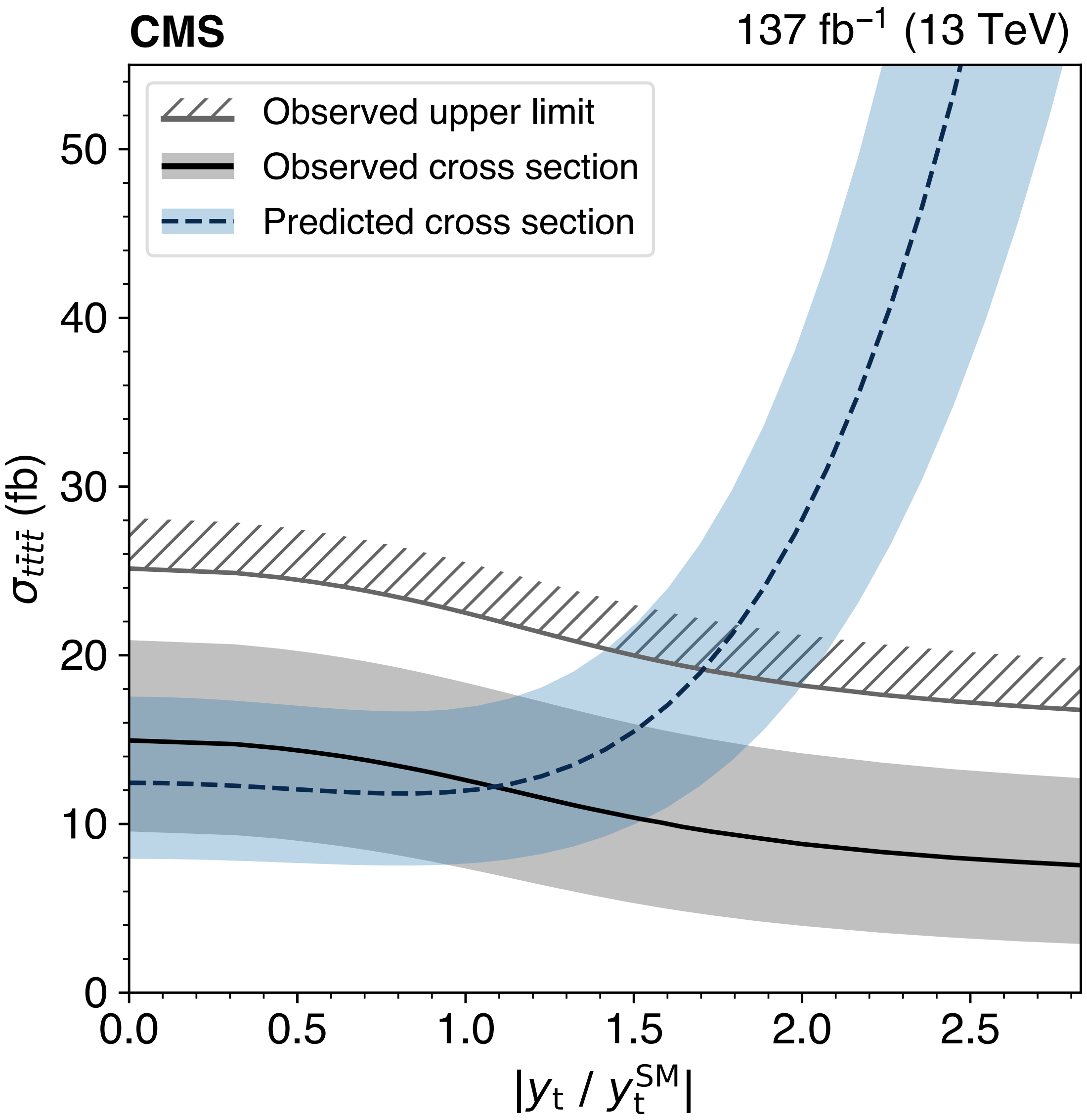
Higgs interpretation without a (on-shell) Higgs?

Search for 4-tops provides complementary approach to constraining **Higgs-top Yukawa coupling!**



Phys. Lett. B 847 (2023) 138290



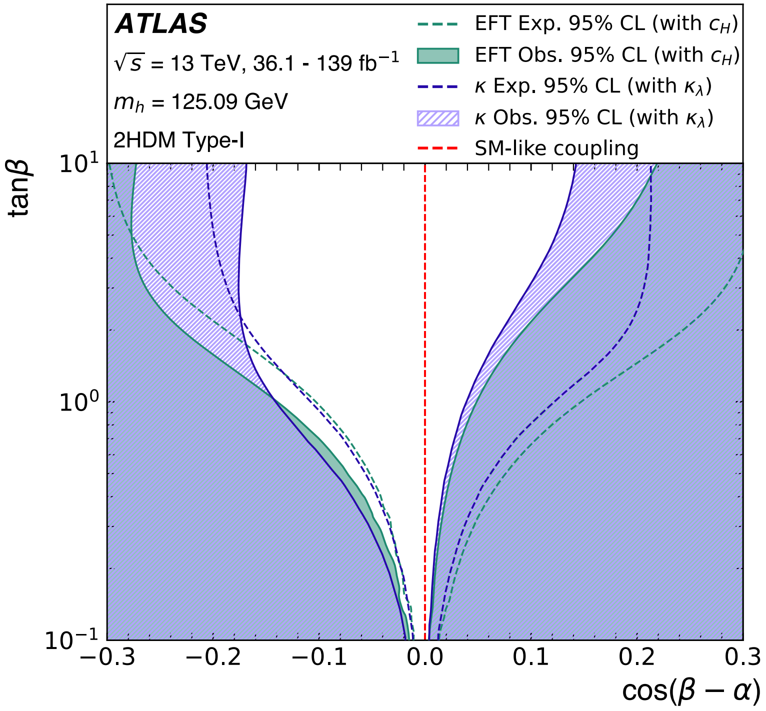


CMS-TOP-18-003

Full Run-2 analysis yields 4-top observation

Combined significance **5.9σ (5.1σ) !**

2HDM (κ vs EFT)



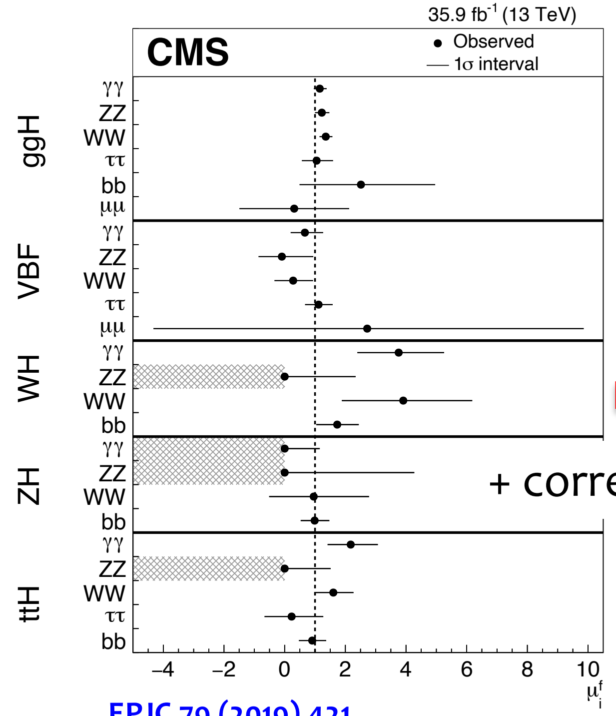
A comparison between the excluded regions from the two approaches is shown in Figure 21. In the regions where the assumptions used in this study are valid, the excluded regions are very similar in the two approaches. In the Type-I model for large values of $\tan\beta$, the EFT-based approach leads to looser constraints on $\cos(\beta-\alpha)$ than the κ-framework-based approach. This difference stems from the fact that the EFT-based approach (i) does not exploit the constraints from the *HVV* couplings, that only enter at dimension-8 in the SMEFT expansion and are not considered here, and (ii) retains only terms of $\mathcal{O}(\cos(\beta-\alpha))$ in the expansion of κ_λ, while in the κ-framework-based approach the constraint $\kappa_V = \sin(\beta-\alpha)$ and the full dependence of κ_λ on $\cos(\beta-\alpha)$ are considered. However, part of the region of Type-I model parameter space allowed in the EFT-based approach but not in that based on the κ-framework is inconsistent with the alignment limit hypothesis of $|\cos(\beta-\alpha)| \ll 1$.

Non-Gaussian likelihoods

In Higgs physics, often find "signal-strength" measurements

→ $\mu_i = \frac{\sigma_i}{(\sigma_i)_{\rm SM}}$ and $\mu^f = \frac{{\rm BR}^f}{({\rm BR}^f)_{\rm SM}}.$ Standard model defined by $\boldsymbol{\mu}_i = \boldsymbol{\mu}^f = 1$

→ **Assume** only total rate of ii→H→ff is modified by new physics (ok in certain models)



EPJC 79 (2019) 421

+ correlations

Re-construct *profile* likelihood

$$-2\log L(\boldsymbol{\mu}) = (\boldsymbol{\mu}-\hat{\boldsymbol{\mu}})^T C^{-1}(\boldsymbol{\mu}-\hat{\boldsymbol{\mu}})$$

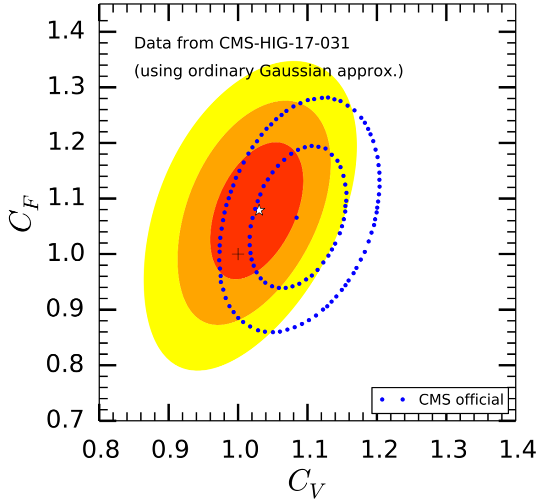
Extend Gaussian approximation with *"variable Gaussian"*

$$C = \boldsymbol{\Sigma}(\boldsymbol{\mu}).\rho.\boldsymbol{\Sigma}(\boldsymbol{\mu})$$

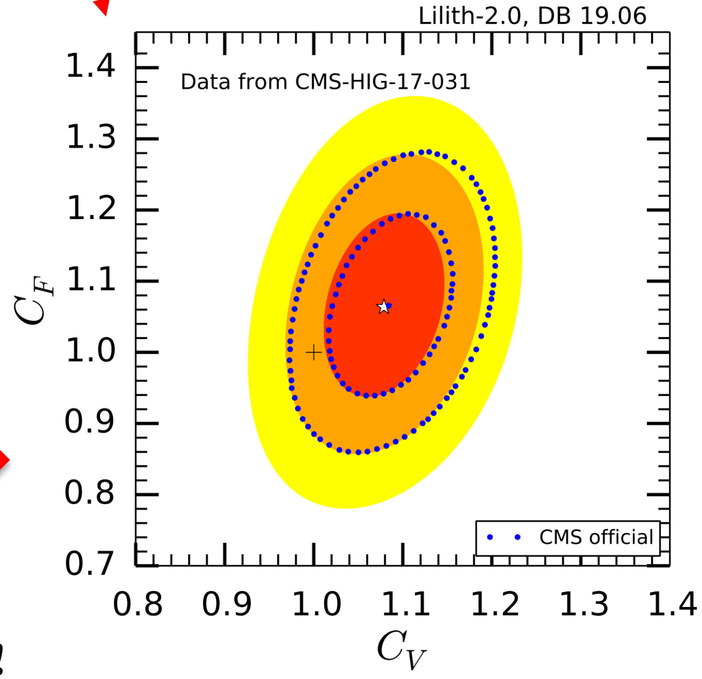
$$\Sigma_i = \sqrt{\sigma_i^+\sigma_i^- + (\sigma_i^+ - \sigma_i^-)(\mu_i - \hat{\mu}_i)},$$

Re-parameterize in terms of coupling modifiers

$$\mu_i,\ \mu^f \rightarrow \mu_i(C_V, C_F),\ \mu^f(C_V, C_F)$$



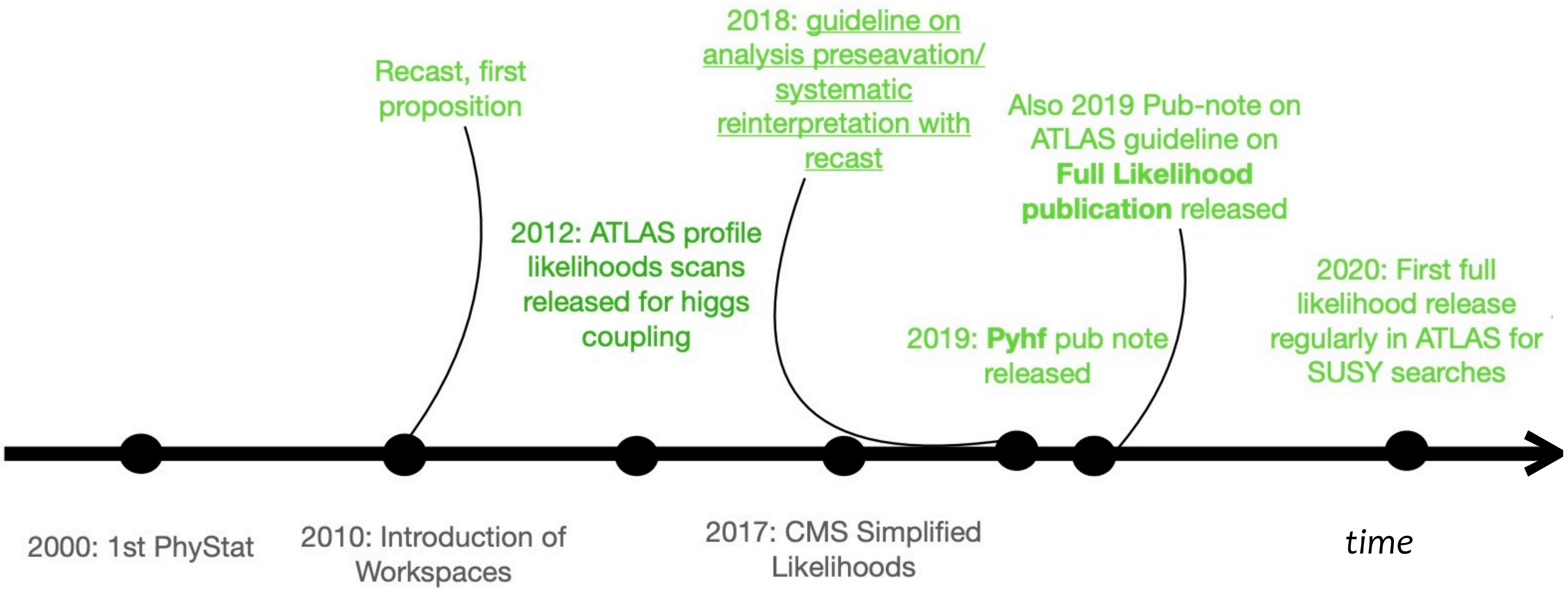
Add variable Gaussian



S. Kraml, T.Q. Loc, D.T. Nhung, L.D. Ninh

Non-Gaussian effects matter!

Timeline for public likelihoods



2024 CMS public statistics models + Combine

L. Heinrich

Recommendations for re-interpretations

https://twiki.cern.ch/twiki/bin/view/LHCPhysics/InterpretingLHCresults

SciPost Phys. 9, 022 (2020)

Reinterpretation of LHC results for new physics: status and recommendations after run 2

The LHC BSM Reinterpretation Forum

Abstract

We report on the status of efforts to improve the reinterpretation of searches and measurements at the LHC in terms of models for new physics, in the context of the LHC Reinterpretation Forum. We detail current experimental offerings in direct searches for new particles, measurements, technical implementations and Open Data, and provide a set of recommendations for further improving the presentation of LHC results in order to better enable reinterpretation in the future. We also provide a brief description of existing software reinterpretation frameworks and recent global analyses of new physics that make use of the current data.

Received 02-04-2020
Accepted 06-08-2020
Published 21-08-2020
doi:10.21468/SciPostPhys.9.2.022

Check for updates

SciPost Phys. 12, 037 (2022)

Publishing statistical models: Getting the most out of particle physics experiments

Kyle Cranmer[1†⋆], Sabine Kraml[2‡⋆], Harrison B. Prosper [3∘⋆], Philip Bechtle[4], Florian U. Bernlochner[4], Itay M. Bloch[5], Enzo Canonero[6], Marcin Chrzaszcz[7], Andrea Coccaro[8], Jan Conrad[9], Glen Cowan[10], Matthew Feickert[11], Nahuel F. Iachellini[12,13], Andrew Fowlie[14], Lukas Heinrich[15], Alexander Held[1], Thomas Kuhr[13,16], Anders Kvellestad[17], Maeve Madigan[18], Farvah Mahmoudi[15,19], Knut D. Morå[20], Mark S. Neubauer[11], Maurizio Pierini[15], Juan Rojo[8], Sezen Sekmen[22], Luca Silvestrini[23], Veronica Sanz[24,25], Giordon Stark[26], Riccardo Torre[8], Robert Thorne[27], Wolfgang Waltenberger[28], Nicholas Wardle[29] and Jonas Wittbrodt[30]

Abstract

The statistical models used to derive the results of experimental analyses are of incredible scientific value and are essential information for analysis preservation and reuse. In this paper, we make the scientific case for systematically publishing the full statistical models and discuss the technical developments that make this practical. By means of a variety of physics cases — including parton distribution functions, Higgs boson measurements, effective field theory interpretations, direct searches for new physics, heavy flavor physics, direct dark matter detection, world averages, and beyond the Standard Model global fits — we illustrate how detailed information on the statistical modelling can enhance the short- and long-term impact of experimental results.

Received 16-09-2021
Accepted 08-12-2021
Published 25-01-2022
doi:10.21468/SciPostPhys.12.1.037

Check for updates

Public Statistical Models

ATLAS has been releasing public versions of their statistical models for several years

Limited to simple histogram-based models but very welcomed by the pheno community for reinterpretations

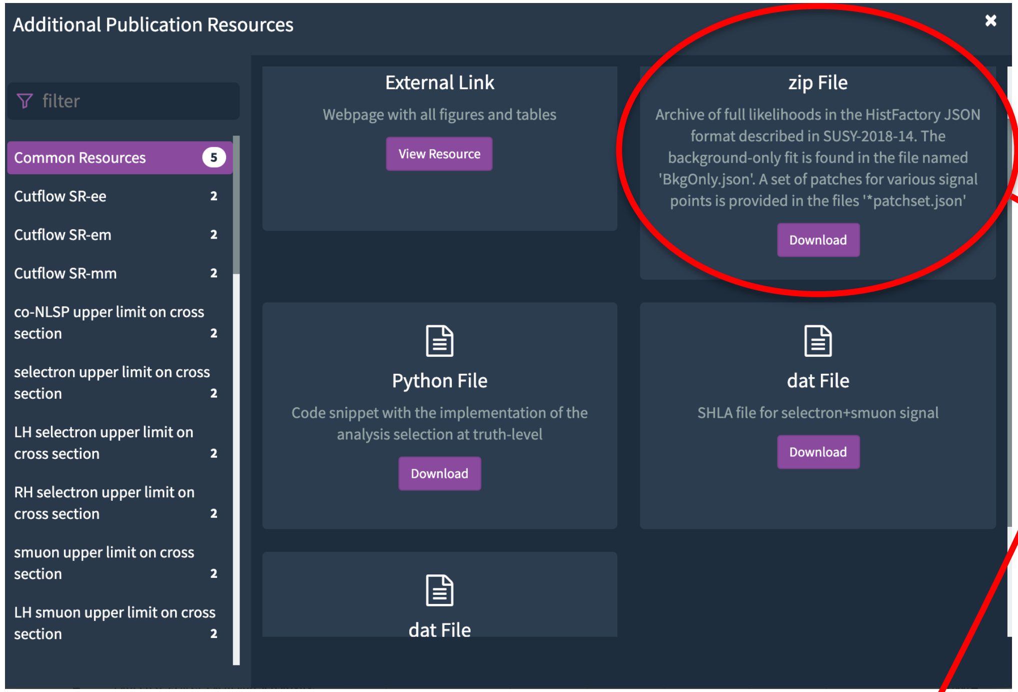
ATLAS analyses with pyHF compatible public models:

Observation of the tgamma production	TOPQ	Accepted by PRL	2023-02-02	13	140 fb^{-1}	Documents | 2302.01283 Inspire | HepData Internal
Search for gluinos in multi-b final states	SUSY	Eur. Phys. J. C 83 (2023) 561	2022-11-15	13	139 fb^{-1}	Documents | 2211.08028 Inspire | HepData Internal
Measurement of the s-channel single top cross-section at 13 TeV	TOPQ	JHEP 06 (2023) 191	2022-09-19	13	139 fb^{-1}	Documents | 2209.08990 Inspire | HepData Internal
Search for flavor-changing neutral-current couplings between the top-quark and the photon at 13 TeV	TOPQ	Phys. Lett. B 842 (2023) 137379	2022-05-05	13	139 fb^{-1}	Documents | 2205.02537 Inspire | HepData Internal
Search for SUSY in events with 2 leptons, jets and MET	SUSY	Eur. Phys. J. C 83 (2023) 515	2022-04-27	13	139 fb^{-1}	Documents | 2204.13072 Inspire | HepData Internal
Search BSM H→hh→bb gamma gamma and hh→bb gamma gamma	HDBS	Phys. Rev. D 106 (2022) 052001	2021-12-22	13	139 fb^{-1}	Documents | 2112.11876 Inspire | HepData Internal
Search for charginos and neutralinos in all-hadronic final states	SUSY	Phys. Rev. D 104 (2021) 112010	2021-08-17	13	139 fb^{-1}	Documents | 2108.07586 Inspire | HepData Briefing | Internal
4-top xsec measurement	TOPQ	JHEP 11 (2021) 118	2021-06-22	13	139 fb^{-1}	Documents | 2106.11683 Inspire | HepData Internal
Search for gluinos, stops and electroweakinos in RPV models in final states with 1L and many jets	SUSY	Eur. Phys. J. C 81 (2021) 1023	2021-06-17	13	139 fb^{-1}	Documents | 2106.09609 Inspire | HepData Briefing | Internal

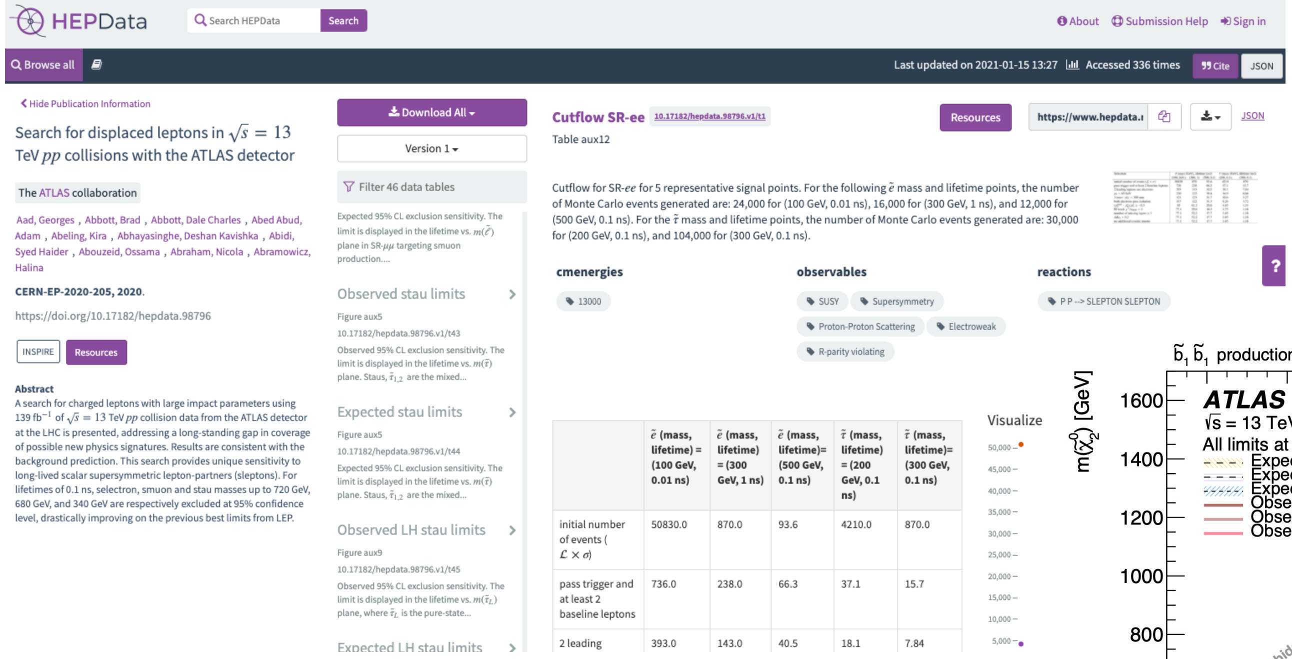
Likelihoods are an essential link between theory and ATLAS data. (Image: K. Cranmer/ATLAS Collaboration)

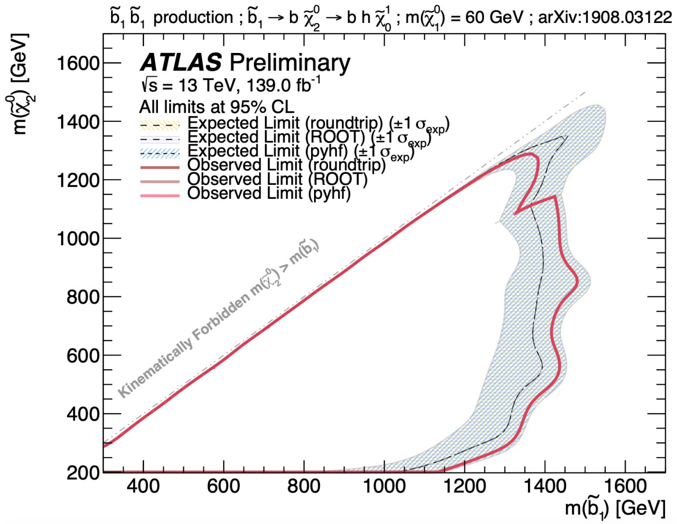
HepData for published likelihoods

https://www.hepdata.net/record/ins1748602



Search for bottom-squark pair production with the ATLAS detector in final states containing Higgs bosons, b-jets and missing transverse momentum





ATL-PHYS-PUB-2019-029

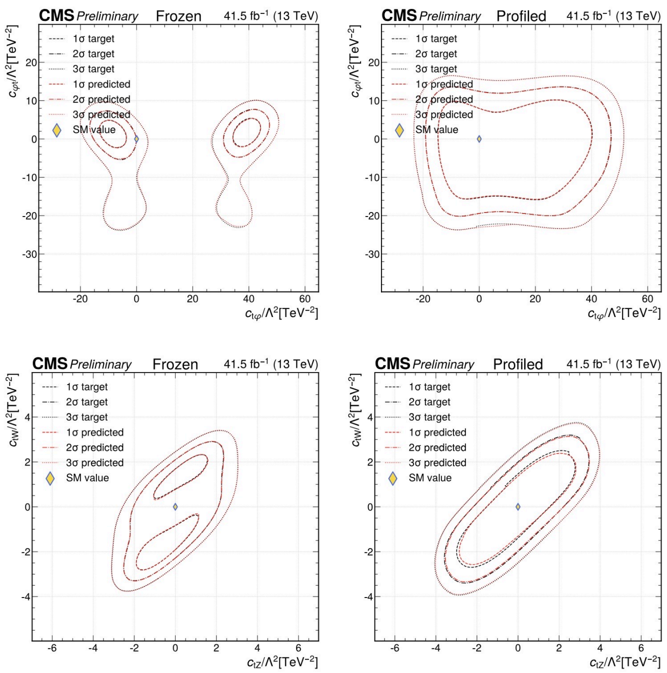
pyhf ∂ifferentiable ℒikelihoods

DOI 10.5281/zenodo.4484948

Publishing *profiled* likelihoods

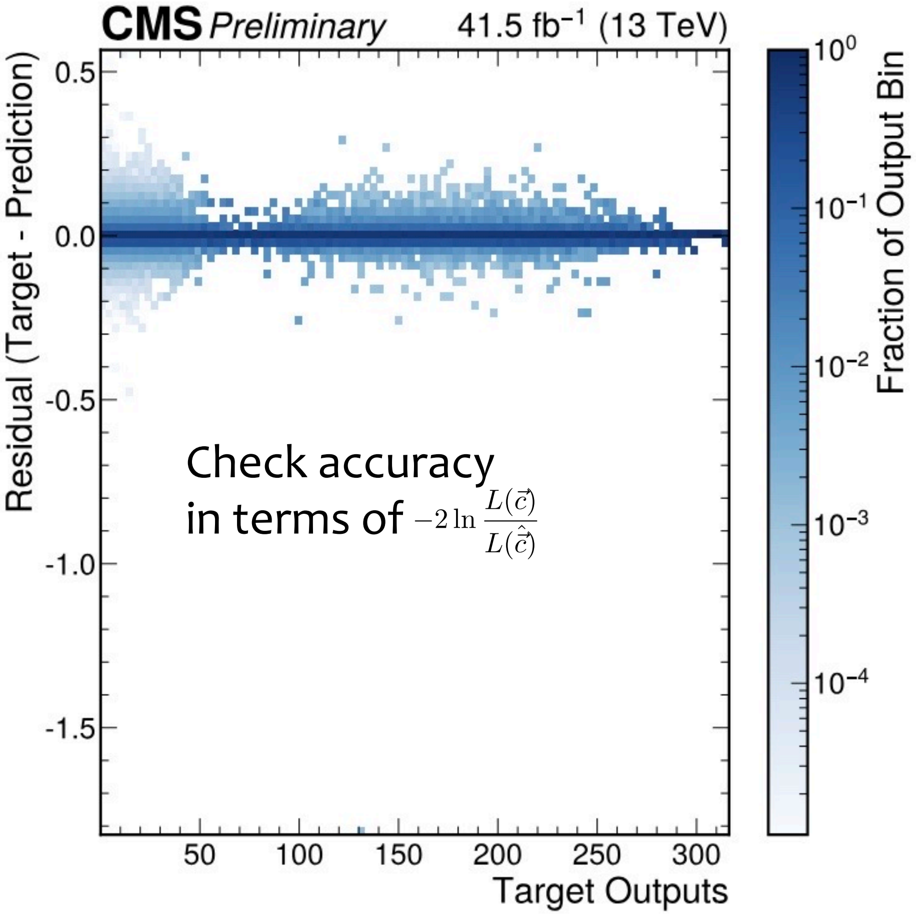
In between simple and full likelihoods
→ Communicating ***profiled likelihoods*** in EFT

Train a DNN to learn $-2\ln\frac{L(\vec{c})}{L(\hat{\vec{c}})}$



DNN can cope with high dimensional space (here example is t-quark measurement with 16 WCs!

Publish DNN for 16D likelihood surface



Check accuracy in terms of $-2\ln\frac{L(\vec{c})}{L(\hat{\vec{c}})}$

☺ Much faster as interpretation already performed
→ can re-interpret (re-profile) very quickly as underlying model is a DNN!
☹ Parameterization baked in
→ can't incorporate developments in EFT, systematics embedded into results

S. Liu (CHEP 2023)

Unfolded vs Full sim

A. Gilbert

Fiducial/differential measurements with EFT interpretation

Unfolding, with **likelihood fit** … or **matrices**

L(data | σ_i)

L(data | $\sigma_i(c_j)$)

$$(\vec{y} - \mathbf{K}\vec{\sigma})^{\mathrm{T}}\mathbf{V}^{-1}(\vec{y} - \mathbf{K}\vec{\sigma}) + \delta P(\vec{\sigma})$$

Can recast from $\boldsymbol{\sigma_i}$ to give $\mathbf{c_j}$, or other parametrization of $\boldsymbol{\sigma_i}$

Direct EFT constraints (w/ optimised analysis) aka "full-sim"

One direct fit:

L(data | c_j)

Can (in principle) recast from c_j to other congruent EFT basis

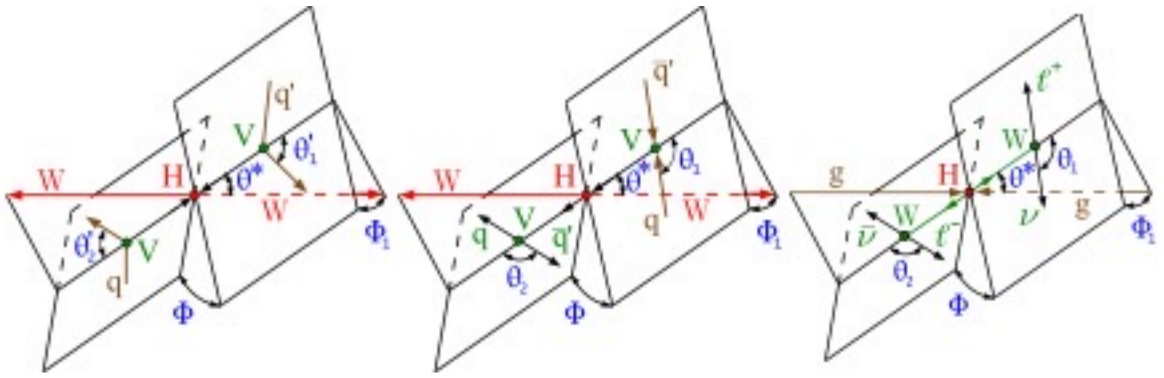
Often simplified info. made public:

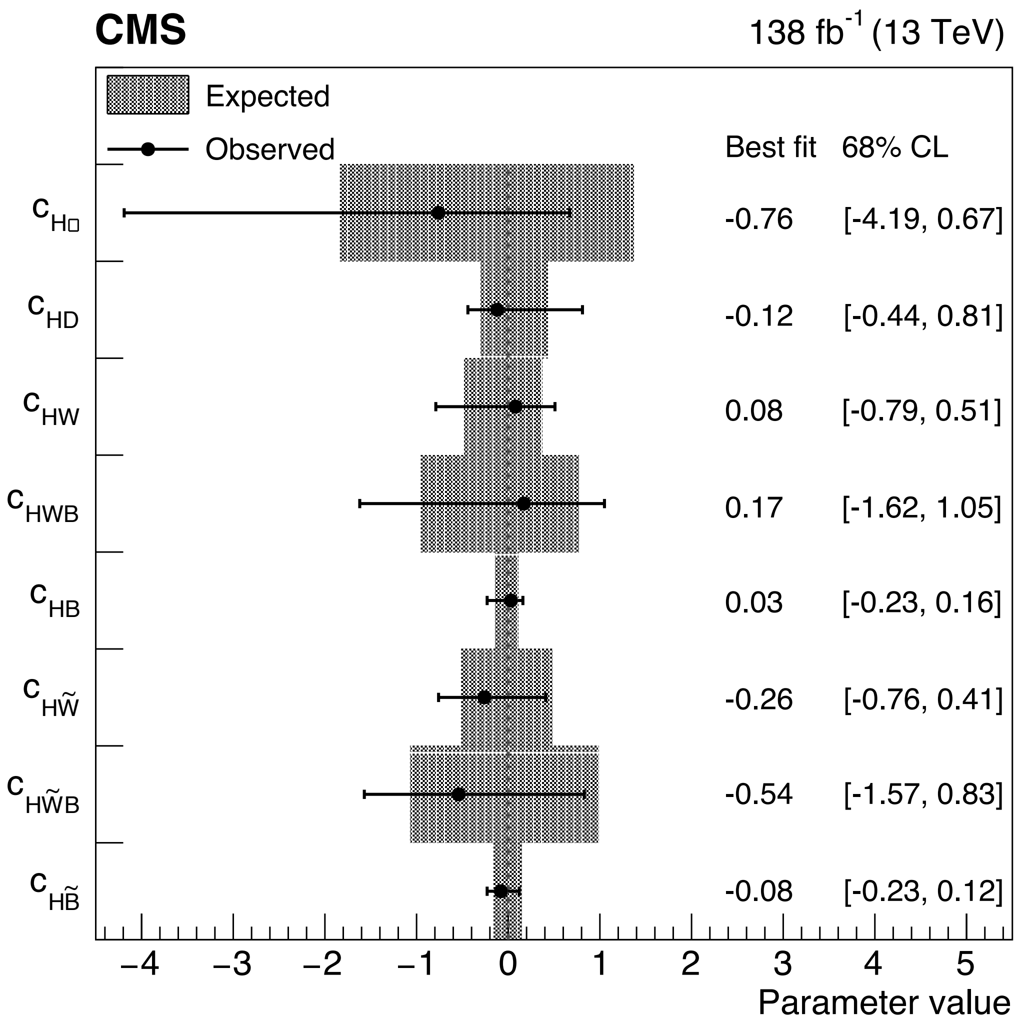
$X^2(\sigma_i\,;\,\sigma_i^{meas.})$

Used in Top/Higgs-land (anomalous couplings)

EFT H→WW (HIG-22-008)

AC H→VV





Direct parameterization of H-VV vertices in terms of EFT couplings → directly measure from terms in LH

Only one of c_{HW}, c_{HWB}, and c_{HB} is independent, the same is also true for cp-odd versions.

EFT parameterizations in `Combine`

Slide from G. Boldrini

EFT quadratic parametrization can be seen as matrix multiplication. **Use optimized matrix math libraries (Eigen) to compute products** → **Factor 100x speed up in minimization**

$$y = y_{SM}\left(1 + \sum_i c_i A_i + \sum_{i\leq j} c_i c_j B_{ij}\right) \Leftrightarrow y = y_{SM}\left(\begin{bmatrix}1\\c_1\\c_2\\\vdots\end{bmatrix}^{\top}\begin{bmatrix}1 & A_1/2 & A_2/2 & \cdots\\A_1/2 & B_{11} & B_{12}/2 & \cdots\\A_2/2 & B_{12}/2 & B_{22} & \cdots\\\vdots & \vdots & \vdots & \ddots\end{bmatrix}\begin{bmatrix}1\\c_1\\c_2\\\vdots\end{bmatrix}\right)$$

Pros and Cons

✅ **100x speed** up in profiled fits
✅ **Memory** consumption reduced ~x2
✅ JSON parametrization: **no template proliferation**
❌ **Does not support any syst unc. on EFT**
❌ **Assumes EFT factorized from nuisances**

ctG fit	AnalyticAnomalousCoupling	InterferenceModel
Fast scan time	*103s*	*2.7s*
Fast scan memory	*600 MB*	*307 MB*
Profile nuisance scan time	*1835s (30min)*	*10.6s*
Profile nuisance scan memory	*602 MB*	*311 MB*
Profile scan time	*2604s (40min)*	*27s*
Profile scan memory	*600 MB*	*311 MB*

Comparison from Nick Smith

STXS → EFT Parameterizations in `Combine`

STXStoSMEFTModel.py

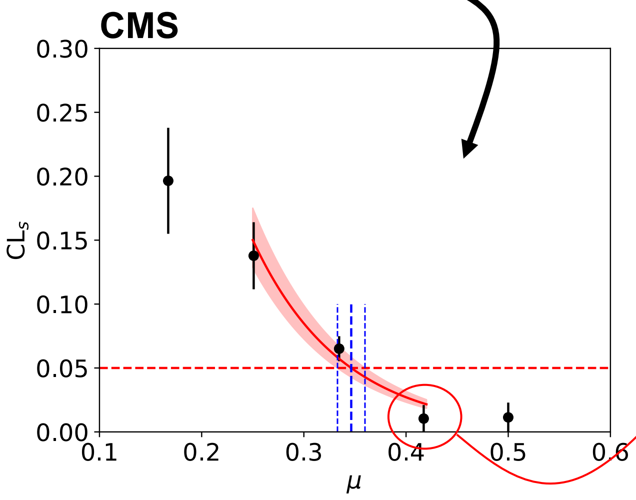
```python
# ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
# Global function to extract reco category, STXS bin, decay mode and energy from process name
def getProcessInfo(bin, process):
    foundRecoCategory = bin
    foundSTXSBin = process
    foundDecay = None
    foundEnergy = "13TeV"
    # Iterate over Higgs decays
    matchedDecayString = False
    for D in ALL_HIGGS_DECAYS:
        if matchedDecayString:
            continue
        if "_%s" % D in foundSTXSBin:
            foundSTXSBin = re.sub("_%s" % D, "", foundSTXSBin)
            foundDecay = D
            matchedDecayString = True
    # Also drop year tag in STXS bin name if present
    for Y in ["2016", "2017", "2018"]:
        if "_%s" % Y in foundSTXSBin:
            foundSTXSBin = re.sub("_%s" % Y, "", foundSTXSBin)

    # Catch for H->Zgam
    if (foundDecay == "hzg") | ("bkg" in foundSTXSBin):
        foundSTXSBin = foundSTXSBin.split("_")[0]

    if not matchedDecayString:
        raise RuntimeError("Validation error: no supported decay found in process")

    return (foundRecoCategory, foundSTXSBin, foundDecay, foundEnergy)
```

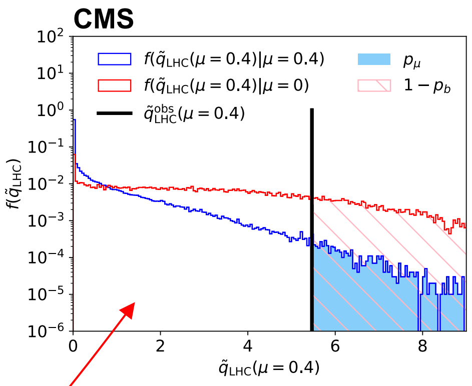
```python
    # Overwrite getYieldScale to extract (RECO-category,STXS bin,decay,energy)
    def getYieldScale(self, bin, process):
        if not self.DC.isSignal[process]:
            return 1.0

        # Extract process line info
        (recocat, stxsbin, decay, energy) = getProcessInfo(bin, process)

        # Return 1 (no scaling) for fixed processes and scaling for non-fixed
        if stxsbin in self.fixProcesses:
            return 1.0
        else:
            procStr = stxsbin
            return self.getHiggsSignalYieldScale(procStr, decay, energy)

    # ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
    # Extract pois from yaml file
    def extractPOIs(self, filename):
        with open(filename, "r") as fpois:
            try:
                self.pois = yaml.safe_load(fpois)
                # Apply eigenvector threshold if set
                if self.eigenvalueThreshold != -1.0:
                    pois_to_keep = {}
                    for poi, v in self.pois.items():
                        if "eigenvalue" in v:
                            if v["eigenvalue"] > float(self.eigenvalueThreshold):
                                pois_to_keep[poi] = v
                        else:
                            pois_to_keep[poi] = v
                    self.pois = pois_to_keep

            except yaml.YAMLERROR as exc:
                print(exc)

    # Function to extract STXS scaling terms from json file
    def extractSTXSScalingTerms(self, filename=""):
        if filename != "":
            with open(filename, "r") as jf:
                self.STXSScalingTerms = json.load(jf)
        else:
            self.STXSScalingTerms = {}
```

Other features in `Combine`

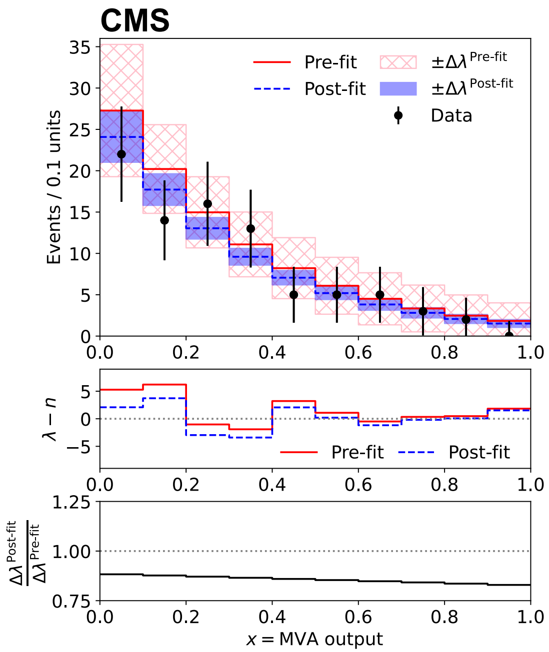
Many other statistical routines available :
see [`Combine`]() paper and [online documentation]()

Determine upper/confidence limits

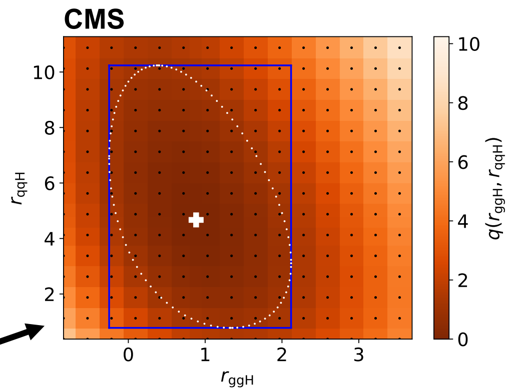


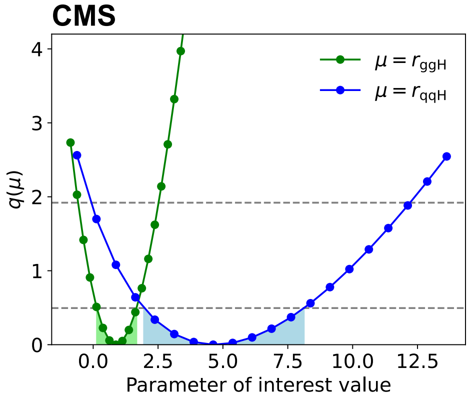


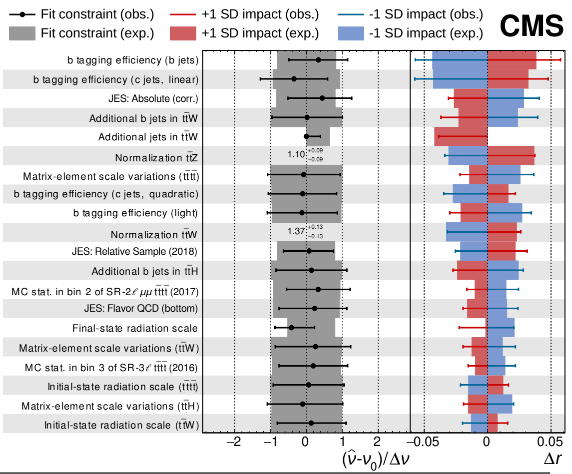
Fit diagnostics tools



Multi-dimensional profile likelihood scans/contours







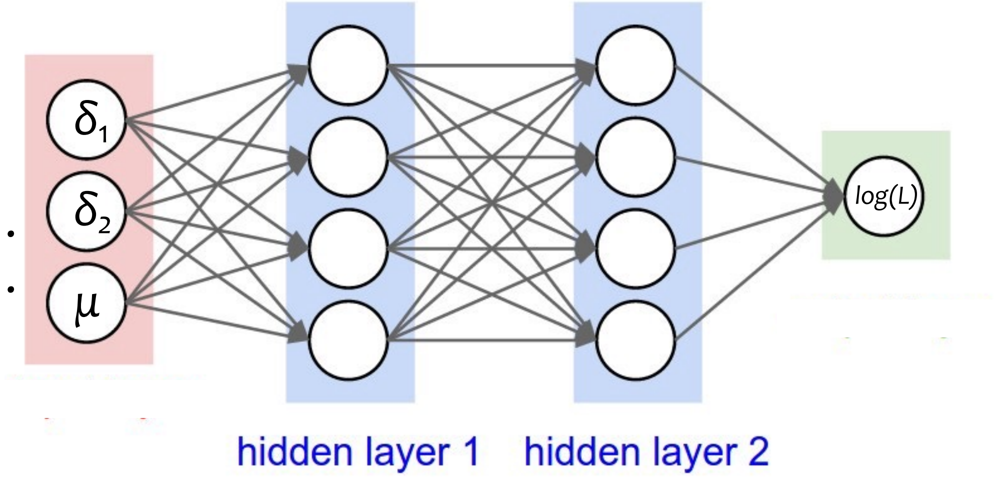
Installation via pre-compiled container image:

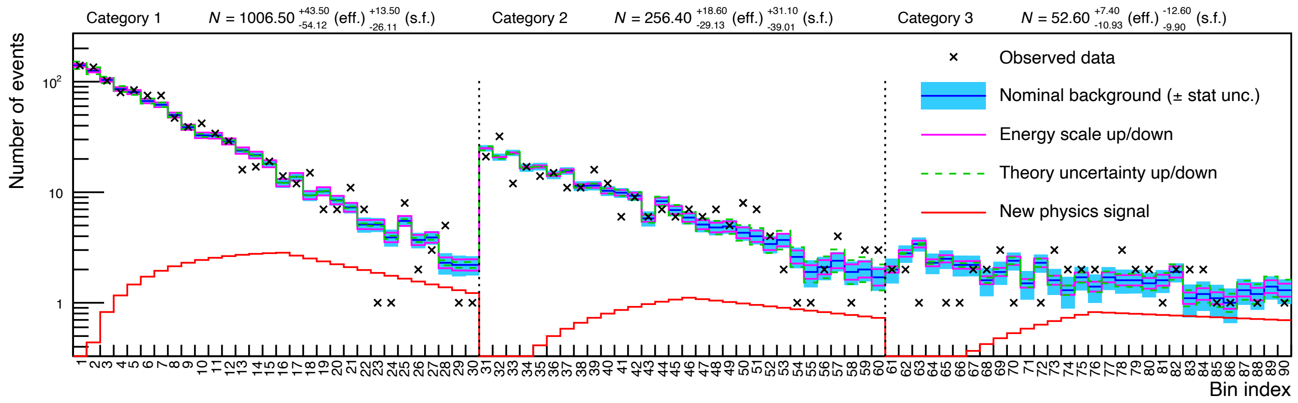
```
> docker run --name combine -it gitlab-registry.cern.ch/cms-cloud/combine-standalone:v9.2.1
```

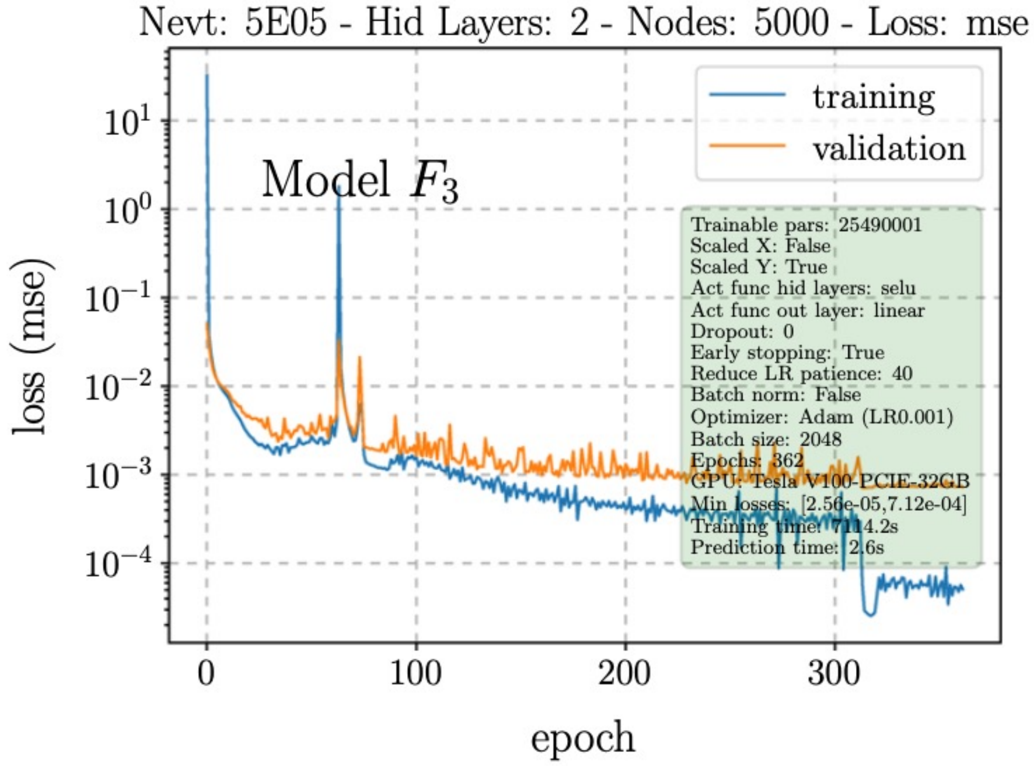
Getting to the “full” likelihood

DNN based likelihoods

Random samples from the toy search experimental likelihood serve as training data for a Deep Neural Network [1]







- 2 hidden layer NN, with SELU activation functions between layers – tested different #nodes in hidden layers.
- Adam optimizer with MSE as loss function to train the NN parameters.
- Sampling based on p(**x**) – in this case known from the expt. LH

[1] A. Coccaro, M. Pierini, L. Silvestrini, R. Torre: *Eur. Phys. J. C* **80,** 664 (2020).

ML-based likehood(ratios)

In some cases, it may be more challenging than necessary to learn the likelihood directly
→ if $p(\mathbf{x}|\alpha)$ must be obtained from some complex simulation, but can still generate from p

If you can find a function $s(\mathbf{x})$ that is monotonic with $r(\mathbf{x};\alpha_0,\alpha_1)$ [1], then;

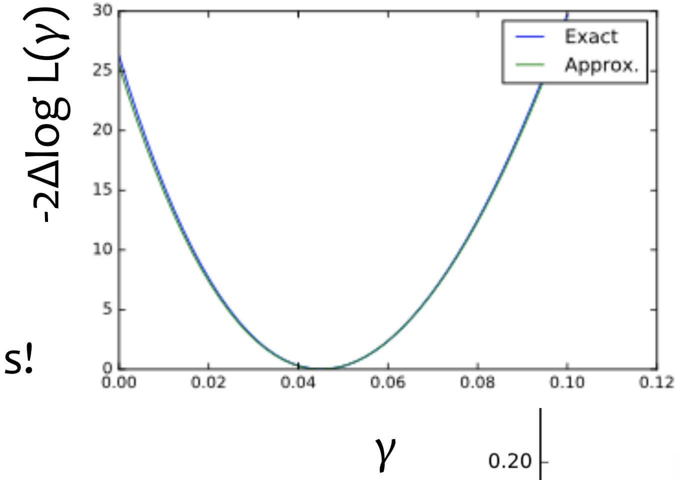
$$r(\mathbf{x}|\alpha_0,\alpha_1) = \frac{p(\mathbf{x}|\alpha_0)}{p(\mathbf{x}|\alpha_1)} = \frac{p(s(\mathbf{x})|\alpha_0)}{p(s(\mathbf{x})|\alpha_1)}$$

e.g $s(\mathbf{x})$ can be a classifier trained to separate α_0 vs α_1

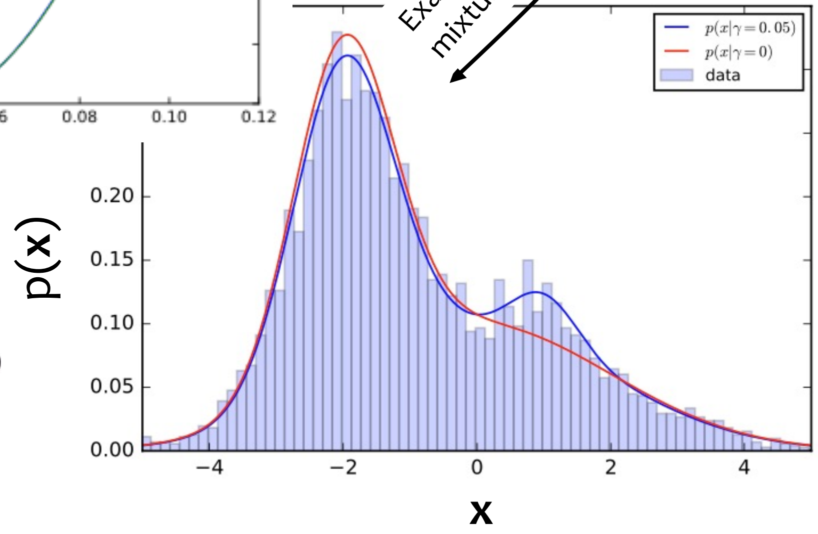
Here **x** can be anything → not restricted to binned likelihoods!

likelihood-free based inference or Approximate Bayesian Computation (ABC) more common outside HEP - See [2] for a very nice review of applications in HEP!

$$p(\mathbf{x}|\gamma) = (1-\gamma)\frac{p_{c_0}(\mathbf{x}) + p_{c_1}(\mathbf{x})}{2} + \gamma\, p_{c_2}(\mathbf{x})$$



Example gaussian mixture model [1]



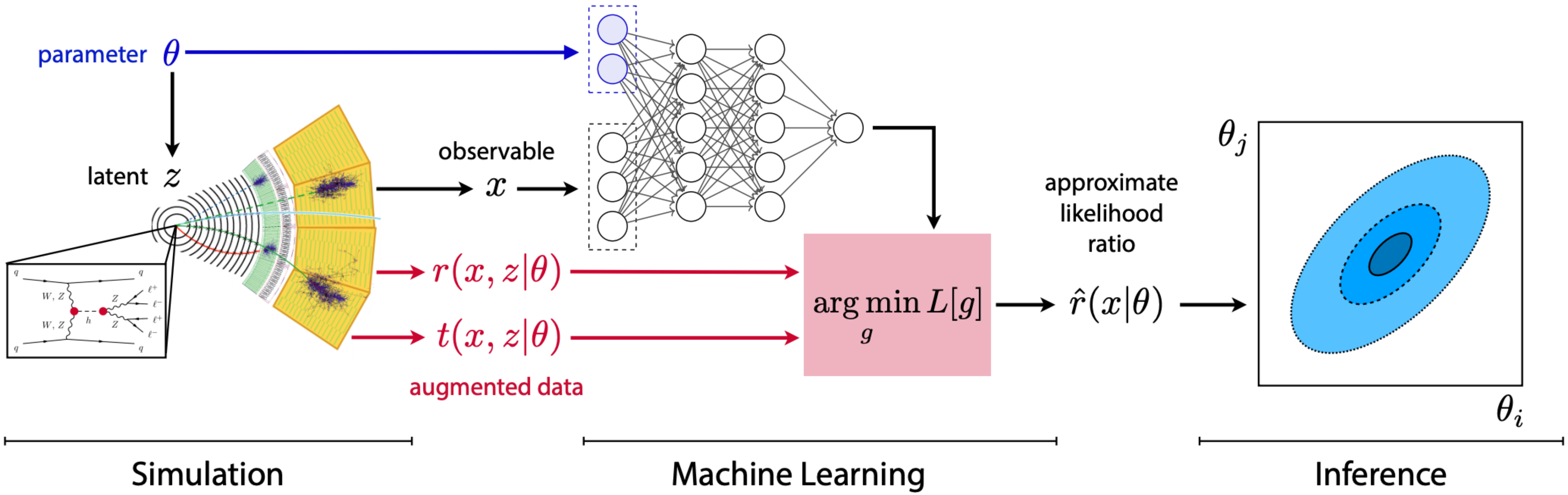
CARL GitHub

[1] arXiv:1506.02169
[2] arXiv:2010.06439

See the PhyStat seminar from Kyle Cranmer for more ML based approaches and MadMiner

MadMiner

Full likelihood with SBI



"Mining gold": Extract additional information from simulator

Use this information to train estimator for likelihood ratio

Limit setting with standard hypothesis tests

Sourced from https://github.com/diana-hep/madminer.
Excellent tutorial by K. Cramner: https://indico.cern.ch/event/982553/contributions/4220018/attachments/2185603/3706682/MadMiner-tutorial-reinterp-2021.pdf

Nuisance parameters description

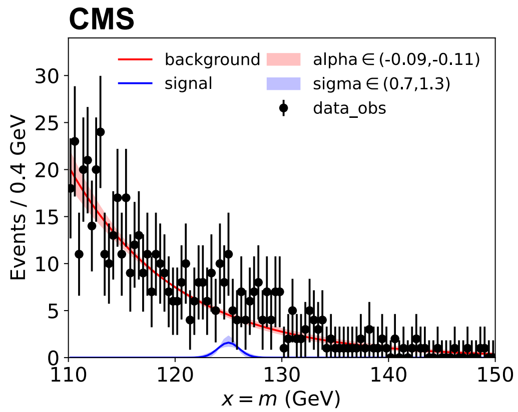
Public statistical model comes with nuisance parameter naming + description as .html files

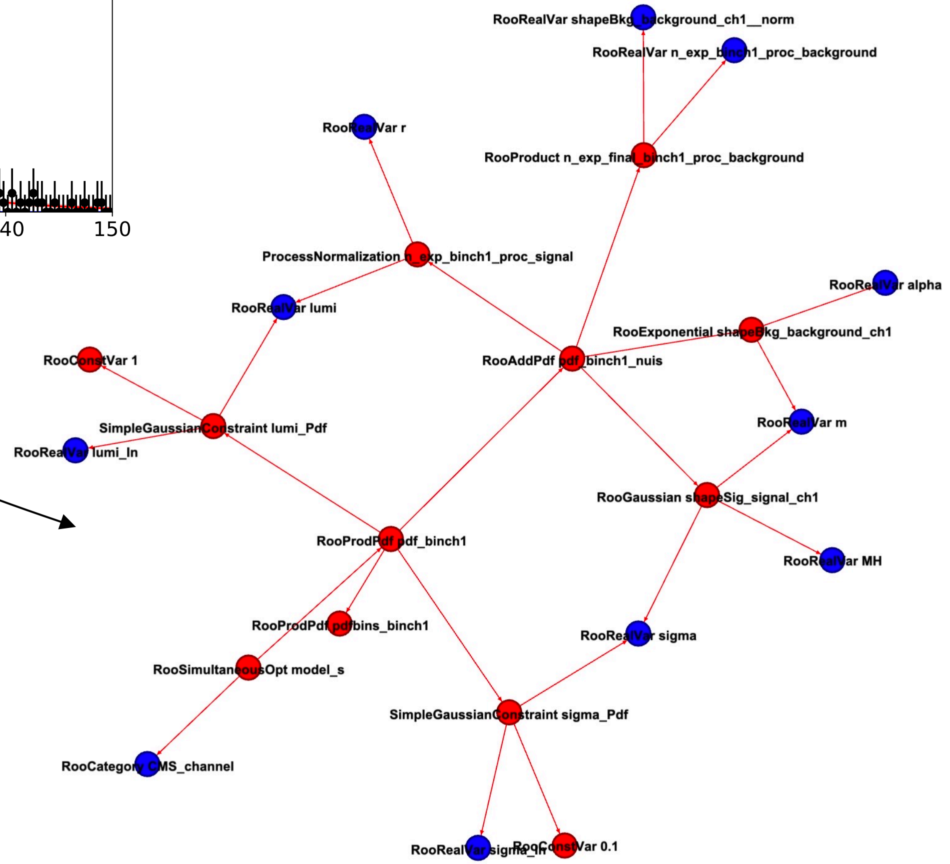
e.g $H \rightarrow \tau\tau$ channel nuisance parameters in `systematics_higgs_htt.html`

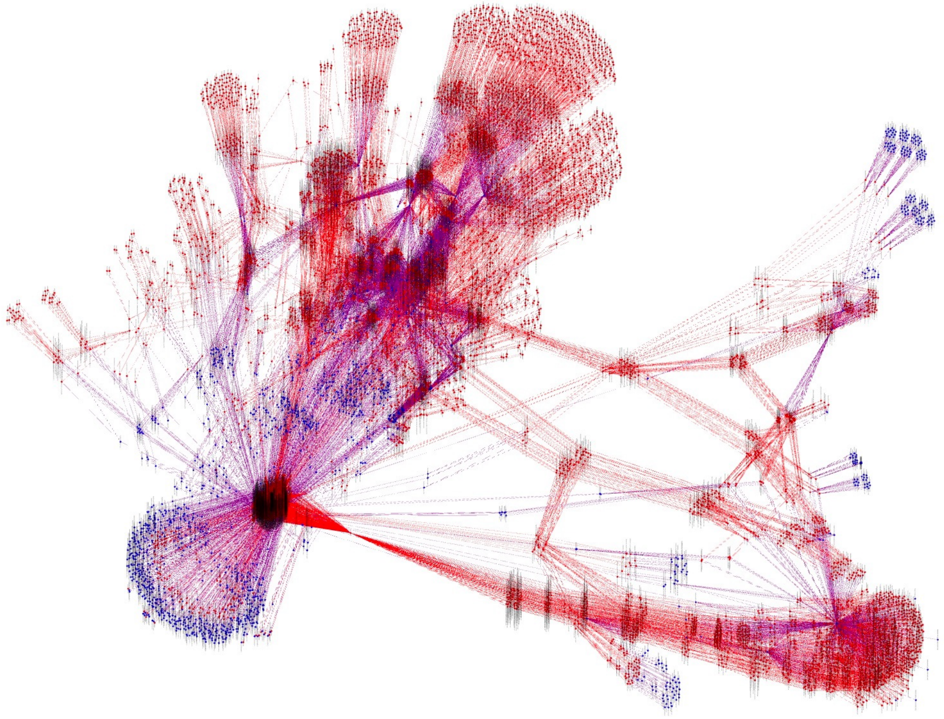
	class	description
CMS_scale_met_8TeV	MET_scale	single overall met energy scale uncertainty
CMS_scale_met_7TeV	MET_scale	single overall met energy scale uncertainty
CMS_eff_b_7TeV	custom	efficiency uncertainty for b jets in 7 and 8 TeV analyses
CMS_vhtt_7TeV_emt_fakeshape_fakes_bin_3	custom	shape uncertainties from the lepton-jet misidentification probabilities
CMS_vhtt_7TeV_emt_fakeshape_fakes_bin_2	custom	shape uncertainties from the lepton-jet misidentification probabilities
CMS_vhtt_7TeV_emt_fakeshape_fakes_bin_1	custom	shape uncertainties from the lepton-jet misidentification probabilities
CMS_trigger_m_7TeV	custom	muon trigger efficiency uncertainty in 7 TeV analysis
CMS_trigger_e_7TeV	custom	electron trigger efficiency uncertainty in 7 TeV analysis
CMS_hww_fakes_em_8TeV	custom	uncertainty on misidentified W+jets and QCD background estimated from the control region with relaxed lepton selection requirements
CMS_hww_fakes_em_7TeV	custom	uncertainty on misidentified W+jets and QCD background estimated from the control region with relaxed lepton selection requirements
CMS_htt_zttNorm_8TeV	custom	inclusive normalisation uncertainty applied for ztt and ttbar backgrounds processes
CMS_htt_zttNorm_7TeV	custom	inclusive normalisation uncertainty applied for ztt and ttbar backgrounds processes
CMS_htt_ttbarNorm_8TeV	custom	inclusive normalisation uncertainty applied for ztt and ttbar backgrounds processes
CMS_htt_ttbarNorm_7TeV	custom	inclusive normalisation uncertainty applied for ztt and ttbar backgrounds processes
CMS_htt_mm_zttLikelihood_8TeV	custom	uncertainty on the di-tau invariant mass reconstruction method
CMS_vhtt_7TeV_emt_fakeshape_fakes_bin_4	custom	shape uncertainties from the lepton-jet misidentification probabilities
CMS_htt_mm_zttLikelihood_7TeV	custom	uncertainty on the di-tau invariant mass reconstruction method
CMS_htt_mm_zmm_extrap_vbf_7TeV	custom	extrapolation uncertainty for Z -> mu mu background
CMS_htt_mm_zmm_extrap_boost_8TeV	custom	extrapolation uncertainty for Z -> mu mu background
CMS_htt_mm_zmm_extrap_boost_7TeV	custom	extrapolation uncertainty for Z -> mu mu background
CMS_htt_mm_zmmNorm_8TeV	custom	uncertainty on Z -> mu mu background estimation
CMS_htt_mm_zmmNorm_7TeV	custom	uncertainty on Z -> mu mu background estimation

Simple Model vs Combination

Simple parametric statistical model

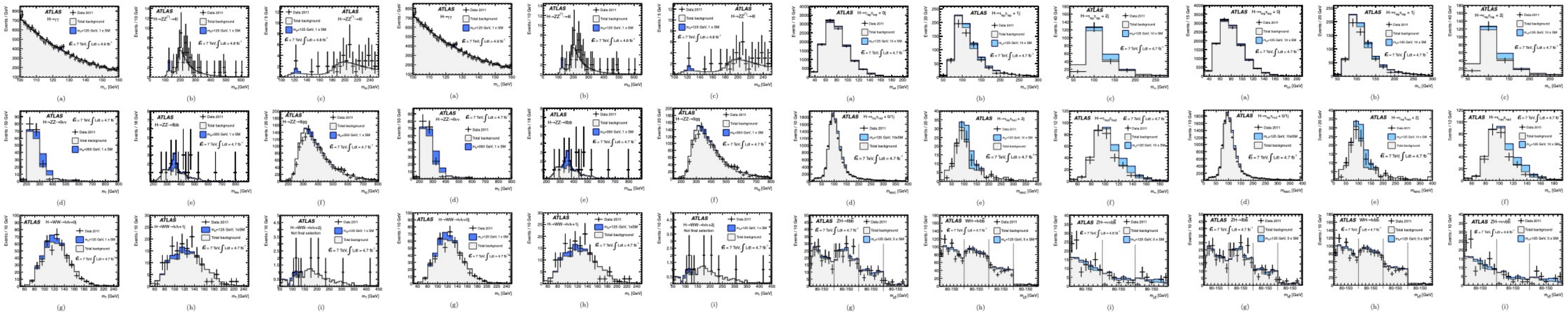


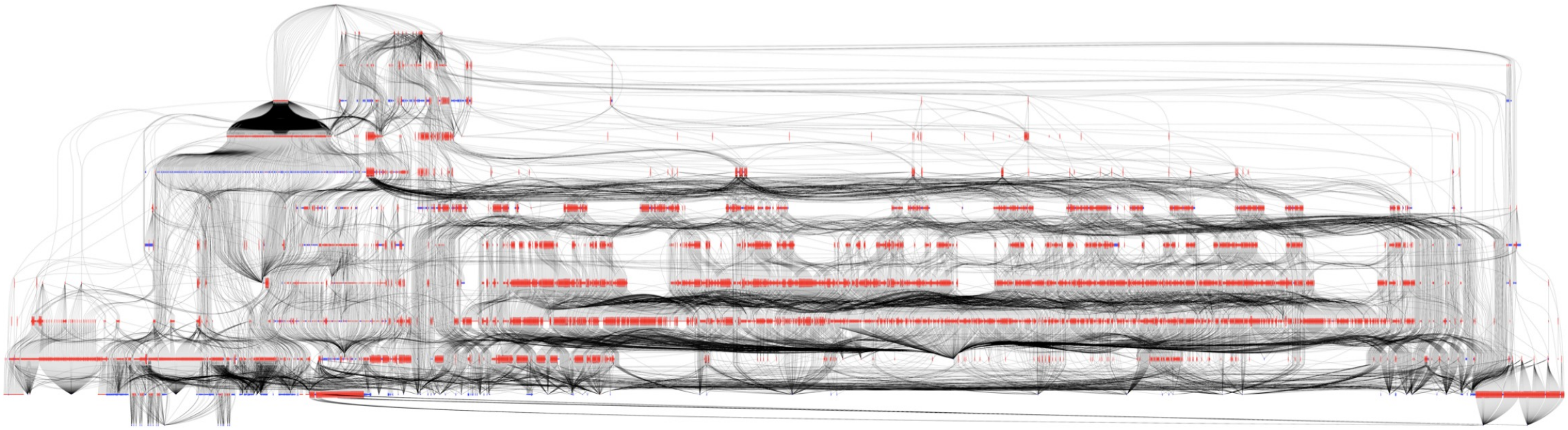




CMS Higgs observation combination statistical model

ATLAS Run-1 Higgs model





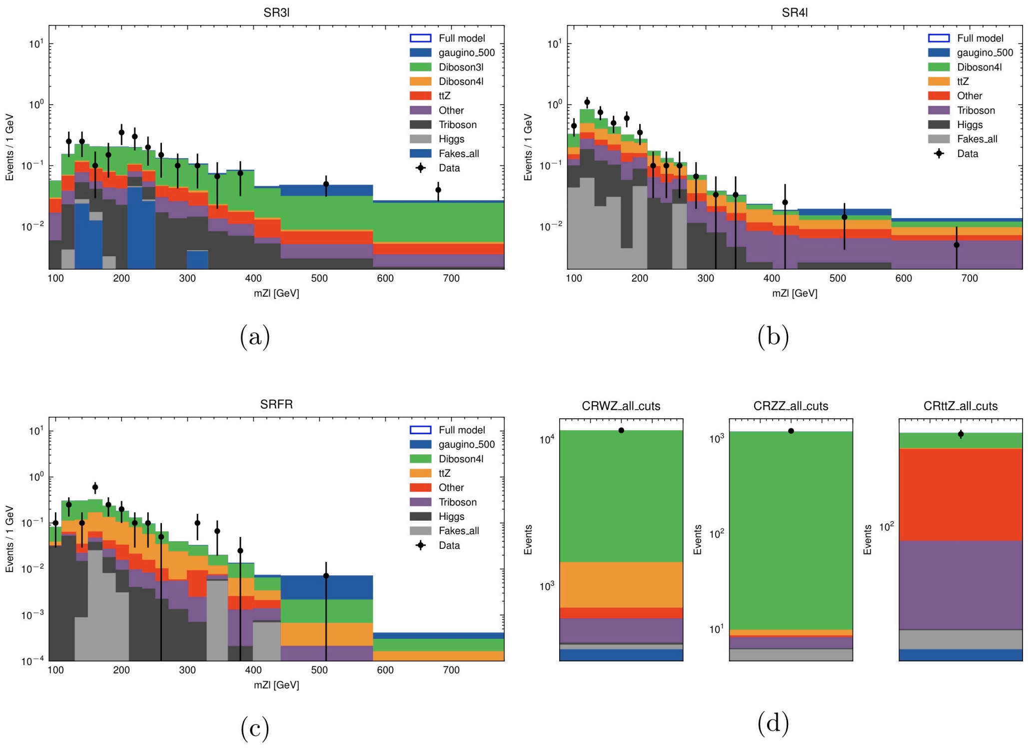
Linearized Simplified Likelihoods

$$L(\boldsymbol{\mu}, \boldsymbol{\theta}) = \prod_{c=1}^{N_{\text{channels}}} \prod_{b=1}^{N_{\text{bins},c}} \text{Pois}\left(n_{cb}, \sum_{s=1}^{N_{\text{samples},c}} \nu_{cbs}(\boldsymbol{\mu}, \boldsymbol{\theta}) \right) \prod_{l=1}^{N_{\text{constraints}}} C_l(\tilde{\theta}_l, \theta_l)$$

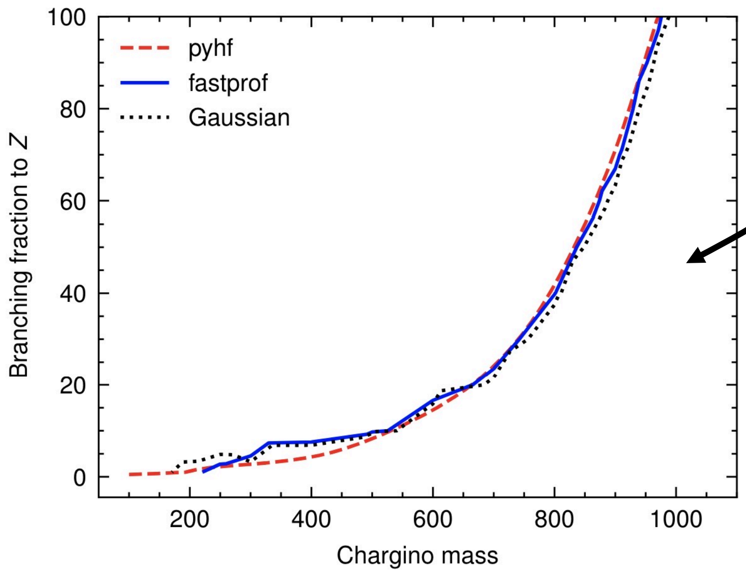
Simplify template-based likelihood functions by linearizing systematic variations

→ Simplified Likelihoods with Linearized Systematics (SLLS)

$$\nu_{cbs}(\boldsymbol{\mu}, \boldsymbol{\theta}) = \nu_{cbs}^{\text{nom}}(\boldsymbol{\mu}) \left[1 + \sum_{k=1}^{N_{\text{NP}}} \Delta_{cbsk}(\theta_k - \theta_k^{\text{nom}}) \right]$$



Example from ATLAS chargino and neutralino pair search (Phys. Rev. D 103 (2021))



Maintains information on NP correlation schemes → combinations of SLLS models also possible!

N. Berger [arXiv:2301.05676](https://arxiv.org/abs/2301.05676)

Implemented in *fastprof* (compatible with pyHF or RooFit models): https://github.com/fastprof-hep/fastprof