Phenomenology of multi-Higgs final states

$\cong$ double

Matthias Kerner

Institute for Theoretical Physics, KIT

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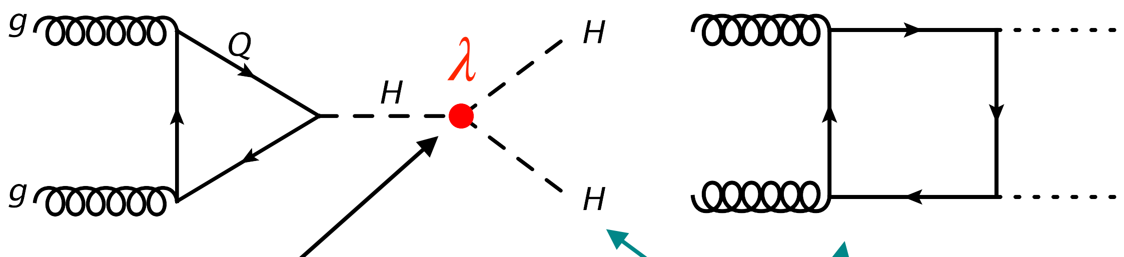
Introduction

Higgs boson pair production is sensitive to Higgs self-interaction λ

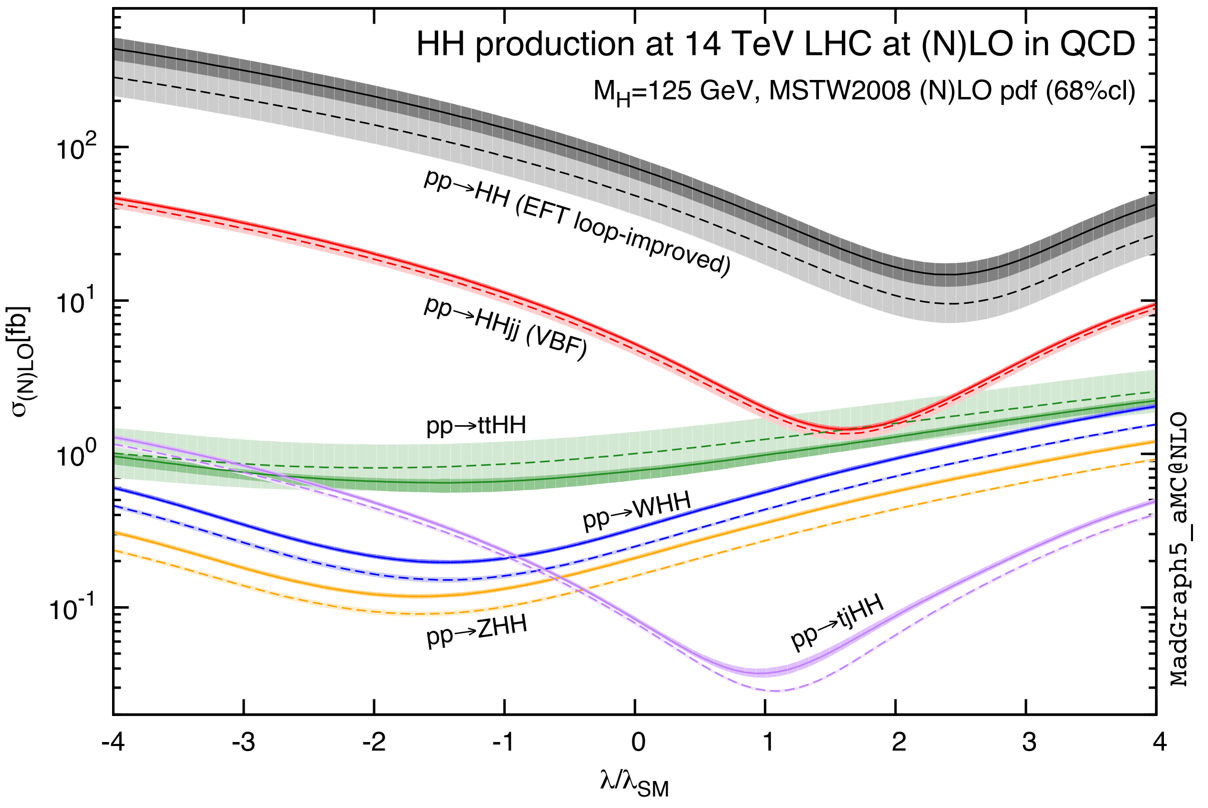
→ direct relation to Higgs potential

→ test mechanism of EW symmetry breaking



$$V(\Phi)=\frac{1}{2}\mu^2\Phi^2+\frac{1}{4}\lambda\Phi^4 \xrightarrow[\text{breaking}]{\text{EW symmetry}} \frac{m_H^2}{2}H^2+\boxed{\frac{m_H^2}{2v}H^3}+\frac{m_H^2}{8v^2}H^4$$

destructive interference



Frederix, Frixione, Hirschi, Maltoni,
Mattelaer, Torrielli, Vryonidou, Zaro `14

current experimental limits on self-coupling λ:

→ talk by Elena Vernazza

focus of this talk:

progress in theory predictions

Gluon Fusion @ NLO — SM vs. HTL

LO (full SM)

Glover, van der Bij `88

$\sigma_{LO} \approx 20\,\text{fb}$

since process loop-induced: higher-order corrections challenging

simplification:

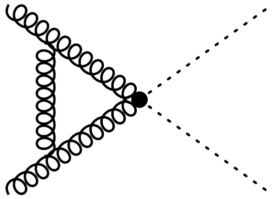
heavy-top limit (HTL) ↓ $m_t \to \infty$

HTL only justified for $m_{HH} \ll 2m_t$

NLO QCD HTL (Born-improved)

Plehn, Spira, Zerwas `96, `98;
Dawson, Dittmaier, Spira `98

K-factor ≈ 2

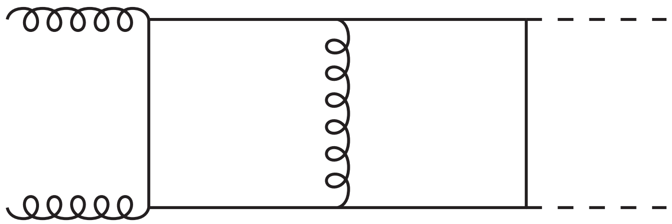


$$\mathrm{d}\sigma^{B.i.-HTL}_{NLO} = \frac{\mathrm{d}\sigma^{HTL}_{NLO}}{\mathrm{d}\sigma^{HTL}_{LO}}\,\mathrm{d}\sigma^{SM}_{LO}$$

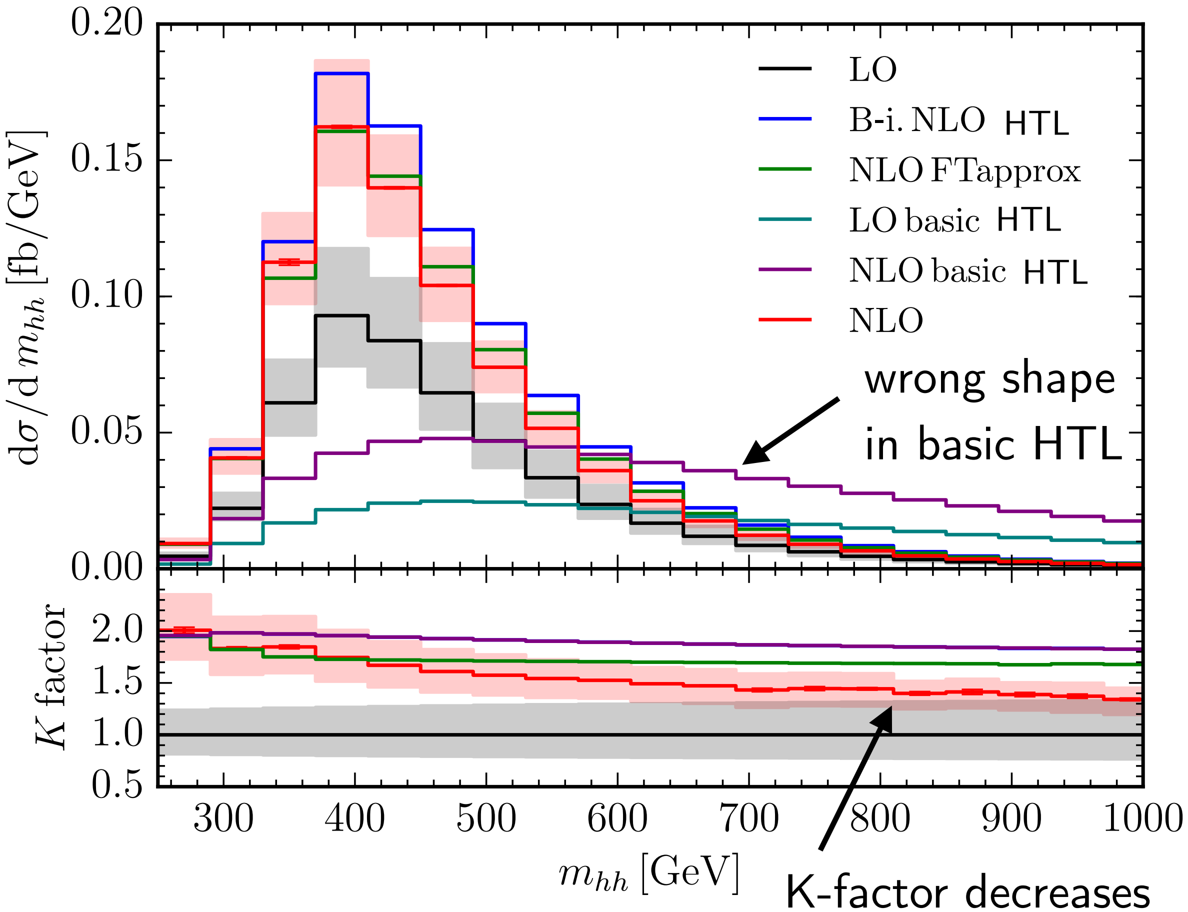
NLO QCD SM

using numerical methods for 2-loop amplitude

Borowka, Greiner, Heinrich, Jones, MK, Schlenk, Schubert, Zirke `16
Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher `18



K-factor 1.7



wrong shape in basic HTL

K-factor decreases for large m_{HH}

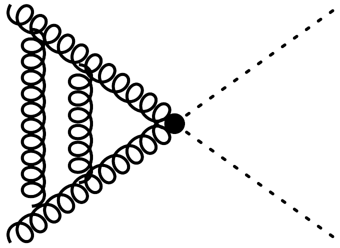
Gluon Fusion @ NNLO

NNLO HTL predictions

de Florian, Mazzitelli `13

Grigo, Melnikov, Steinhauser `14

de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev `16



K-factor 1.2

can be combined with exact m_t dependence at NLO

Grazzini, Heinrich, Jones, Kallweit, MK, Lindert, Mazzitelli `18

3 methods to approximate NNLO m_t dependence:

1) NNLO$_{\text{NLO-i}}$

   rescale NLO by $K_{\text{NNLO}} = \text{NNLO}_{\text{HEFT}}/\text{NLO}_{\text{HEFT}}$

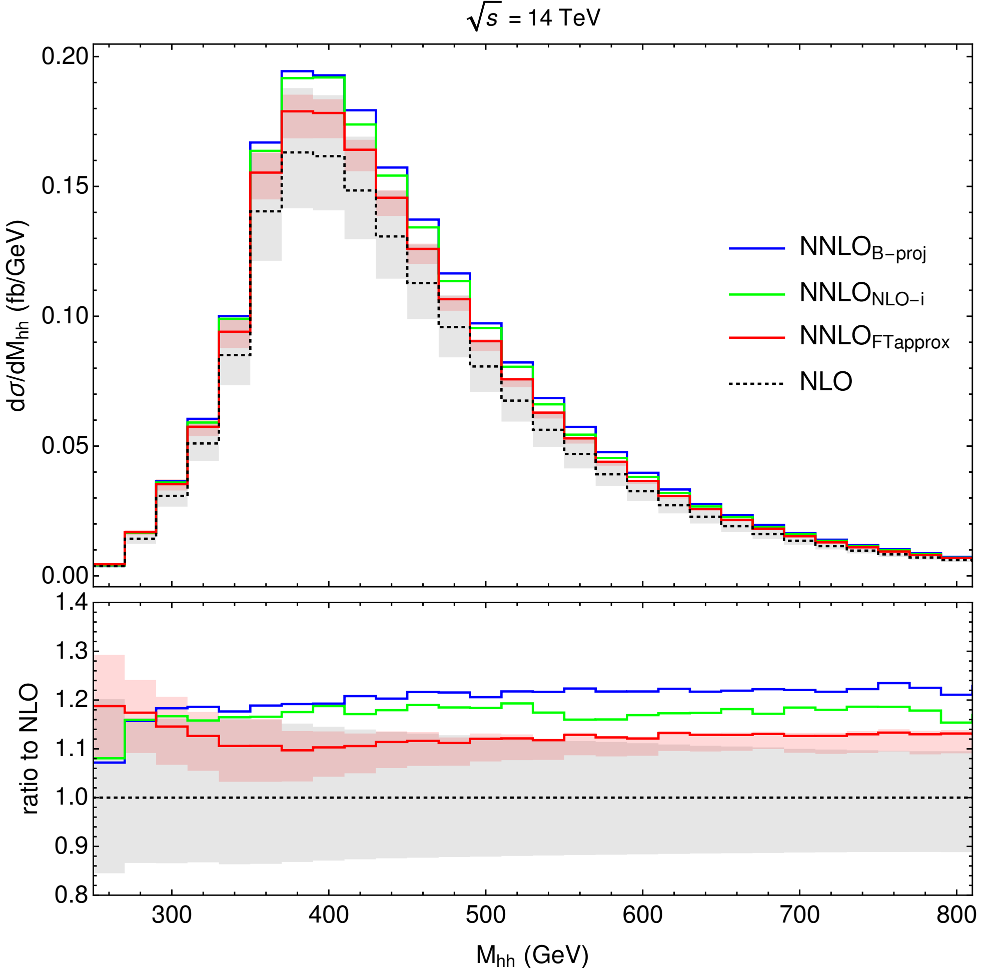
2) NNLO$_{\text{B-proj}}$

   project all real radiation contributions to Born configuration, rescale by LO/LO$_{\text{HEFT}}$

3) NNLO$_{\text{FTapprox}}$

   calculate NNLO$_{\text{HEFT}}$ and for each multiplicity rescale by

$$\mathcal{R}(ij \to HH+X) = \frac{\mathcal{A}^{\text{Born}}_{\text{Full}}(ij \to HH+X)}{\mathcal{A}^{(0)}_{\text{HEFT}}(ij \to HH+X)}$$



$\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
NLO [fb]	$27.78^{+13.8\%}_{-12.8\%}$	$32.88^{+13.5\%}_{-12.5\%}$	$127.7^{+11.5\%}_{-10.4\%}$	$1147^{+10.7\%}_{-9.9\%}$
NLO$_{\text{FTapprox}}$ [fb]	$28.91^{+15.0\%}_{-13.4\%}$	$34.25^{+14.7\%}_{-13.2\%}$	$134.1^{+12.7\%}_{-11.1\%}$	$1220^{+11.9\%}_{-10.6\%}$
NNLO$_{\text{NLO-i}}$ [fb]	$32.69^{+5.3\%}_{-7.7\%}$	$38.66^{+5.3\%}_{-7.7\%}$	$149.3^{+4.8\%}_{-6.7\%}$	$1337^{+4.1\%}_{-5.4\%}$
NNLO$_{\text{B-proj}}$ [fb]	$33.42^{+1.5\%}_{-4.8\%}$	$39.58^{+1.4\%}_{-4.7\%}$	$154.2^{+0.7\%}_{-3.8\%}$	$1406^{+0.5\%}_{-2.8\%}$
NNLO$_{\text{FTapprox}}$ [fb]	$31.05^{+2.2\%}_{-5.0\%}$	$36.69^{+2.1\%}_{-4.9\%}$	$139.9^{+1.3\%}_{-3.9\%}$	$1224^{+0.9\%}_{-3.2\%}$
M_t unc. NNLO$_{\text{FTapprox}}$	±2.6%	±2.7%	±3.4%	±4.6%
NNLO$_{\text{FTapprox}}$/NLO	1.118	1.116	1.096	1.067

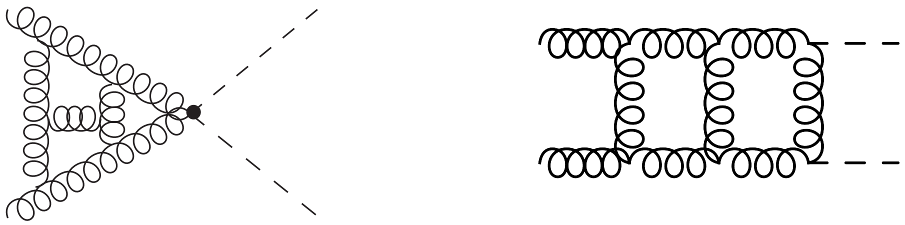
K-factor after inclusion of m_t dependence

Gluon Fusion @ N^3LO + N^3LL

going even higher in perturbation theory:

N^3LO Chen, Li, Shao, Wang 19

requires:
- triangle contributions up to 3 loop (similar to H @ N^3LO)
- box contributions up to 2 loop
  Banerjee, Borowka, Dhani, Gehrmann, Ravindran 18



most accurate prediction:

$(N^3LO + N^3LL) \otimes NLO_{m_t}$

N^3LL threshold resummation Ajjath, Shao `22

$\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
NLO_{m_t}	$27.56^{+13.9\%}_{-12.7\%}$	$32.64^{+13.5\%}_{-12.47\%}$	$126.1^{+11.5\%}_{-10.4\%}$	$1119^{+10.7\%}_{-9.9\%}$
$(\mathrm{NNLO}+\mathrm{NNLL}) \otimes \mathrm{NLO}_{m_t}$	$33.33^{+3.0\%}_{-3.3\%}$	$39.42^{+3.0\%}_{-3.4\%}$	$150.8^{+2.7\%}_{-3.4\%}$	$1320^{+2.4\%}_{-3.4\%}$
$\mathrm{N^3LO} \otimes \mathrm{NLO}_{m_t}$	$33.43^{+0.50\%}_{-2.8\%}$	$39.56^{+0.50\%}_{-2.7\%}$	$151.7^{+0.46\%}_{-2.3\%}$	$1333^{+0.51\%}_{-1.8\%}$
$(\mathrm{N^3LO}+\mathrm{N^3LL}) \otimes \mathrm{NLO}_{m_t}$	$33.47^{+0.88\%}_{-0.85\%}$	$39.60^{+0.85\%}_{-0.87\%}$	$151.9^{+0.63\%}_{-0.94\%}$	$1335^{+0.35\%}_{-1.0\%}$

→ 1% remaining scale (μ_R, μ_F) uncertainty

additional uncertainties:
- 3% m_t-effects beyond NLO
- $\mathcal{O}(10\%)$ mass-scheme
- EW corrections

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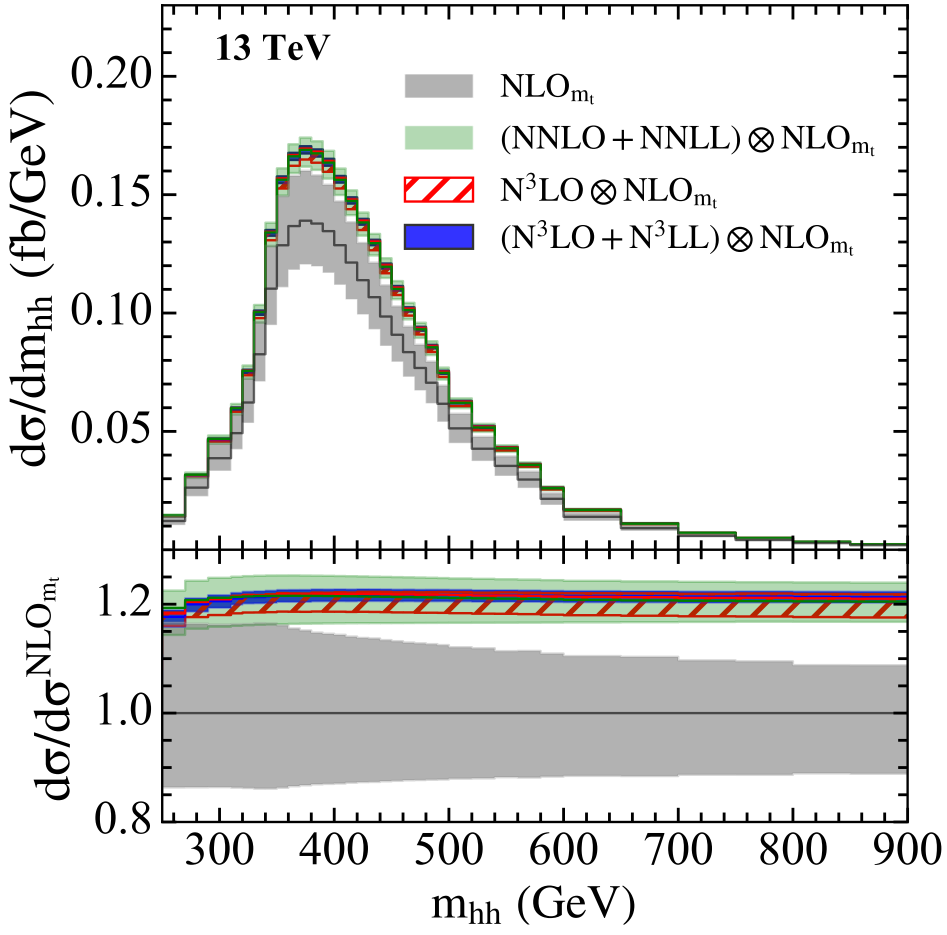


Mass Scheme Uncertainties

So far, all results used OS renormalization of m_t,

but also other schemes, e.g. $\overline{\text{MS}}$ valid → additional mass scheme uncertainty

NLO predictions in $\overline{\text{MS}}$ scheme

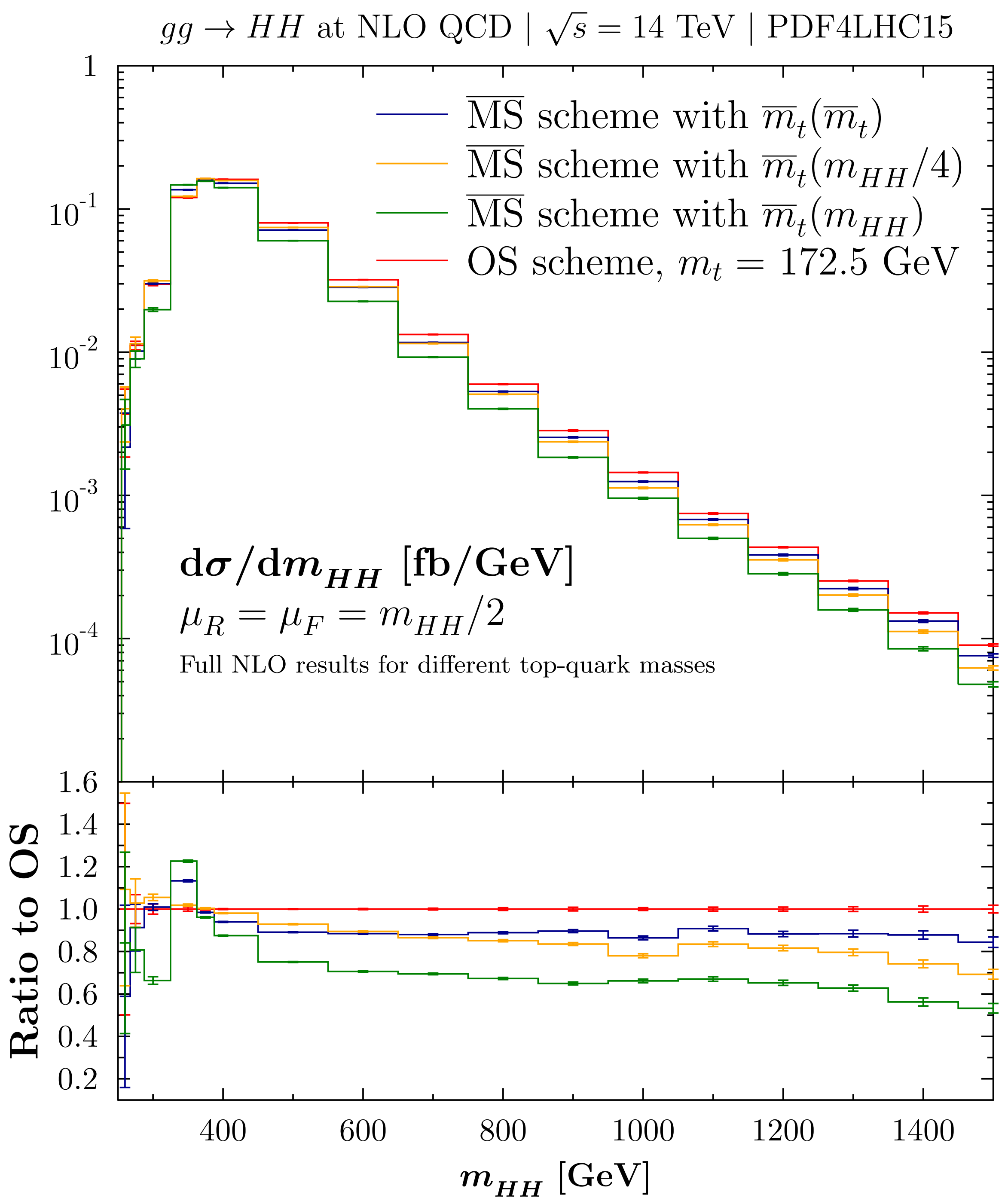
Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 19,20

$$\frac{d\sigma(gg\to HH)}{dQ}\bigg|_{Q=400\ \text{GeV}} = 0.1609(4)^{+0\%}_{-13\%}\ \text{fb/GeV}$$

$$\frac{d\sigma(gg\to HH)}{dQ}\bigg|_{Q=1200\ \text{GeV}} = 0.000435(4)^{+0\%}_{-35\%}\ \text{fb/GeV}$$

large scheme uncertainties at large m_{HH}
(larger than μ_R, μ_F dependence)

$$\sigma_{tot} = 32.81(7)^{+4\%}_{-18\%}\ \text{fb}$$



Mass Scheme Uncertainties

HH mass scheme uncertainties:

$$\left.\frac{d\sigma(gg\to HH)}{dQ}\right|_{Q=400\text{ GeV}} = 0.1609(4)^{+0\%}_{-13\%}\,\text{fb/GeV}$$

$$\left.\frac{d\sigma(gg\to HH)}{dQ}\right|_{Q=1200\text{ GeV}} = 0.000435(4)^{+0\%}_{-35\%}\,\text{fb/GeV}$$

Similar effects for (off-shell) H production:

$$\left.\sigma(gg\to H^*)\right|_{Q=125\text{ GeV}} = 42.17^{+0.4\%}_{\underline{-0.5\%}}\,\text{pb}$$

$$\left.\sigma(gg\to H^*)\right|_{Q=600\text{ GeV}} = 1.97^{+0.0\%}_{\underline{-15.9\%}}\,\text{pb}$$

Is there any preferred scheme choice?

- Leading contributions in high-energy expansion ($\sqrt{\hat{s}} = m_{HH} \gg m_t$) at NLO

  Jones, Spira (Les Houches 2019); Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 20; based on Davies, Mishima, Steinhauser, Wellmann 18

  OS: $\quad F^{\text{NLO}}_{\text{box,i}} = 2F^{\text{LO}}_{\text{box,i}} \log\frac{m_t^2}{s} + \frac{m_t^2}{s}c_{1,i} + \mathcal{O}\left(\frac{1}{s^2}\right)$ $\qquad$ $\overline{MS}$: $\quad F^{\text{NLO}}_{\text{box,i}} = 2F^{\text{LO}}_{\text{box,i}}\left[\log\frac{\mu_t^2}{s} + \frac{4}{3}\right] + \frac{\overline{m}_t^2(\mu_t)}{s}c_{1,i} + \mathcal{O}\left(\frac{1}{s^2}\right)$

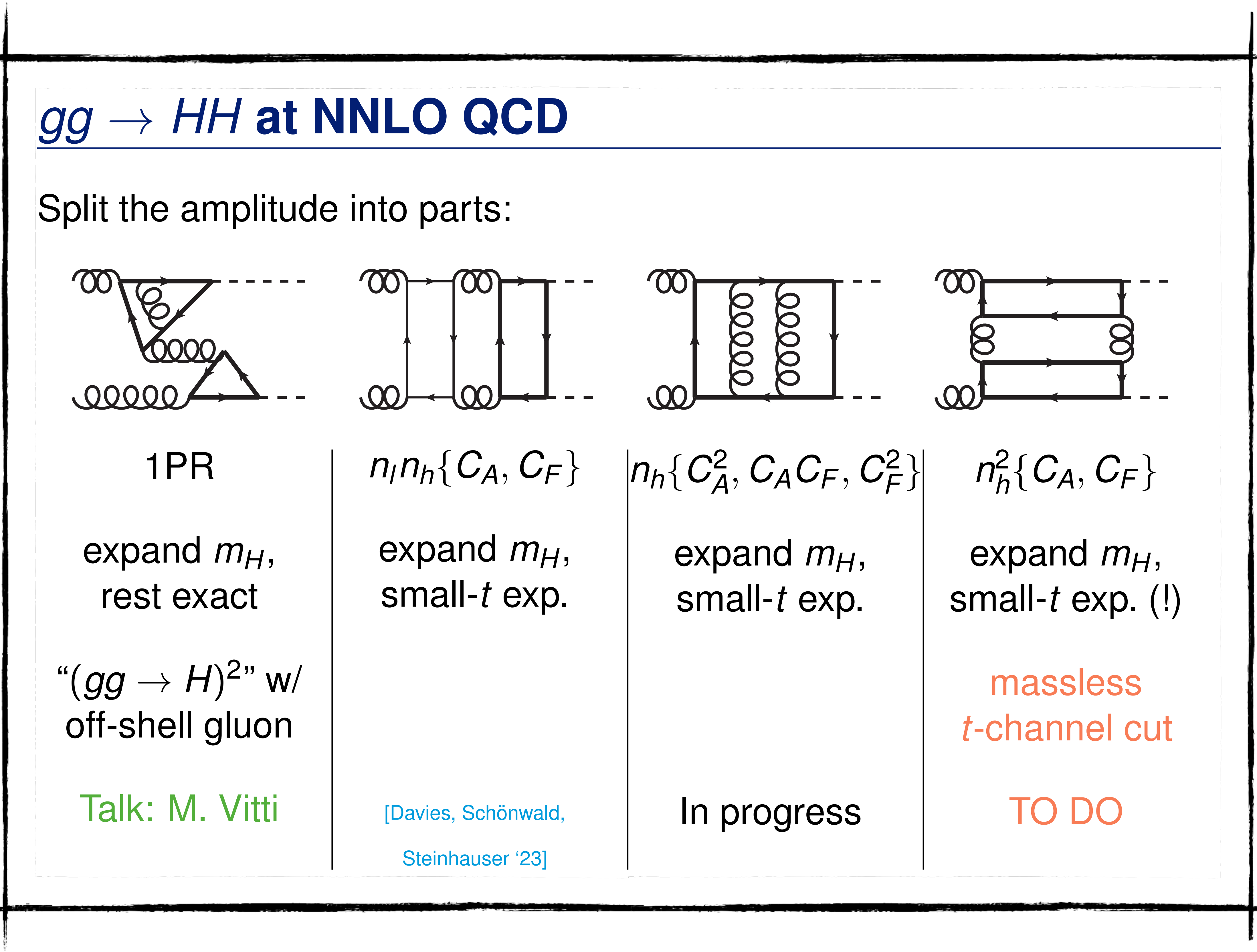
  → preferred choice $\mu_t^2 = s$
- Matching to HTL at low energies: preferred choice $\mu_t = m_t$
- Better convergence using OS scheme in H* @ NNLO J. Mazzitelli 16

→ Need NNLO predictions with full m_t dependence

Towards NNLO QCD with m_t-dependence

Full NNLO QCD predictions with m_t-dependence out of reach,

but can be approximated using expansions!



$gg \to HH$ at NNLO QCD

Split the amplitude into parts:

1PR	$n_l n_h\{C_A, C_F\}$	$n_h\{C_A^2, C_A C_F, C_F^2\}$	$n_h^2\{C_A, C_F\}$
expand m_H, rest exact	expand m_H, small-t exp.	expand m_H, small-t exp.	expand m_H, small-t exp. (!)
"$(gg \to H)^2$" w/ off-shell gluon			massless t-channel cut
Talk: M. Vitti	[Davies, Schönwald, Steinhauser '23]	In progress	TO DO

J. Davies, Loops & Legs 2024

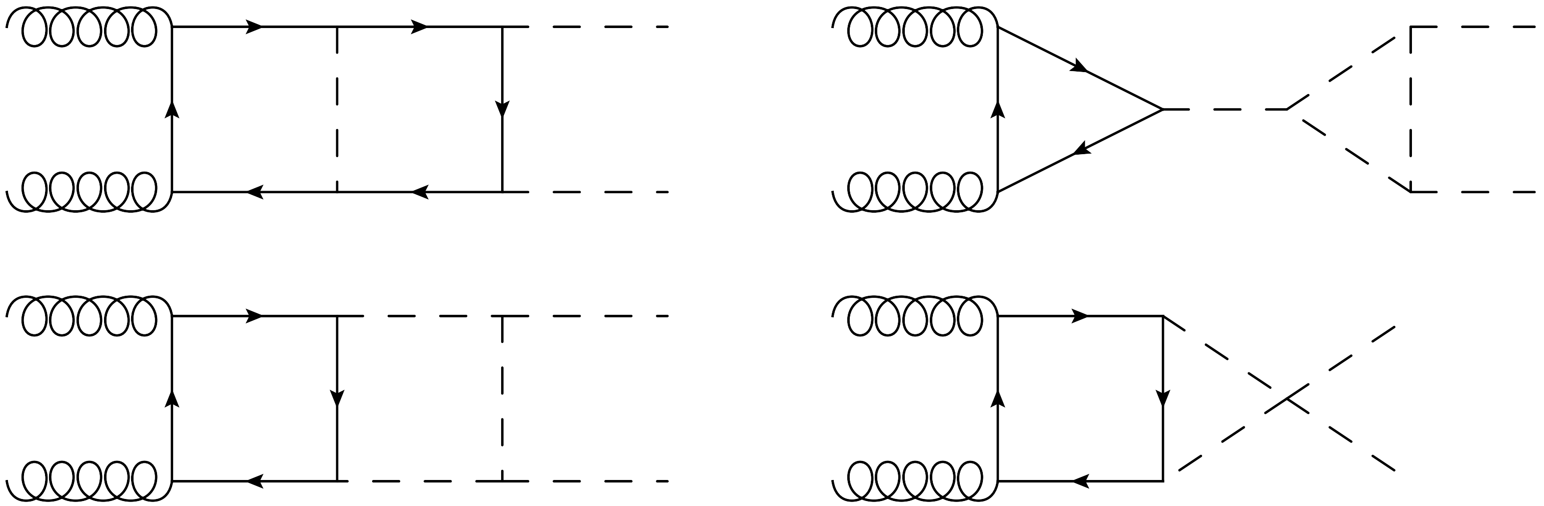
small-t expansion valid for $p_T \lesssim 200\,\mathrm{GeV}$

NLO EW corrections to $gg \to HH$

NLO EW corrections needed in addition to QCD corrections

Recently, huge progress:

- partial results (Yukawa-/Higgs- interactions):
  Bizoń, Haisch, Rottoli 18,24; Borowka, Duhr, Maltoni, Pagani, Shivaji, Zhao 19;
  Mühlleitner, Schlenk, Spira 22; Xiao Zhang et.al. Higgs 2023; MK et.al. Loops & Legs 2024

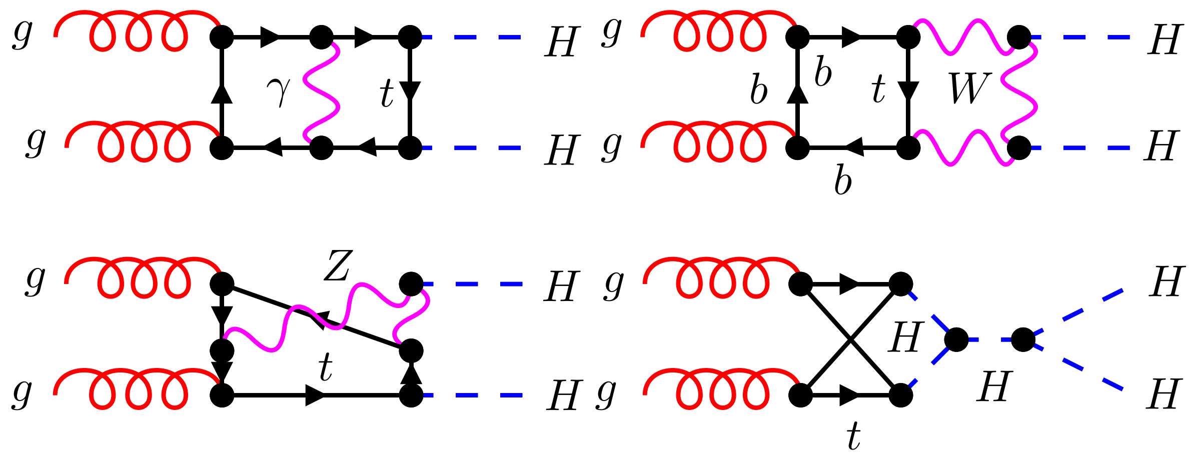


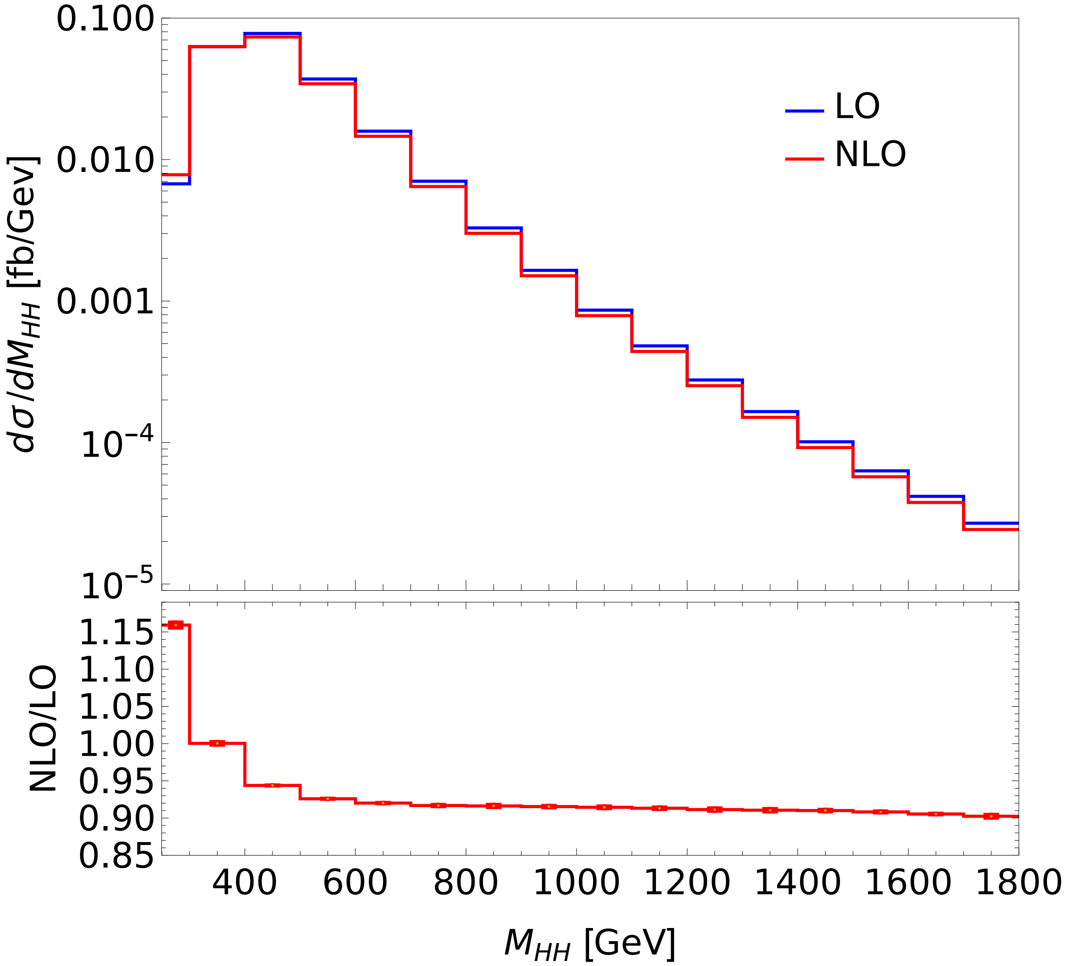
- approximate results:
  Top-Yukawa corrections in the high-energy limit [Davies, Mishima, Schönwald, Steinhauser, Zhang, 22]
  EW corrections in large-m_t limit [Davies, Schönwald, Steinhauser, Zhang, 23]
- full EW corrections [Bi, Huang, Huang, Ma, Yu 23]

NLO EW corrections to $gg \to HH$

Karlsruhe Institute of Technology

Full EW corrections Bi, Huang, Huang, Ma, Yu 23





Method: Auxiliary Mass Flow (AMFlow)
Liu, Ma, Wang 17; Liu, Ma, Tao et.al. 20; Liu, Ma 22

Solve Loop Integrals

$$I \propto \lim_{\eta \to 0^+} \int \prod_l^L d^d k_i \prod_i^N \frac{1}{[q_i^2 - (m_i^2 - i\eta)]^{\nu_i}}$$

via differential equations in $x = -i\eta$

$$\partial_x I = MI$$

- Start from boundary point $x = -i\infty$
  → easy to calculate
  (massive vacuum graphs) x (massless graphs)
- Transfer to x=0 using power-log expansions
  Moriello 19

-4% correction w.r.t σ_{LO}
$\mathcal{O}(10\%)$ differentially

EFT Studies

Heinrich, Jones, MK, Luisoni, Scyboz 19, 20
De Florian, Fabre, Heinrich, Mazzitelli, Scyboz 21
Heinrich, Lang, Scyboz 22, 23

HEFT:

- Higgs h(x) is EW singlet
  → $\mathscr{L}$ can be polynomial in h/v → independent couplings c_t, c_{tt},…
- UV completion can be strongly coupled

$$\Delta\mathcal{L}_{\mathrm{HEFT}} = -m_t\left(c_t\frac{h}{v}+c_{tt}\frac{h^2}{v^2}\right)\bar{t}\,t - c_{hhh}\frac{m_h^2}{2v}h^3+\frac{\alpha_s}{8\pi}\left(c_{ggh}\frac{h}{v}+c_{gghh}\frac{h^2}{v^2}\right)G^a_{\mu\nu}G^{a,\mu\nu}$$

SMEFT:

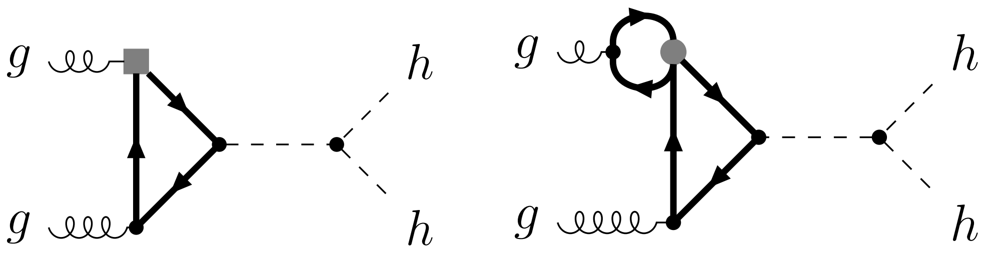
- Higgs doublet $\Phi(x)$ transforms linearly under SU(2)$_\mathrm{L}$
- Canonical expansion in $1/\Lambda$

$$\begin{aligned}\Delta\mathcal{L}_{\mathrm{Warsaw}} &= \frac{C_{H,\Box}}{\Lambda^2}(\phi^\dagger\phi)\Box(\phi^\dagger\phi)+\frac{C_{HD}}{\Lambda^2}(\phi^\dagger D_\mu\phi)^*(\phi^\dagger D^\mu\phi)+\frac{C_H}{\Lambda^2}(\phi^\dagger\phi)^3\\ &+\left(\frac{C_{uH}}{\Lambda^2}\phi^\dagger\phi\bar{q}_L\phi^c t_R+h.c.\right)+\frac{C_{HG}}{\Lambda^2}\phi^\dagger\phi G^a_{\mu\nu}G^{\mu\nu,a}\,.\end{aligned}$$

- Chromomagnetic operator sub-leading (assuming renormalizable, weakly coupled UV completion)
  → loop counting Buchalla, Heinrich, Müller-Salditt, Pandler 22

$$\mathcal{L}_{tG} = \frac{C_{tG}}{\Lambda^2}\left(\bar{Q}_L\sigma^{\mu\nu}T^aG^a_{\mu\nu}\tilde{\phi}t_R+\mathrm{h.c.}\right)$$

- Chromomagnetic and 4-top operators depend on γ_5-scheme
  Scheme conversion relates both types of operators Di Noi, Gröber, Heinrich, Lang, Vitti 23; Heinrich, Lang 23



$$C_{tG}^{\mathrm{BMHV}} = C_{tG}^{\mathrm{NDR}} - \frac{\sqrt{2}m_tg_s}{16\pi^2v}\left(C_{Qt}^{(1)}+\left(c_F-\frac{c_A}{2}\right)C_{Qt}^{(8)}\right)$$

EFT Studies

Heinrich, Lang, Scyboz 22

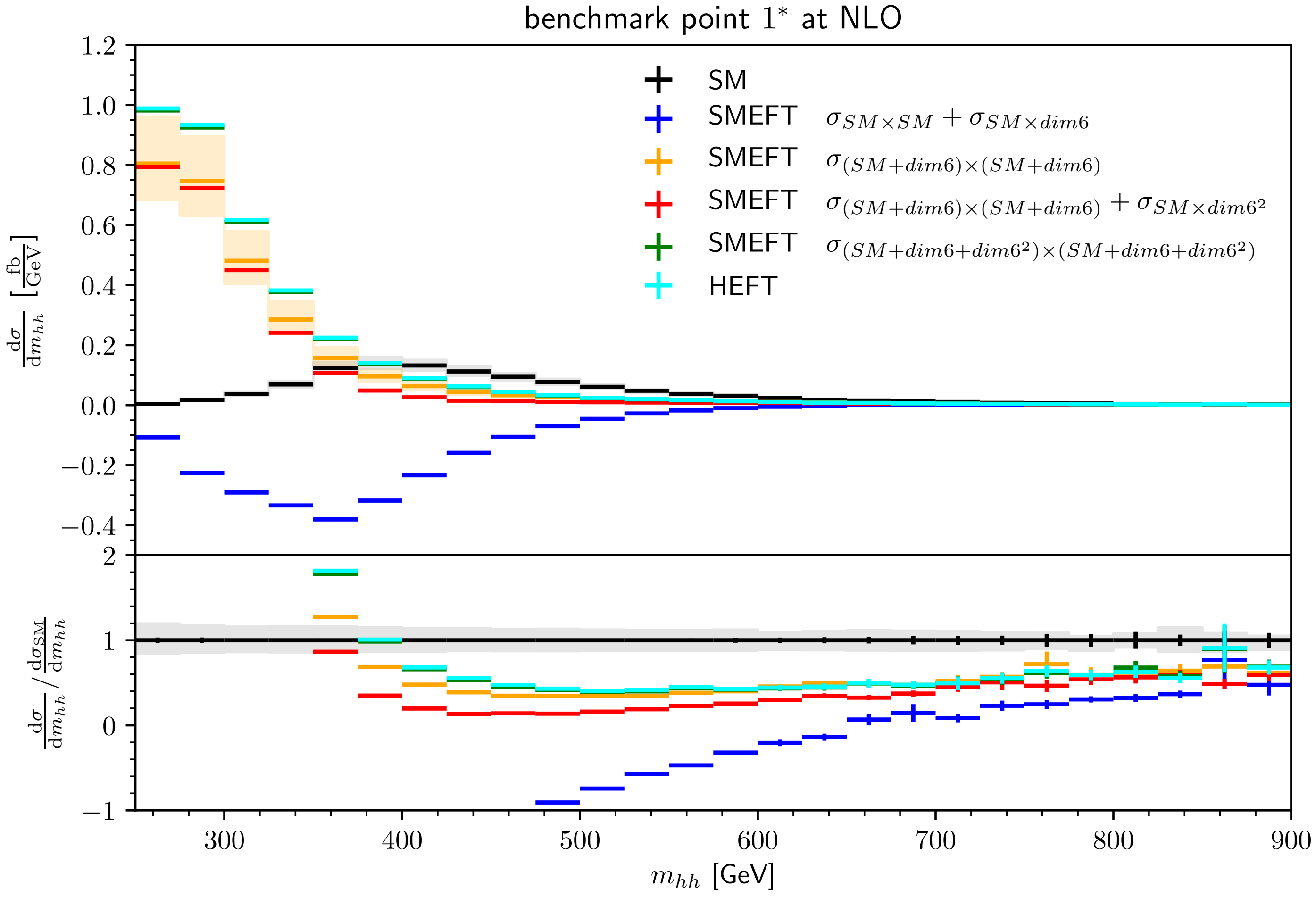
Naive conversion HEFT → SMEFT:

HEFT	Warsaw
c_{hhh}	$1 - 2\frac{v^2}{\Lambda^2}\frac{v^2}{m_h^2} C_H + 3\frac{v^2}{\Lambda^2} C_{H,\mathrm{kin}}$
c_t	$1 + \frac{v^2}{\Lambda^2} C_{H,\mathrm{kin}} - \frac{v^2}{\Lambda^2}\frac{v}{\sqrt{2}m_t} C_{uH}$
c_{tt}	$-\frac{v^2}{\Lambda^2}\frac{3v}{2\sqrt{2}m_t} C_{uH} + \frac{v^2}{\Lambda^2} C_{H,\mathrm{kin}}$
c_{ggh}	$\frac{v^2}{\Lambda^2}\frac{8\pi}{\alpha_s} C_{HG}$
c_{gghh}	$\frac{v^2}{\Lambda^2}\frac{4\pi}{\alpha_s} C_{HG}$

$$C_{H,\mathrm{kin}} := C_{H,\Box} - \frac{1}{4} C_{HD}$$

Benchmark 1*

c_{hhh}	c_t	c_{tt}	c_{ggh}	c_{gghh}	$C_{H,\mathrm{kin}}$	C_H	C_{uH}	C_{HG}
5.105	1.1	0	0	0	4.95	−6.81	3.28	0



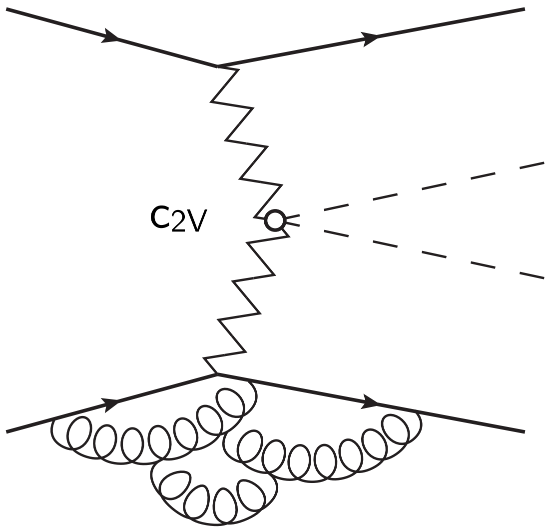
- → not guaranteed to work
  - different model assumptions
  - can lead out of validity range for $1/\Lambda$ expansion
- → depends on truncation of SMEFT predictions

VBF HH + 2-jet production

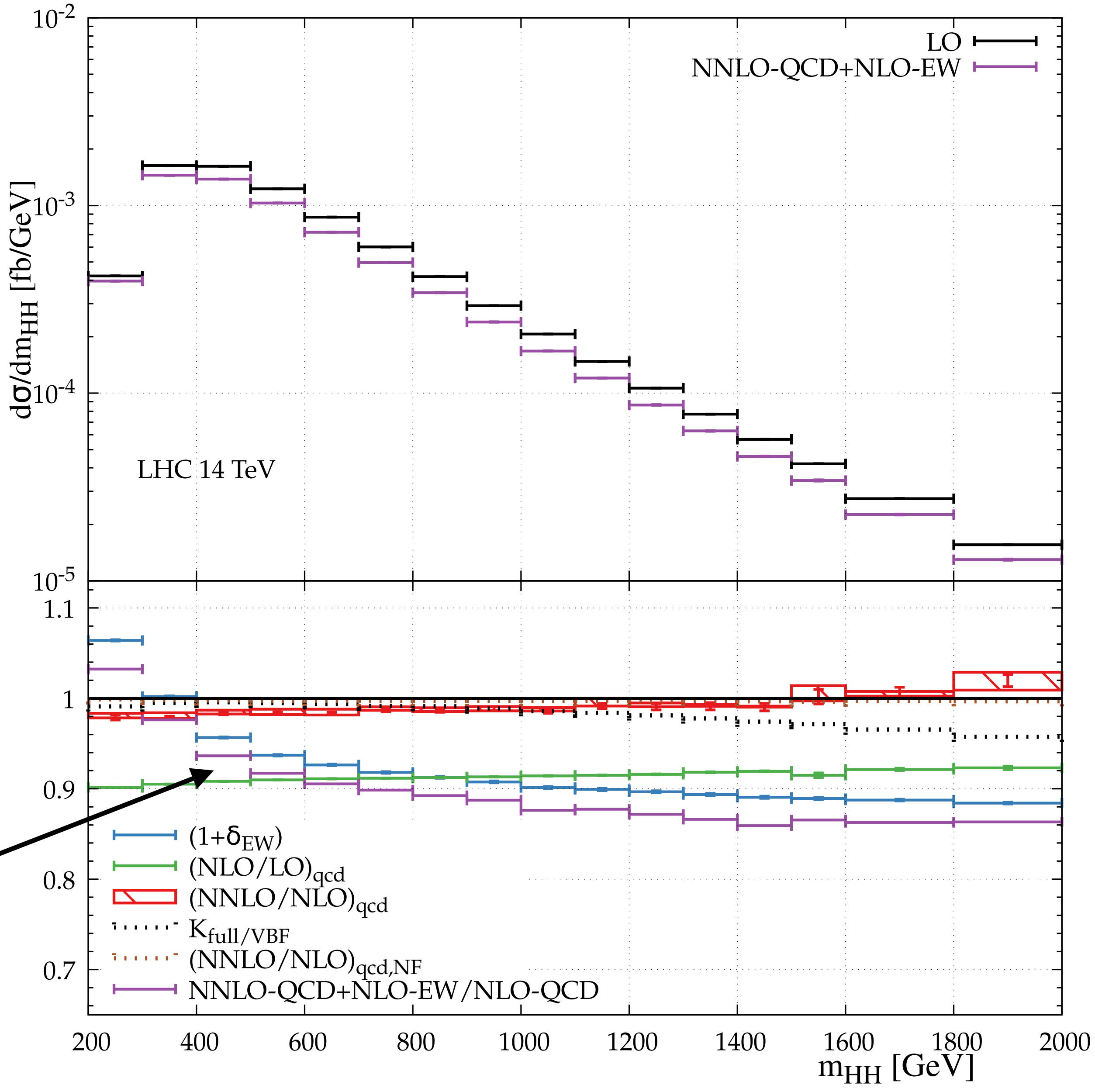
- sensitivity to couplings λ and c_{2V}
- known with high accuracy using VBF approximation

  → no color exchange between quark-lines

  - N^3LO QCD Dreyer, Karlberg 18
  - NNLO QCD + NLO EW Dreyer, Karlberg, Lang, Pellen 20

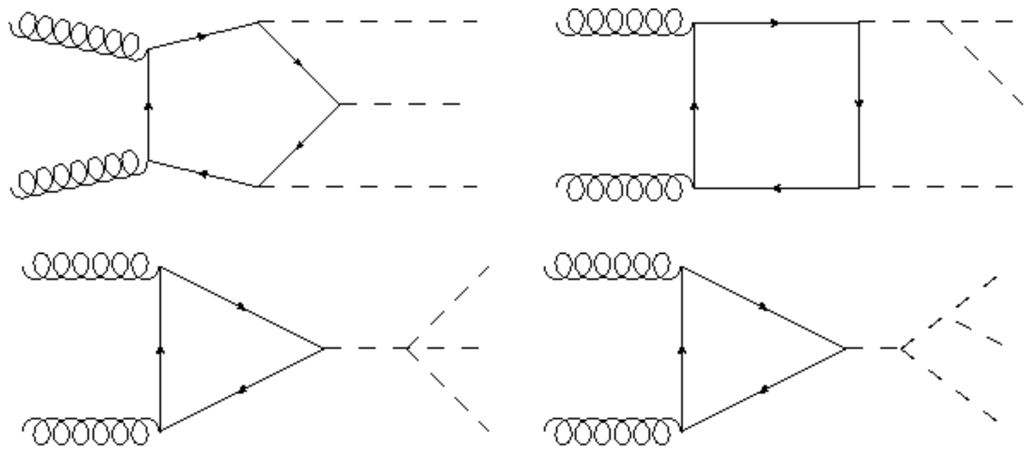


	$\sigma^{(14\ \text{TeV})}$ [fb]
LO	$2.079^{+0.177}_{-0.152}$
NLO	$2.065^{+0.022}_{-0.018}$
NNLO	$2.056^{+0.003}_{-0.005}$
N^3LO	$2.055^{+0.001}_{-0.001}$



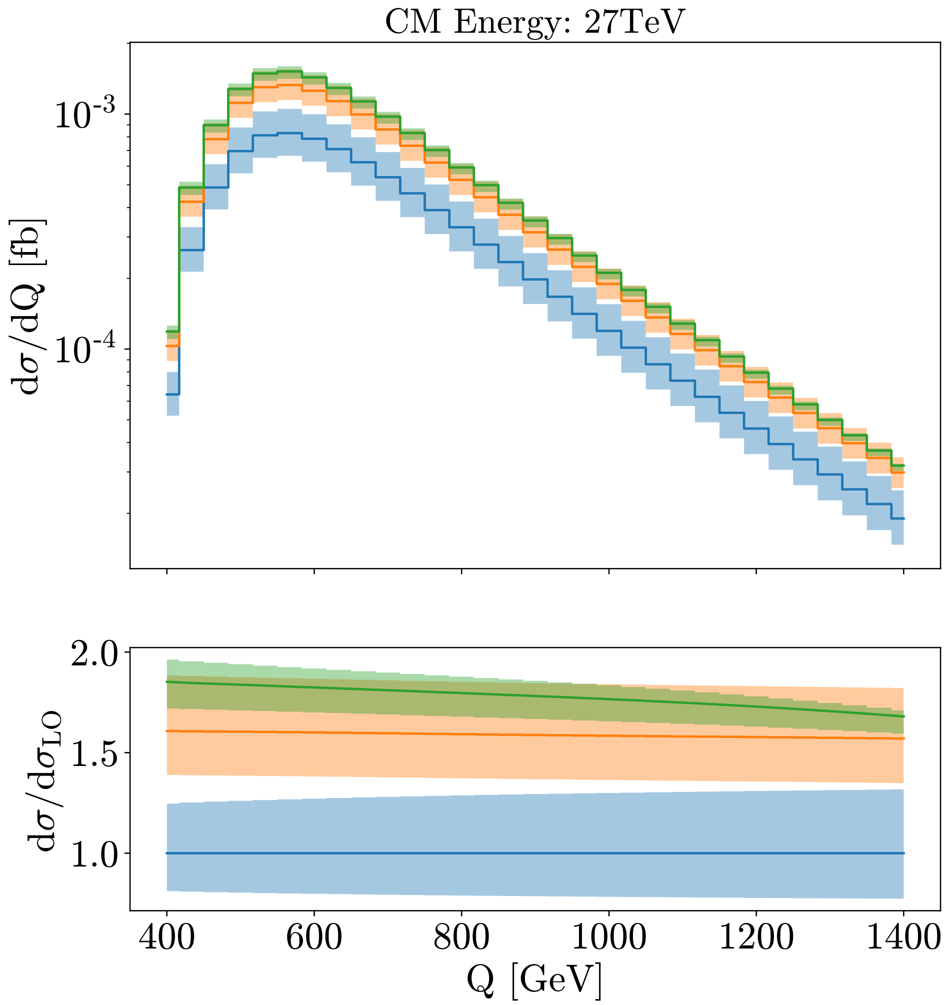
similar size of
NLO QCD & NLO EW

Triple-H Production



$\mu_0 = Q/2$	14 TeV	27 TeV	100 TeV
LO	$0.0605^{+34\%}_{-24\%}$	$0.295^{+28\%}_{-20\%}$	$3.88^{+21\%}_{-16\%}$
$\mathrm{NLO_{Bi}}$	$0.0983^{+18\%}_{-15\%}$	$0.473^{+16\%}_{-14\%}$	$5.75^{+15\%}_{-12\%}$
$\mathrm{NLO_{dBi}}$	$0.0982^{+18\%}_{-15\%}$	$0.471^{+17\%}_{-14\%}$	$5.72^{+15\%}_{-12\%}$
$\mathrm{NNLO_{Bi}}$	$0.114^{+5\%}_{-8\%}$	$0.540^{+5\%}_{-7\%}$	$6.47^{+5\%}_{-6\%}$
$\mathrm{NNLO_{dBi}}$	$0.113^{+5\%}_{-8\%}$	$0.534^{+5\%}_{-7\%}$	$6.36^{+5\%}_{-6\%}$
$\mathrm{NNLO_{Best}}$	$0.103^{+5\%}_{-8\%}$	$0.501^{+5\%}_{-7\%}$	$5.56^{+5\%}_{-6\%}$

- NLO $\mathrm{FT_{approx}}$ Maltonia, Vryonidoua, Zaro 14
- NNLO HTL de Florian, Mazzitelli 16
- NNLO HTL $\otimes$ NLO $\mathrm{FT_{approx}}$ de Florian, Fabre, Mazzitelli 19



CM Energy: 27TeV

Summary

Theory Predictions for gg $\rightarrow$ HH Production

- $(\mathrm{N^3LO} + \mathrm{N^3LL}) \otimes \mathrm{NLO}_{m_t}$

  remaining scale-dependence 1%
  3% uncertainty due to m_t-effects beyond NLO

- 10-20% Mass-Scheme Uncertainties

  m_t effects at NNLO required

  possibly in reach, using expansions

- -4% EW NLO Corrections

  $\mathcal{O}(10\,\%)$ differential corrections

- EFT predictions in HEFT and SMEFT

  γ_5-scheme dependence of chromomagnetic and 4-top operators