# Perturbative Inputs to Event Generators 

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## Colliders in the real world



## Colliders for theorists

- Event simulation factorised into
- Hard Process
- Parton Shower
- PDF/Underlying event
- Hadronisation
- QED radiation
- Hadron Decays



## Colliders for theorists

- Event simulation factorised into - Event simulation factorised into
- Hard Process
- Parton Shower
- PDF/Underlying event
- Hadronisation
- perturbative QCD / SM
- resummation based on pQCD
- connection to factorisation [see talk by T. Cridge]
- NP modelling [see talk by S.Gieseke]


## Colliders for theorists

- Event simulation factorised into - Event simulation factorised into
- Hard Process
- Parton Shower
- PDF/Underlying event
- perturbative QCD / SM
- resummation based on pQCD
- non-perturbative, connection to factorisation


## Matching

- Event simulation factorised into
- Hard Process
- Parton Shower

Standard for LHC SM pheno:

- matching to NLO QCD, 2 main schemes: Powheg [Nason '04] and MC@NLO [Frixione, Webber ‘02]



## Selected developments

- matching to NNLO (inclusive for some processes) in principle available in several approaches:
- Geneva [Alioli, Bauer, Berggen, Tackmann, Walsh, Zuberi ' 13 ] ...
- MINNLO [Monni, Nason, Re, Wiesemann, Zanderighi ' 19$]$...
- UNLOPS [Höche, Li, Prestel '15] ...
- first steps towards differential NNLO in Vincia, "Powheg-style" matching based on matrix element corrections [Skands,

[Talk by C. Preuss HP2 '22]
Preuss '23]


## Selected developments

- including electroweak corrections, in various approximations [see talk by S. Schumann]
- advancements in available fixed order calculations [see talk by M. Grazzini + specialised talks]
- efficient implementation LO/NLO calculation,
- see e.g.
[Bothmann et. al. '22]
- improvements by
$\mathcal{O}(10)$ factors in
Sherpa event generation

$$
p p \rightarrow e^{+} e^{-}+0,1,2 j @ \mathrm{NLO}+3,4,5 j @ \mathrm{LO}
$$



## Parton Showers

- Event simulation factorised into
- Hard Process
- Parton Shower

Standard for LHC SM pheno:

- angular ordered parton shower in Herwig
- dipole/antenna showers in Pythia 8 (default showers, Dire, Vincia), Herwig 7, Sherpa (default CS shower, Dire)



## New Parton Showers - NLL accuracy

- typical claim based on accuracy of splitting functions etc.
- parton showers $\sim$ NLL accurate if CMW schem $\epsilon$ for strong coupling is used
- observation in [Dasgupta, Dreyer, Hamilton, Monni, Salam '18] (PanScales collaboration):
- subtleties arise in distribution of recoil for subsequent emissions $\Rightarrow$ phase space where accuracy is spoiled if soft gluon absorbs recoil
-     + in colour assignment
- also: set of tests for shower accuracy [Dasgupta,
 Dreyer, Hamilton, Monni, Salam '20]


## Compare: resummation e.g. in CAESAR

- factorisation of matrix elements in soft collinear limit well known
- how to extract NLL observable independent (i.e. without additional information)?
- method from [Banfi, Salam, Zanderighi ' ${ }^{\circ} 5$ ]: need explicit implementation of soft-collinear limit*:

$$
\begin{aligned}
& k_{t}^{\rho}=k_{t} \rho \\
& \eta^{\rho}=\eta-\xi \ln \rho
\end{aligned} \quad \xi=\frac{\eta}{\eta_{\max }}
$$

and assume
$V\left(k_{i}^{\rho}\right)=\rho V\left(k_{i}\right)$
$\rightarrow$ numerically evaluate phase space integrals in this limit

$$
\left(+,-, k_{t}\right) \sim\left(k_{t} e^{\eta}, k_{t} e^{-\eta}, k_{t}\right)
$$

$$
\sim\left(\rho^{1-\xi}, \rho^{1+\xi}, \rho\right)
$$



* example assuming $V\left(k_{t}, \eta\right) \sim k_{t} / Q$ for brevity


## Effect of recoil on accuracy

- question: do recoil effects indeed vanish in soft limit (i.e. $\rho \rightarrow 0$ )?* [Dasgupta,Dreyer,Hamilton,Monni,Salam '18]
- consider situation where we first emit $\tilde{p}_{i j}$ from $p_{a}, p_{b}$, then emit $p_{j}$, $\tilde{p}_{i j} \rightarrow p_{i}, p_{j}$
- transverse momentum of $p_{i}$ will be

$$
k_{t}^{i} \sim k_{t}^{i j}+k_{t}^{j} \rightarrow k_{t}^{i j} \text { as } \frac{k_{t}^{j}}{k_{t}^{i}} \rightarrow 0
$$

- but, relevant limit is $\frac{\Delta k_{t}^{i}}{k_{t}^{i}} \rightarrow \frac{\rho k_{t}^{j}}{\rho k_{t}^{i}}=\mathcal{O}(1)$

$$
\begin{aligned}
& p_{i}=z \tilde{p}_{i j}+(1-z) y \tilde{p}_{k}+k_{\perp} \\
& p_{j}=(1-z) \tilde{p}_{i j}+z y \tilde{p}_{k}-k_{\perp} \\
& p_{k}=(1-y) \tilde{p}_{k}
\end{aligned}
$$



## New Parton Showers - NLL accuracy

- Several solutions/re-evaluations of parton shower concepts:
- [Dasgupta, Dreyer, Hamilton,Monni, Salam, Soyez '20], [vanBeekveld, Ferrario Ravasio, Hamilton, Salam, Soto-Ontoso,Soyez '22]
- partitioning of splitting functions and appropriate choice of evolution variable can lead to NLL accurate shower for local and global recoil strategies
- [Forshaw, Holguin, Plätzer '20]
- Connections between angular ordered and dipole showers
- [Nagy, Soper '11]
- local transverse, global longitudinal recoil
- [Herren, Krauss, DR, Schönherr, Höche '22]
- global recoil, enables analytic comparison to resummation and proof of NLL accuracy
- [Preuss '24]
- global recoil in antenna shower Vinca


## Example Solution: Alaric

- Before splitting:

$$
k_{i}^{\mu} \rightarrow \Lambda_{\nu}^{\mu} k_{i}^{\nu}
$$

- After splitting:

$$
p_{k}=\tilde{p}_{k}
$$

$$
\sum_{K^{2}=\tilde{K}^{2}}^{\substack{k_{k}}} p_{i}=z \tilde{p}_{i}
$$

$$
\tilde{K}+\tilde{p}_{i}=K+p_{i}+p_{j}
$$

$$
\Lambda_{\nu}^{\mu}=g_{\nu}^{\mu}-\frac{(K+\tilde{K})^{\mu}(K+\tilde{K})_{\nu}}{K \cdot \tilde{K}+\tilde{K}^{2}}+2 \frac{K^{\mu} \tilde{K}_{\nu}}{\tilde{K}^{2}} \rightarrow \Lambda_{\nu}^{\mu} \tilde{K}^{\nu}=K^{\mu}
$$

## Alaric at the LHC - jets

- [Höche, Krauss, DR '24] extend Alaric method to IS evolution
- satisfactory description of inclusive and dijet events
- transverse momentum spectrum of leading jet and ratio 3-to-2 jet rate
- NLL accuracy shown numerically for FS, in addition to analytic proof





## Beyond logarithmic accuracy

- Observations
- LL and NLL accurate showers can be very similar (e.g. failing of NLL accuracy numerically undetectable for Dire in prominent observables like Thrust)
- NLL accurate showers can differ significantly from NLL result away from strict limit
- $\Rightarrow$ subleading effect play a significant role in phenomenological successful parton showers, more systematic understanding
 desirable, see also [Höche, Siegert, DR '17]


## Alaric beyond NLL - subleading effects

assume Sudakov decompose like

$$
\begin{aligned}
p_{i}^{\mu} & =z_{i} \hat{p}_{i j}^{\mu}+\frac{-k_{t}^{2}}{z_{i} 2 p_{i j} \bar{n}} \bar{n}^{\mu}+k_{t}^{\mu}, \\
p_{j}^{\mu} & =z_{j} \hat{p}_{i j}^{\mu}+\frac{-k_{t}^{2}}{z_{j} 2 p_{i j} \bar{n}} \bar{n}^{\mu}-k_{t}^{\mu}
\end{aligned}
$$

actual shower kinematics:
derivation of splitting functions leads to:

$$
\begin{aligned}
& P_{q q \|}^{(\mathrm{F})}\left(p_{i}, p_{j}, \bar{n}\right)=C_{F}(1-\varepsilon)\left(1-z_{i}\right) \\
& P_{g g \|}^{(\mathrm{F})}\left(p_{i}, p_{j}, \bar{n}\right)=2 C_{A} z_{i} z_{j}, \\
& P_{g q \|}^{(\mathrm{F})}\left(p_{i}, p_{j}, \bar{n}\right)=T_{R}\left[1-\frac{2 z_{i} z_{j}}{1-\varepsilon}\right] .
\end{aligned}
$$

$$
p_{i}=z \tilde{p}_{i}
$$

$$
p_{j}=(1-z) \tilde{p}_{i}+v\left(\tilde{K}-(1-z+2 \kappa) \tilde{p}_{i}\right)-k_{\perp}
$$

ultimately, "proper"

$$
K=\tilde{K}-v\left(\tilde{K}-(1-z+2 \kappa) \tilde{p}_{i}\right)+k_{\perp},
$$

splitting variables:

$$
p_{i}=\frac{z}{1-v(1-z+\kappa)} \hat{p}_{i j}+\frac{z}{1-v(1-z+\kappa)} k_{\perp}+\mathcal{O}\left(\frac{k_{\perp}^{2}}{2 \tilde{p}_{i} \tilde{K}}\right)
$$

$$
p_{j}=\frac{(1-z)(1-v)-v \kappa}{1-v(1-z+\kappa)} \hat{p}_{i j}-\frac{z}{1-v(1-z+\kappa)} k_{\perp}+\mathcal{O}\left(\frac{k_{\perp}^{2}}{2 \tilde{p}_{i} \tilde{K}}\right)
$$

$$
\begin{aligned}
z_{i} & =\frac{z}{1-v(1-z+\kappa)} \\
z_{j} & =1-\frac{z}{1-v(1-z+\kappa)}
\end{aligned}
$$

## Alaric — subleading effects in Z+jets

- effects/choices beyond NLL accuracy:
- choice of evolution variable (up to factors of $z \sim 1$ )
- identify PS parameter $z$ with $z_{i}, z_{j}$
- choice of recoil momentum $K$ (NLL accuracy needs "hard" $K$ )



## Towards NNLL - Shower Evolution at NLO

- NLO splitting kernels implemented in a parton shower [Höche, Prestel '17], [Dulat, Höche, Prestel '18]
- not in a NLL safe framework, but conceptual problems are largely solved
- more recent work on the precise relation to NNLL resummation and in other showers [Dasgupta, El-Menoufi '21] [Braun-White, Glover, Preuss '23] [Ferraro Ravasio, Hamilton, Karlberg, Salam, Skybox '23], [van Beekveld, Dasgupta, El-Menoufi, Helliwell, Monni '23] ...

[Höche, Krauss, Prestel '17]


## Summary \& other topics

- perturbative inputs to event generation, focus on parton showers and their log accuracy
- many additional topics not discussed:
- improvements to color evolution
- accurate matching already at NLO to achieve NLL' accuracy
- QED precision physics / photon resummation
- massive Quark effects
-...

