UNIVERSITÀ DEGLI STUDI DI MILANO

Istituto Nazionale di Fisica Nucleare

Mixed QCD-EW corrections to neutral- and charged-current Drell-Yan processes

Alessandro Vicini

University of Milano, INFN Milano

Roma SM@LHC, May 9th 2024

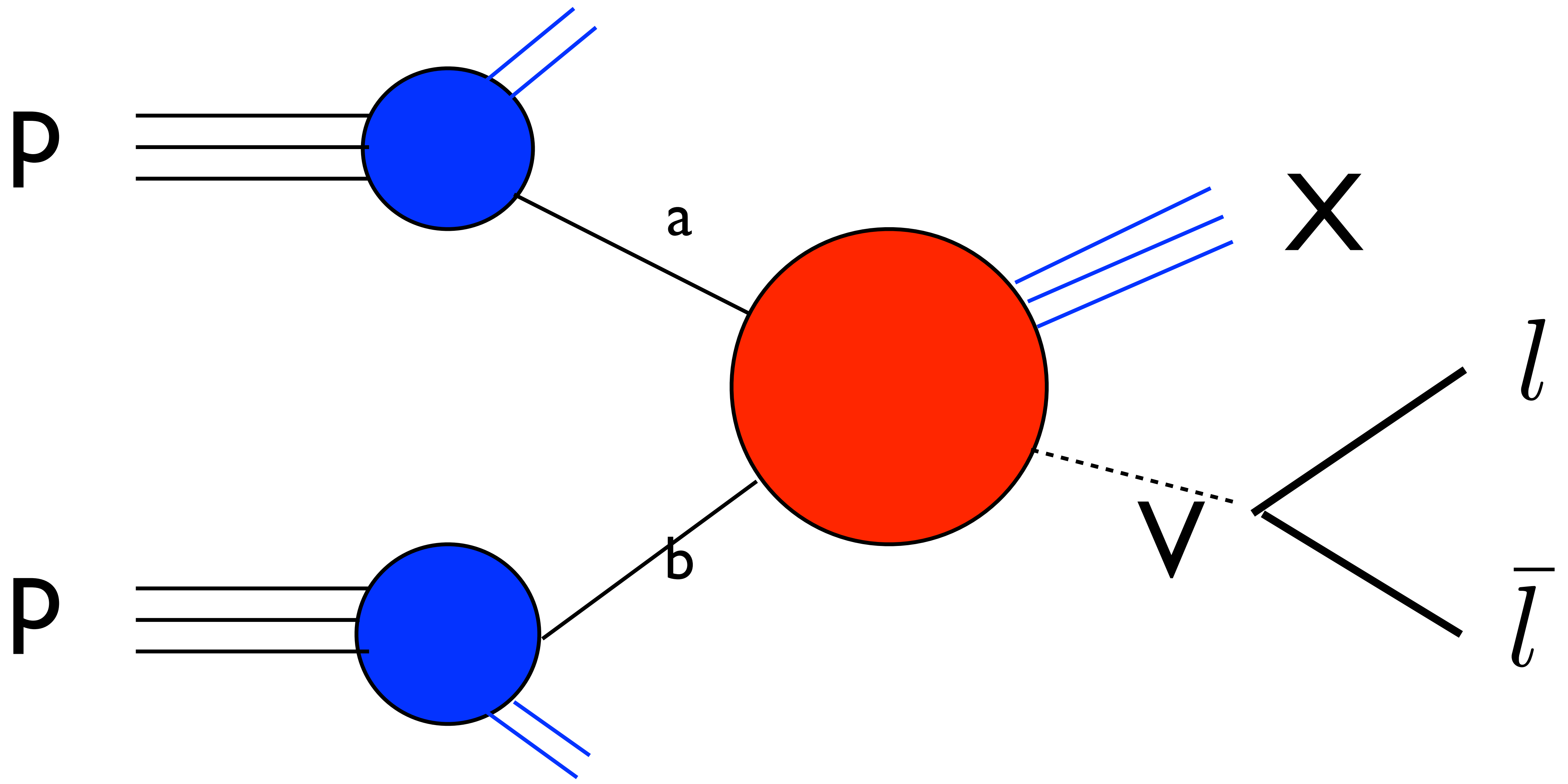
based on: R.Bonciani, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, arXiv:2106.11953

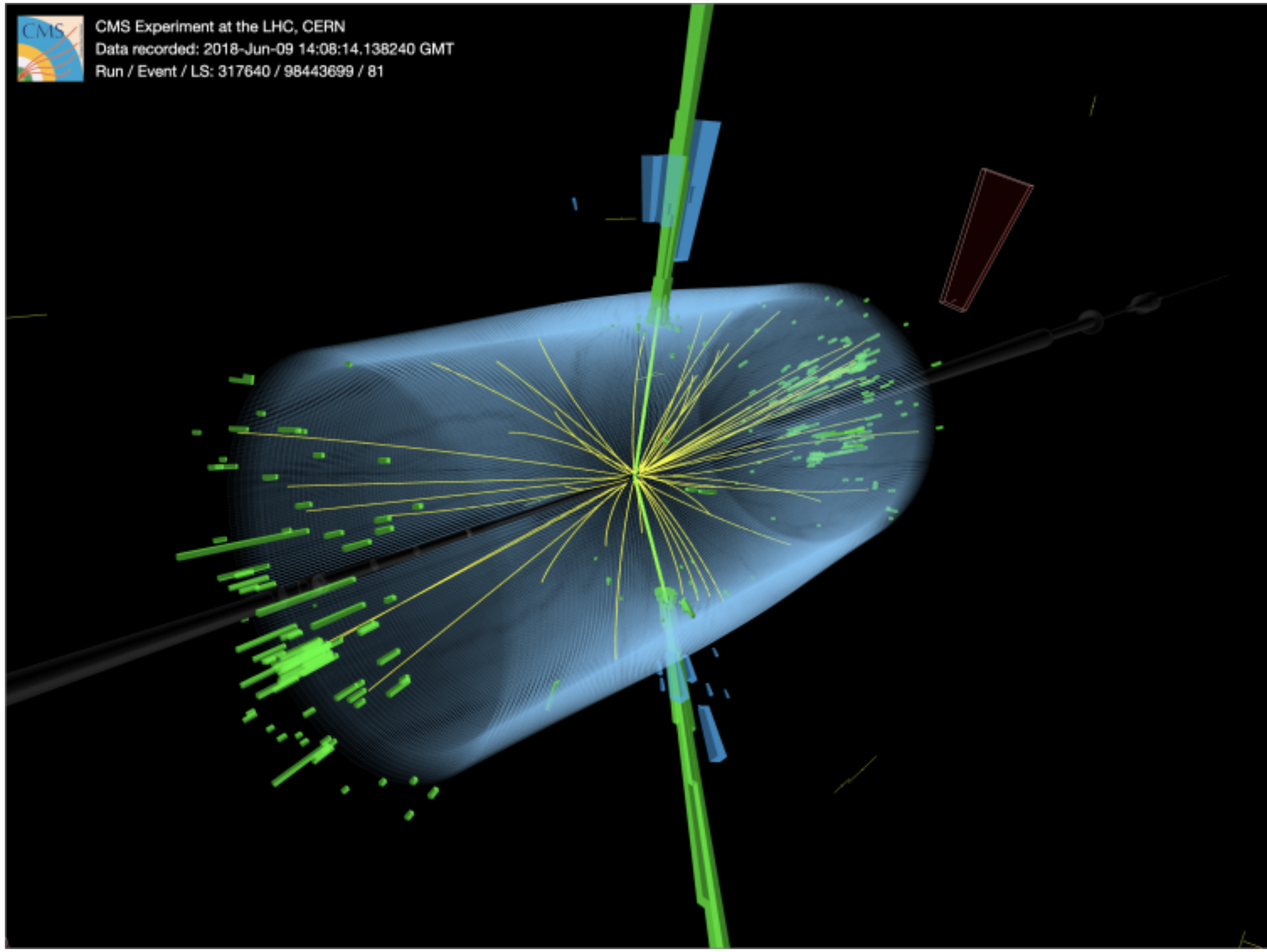
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV: 2201.01754, 2205.03345, 2405.00612

Outline of the talk

- Precision Physics with the Drell-Yan processes at hadron colliders
- Precision predictions for the Drell-Yan processes in the Standard Model

Lepton-pair Drell-Yan production at hadron colliders

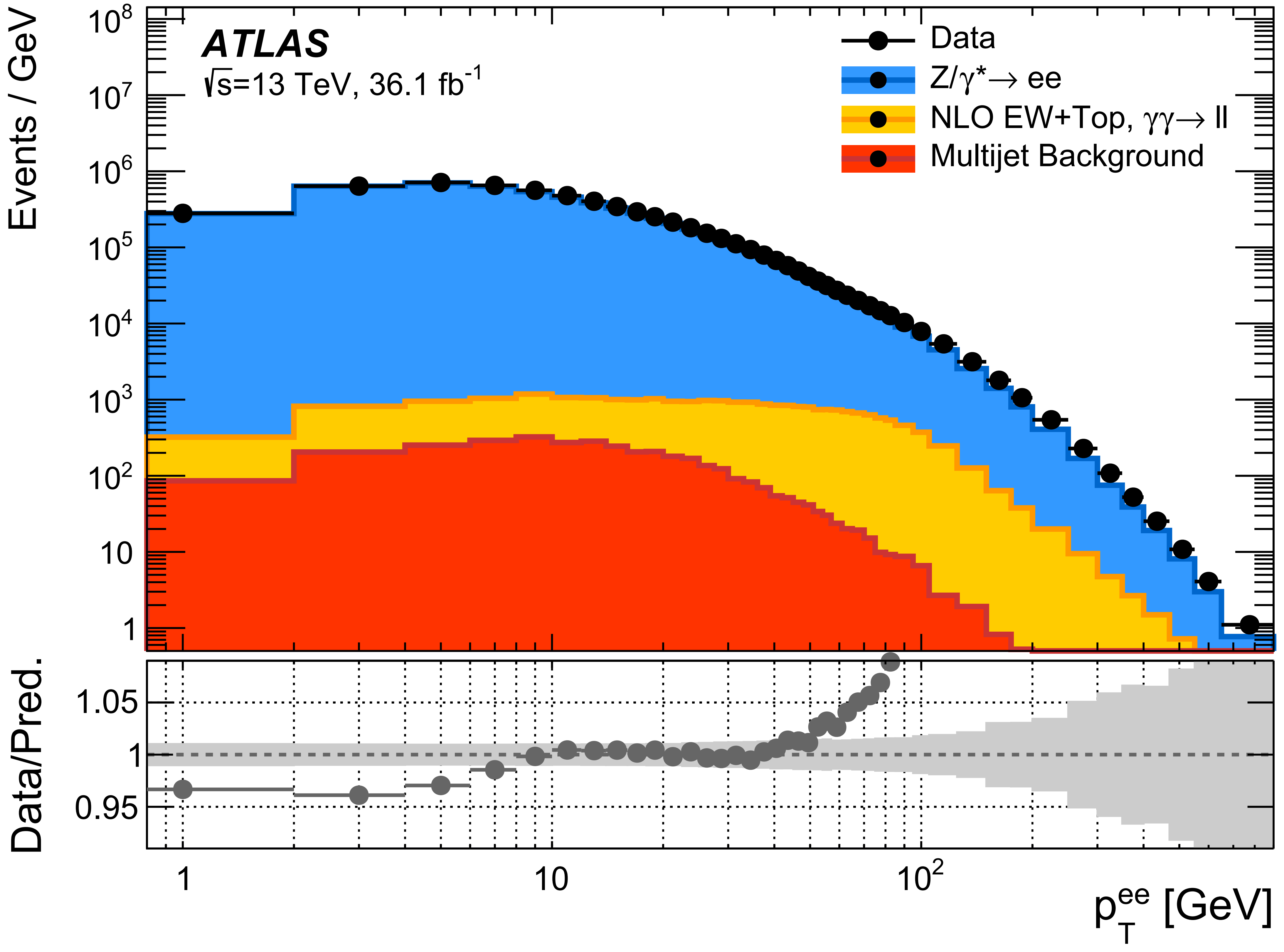


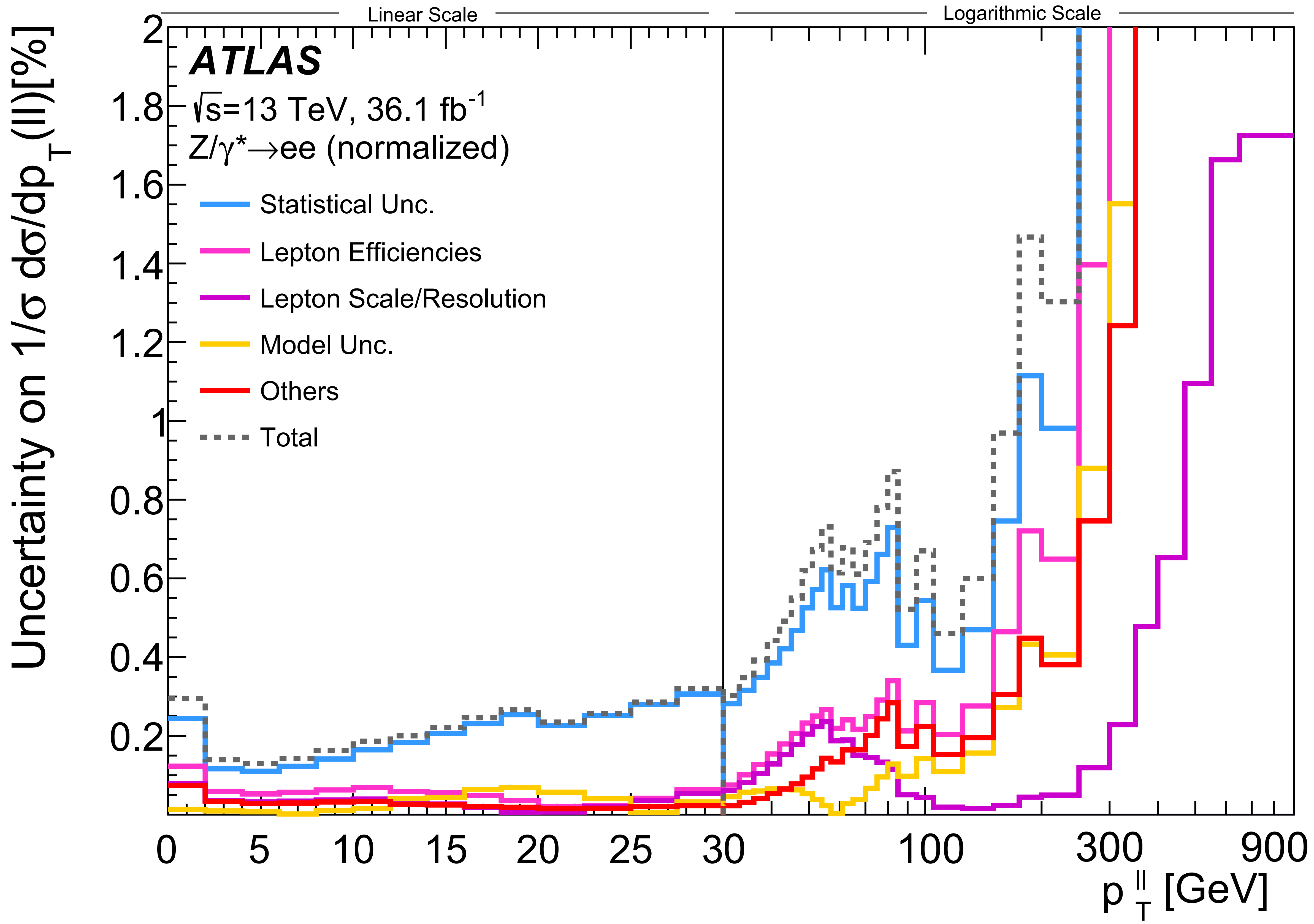


- Test of perturbative QCD
- Determination of the proton structure
- Discovery of W and Z bosons (1983)
- High-precision determination of W and Z properties
- Background to New Physics searches

Lepton-pair transverse momentum distribution

- A crucial role in QCD tests and precision EW measurements (m_W in particular) is played by the $p_\perp^{\ell^+\ell^-}$ distribution
- The impressive experimental precision is a formidable test of the theory predictions, QCD in first place
- At per mille level higher-order QCD resummation matched with fixed order corrections
  non-perturbative QCD effects and heavy quarks corrections — are relevant
  EW corrections

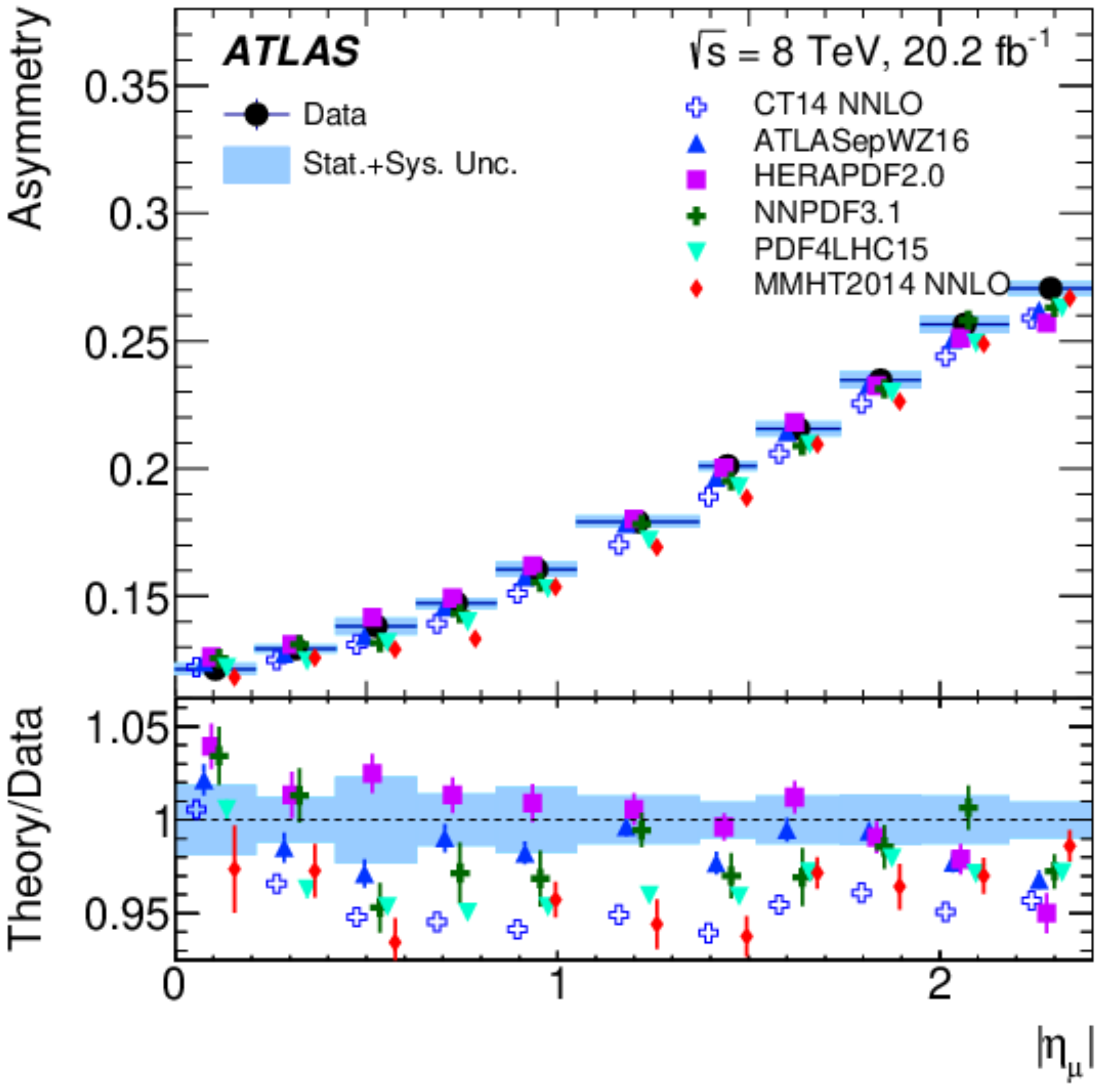


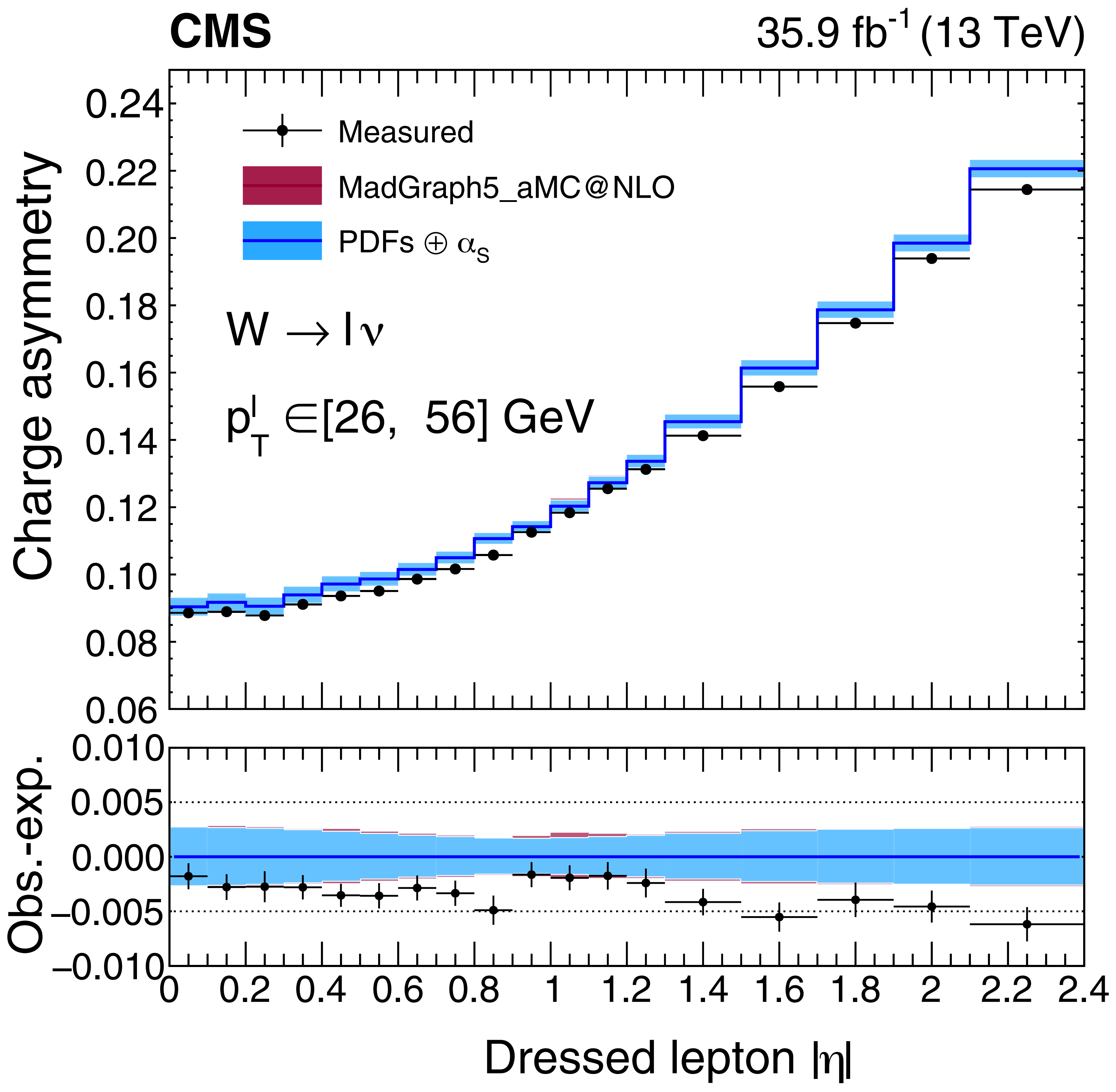


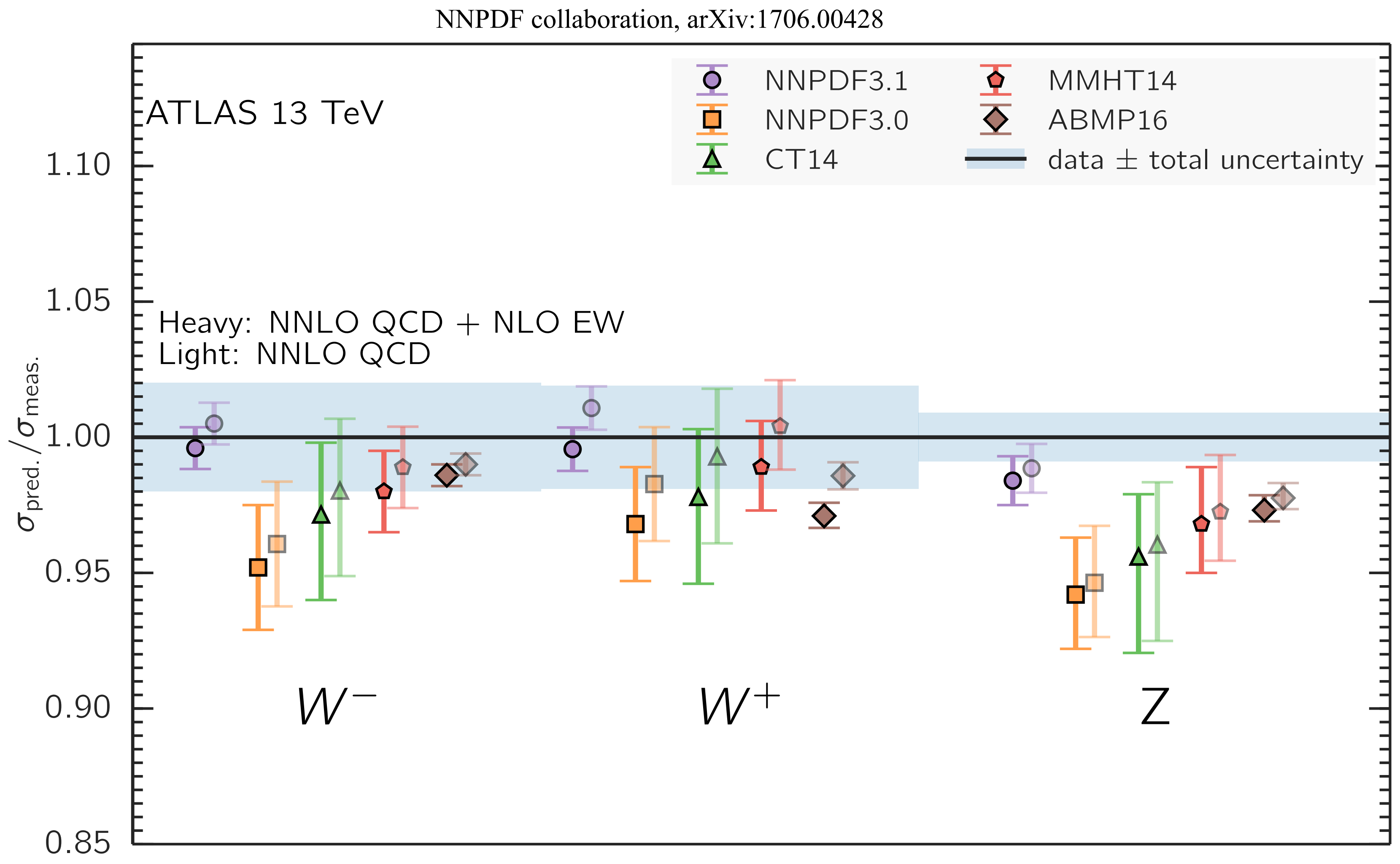
At CERN the EW WG has a subgroup scrutinising the predictions of this observable by different collaborations

Charge asymmetry in charged-current Drell-Yan

- An important role in the determination of the proton structure is played by the charge-asymmetry rapidity distribution
  - ▻ needed to improve the flavour separation
  - ▻ precise results at parton level for this quantity make its contribution to the PDF fit more significant
    - → importance of NNLO and N3LO calculations
  - ▻ in a fiducial volume the rapidity and transverse momentum dependencies are connected by kinematics
    - → impact on the m_W determination



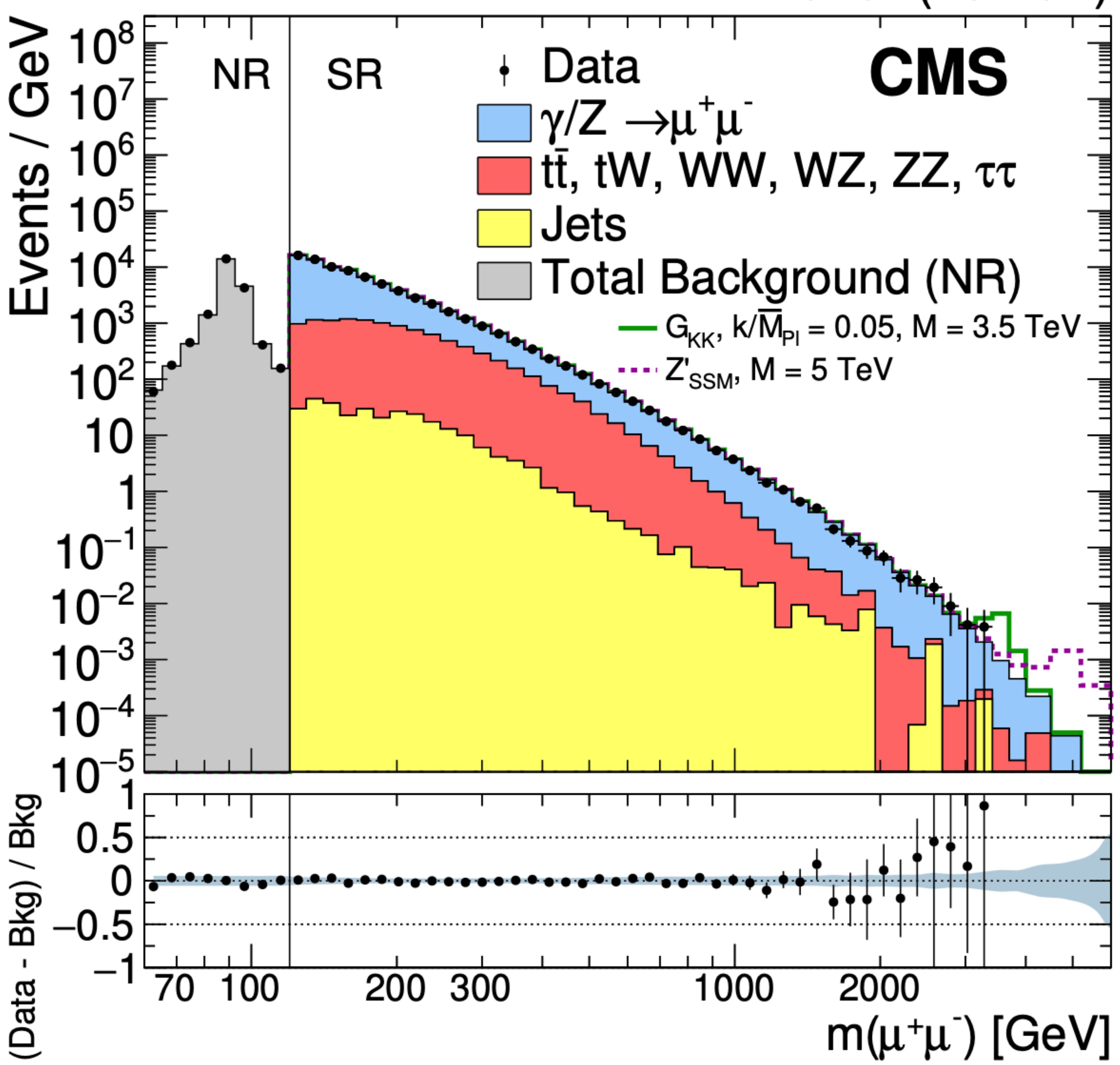




on-shell gauge boson production as a PDF benchmark

Relevance of Neutral Current Drell-Yan measurements: searches for New Physics signals

arXiv:2103.02708



mass window [GeV]	stat. unc. $140fb^{-1}$	stat. unc. $3ab^{-1}$
$600<m_{\mu\mu}<900$	1.4%	0.2%
$900<m_{\mu\mu}<1300$	3.2%	0.6%

At the end of High-Luminosity LHC we will be able to test the TeV region with data at per mille level
i.e.
to test the SM at the level of its quantum corrections

Is the SM prediction under control at the O(0.5%) level in the TeV region of the $m_{\ell\ell}$ distribution ?

Do we precisely know what is the SM,
so that we can significantly claim to observe a discrepancy ?

Testing the Standard Model with the W-boson mass

The W boson mass can be predicted

in terms of the input parameters of the model,

including the quantum effects Standard Model or beyond

$$m_W^2 = \frac{m_Z^2}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha}{G_\mu \sqrt{2}\, m_Z^2}\,(1+\Delta r)}\right)$$

Testing the Standard Model with the W-boson mass

The W boson mass can be predicted

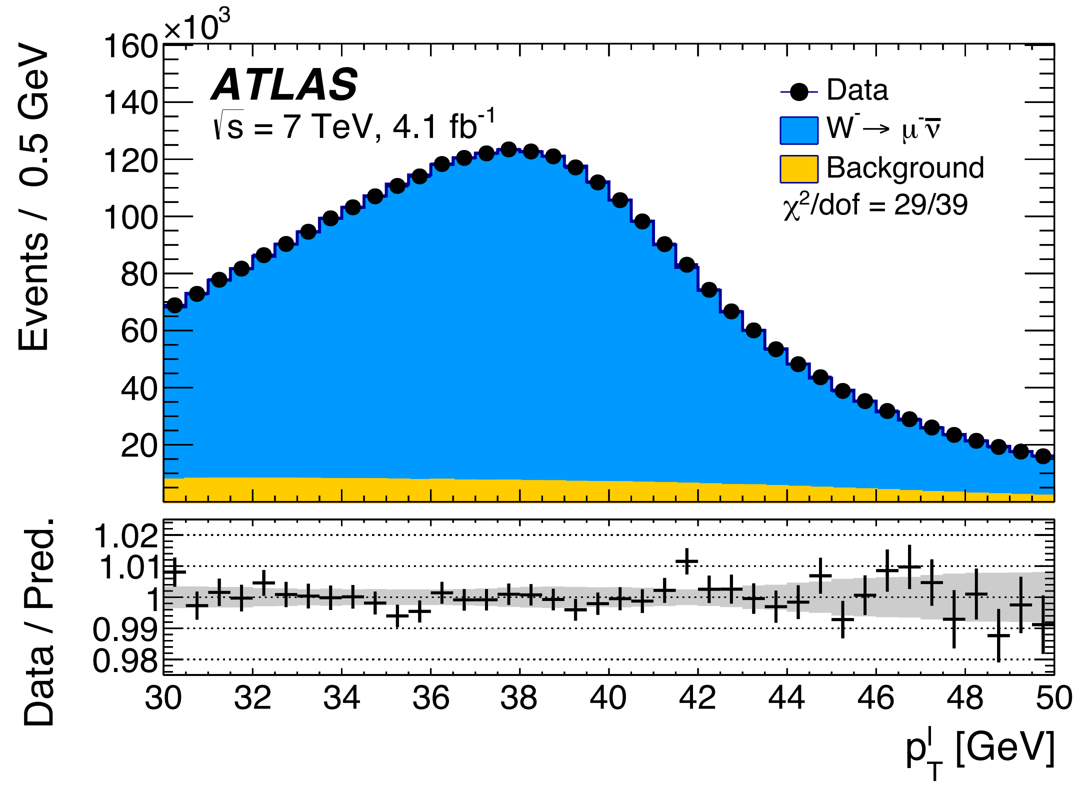
in terms of the input parameters of the model,

including the quantum effects Standard Model or beyond

$$m_W^2 = \frac{m_Z^2}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha}{G_\mu \sqrt{2}\, m_Z^2}(1+\Delta r)}\right)$$

The W boson mass can be determined from the data

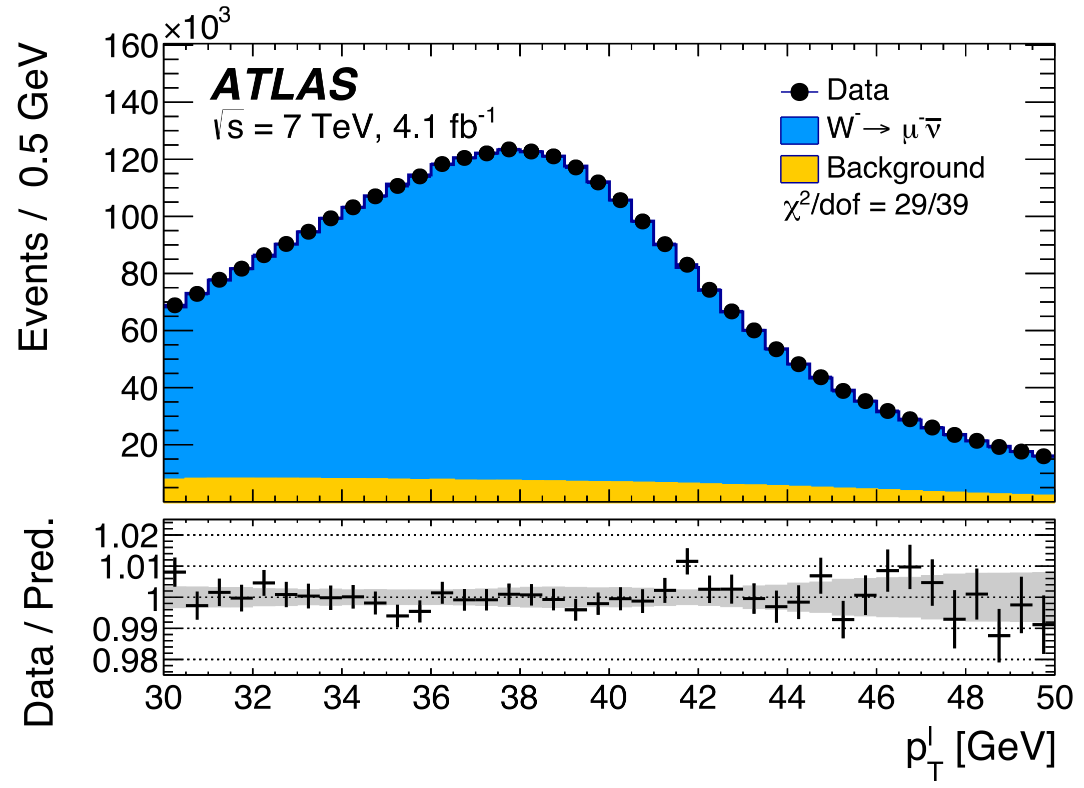
fitting the kinematic distributions of charged-current Drell-Yan



Testing the Standard Model with the W-boson mass

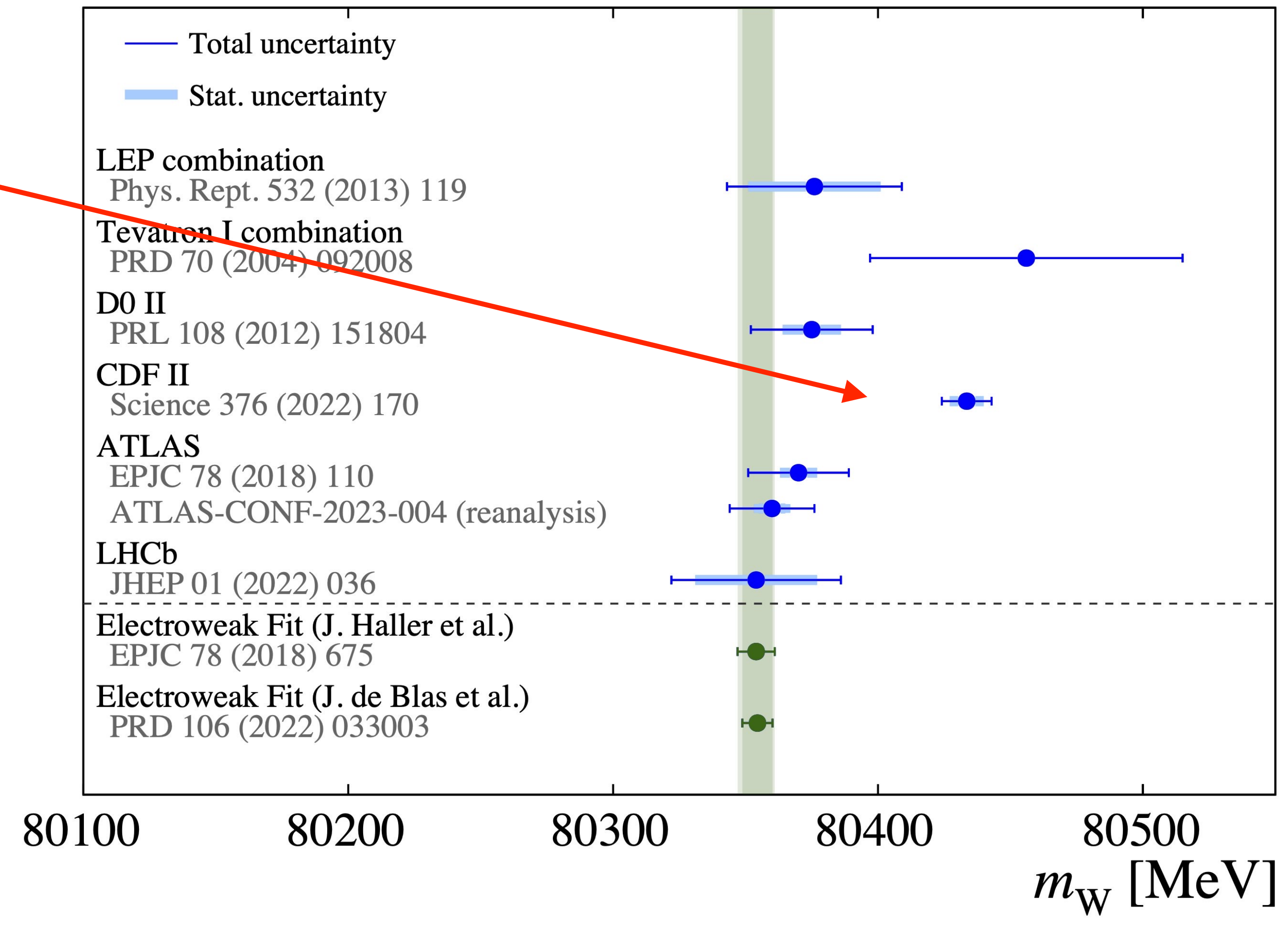
The W boson mass can be predicted
in terms of the input parameters of the model,
including the quantum effects Standard Model or beyond

$$m_W^2 = \frac{m_Z^2}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha}{G_\mu\sqrt{2}\, m_Z^2}(1+\Delta r)}\right)$$



The W boson mass can be determined from the data
fitting the kinematic distributions of charged-current Drell-Yan

A discrepancy between the Standard Model and experimental values
may hint about the presence of New Physics:
new BSM particles contributing to Δr could explain the difference



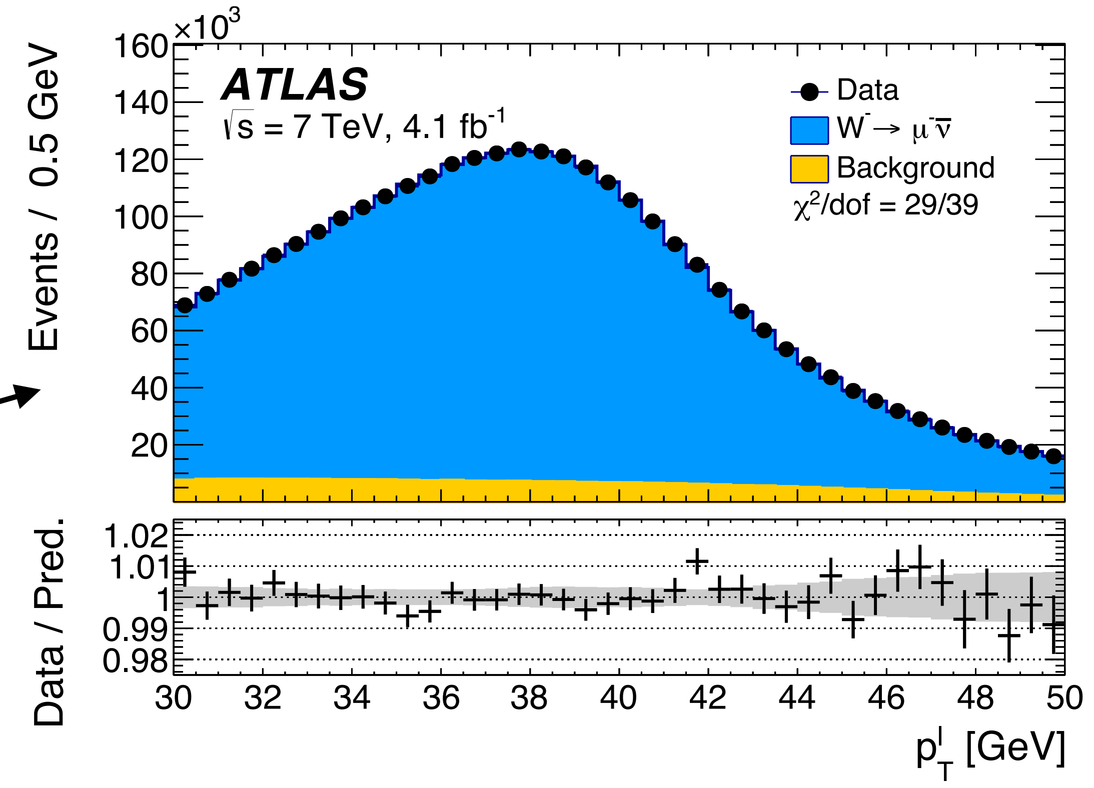
Testing the Standard Model with the W-boson mass

The W boson mass can be predicted
in terms of the input parameters of the model,
including the quantum effects Standard Model or beyond

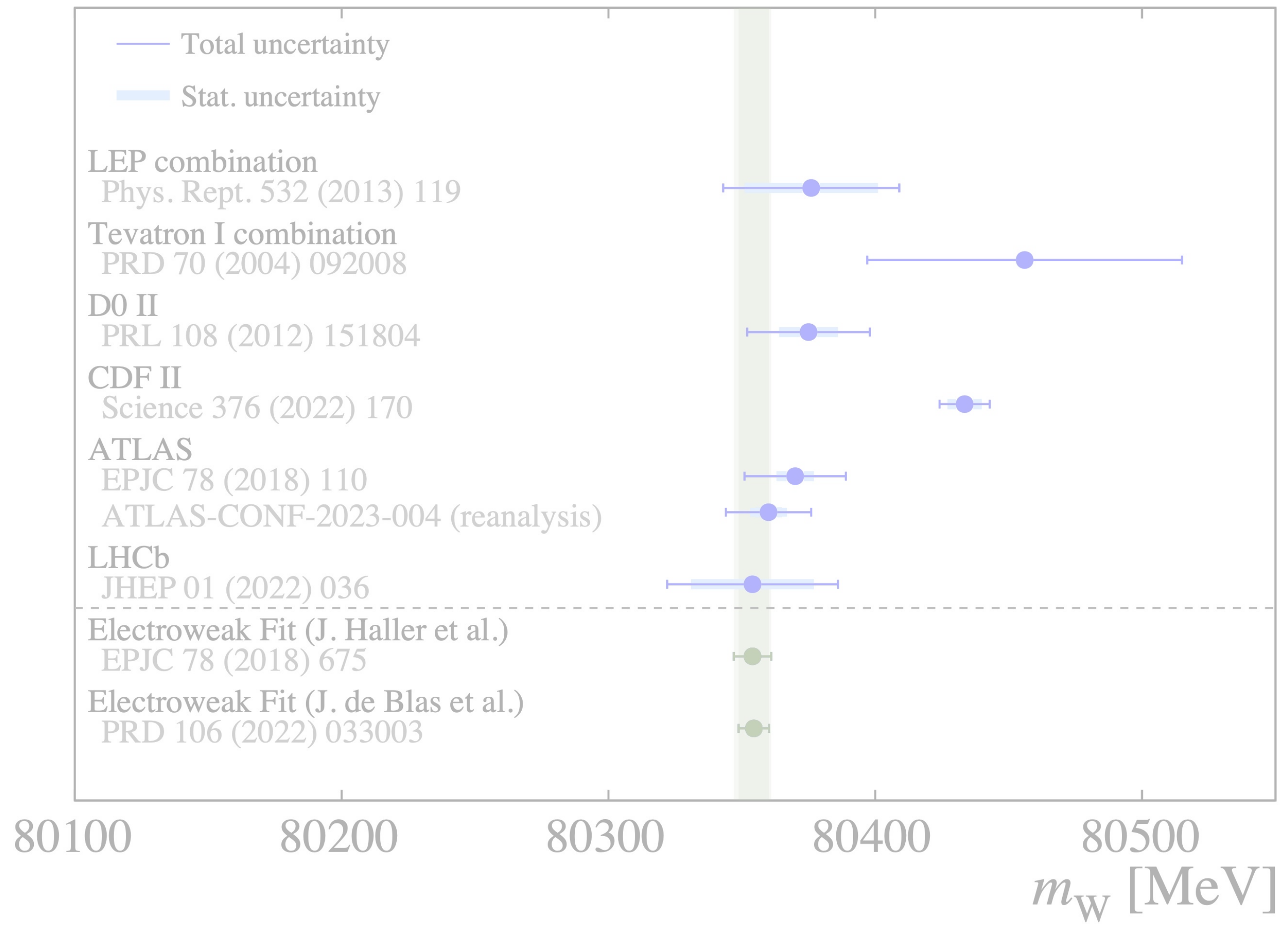
$$m_W^2 = \frac{m_Z^2}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha}{G_\mu \sqrt{2}\, m_Z^2}(1+\Delta r)}\right)$$

The W boson mass can be determined from the data
fitting the kinematic distributions of charged-current Drell-Yan

Challenging theoretical calculations are needed for both:
the theoretical predictions and the distributions used to fit the data



A discrepancy between the Standard Model and experimental values
may hint about the presence of New Physics:
new particles contributing to Δr could explain the difference



The W boson mass: theoretical prediction

Sirlin, 1980, 1984; Marciano, Sirlin, 1980, 1981;
van der Bij, Veltman, 1984; Barbieri, Ciafaloni, Strumia 1993;
Djouadi, Verzegnassi 1987; Consoli, Hollik, Jegerlehner, 1989;
Chetyrkin, Kühn, Steinhauser, 1995;
Barbieri, Beccaria, Ciafaloni, Curci, Viceré, 1992, 1993; Fleischer, Tarasov, Jegerlehner, 1993;
Degrassi, Gambino, AV, 1996; Degrassi, Gambino, Sirlin, 1997;
Freitas, Hollik, Walter, Weiglein, 2000, 2003;
Awramik, Czakon, 2002; Awramik, Czakon, Onishchenko, Veretin, 2003; Onishchenko, Veretin, 2003

The best available prediction includes
the full 2-loop EW result, leading higher-order EW and QCD corrections,
resummation of reducible terms
Missing 3-loop and 4-loop terms needed to reduce the uncertainties.

$$m_W = w_0 + w_1 dH + w_2 dH^2 + w_3 dh + w_4 dt + w_5 dH dt + w_6 da_s + w_7 da^{(5)}$$

$$dt = [(M_t/173.34\,\mathrm{GeV})^2 - 1]$$

$$da^{(5)} = [\Delta\alpha^{(5)}_{\mathrm{had}}(m_Z^2)/0.02750 - 1]$$

$$dH = \ln\left(\frac{m_H}{125.15\,\mathrm{GeV}}\right)$$

$$dh = [(m_H/125.15\ \mathrm{GeV})^2 - 1].$$

$$da_s = \left(\frac{\alpha_s(m_Z)}{0.1184} - 1\right)$$

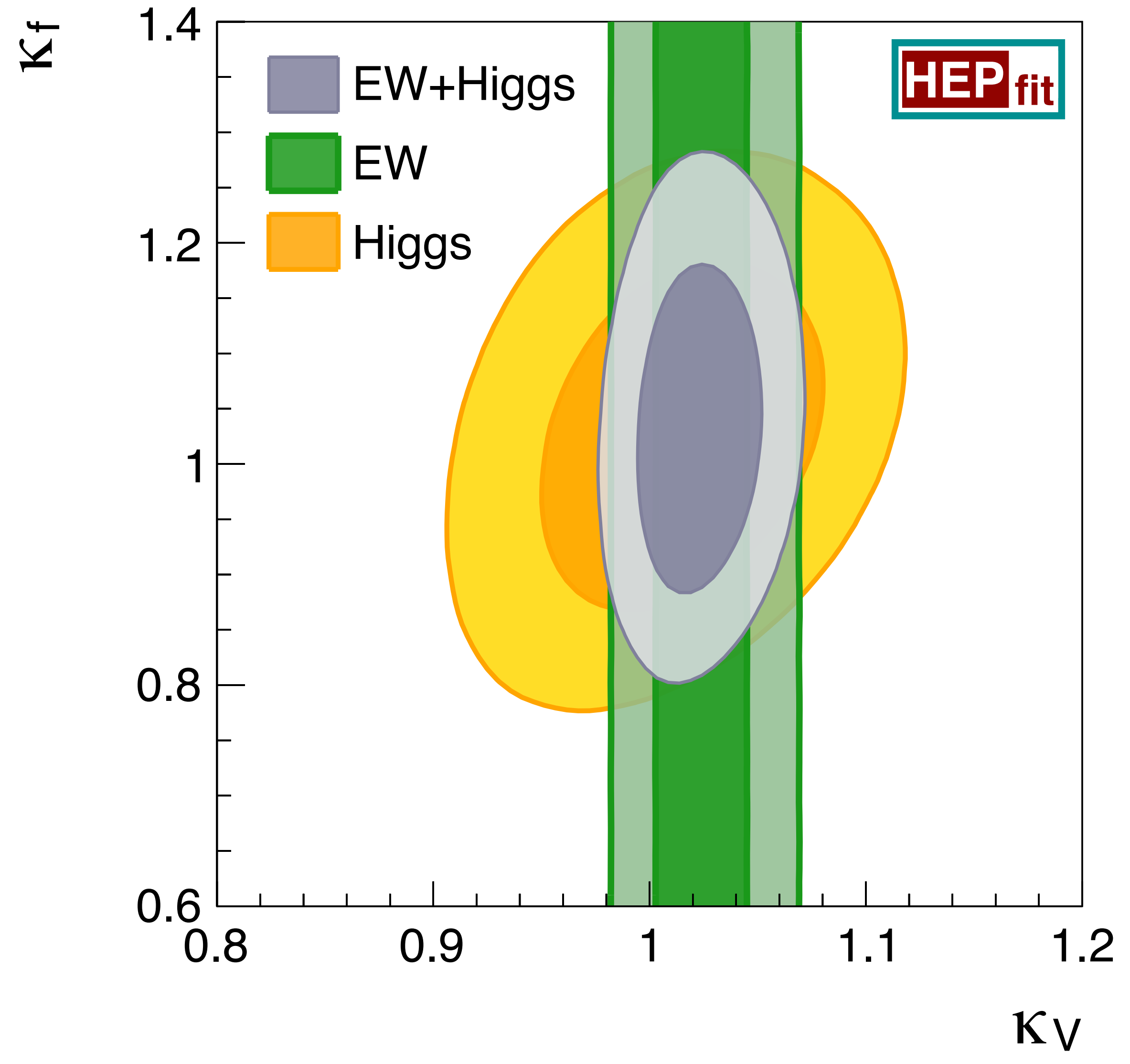
	$124.42 \le m_H \le 125.87$ GeV	$50 \le m_H \le 450$ GeV
w_0	80.35712	80.35714
w_1	-0.06017	-0.06094
w_2	0.0	-0.00971
w_3	0.0	0.00028
w_4	0.52749	0.52655
w_5	-0.00613	-0.00646
w_6	-0.08178	-0.08199
w_7	-0.50530	-0.50259

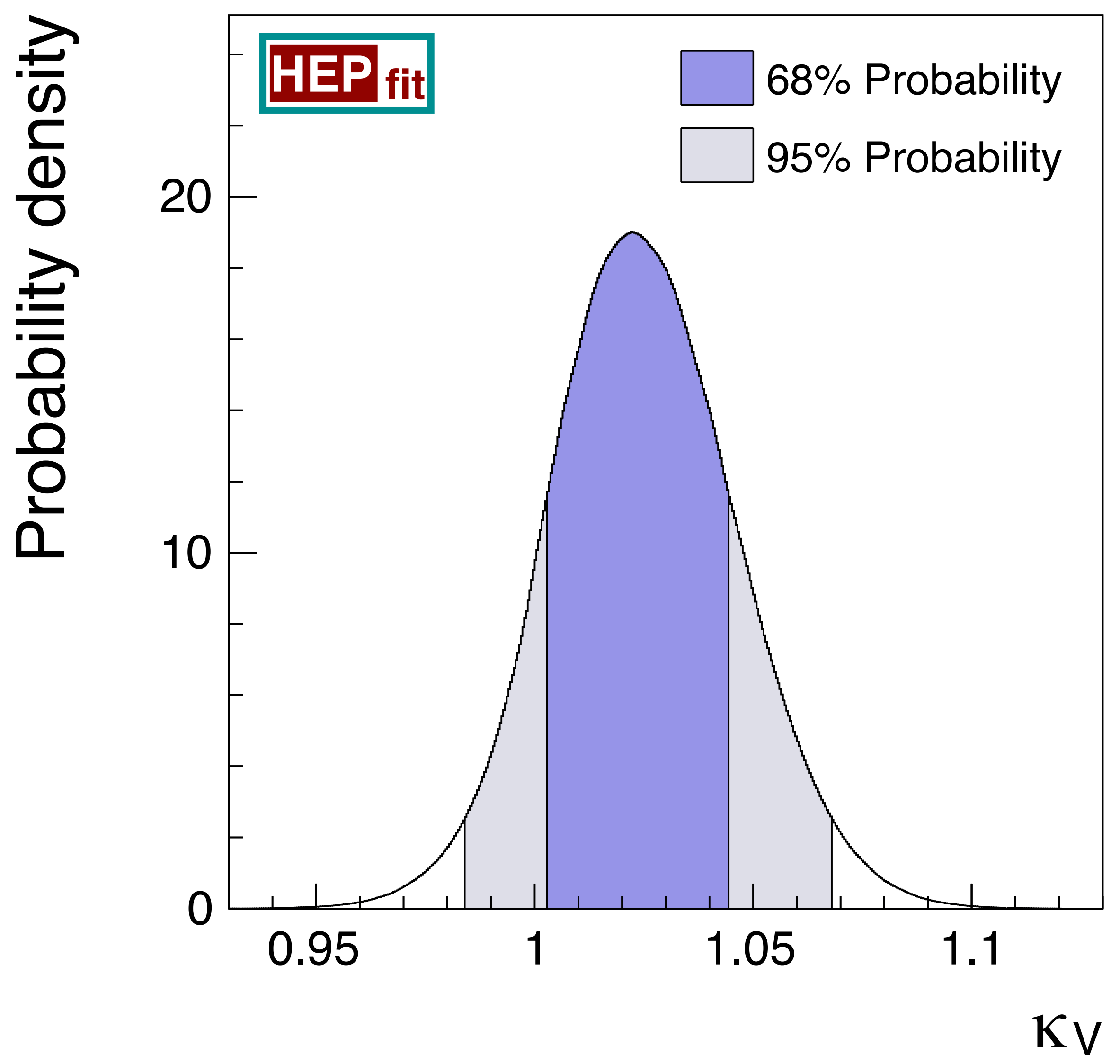
on-shell scheme $m_W^{os} = 80.353 \pm 0.004$ GeV (Freitas, Hollik, Walter, Weiglein)
MSbar scheme. $m_W^{\overline{MS}} = 80.351 \pm 0.003$ GeV (Degrassi, Gambino, Giardino)

parametric uncertainties $\delta m_W^{par} = \pm 0.005$ GeV due to the $(\alpha, G_\mu, m_Z, m_H, m_t)$ values

Relevance of new high-precision measurement of EW parameters

de Blas et al, arXiv:1608.01509





$$\mathcal{L}_{\rm Eff} = \sum_{d=4}^{\infty} \frac{1}{\Lambda^{d-4}} \mathcal{L}_d = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \cdots$$

$$\mathcal{L}_d = \sum_i C_i^d \mathcal{O}_i \qquad [\mathcal{O}_i] = d \longrightarrow \left(\frac{q}{\Lambda}\right)^{d-4}$$

Effects suppressed by $q = v, E < \Lambda$

Λ: Cut-off of the EFT

$$\mathcal{O}_{\phi WB} = \phi^\dagger \sigma_a \phi B^{\mu\nu} W^a_{\mu\nu}$$

EWSB → $v^2 B^{\mu\nu} W^3_{\mu\nu}$ gauge boson masses

EWSB → $vhB^{\mu\nu} W^3_{\mu\nu}$ h→ZZ, γ γ

$$M_W^2 = M_Z^2 c^2 \left[1 - \frac{c^2}{c^2 - s^2} \left(\frac{1}{2} C_{\phi D} + 2\frac{s}{c} C_{\phi WB} + \frac{s^2}{c^2} \Delta_{G_\mu} \right) \frac{v^2}{\Lambda^2} \right]$$

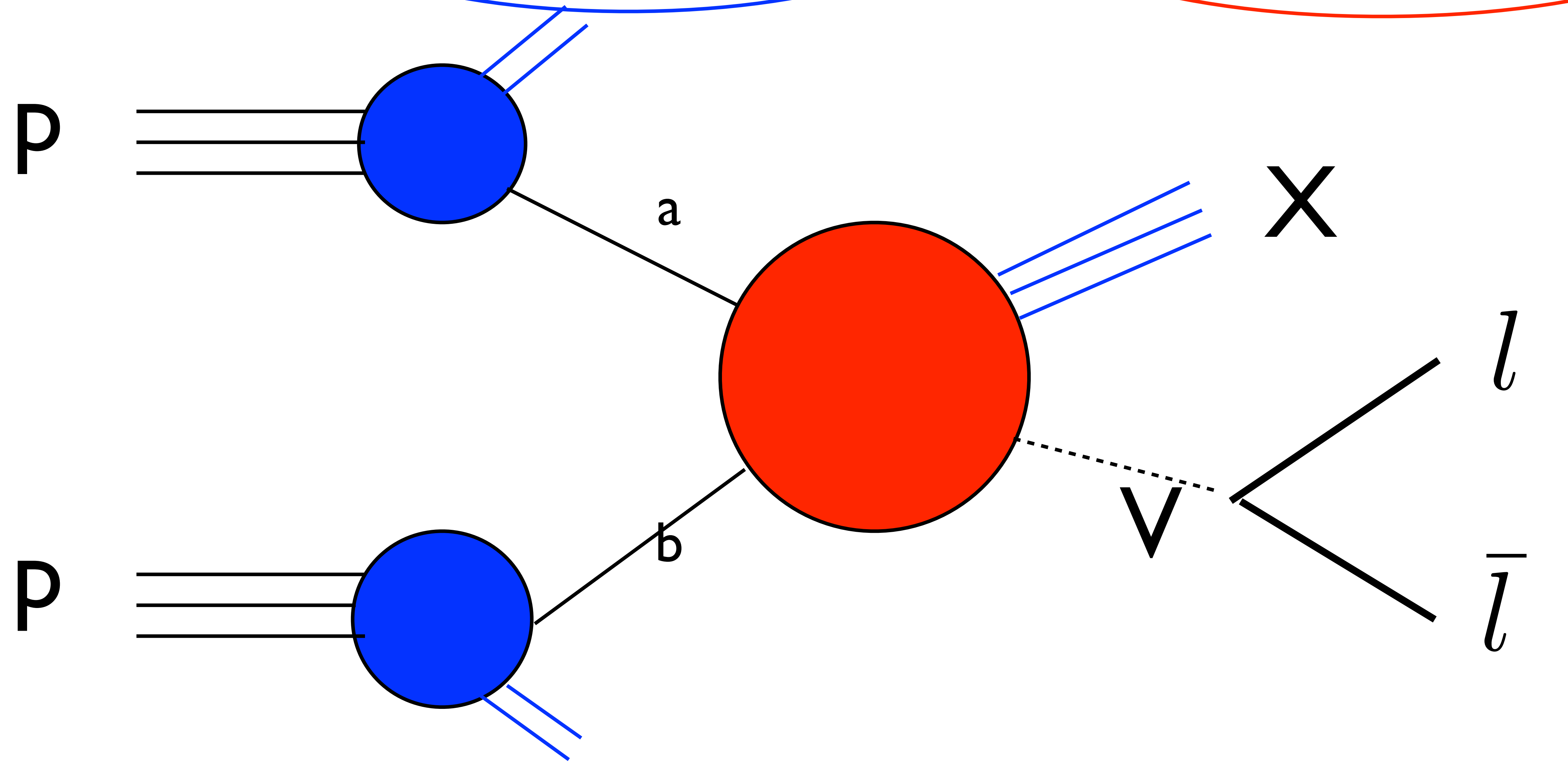
A precise measurement of m_W and $\sin^2\theta_{eff}$ constrains several dim-6 operators contributing to Higgs and gauge interaction vertices.

Today still one of the strongest constraints

Theoretical predictions for the Drell-Yan processes

Lepton-pair Drell-Yan production at hadron colliders

$$\sigma(P_1, P_2; m_V) = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_{h_1,a}(x_1, M_F) f_{h_2,b}(x_2, M_F) \, \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2, \alpha_s(\mu), M_F)$$



The factorisation theorems guarantee the validity of the above picture up to power correction effects

The interplay of QCD and EW interactions appears both in the partonic cross section and in the proton PDFs

The Drell-Yan cross section in a fixed-order expansion

Drell-Yan (1970)

Baur, Brein, Hollik, Schappacher, Wackeroth (2001)

$$\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} + \alpha_s\,\sigma^{(1,0)} + \alpha\,\sigma^{(0,1)} + \alpha_s^2\,\sigma^{(2,0)} + \alpha\,\alpha_s\,\sigma^{(1,1)} + \alpha^2\,\sigma^{(0,2)} + \alpha_s^3\,\sigma^{(3,0)} + \ldots$$

Altarelli, Ellis, Martinelli (1979)

Hamberg, Matsuura, van Nerveen, (1991)
Anastasiou, Dixon, Melnikov, Petriello, (2003)
Catani, Cieri, Ferrera, de Florian, Grazzini (2009)

C.Duhr, B.Mistlberger, arXiv:2111.10379

still missing
Sudakov high-energy approximations

Neutral Current

R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, (2021)
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, (2022)
F.Buccioni, F.Caola, H.Chawdhry, F.Devoto, M.Heller, A.von Manteuffel, K.Melnikov, R.Röntsch, C.Signorile-Signorile, (2022)

New!!! Charged-current 2-loop amplitude

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, (2024)

The Drell-Yan cross section in a fixed-order expansion

Drell-Yan (1970)

Baur, Brein, Hollik, Schappacher, Wackeroth (2001)

$$\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} +$$
$$\alpha_s\,\sigma^{(1,0)} + \alpha\,\sigma^{(0,1)} +$$
$$\alpha_s^2\,\sigma^{(2,0)} + \alpha\,\alpha_s\,\sigma^{(1,1)} + \alpha^2\,\sigma^{(0,2)} +$$
$$\alpha_s^3\,\sigma^{(3,0)} + \ldots$$

Altarelli, Ellis, Martinelli (1979)

Hamberg, Matsuura, van Nerveen, (1991)
Anastasiou, Dixon, Melnikov, Petriello, (2003)
Catani, Cieri, Ferrera, de Florian, Grazzini (2009)

C.Duhr, B.Mistlberger, arXiv:2111.10379

still missing
Sudakov high-energy approximations

Neutral Current

R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, (2021)
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, (2022)
F.Buccioni, F.Caola, H.Chawdhry, F.Devoto, M.Heller, A.von Manteuffel, K.Melnikov, R.Röntsch, C.Signorile-Signorile, (2022)

New!!! Charged-current 2-loop amplitude

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, (2024)

The resummation of QCD and QED corrections is another crucial topic → see P.Torrielli's talk

Mixed QCD-EW corrections to the Drell-Yan processes

Strong boost of the activities in the theory community in the last 4 years! (references not covering the Monte Carlo developments)

→ mathematical and theoretical developments and computation of universal building blocks

- 2-loop virtual Master Integrals with internal masses

U. Aglietti, R. Bonciani, arXiv:0304028, arXiv:0401193, R. Bonciani, S. Di Vita, P. Mastrolia, U. Schubert, arXiv:1604.08581, M.Heller, A.von Manteuffel, R.Schabinger arXiv:1907.00491, S.Hasan, U.Schubert, arXiv:2004.14908, M.Long,R,Zhang,W.Ma,Y,Jiang,L.Han,,Z.Li,S.Wang, arXiv:2111.14130

- New methods to solve the Master Integrals

M.Hidding, arXiv:2006,05510, D.X.Liu, Y.-Q. Ma, arXiv:2201.11669, T.Armadillo, R.Bonciani, S.Devoto, N.Rana,AV, arXiv: 2205.03345

- Altarelli-Parisi splitting functions including QCD-QED effects

D. de Florian, G. Sborlini, G. Rodrigo, arXiv:1512.00612

- renormalization

G.Degrassi, AV, hep-ph/0307122, S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229 S.Dittmaier, arXiv:2101.05154

→ on-shell Z and W production as a first step towards full Drell-Yan

- pole approximation of the NNLO QCD-EW corrections

S.Dittmaier, A.Huss, C.Schwinn, arXiv:1403.3216, 1511.08016, 2401.15682

- analytical total cross section including NNLO QCD-QED and NNLO QED corrections

D. de Florian, M.Der, I.Fabre, arXiv:1805.12214

- ptZ distribution including QCD-QED analytical transverse momentum resummation

L. Cieri, G. Ferrera, G. Sborlini, arXiv:1805.11948

- fully differential on-shell Z production including exact NNLO QCD-QED corrections

M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:1909.08428

- total Z production cross section in fully analytical form including exact NNLO QCD-EW corrections

R. Bonciani, F. Buccioni, R.Mondini, AV, arXiv:1611.00645, R. Bonciani, F. Buccioni, N.Rana, I.Triscari, AV, arXiv:1911.06200, R. Bonciani, F. Buccioni, N.Rana, AV, arXiv:2007.06518, arXiv:2111.12694

- fully differential on-shell Z and W production including exact NNLO QCD-EW corrections

F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2005.10221, A. Behring, F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2009.10386, 2103.02671,

Mixed QCD-EW corrections to the Drell-Yan processes

Strong boost of the activities in the theory community in the last 4 years! (references not covering the Monte Carlo developments)

→ complete Drell-Yan

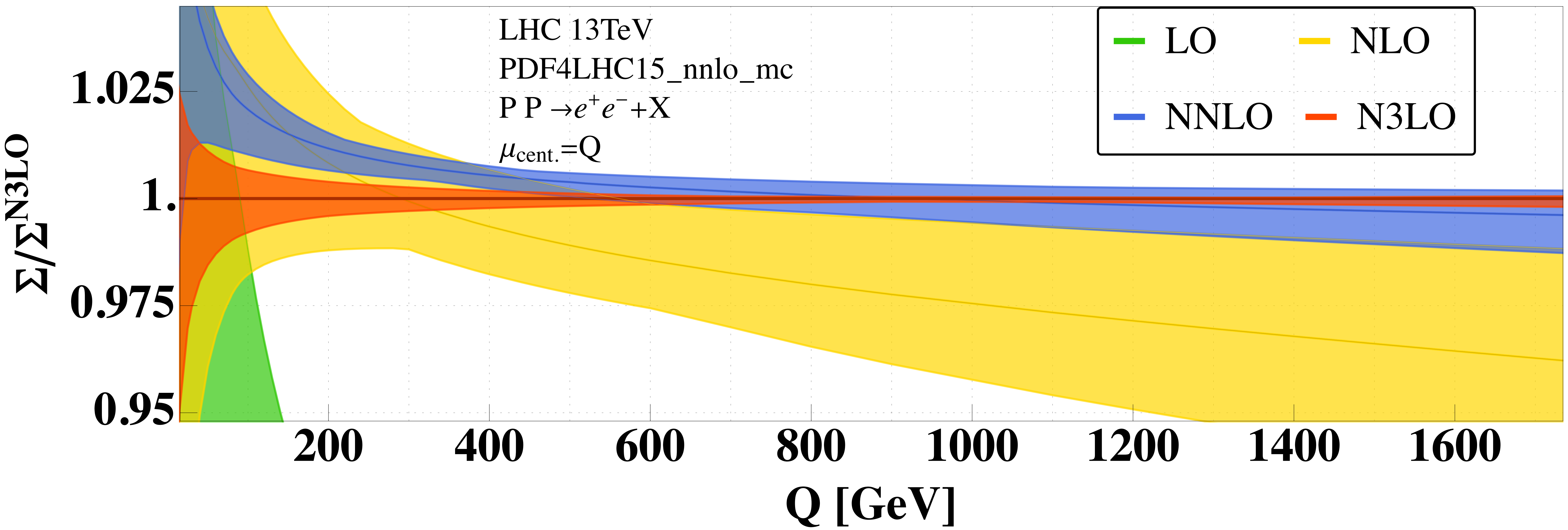
- neutrino-pair production including NNLO QCD-QED corrections
  L. Cieri, D. de Florian, M.Der, J.Mazzitelli, arXiv:2005.01315
- 2-loop NC and CC amplitudes
  M.Heller, A.von Manteuffel, R.Schabinger, arXiv:2012.05918 , T.Armadillo, R.Bonciani, S.Devoto, N.Rana,AV, arXiv: 2201.01754, 2405.00612
- NNLO QCD-EW corrections to charged-current DY (2-loop contributions in pole approximation).
  L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, arXiv:2102.12539
- NNLO QCD-EW corrections to neutral-current DY
  R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, N.Rana, F.Tramontano, AV, arXiv:2102.12539, F. Buccioni, F. Caola, H.A.Chawdhry, F.Devoto, M.Heller, A.V.Manteuffel, K.Melnikov, R.Roentsch, C.Signorile-Signorile, arXiv:2203.11237

→ mixed QCD-QED resummation

- initial-state corrections
  L. Cieri,G.Ferrera, G.Sborlini,, arXiv:1805.11948, A.Autieri, L. Cieri,G.Ferrera, G.Sborlini,, arXiv:2302.05403
- initial and final state corrections
  L.Buonocore, L'Rottoli, P.Torrielli, arXiv:2404.15112

QCD results: lepton-pair invariant mass

C.Duhr, B.Mistlberger, arXiv:2111.10379



Thanks to the N3LO-QCD results for the Drell-Yan cross section, scale variation band at the few per mille level at any Q

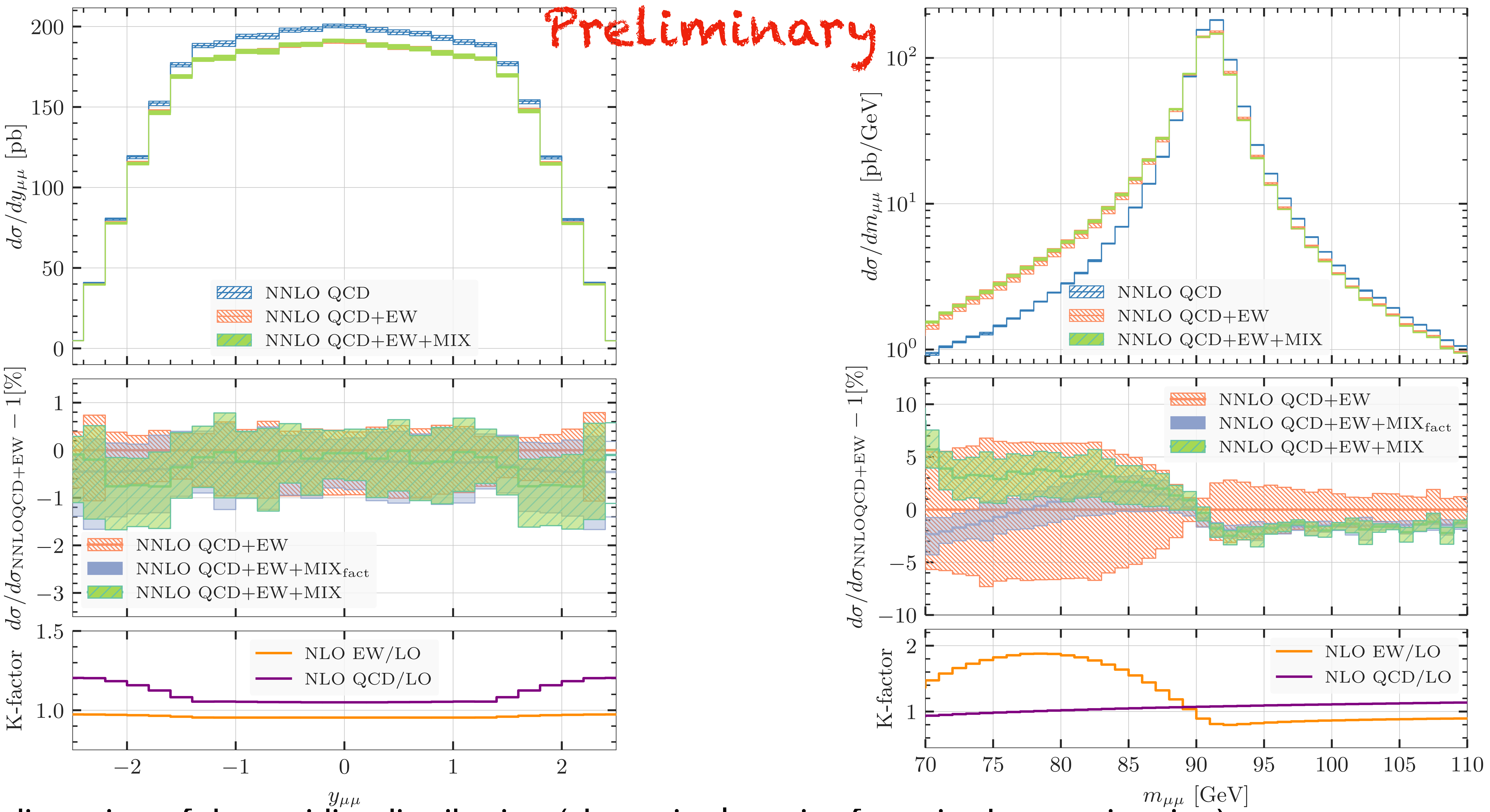
The PDFs are not yet at N3LO

This is promising, in view of the program of searches for deviation from the SM in the TeV range

What about NNLO QCD-EW and NNLO-EW corrections ?

Phenomenology of Neutral Current Drell-Yan including exact NNLO QCD-EW corrections

R.Bonciani, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, arXiv:2106.11953 , Phys.Rev.Lett. 128 (2022) 1, 012002 and work in preparation



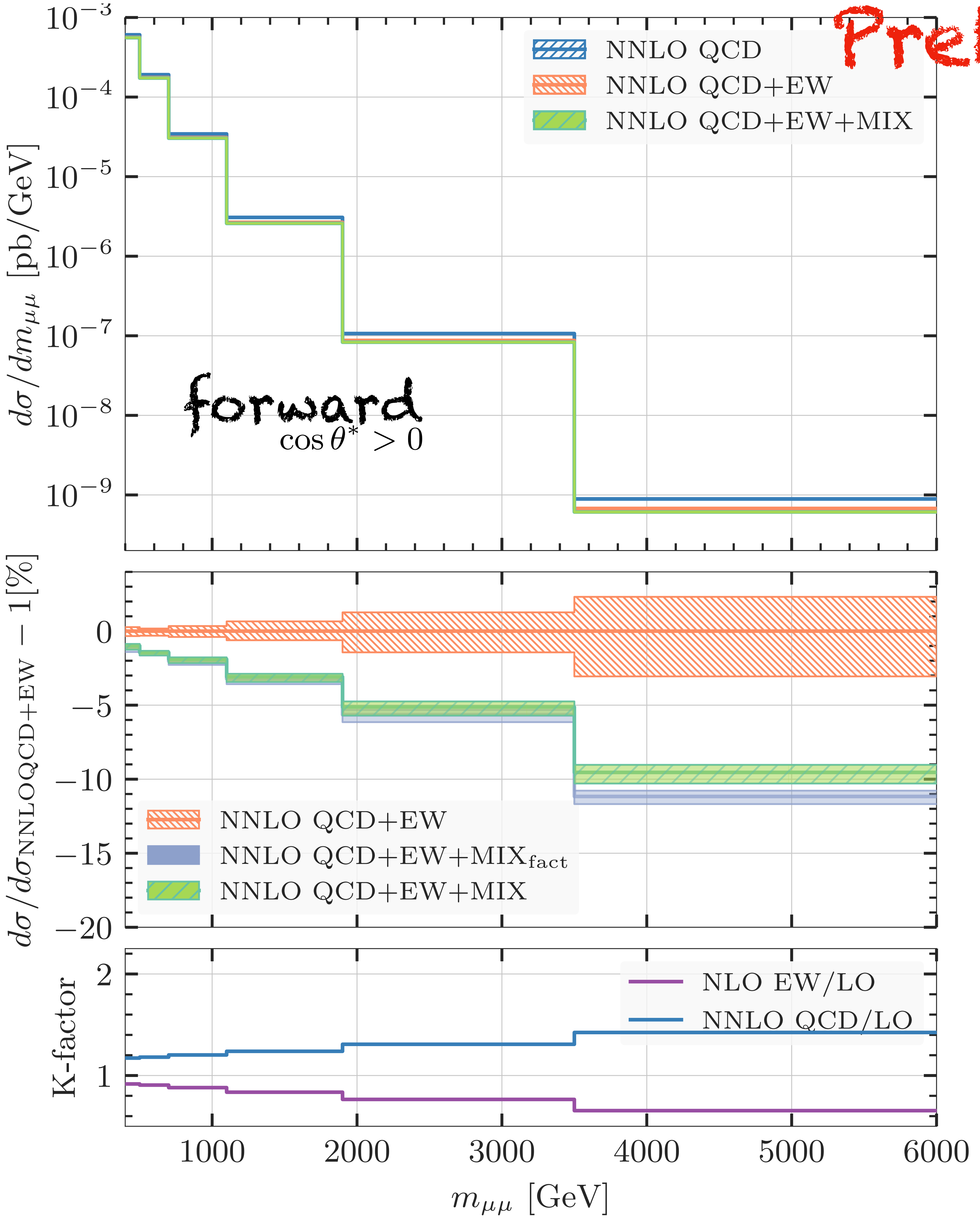
Non-trivial distortion of the rapidity distribution (absent in the naive factorised approximation)

Large effects below the Z resonance (the factorised approximation fails) → impact on the $\sin^2\theta_{eff}$ determination

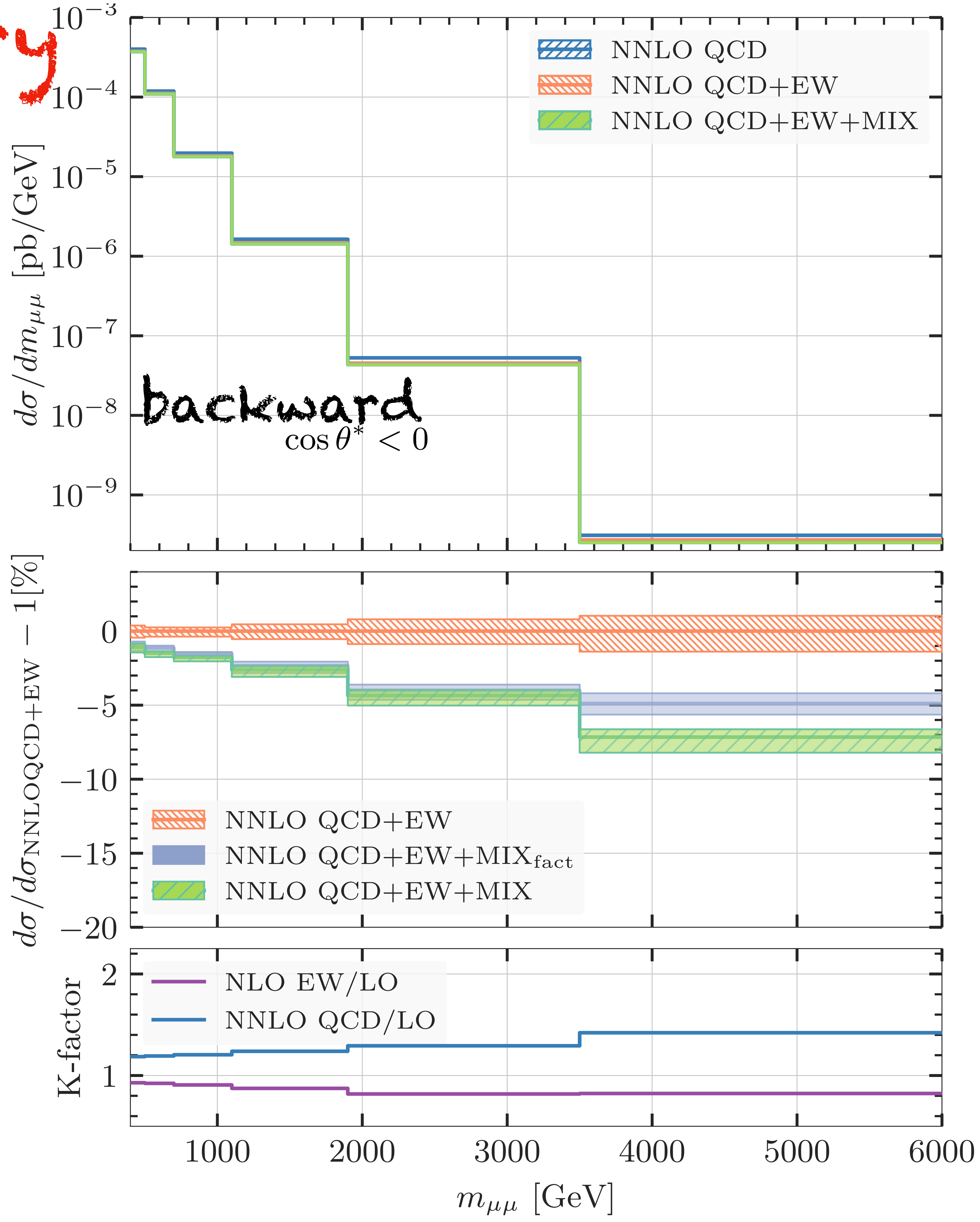
O(-1.5%) effects above the resonance → ongoing precision studies in the CERN EW WG

Phenomenology of Neutral Current Drell-Yan including exact NNLO QCD-EW corrections

R.Bonciani, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, arXiv:2106.11953 , Phys.Rev.Lett. 128 (2022) 1, 012002 and work in preparation



Preliminary



Negative mixed NNLO QCD-EW effects (-3% or more) at large invariant masses,

absent in any additive combination → impact on the searches for new physics

Charged Current Drell-Yan: NNLO QCD-EW results with approximated 2-loop virtual corrections

L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, arXiv:2102.12539

Exact LO, NLO (QCD+EW), NNLO QCD corrections
are combined with mixed QCD-EW corrections

Partonic subprocesses with 1 and 2 additional partons
are evaluated exactly at NLO and LO respectively

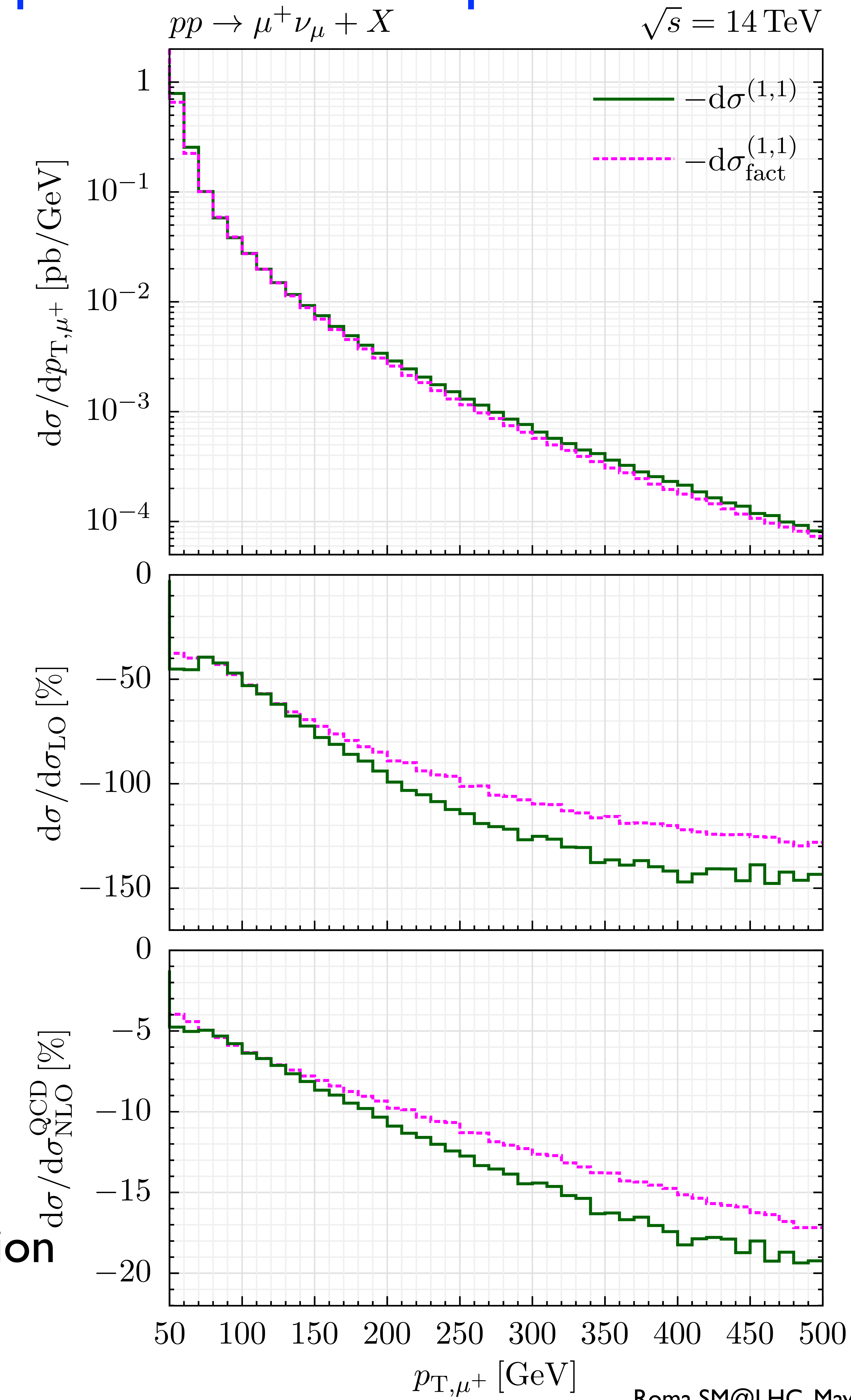
The 2-loop virtual corrections to $q\bar{q}' \to \ell\nu_\ell$ treated in pole approximation

Accurate description of the charged lepton $p_\perp^\ell$ spectrum,
dominated by the (exact) real radiation effects
resonant configurations

The factorisation of QCD and EW corrections is not accurate at large $p_\perp^\ell$

The lepton-pair transverse mass might receive large non-negligible
2-loop virtual corrections at large mass, poorly described in pole approximation

→ new results !



Evaluation of the exact NNLO QCD-EW corrections

Neutral-Current Drell-Yan

The Neutral Current Drell-Yan cross section in the SM: perturbative expansion

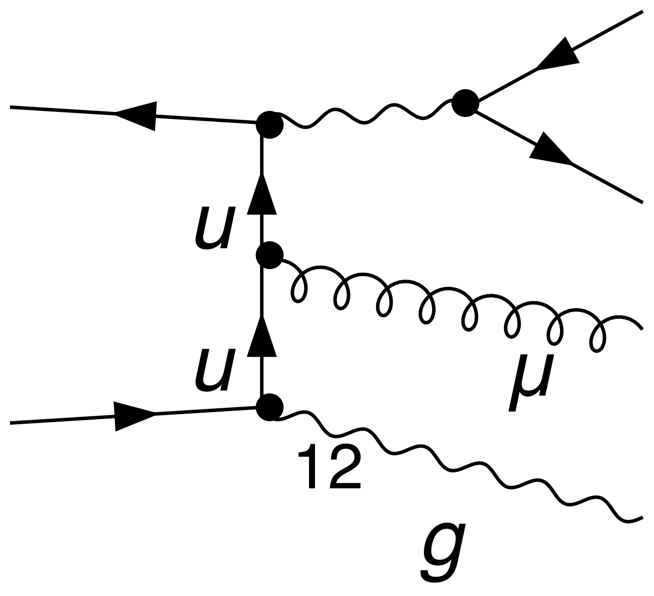
$$
\begin{aligned}
\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} + \\
\alpha_s\,\sigma^{(1,0)} + \alpha\,\sigma^{(0,1)} + \\
\alpha_s^2\,\sigma^{(2,0)} + \alpha\,\alpha_s\,\sigma^{(1,1)} + \alpha^2\,\sigma^{(0,2)} + \\
\alpha_s^3\,\sigma^{(3,0)} + \ldots
\end{aligned}
$$

$$
\sigma(h_1 h_2 \to l\bar{l} + X) = \sum_{i,j=q\bar{q},g,\gamma} \int dx_1\, dx_2\, f_i^{h_1}(x_1, \mu_F) f_j^{h_2}(x_2, \mu_F)\, \hat{\sigma}(ij \to l\bar{l} + X)
$$

$\sigma^{(1,1)}$ requires the evaluation of the xsecs of the following processes, including photon-induced

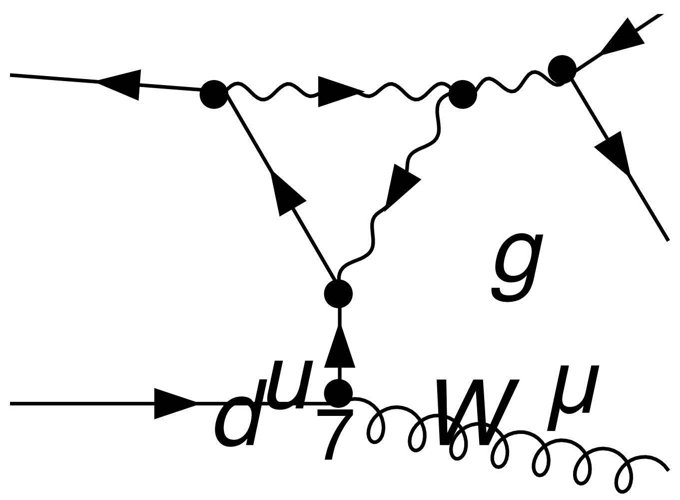
0 additional partons	$q\bar{q} \to l\bar{l},\ \gamma\gamma \to l\bar{l}$	including virtual corrections of $\mathcal{O}(\alpha_s), \mathcal{O}(\alpha), \mathcal{O}(\alpha\alpha_s)$
1 additional parton	$q\bar{q} \to l\bar{l}g,\ qg \to l\bar{l}q$	including virtual corrections of $\mathcal{O}(\alpha)$
	$q\bar{q} \to l\bar{l}\gamma,\ q\gamma \to l\bar{l}q$	including virtual corrections of $\mathcal{O}(\alpha_s)$
2 additional partons	$q\bar{q} \to l\bar{l}g\gamma, qg \to l\bar{l}q\gamma, q\gamma \to l\bar{l}qg, g\gamma \to l\bar{l}q\bar{q}$	
	$q\bar{q} \to l\bar{l}q\bar{q},\ q\bar{q} \to l\bar{l}q'\bar{q}',\ qq' \to l\bar{l}qq',\ q\bar{q}' \to l\bar{l}q\bar{q}',\ qq \to l\bar{l}qq$	at tree level

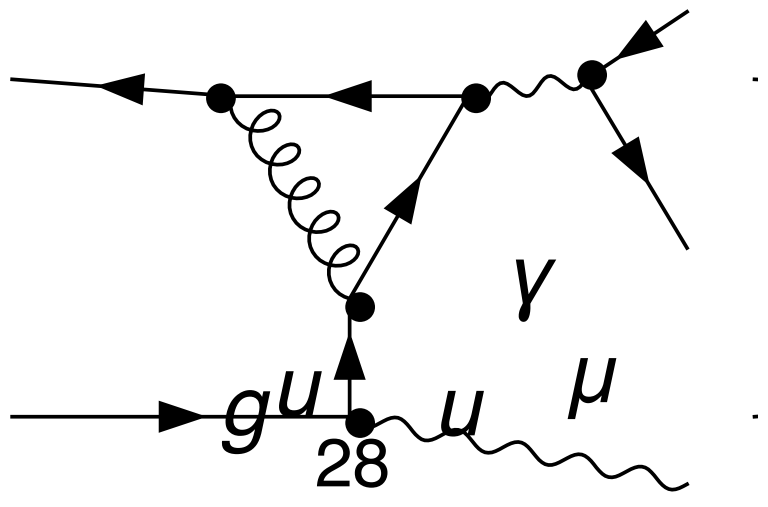
Different kinds of contributions at $\mathcal{O}(\alpha\alpha_s)$ and corresponding problems



double-real contributions

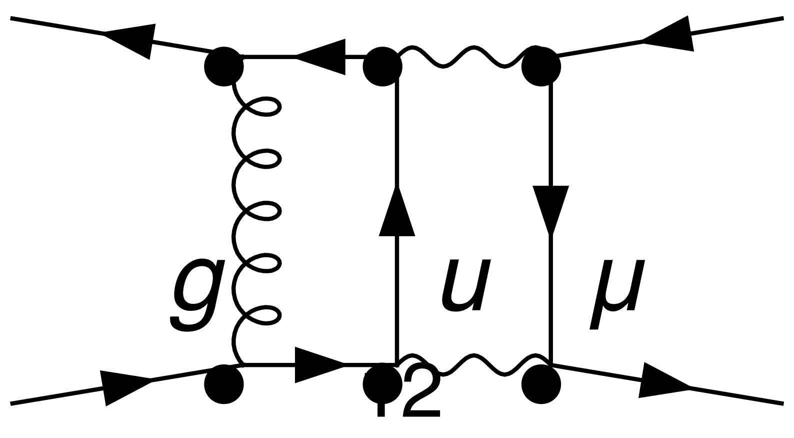
amplitudes are easily generated with OpenLoops
IR subtraction
care about the numerical convergence when aiming at 0.1% precision

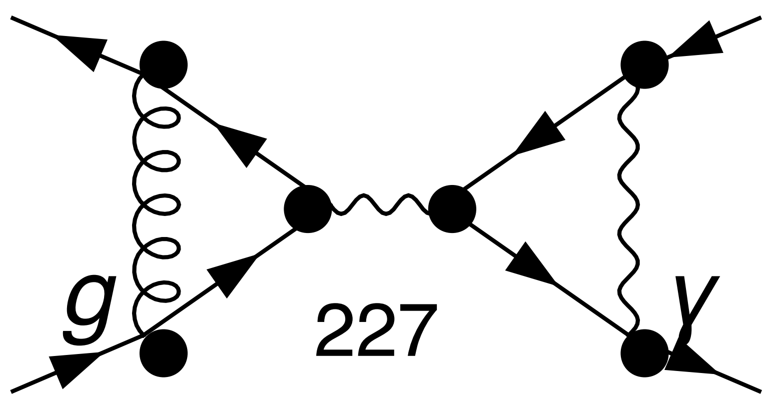




real-virtual contributions

amplitudes are easily generated with OpenLoops or Recola
1-loop UV renormalisation and IR subtraction
care about the numerical convergence when aiming at 0.1% precision





double-virtual contributions

generation of the amplitudes
γ_5 treatment
2-loop UV renormalization
solution and evaluation of the Master Integrals
subtraction of the IR divergences
numerical evaluation of the squared matrix element

General structure of the inclusive cross section and the q_T-subtraction formalism

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

IR structure associated to the QCD-QED part derived from NNLO-QCD results via abelianisation
(de Florian, Rodrigo, Sborlini, 2016, de Florian, Der , Fabre, 2018)

the q_T-subtraction formalism has been extended to the case of final-state emitters (heavy quarks in QCD, leptons in EW)
(Catani, Torre, Grazzini, 2014, Buonocore, Grazzini, Tramontano 2019.)

General structure of the inclusive cross section and the q_T-subtraction formalism

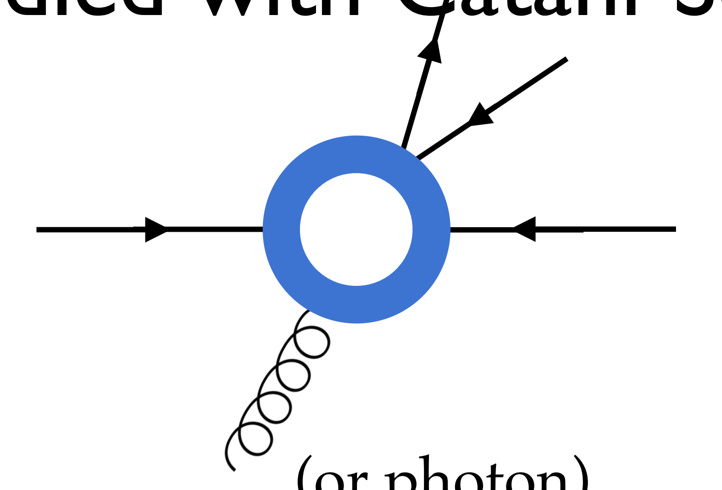
$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

IR structure associated to the QCD-QED part derived from NNLO-QCD results via abelianisation
(de Florian, Rodrigo, Sborlini, 2016, de Florian, Der , Fabre, 2018)

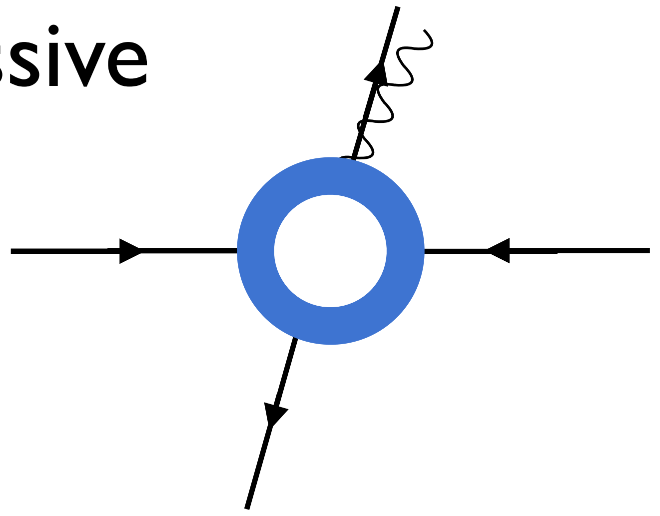
the q_T-subtraction formalism has been extended to the case of final-state emitters (heavy quarks in QCD, leptons in EW)
(Catani, Torre, Grazzini, 2014, Buonocore, Grazzini, Tramontano 2019.)

the gauge-boson phase space is split into $q_T = 0$ and $q_T > 0$ regions $\qquad r_{cut} = q_T^{cut}/Q$

for ISR, if $q_T > 0$ the emitted parton is always resolved
and the process under study receives only NLO corrections
which can be handled with Catani-Seymour dipoles



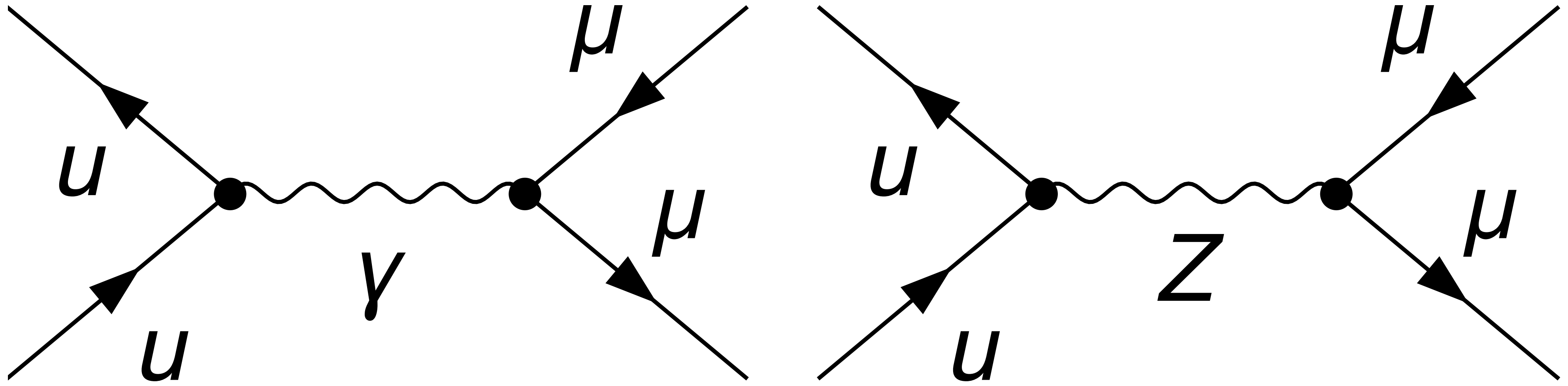
in the FSR case, with $q_T > 0$,
the emitted parton is always resolved
only if the emitter is massive



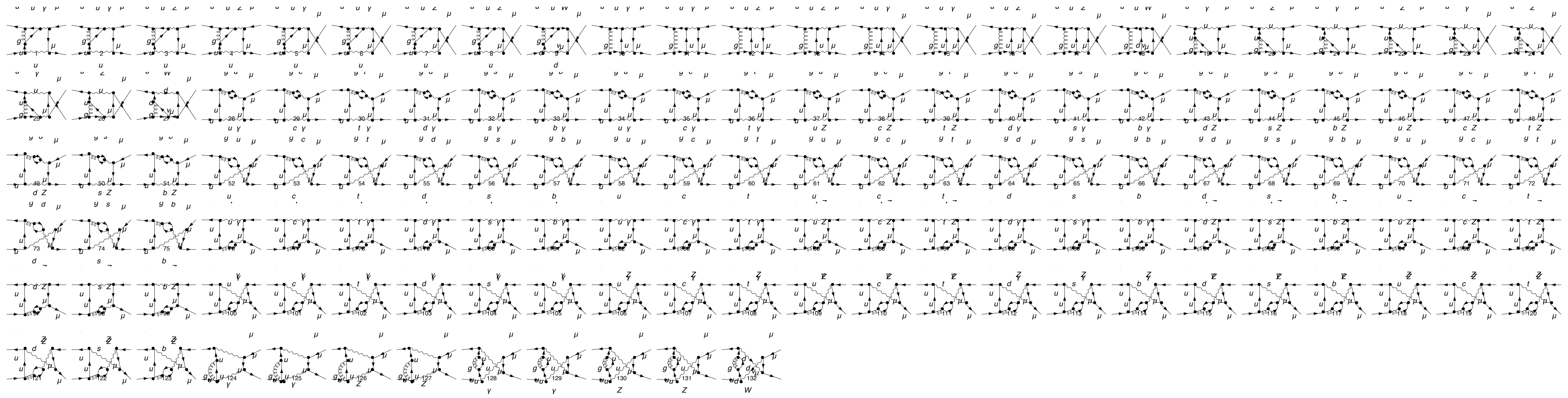
the final state consists of a pair of massive leptons (treated as bare) to regulate the collinear (mass) singularities

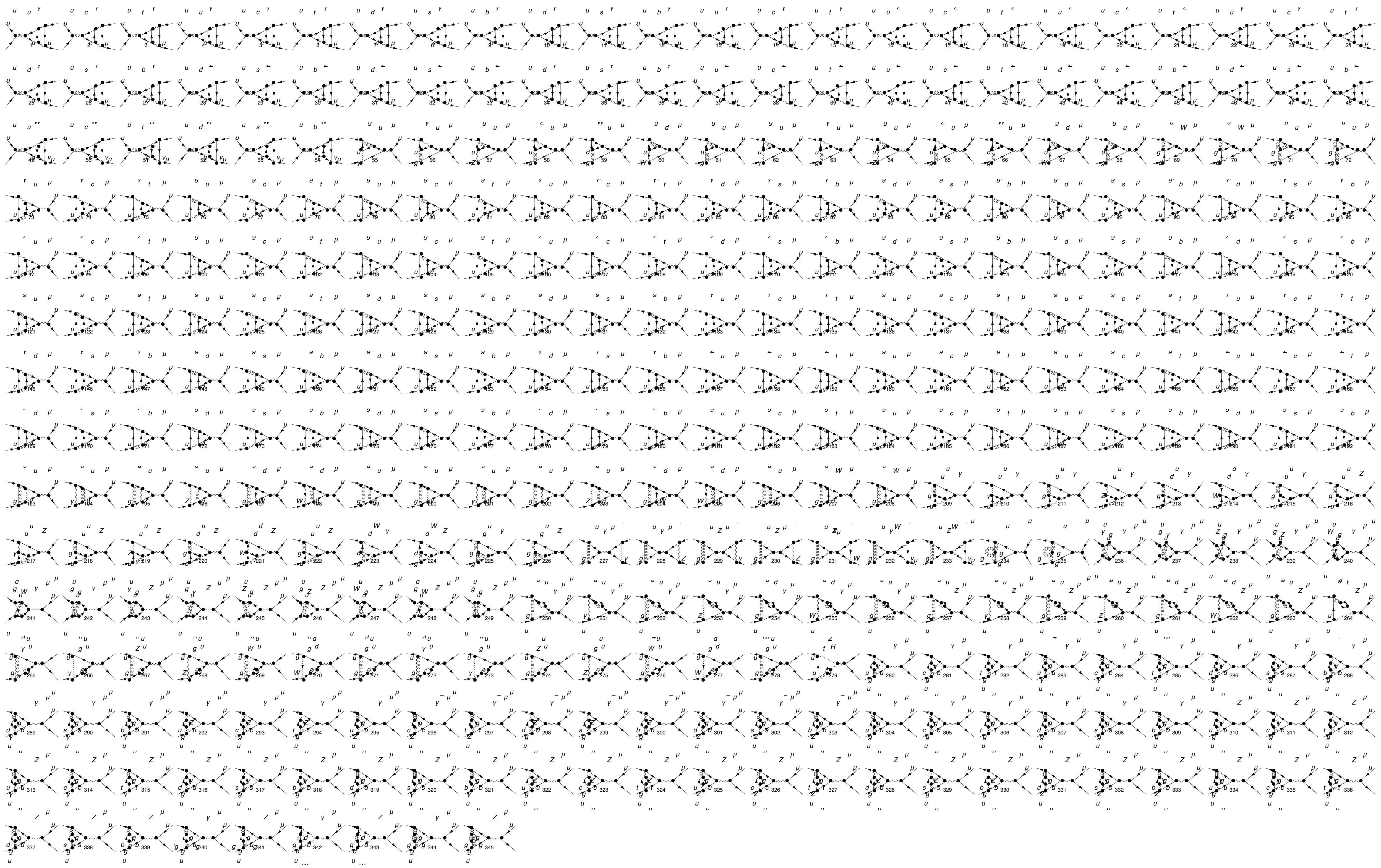
The double virtual amplitude: generation of the amplitude

$\mathcal{M}^{(0,0)}(q\bar{q} \to l\bar{l}) =$



$\mathcal{M}^{(1,1)}(q\bar{q} \to l\bar{l}) =$ O(1000) self-energies + O(300) vertex corrections +O(130) box corrections + 1loop x 1loop
(before discarding all those vanishing for colour conservation, e.g. no fermonic triangles)





Structure of the double virtual amplitude

$$2\mathrm{Re}\left(\mathscr{M}^{(1,1)}(\mathscr{M}^{(0,0)})^\dagger\right) = \sum_{i=1}^{N_{MI}} c_i(s,t,m;\varepsilon)\ \mathscr{I}_i(s,t,m;\varepsilon)$$

Structure of the double virtual amplitude

$$2\mathrm{Re}\left(\mathscr{M}^{(1,1)}(\mathscr{M}^{(0,0)})^{\dagger}\right) = \sum_{i=1}^{N_{MI}} c_i(s,t,m;\varepsilon)\ \mathscr{I}_i(s,t,m;\varepsilon)$$

The coefficients c_i are rational functions of the invariants, masses and of ε

Their size can rapidly "explode" in the GB range

→ careful work to identify the patterns of recurring subexpressions, keeping the total size in the O(1-10 MB) range

Abiss Mathematica package

Structure of the double virtual amplitude

$$2\mathrm{Re}\left(\mathscr{M}^{(1,1)}(\mathscr{M}^{(0,0)})^{\dagger}\right) = \sum_{i=1}^{N_{MI}} c_i(s,t,m;\varepsilon)\ \mathscr{I}_i(s,t,m;\varepsilon)$$

The coefficients c_i are rational functions of the invariants, masses and of ε

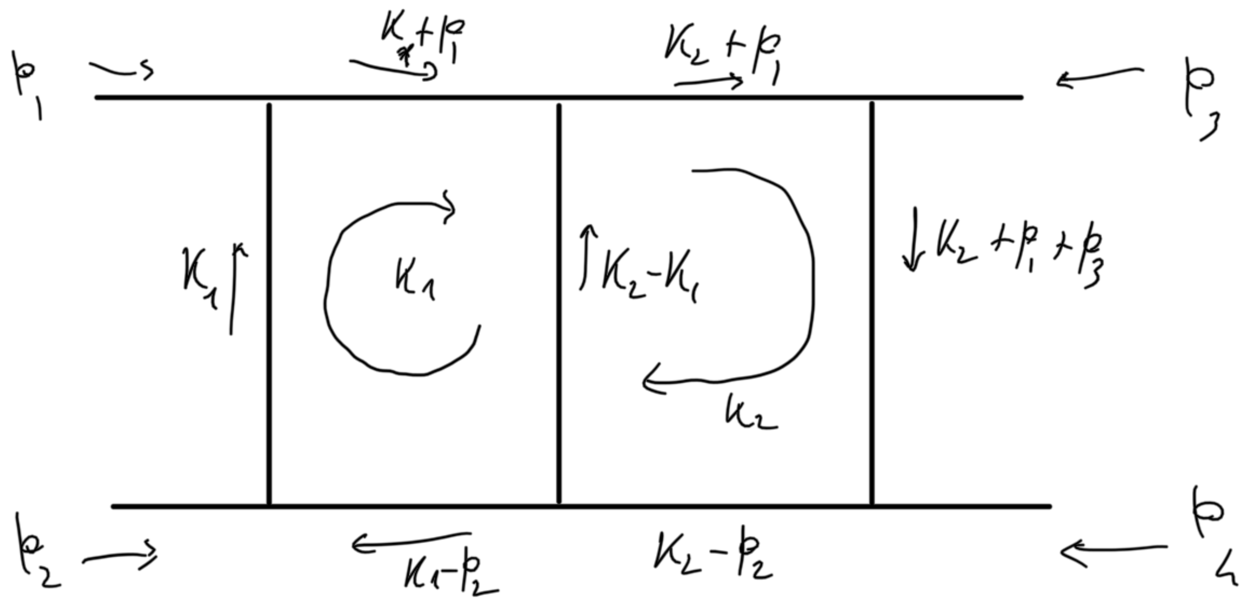
Their size can rapidly "explode" in the GB range

→ careful work to identify the patterns of recurring subexpressions, keeping the total size in the O(1-10 MB) range

Abiss Mathematica package

The Feynman Integrals $\mathscr{I}_i$ are one of the major challenges in the evaluation of the virtual corrections

$$\mathscr{I}(p_i\cdot p_j;\vec{m}) = \int \frac{d^n k_1}{(2\pi)^n}\int\frac{d^n k_2}{(2\pi)^n}\ \frac{1}{[k_1^2-m_0^2]^{\alpha_0}\ [(k_1+p_1)^2-m_1^2]^{\alpha_1}\ \dots\ [(k_1+k_2+p_j)^2-m_j^2]^{\alpha_j}\ \dots\ [(k_2+p_l)^2-m_l^2]^{\alpha_l}}$$



The complexity of the solution grows with the number of energy scales (masses and invariants) upon which it depends

The double virtual amplitude: reduction to Master Integrals

The complexity of the MIs depends on the number of energy scales

MIs relevant for the QCD-QED corrections, with massive final state

Bonciani, Ferroglia,Gehrmann, Maitre, Studerus., arXiv:0806.2301, 0906.3671

MIs with 1or 2 internal mass relevant for the EW form factor

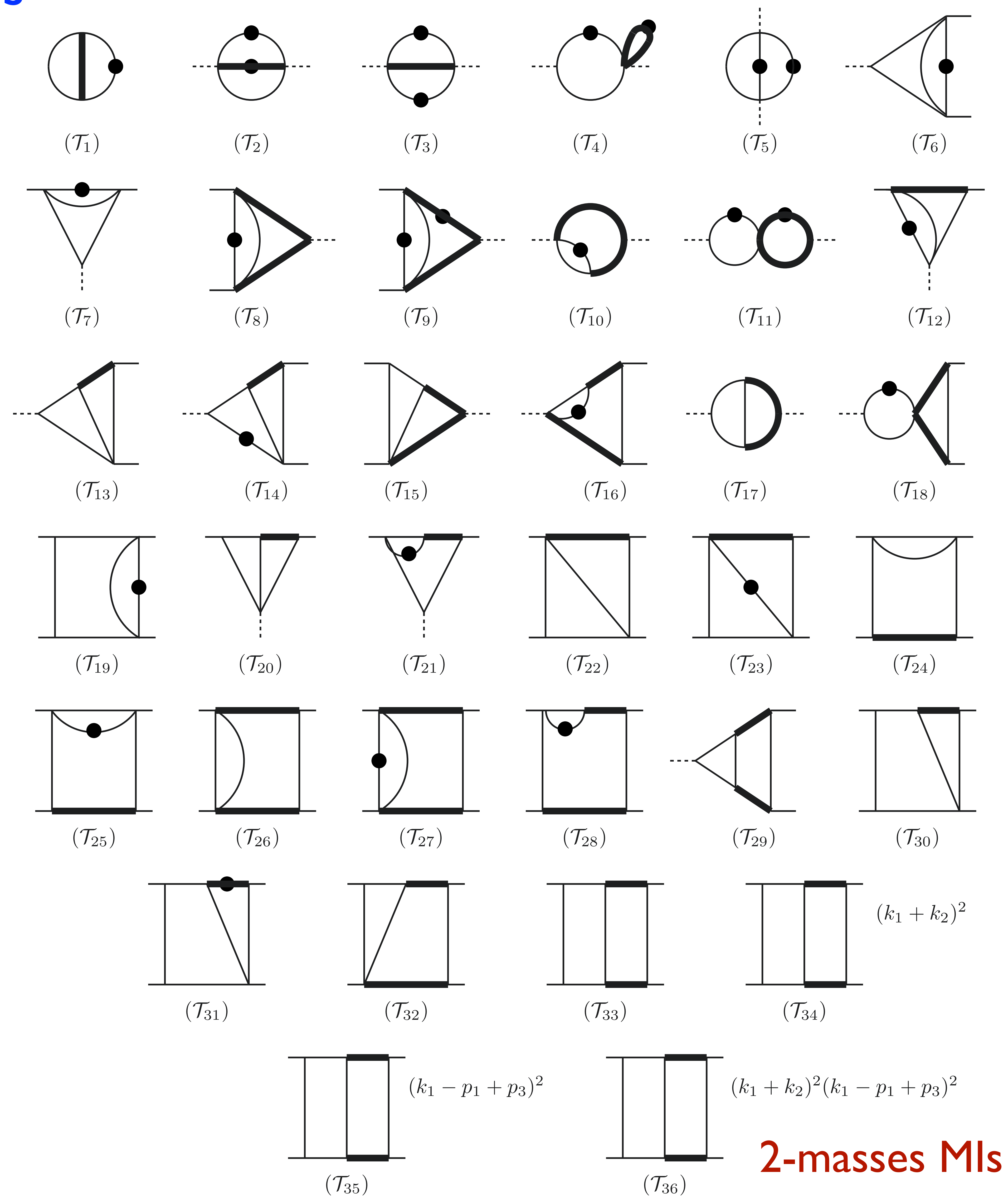
Aglietti, Bonciani, hep-ph/0304028, hep-ph/0401193

31 MIs with 1 mass and 36 MIs with 2 masses including boxes, relevant for the QCD-weak corrections to the full Drell-Yan

Bonciani, Di Vita, Mastrolia, Schubert., arXiv:1604.08581

In the 2-mass case, 5 box integrals in Chen-Goncharov representation

→ problematic numerical evaluation → need an alternative strategy



cfr. also Heller, von Manteuffel, Schabinger,
arXiv:1907.00491 for a representation of the MIs in terms of GPLs
arXiv:2012.05918 for a description of the 2-loop virtual amplitude

Evaluation of the Master Integrals by series expansions

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

The Master Integrals satisfy a system of differential equations.

The MIs are replaced by formal series with unknown coefficients → eqs for the unknown coefficients of the series.

The package `DiffExp` by M.Hidding, arXiv:2006.05510 implements this idea, for real valued masses, with real kinematical vars.

But we need complex-valued masses of W and Z bosons (unstable particles) → we wrote a new package (`SeaSyde`)

Evaluation of the Master Integrals by series expansions

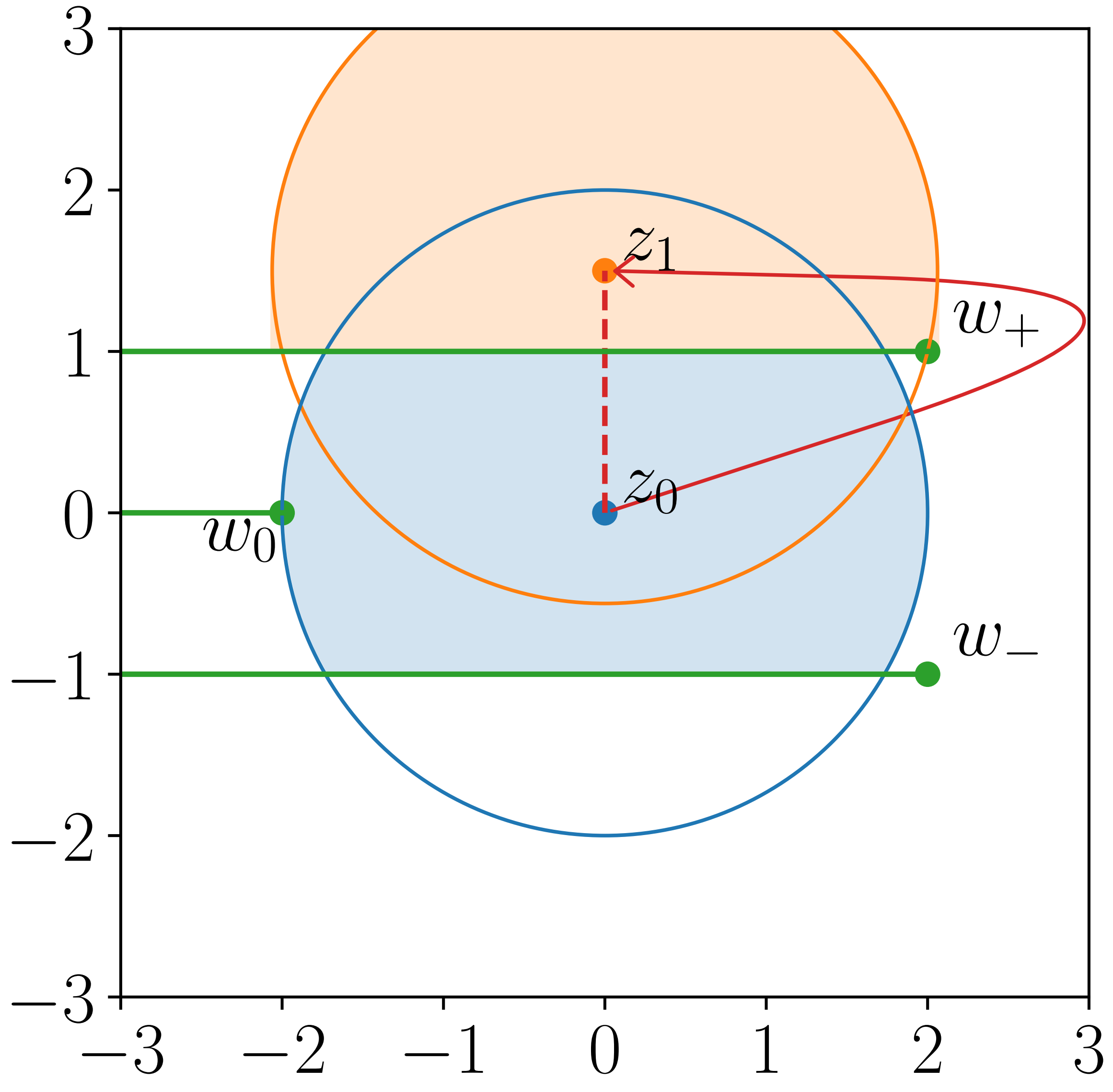
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

The Master Integrals satisfy a system of differential equations.
The MIs are replaced by formal series with unknown coefficients → eqs for the unknown coefficients of the series.
The package DiffExp by M.Hidding, arXiv:2006.05510 implements this idea, for real valued masses, with real kinematical vars.
But we need complex-valued masses of W and Z bosons (unstable particles) → we wrote a new package (SeaSyde)

We implemented the series expansion approach, for arbitrary complex-valued masses,
working in the complex plane of each kinematical variable, one variable at a time

Complete knowledge about the singular structure of the MI
can be read directly from the differential equation matrix

The solution can be computed with an arbitrary number of significant digits,
but not in closed form → semi-analytical



Evaluation of the Master Integrals by series expansions

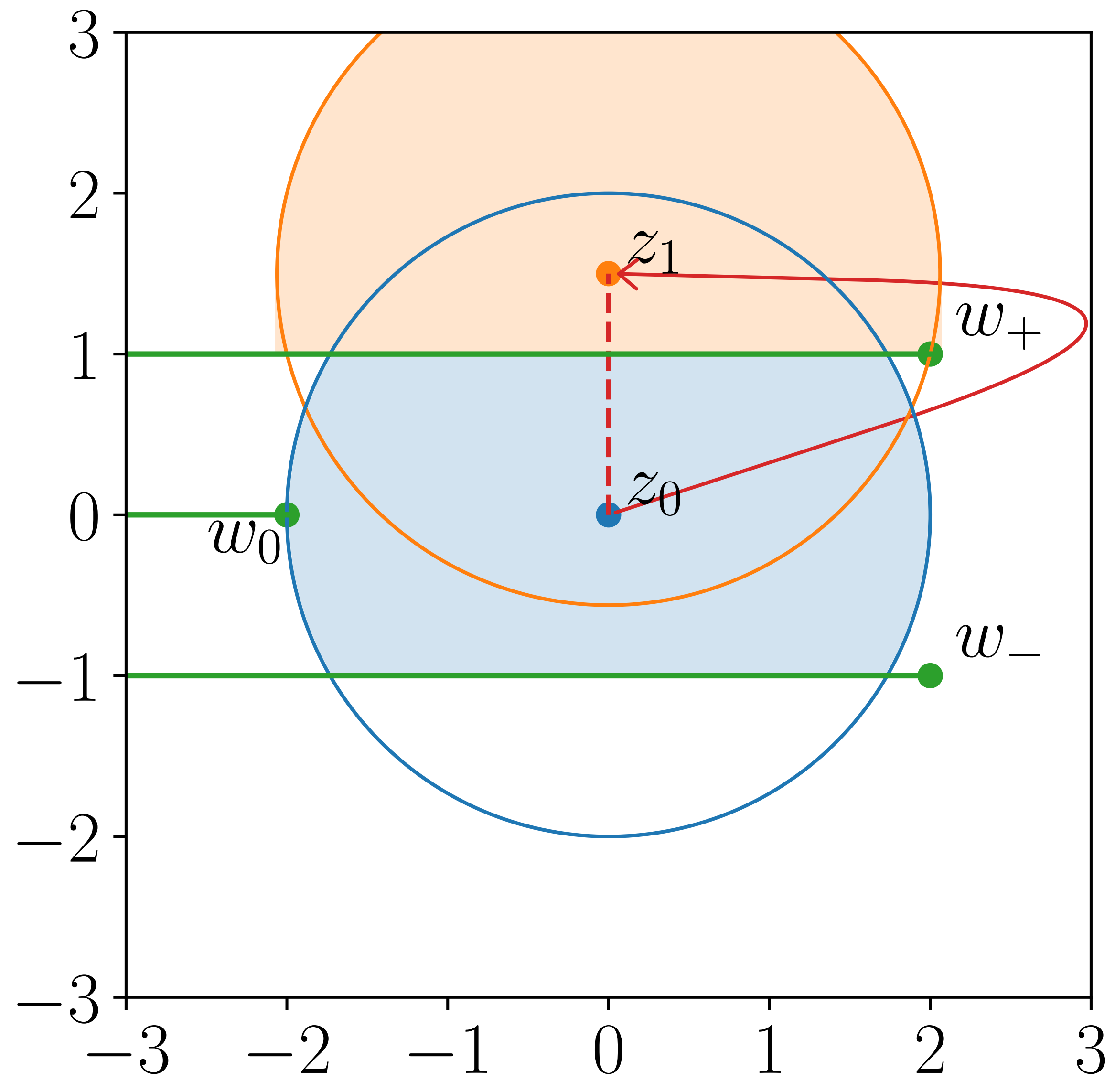
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

The Master Integrals satisfy a system of differential equations.
The MIs are replaced by formal series with unknown coefficients → eqs for the unknown coefficients of the series.
The package `DiffExp` by M.Hidding, arXiv:2006.05510 implements this idea, for real valued masses, with real kinematical vars.
But we need complex-valued masses of W and Z bosons (unstable particles) → we wrote a new package (`SeaSyde`)

We implemented the series expansion approach, for arbitrary complex-valued masses, working in the complex plane of each kinematical variable, one variable at a time

Complete knowledge about the singular structure of the MI can be read directly from the differential equation matrix

The solution can be computed with an arbitrary number of significant digits, but not in closed form → semi-analytical



Numerical evaluation of the hard coefficient function

The interference term $2\mathrm{Re}\langle \mathcal{M}^{(1,1),fin} | \mathcal{M}^{(0,0)} \rangle$ contributes to the hard function $H^{(1,1)}$

After the subtraction of all the universal IR divergences, it is a finite correction

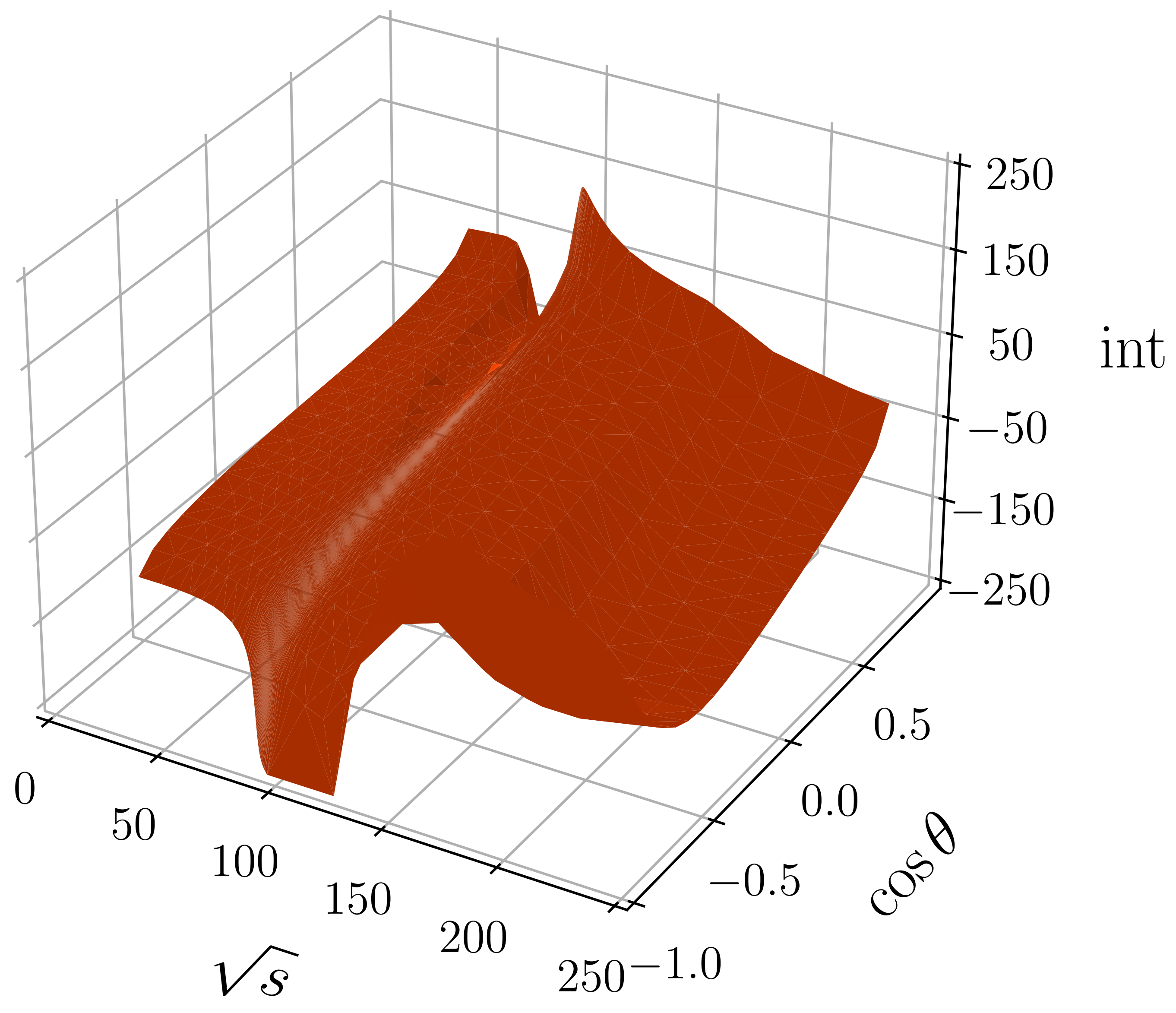
It has been published in arXiv:2201.01754 and is available as a Mathematica notebook

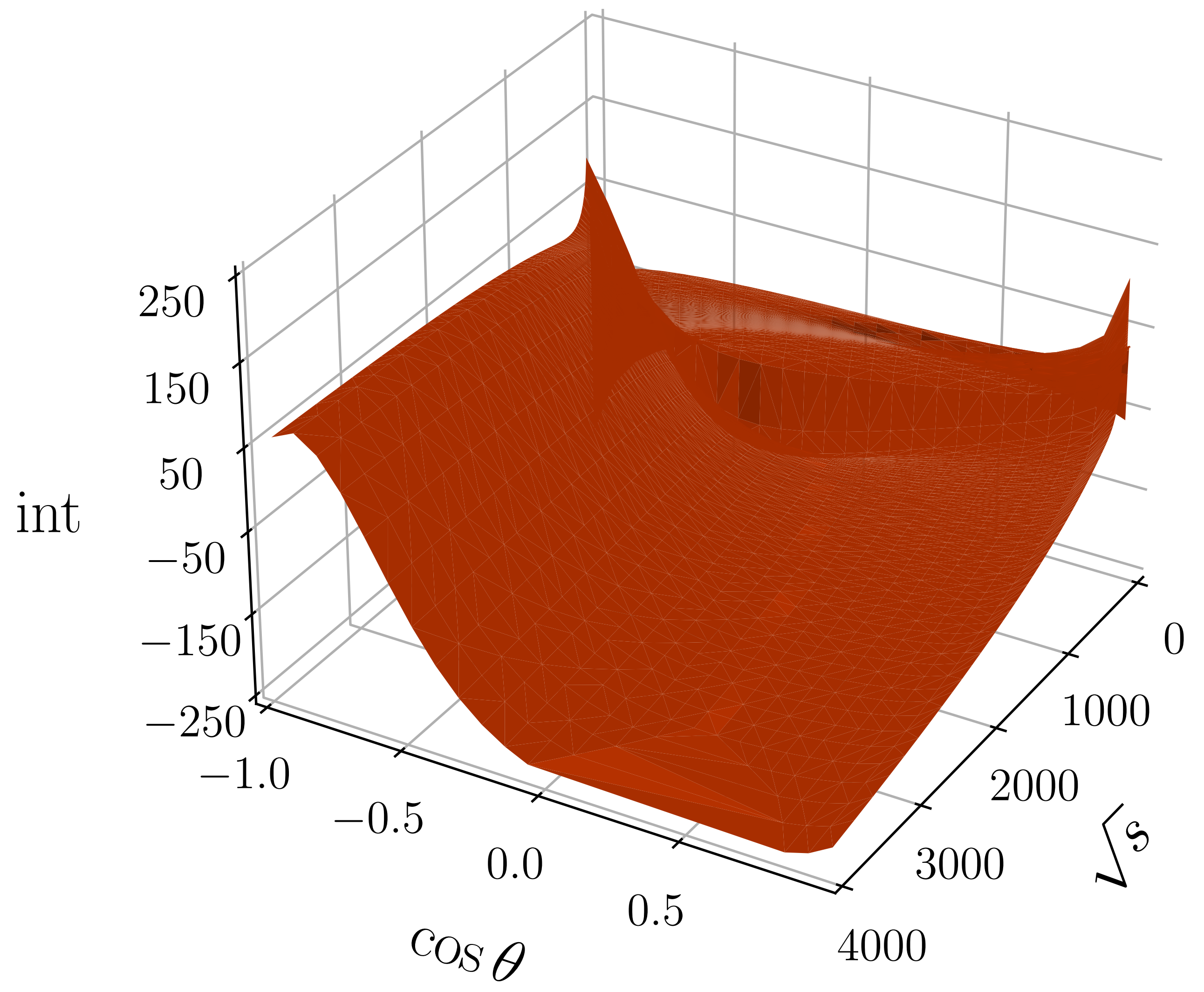
Several checks of the MIs performed with Fiesta, PySecDec and AMFlow

A numerical grid has been prepared for all the 36 MIs, with GiNaC and SeaSyde (T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345) ,
covering the whole $2 \to 2$ phase space in (s,t) (3250 points) ,
in O(12 h) on one 32-cores machine

→ values at arbitrary phase space points obtained with excellent accuracy via interpolation, with negligible evaluation time

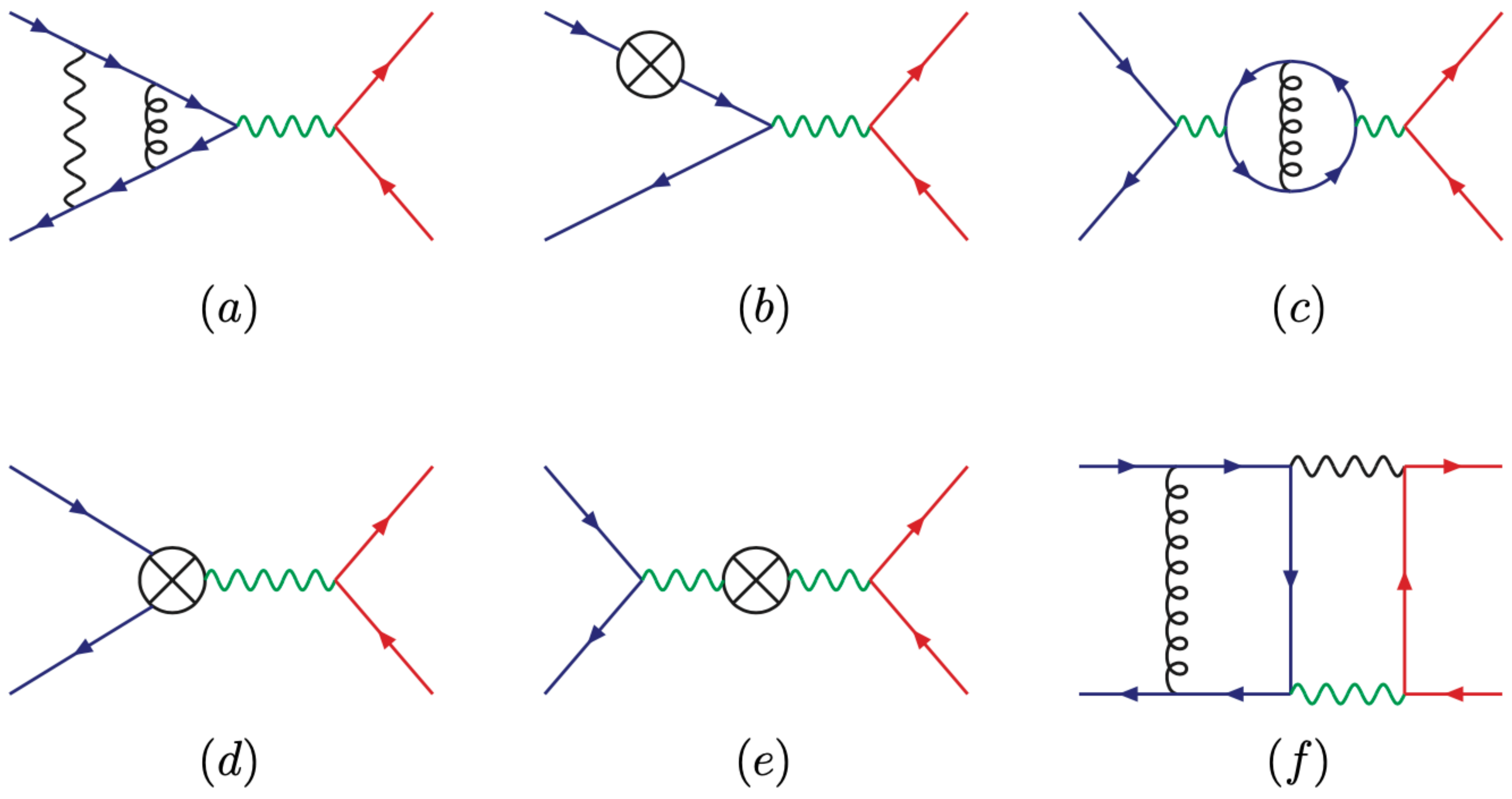
in units $\frac{\alpha}{\pi} \frac{\alpha_s}{\pi} \sigma_0$





Exact 2-loop virtual QCD-EW corrections to Charged-Current Drell-Yan

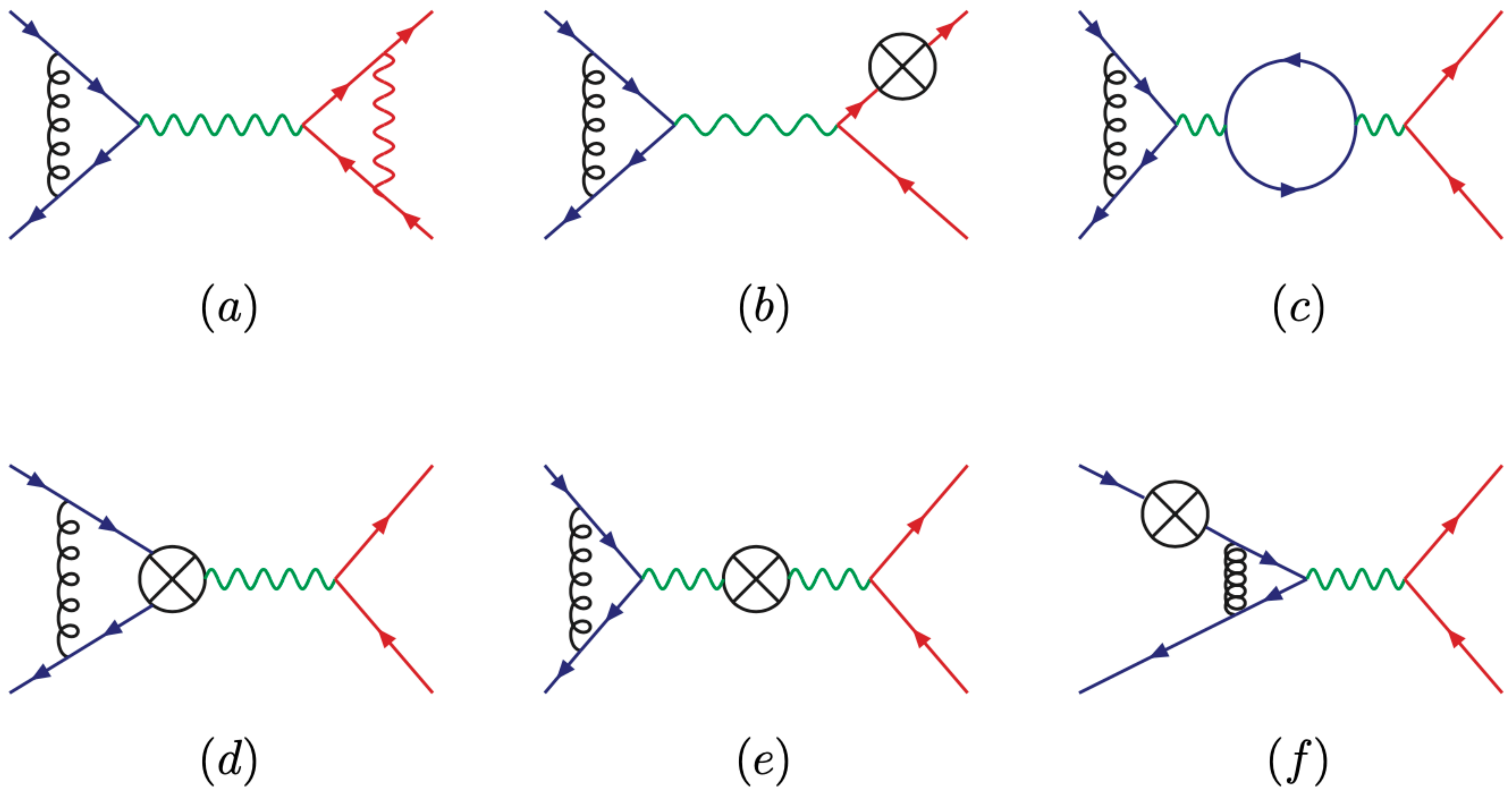
2-loop virtual QCD-EW corrections to the Charged-Current Drell-Yan in the SM



The Charged-Current process is mediated by a W exchange

For a general lepton-pair invariant mass,
there is no general gauge invariant separation of initial- and final-state photonic corrections, at variance with the NC DY case

We consider a massive final-state lepton,
yielding mass logarithms instead of collinear poles in dim.reg.



The presence of two weak bosons with different masses (W and Z) is a new challenge for the solution of the Feynman integrals

Large number of terms → increased automation level

Subtraction of the IR divergences from the 2-loop amplitude

we identify QCD-QED (poles up to $1/\varepsilon^4$) and QCD-weak (poles up to $1/\varepsilon^2$ with cumbersome coefficients) diagrams

$$|\mathcal{M}^{(1,0),fin}\rangle = |\mathcal{M}^{(1,0)}\rangle - \mathcal{I}^{(1,0)}|\mathcal{M}^{(0)}\rangle\,,$$
standard NLO-QCD subtraction

$$|\mathcal{M}^{(0,1),fin}\rangle = |\mathcal{M}^{(0,1)}\rangle - \mathcal{I}^{(0,1)}|\mathcal{M}^{(0)}\rangle\,.$$
NLO-EW subtraction, with massive leptons

$$|\mathcal{M}^{(1,1),fin}\rangle = |\mathcal{M}^{(1,1)}\rangle - \mathcal{I}^{(1,1)}|\mathcal{M}^{(0)}\rangle - \tilde{\mathcal{I}}^{(0,1)}|\mathcal{M}^{(1,0),fin}\rangle - \tilde{\mathcal{I}}^{(1,0)}|\mathcal{M}^{(0,1),fin}\rangle\,.$$

$$\mathcal{I}^{(1,0)} = \left(\frac{s}{\mu^2}\right)^{-\epsilon} C_F\left(-\frac{2}{\epsilon^2}-\frac{1}{\epsilon}(3+2i\pi)+\zeta_2\right)\,,$$

$$\mathcal{I}^{(0,1)} = \left(\frac{s}{\mu^2}\right)^{-\epsilon}\left[\frac{Q_u^2+Q_d^2}{2}\left(-\frac{2}{\epsilon^2}-\frac{1}{\epsilon}(3+2i\pi)+\zeta_2\right)+\frac{4}{\epsilon}\Gamma_l^{(0,1)}\right]$$

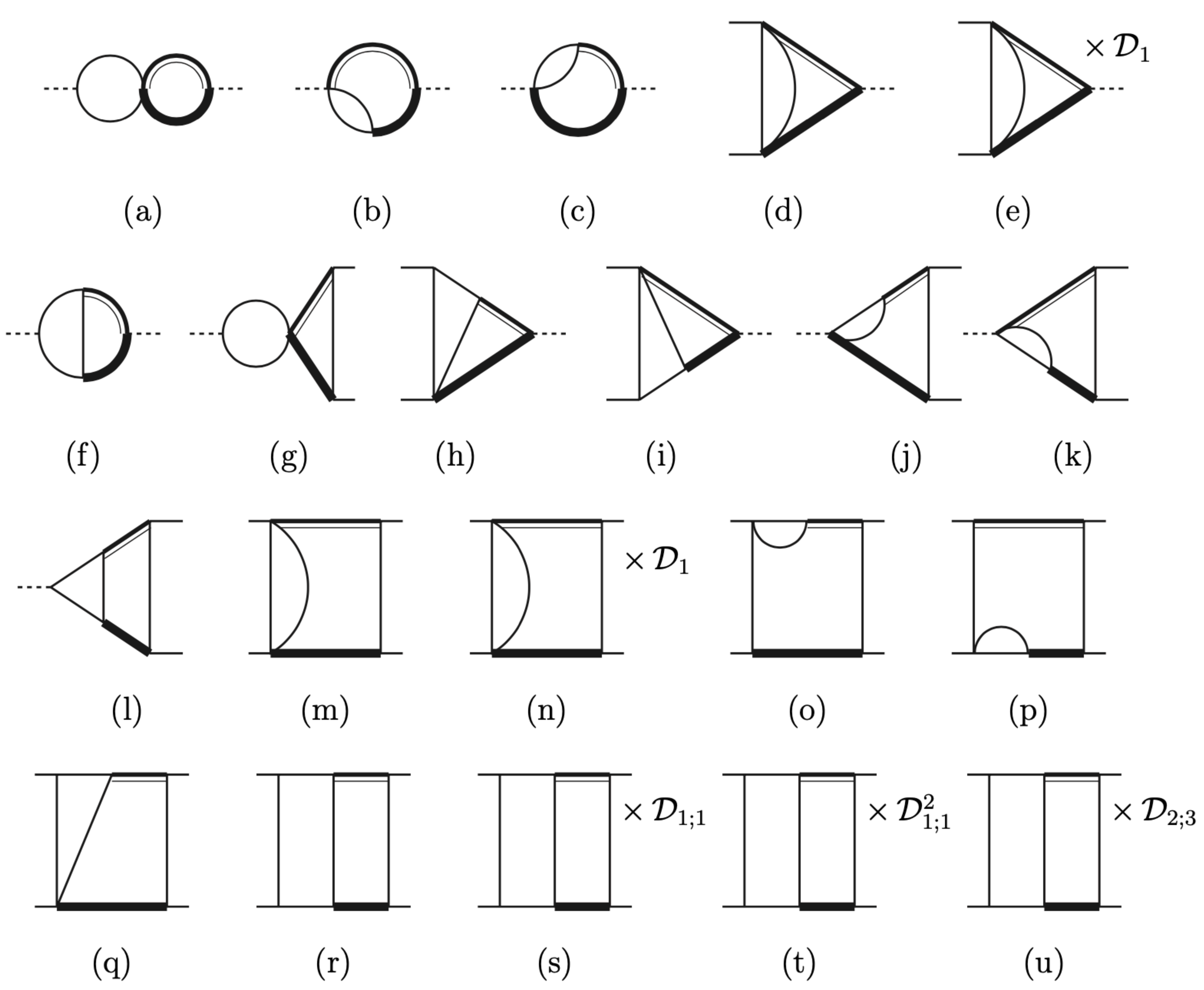
$$\Gamma_l^{(0,1)} = -\frac{1}{4}\left[Q_l^2\,(1-i\pi)+Q_l^2\log\left(\frac{m_l^2}{s}\right)+ + 2Q_uQ_l\log\left(\frac{(2p_1\cdot p_4)}{s}\right)-2Q_dQ_l\log\left(\frac{(2p_2\cdot p_4)}{s}\right)\right]$$

$$\mathcal{I}^{(1,1)} = \left(\frac{s}{\mu^2}\right)^{-2\epsilon} C_F\left[\frac{Q_u^2+Q_d^2}{2}\left(\frac{4}{\epsilon^4}+\frac{1}{\epsilon^3}(12+8i\pi)+\frac{1}{\epsilon^2}(9-28\zeta_2+12i\pi)++\frac{1}{\epsilon}\left(-\frac{3}{2}+6\zeta_2-24\zeta_3-4i\pi\zeta_2\right)\right) + \left(-\frac{2}{\epsilon^2}-\frac{1}{\epsilon}(3+2i\pi)+\zeta_2\right)\frac{4}{\epsilon}\,\Gamma_l^{(0,1)}\right]$$

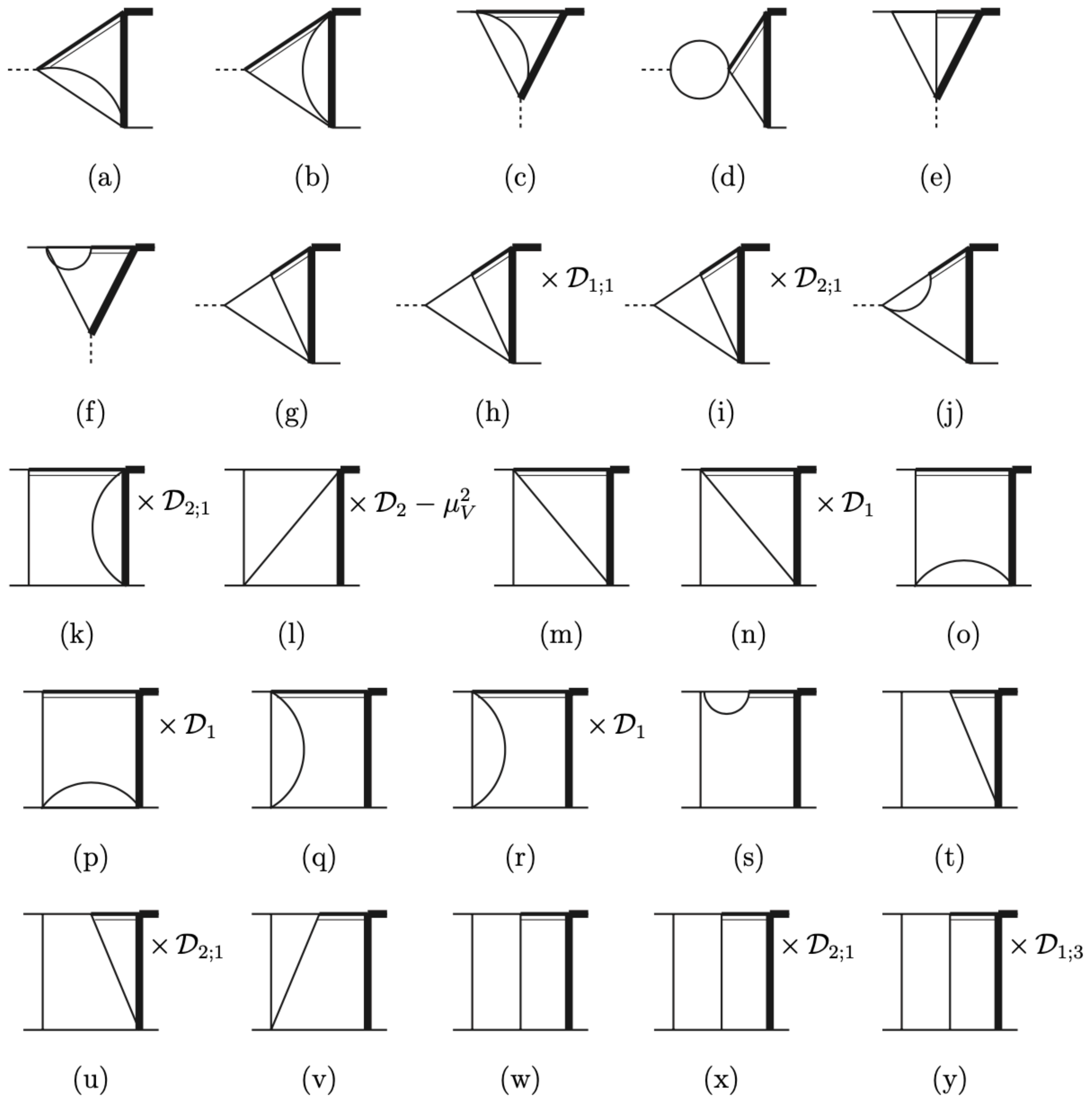
The analytical check of the cancellation of the IR poles in the QCD-weak sector is one very demanding test of the calculation.

In CC-DY for the first time we achieved a completely numerical check of the cancellation of all the IR poles

2-loop virtual QCD-EW corrections to CC DY: new Master Integrals



Master Integrals with two different internal masses

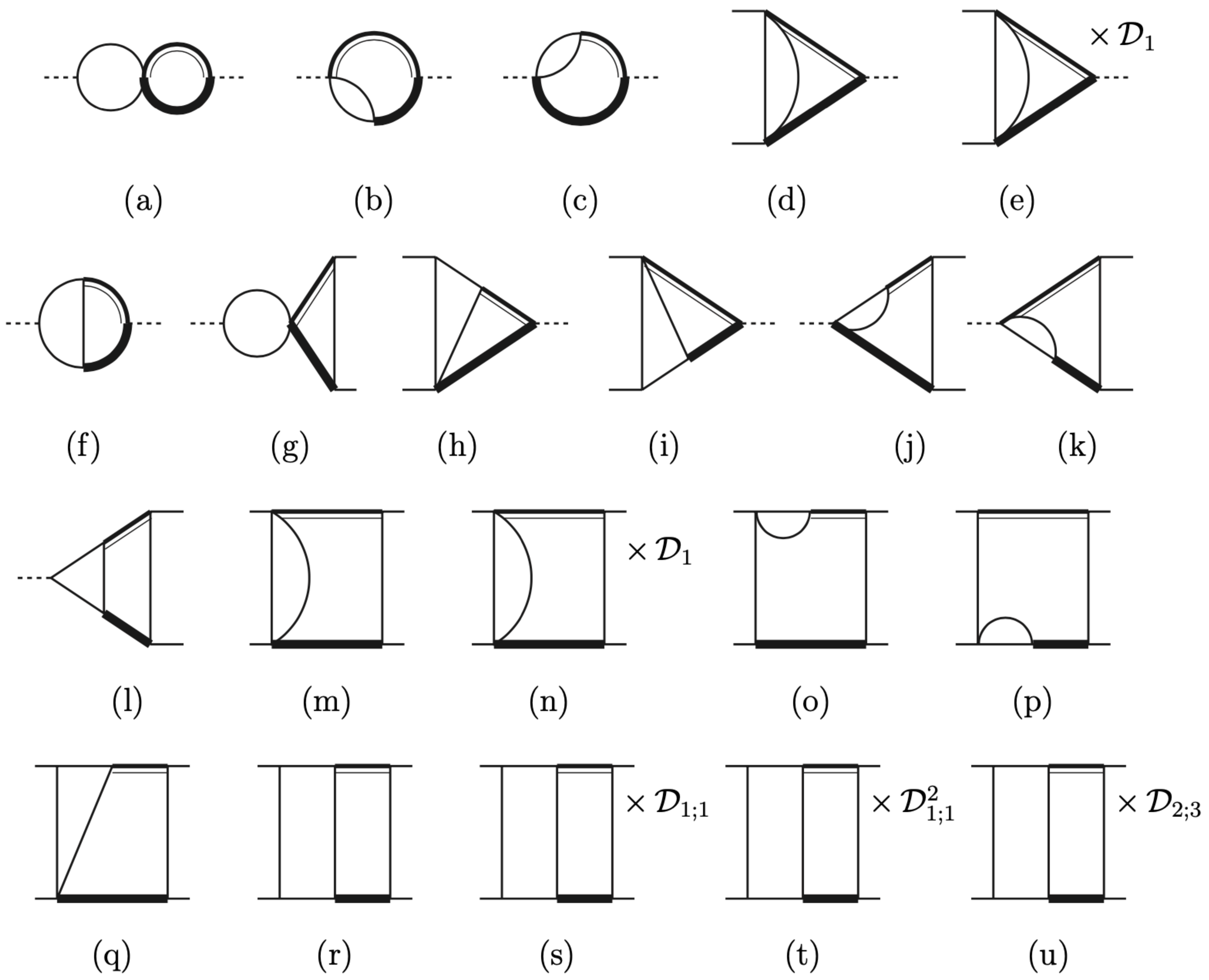


Master Integrals with
one W and one internal massive lepton lines

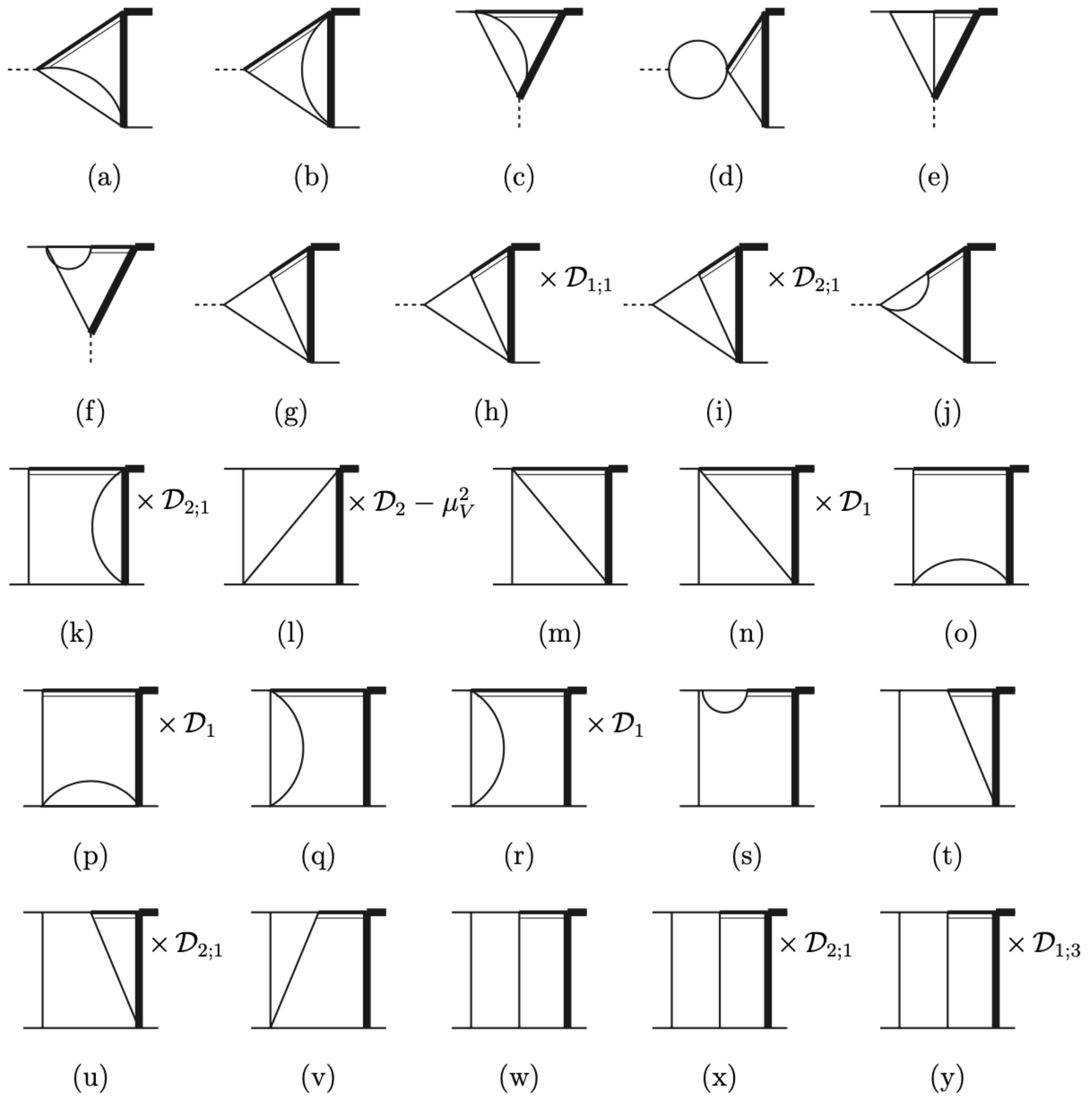
Automated workflow

- All the terms in the amplitude are reduced to Master Integrals with Abiss+KIRA
- The differential equations are written with LiteRed
- The Boundary Conditions are computed with AMFlow
- The Master Integrals are computed with SeaSyde

2-loop virtual QCD-EW corrections to CC DY: new Master Integrals



Master Integrals with two different internal masses



Master Integrals with
one W and one internal massive lepton lines

Automated workflow

- All the terms in the amplitude are reduced to Master Integrals with Abiss+KIRA
- The differential equations are written with LiteRed
- The Boundary Conditions are computed with AMFlow
- The Master Integrals are computed with SeaSyde

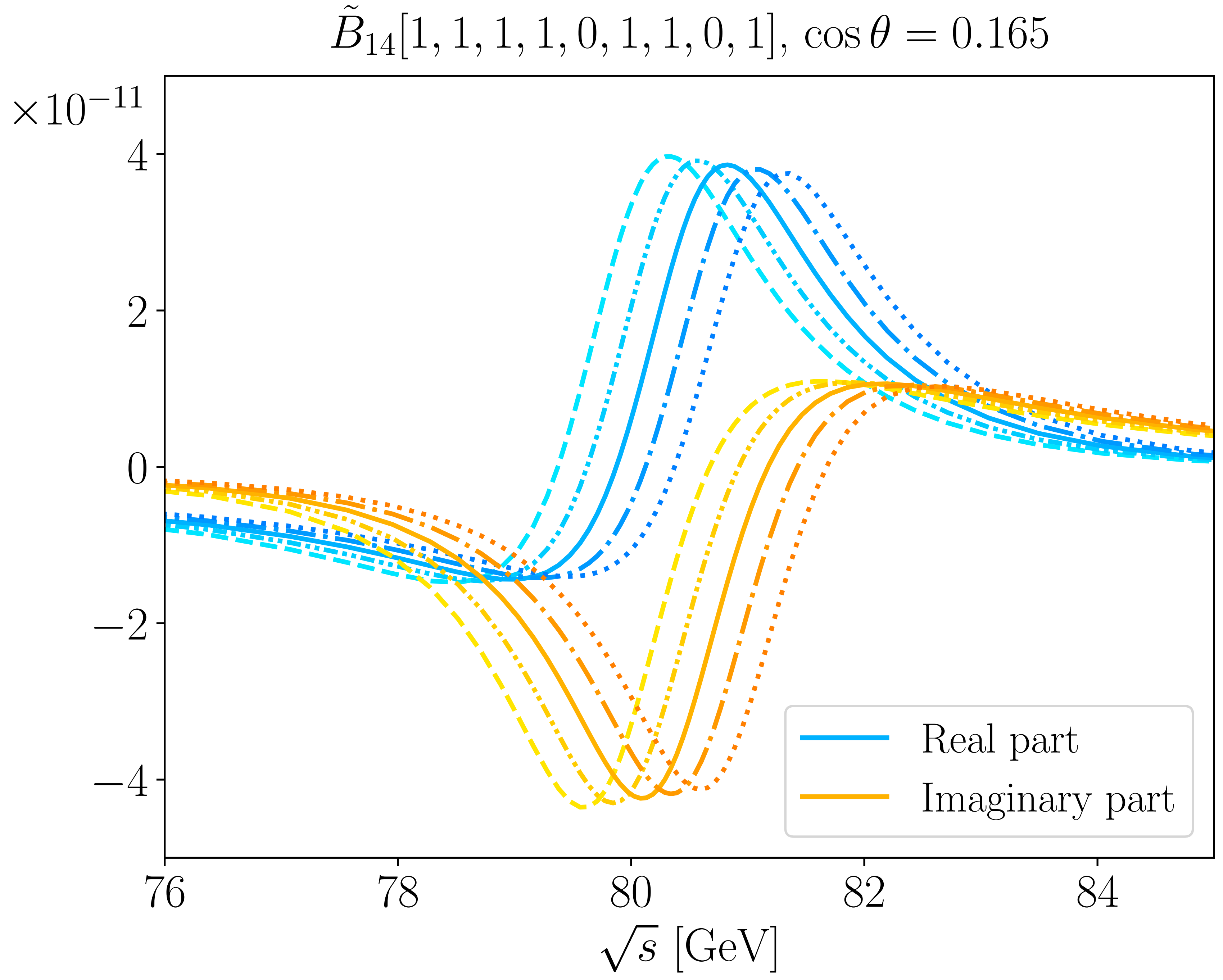
useful to tackle NNLO-EW corrections
→ relevant at LHC and later at FCC-ee

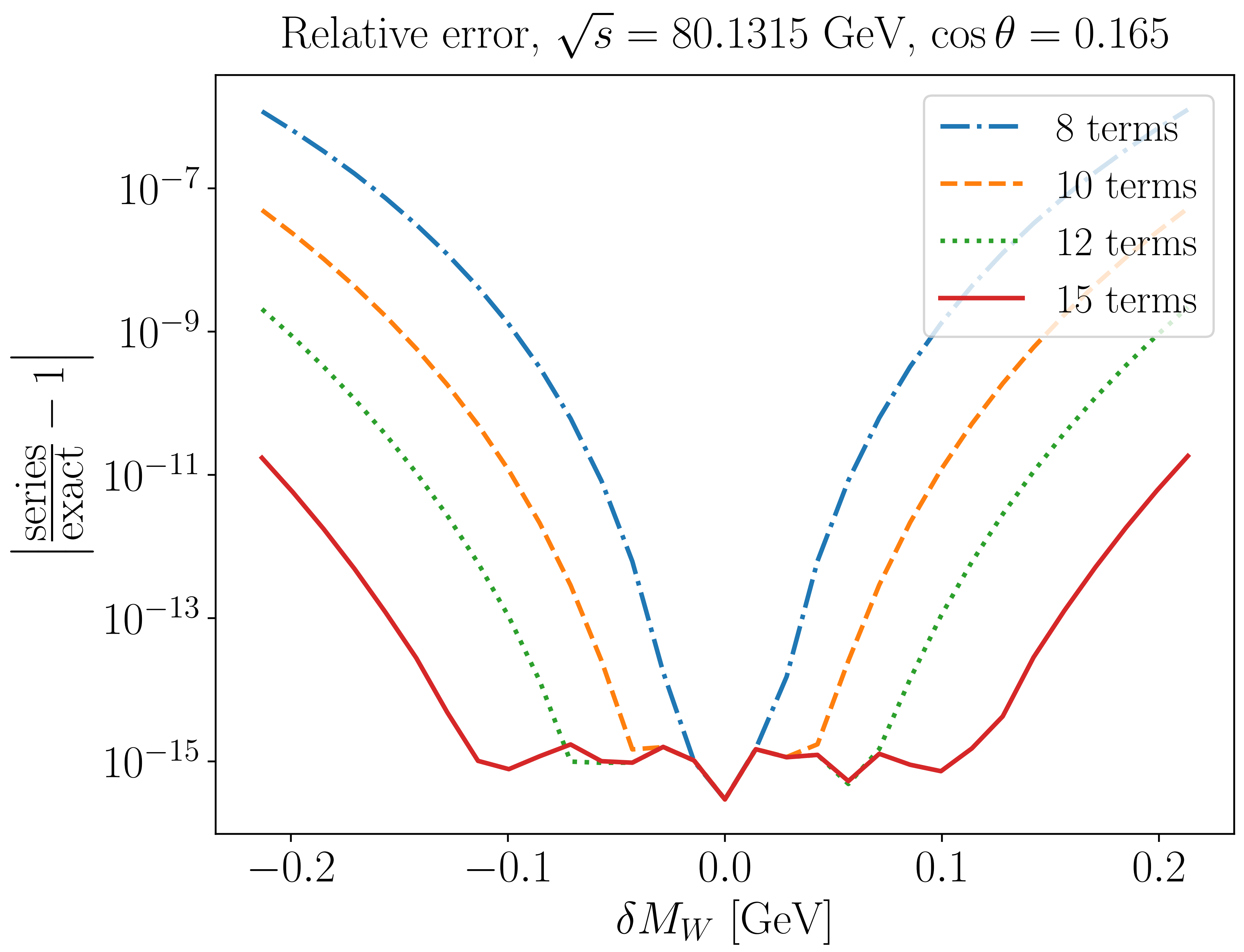
Fast numerical evaluation with arbitrary W-mass values

The Master Integrals can be solved at different (s, t) values, yielding a numerical grid, for a given value $\overline{m}_W$ of the W boson mass.
→ very efficient and accurate in Monte Carlo simulations

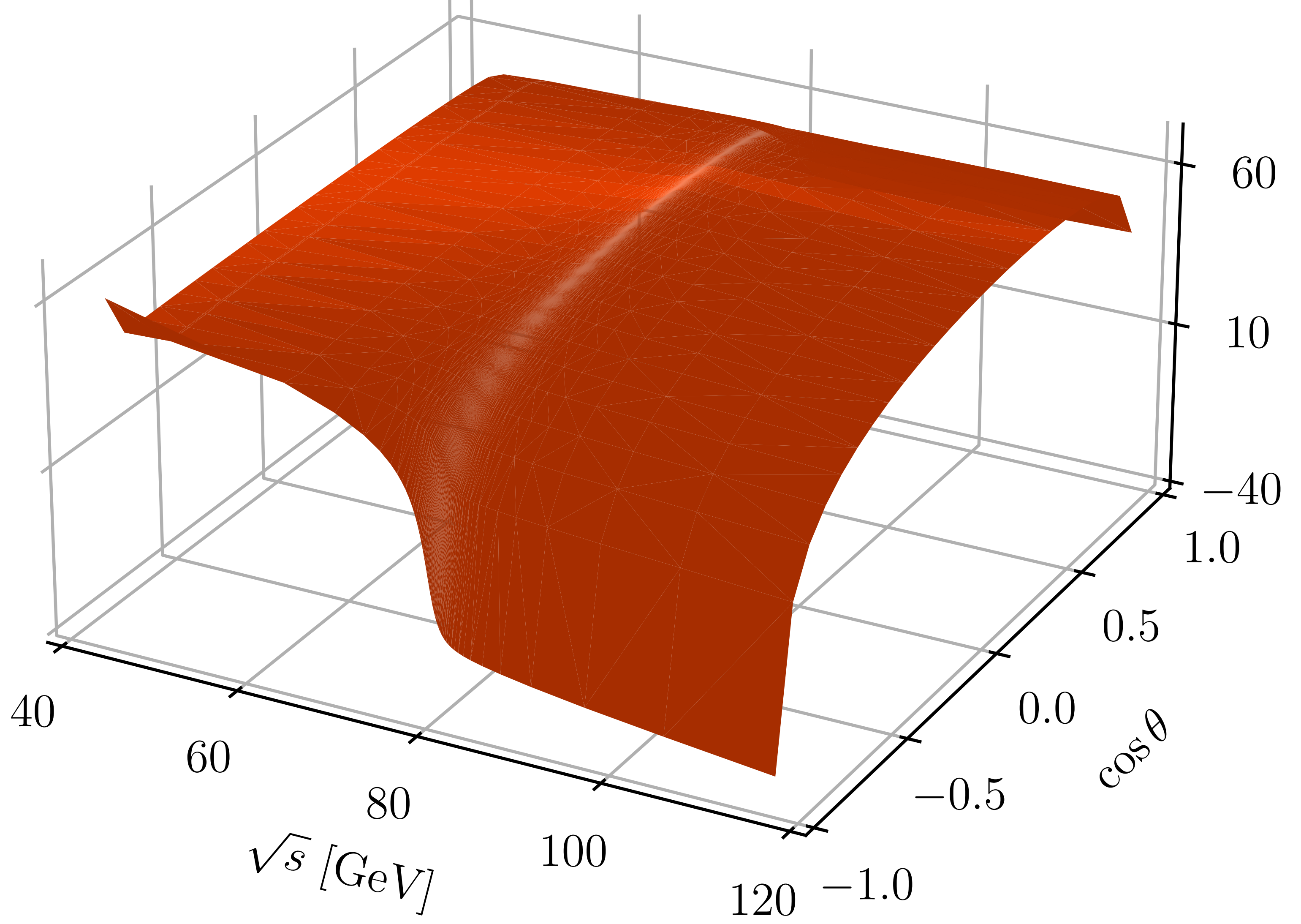
The differential equations with respect to the internal W mass
can be solved via the series expansion approach, yielding as a solution a power series in $\delta m_W = m_W - \overline{m}_W$,
taking as BCs the first grid with $\overline{m}_W$.

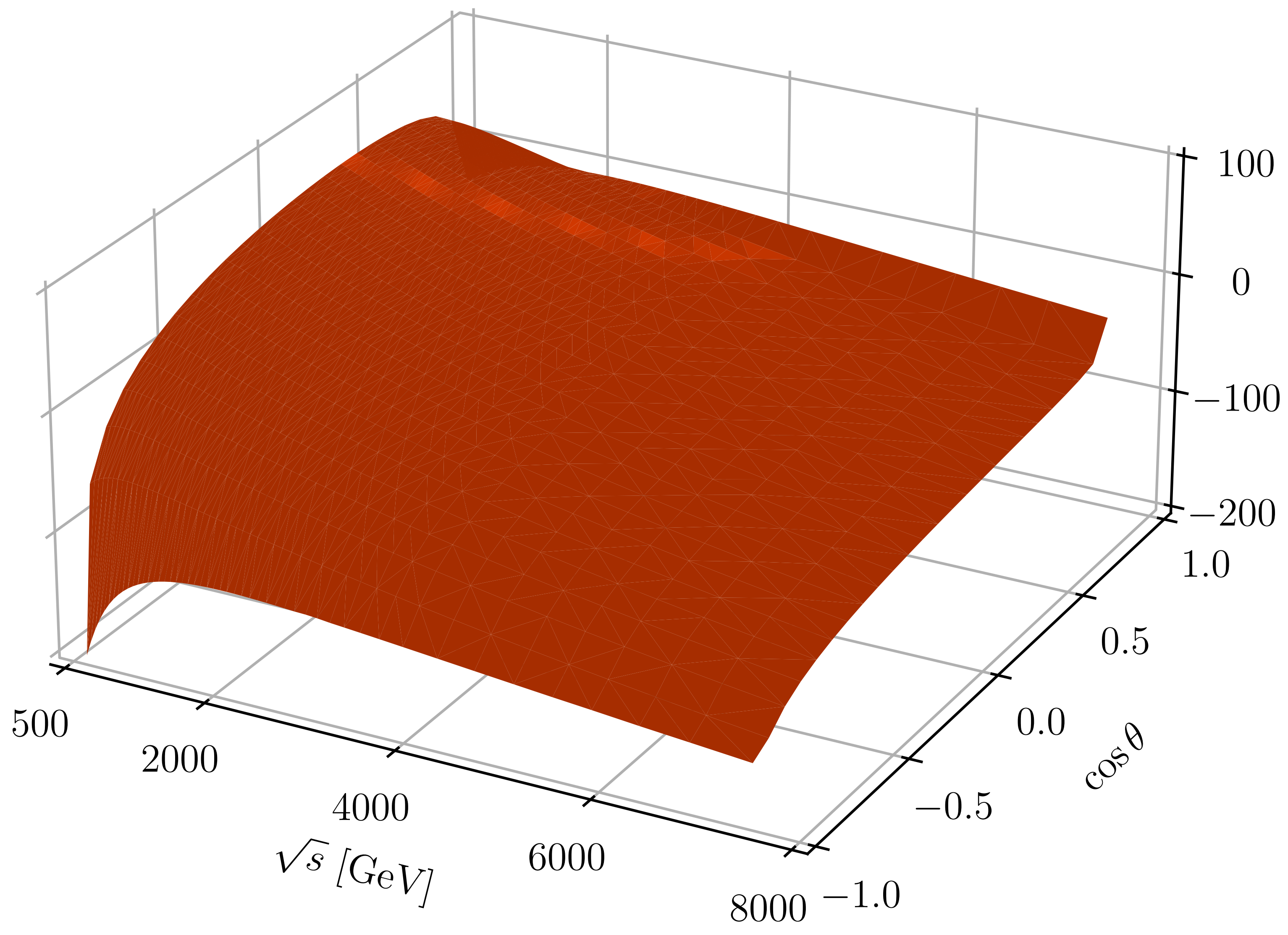
Our final 2-loop virtual result is cast, at every phase-space point, as a power series in δm_W,
which can be evaluated in a negligible amount of time, to give the actual grid, for any m_W choice





Finite 2-loop exact QCD-EW virtual corrections to Charged-Current Drell-Yan

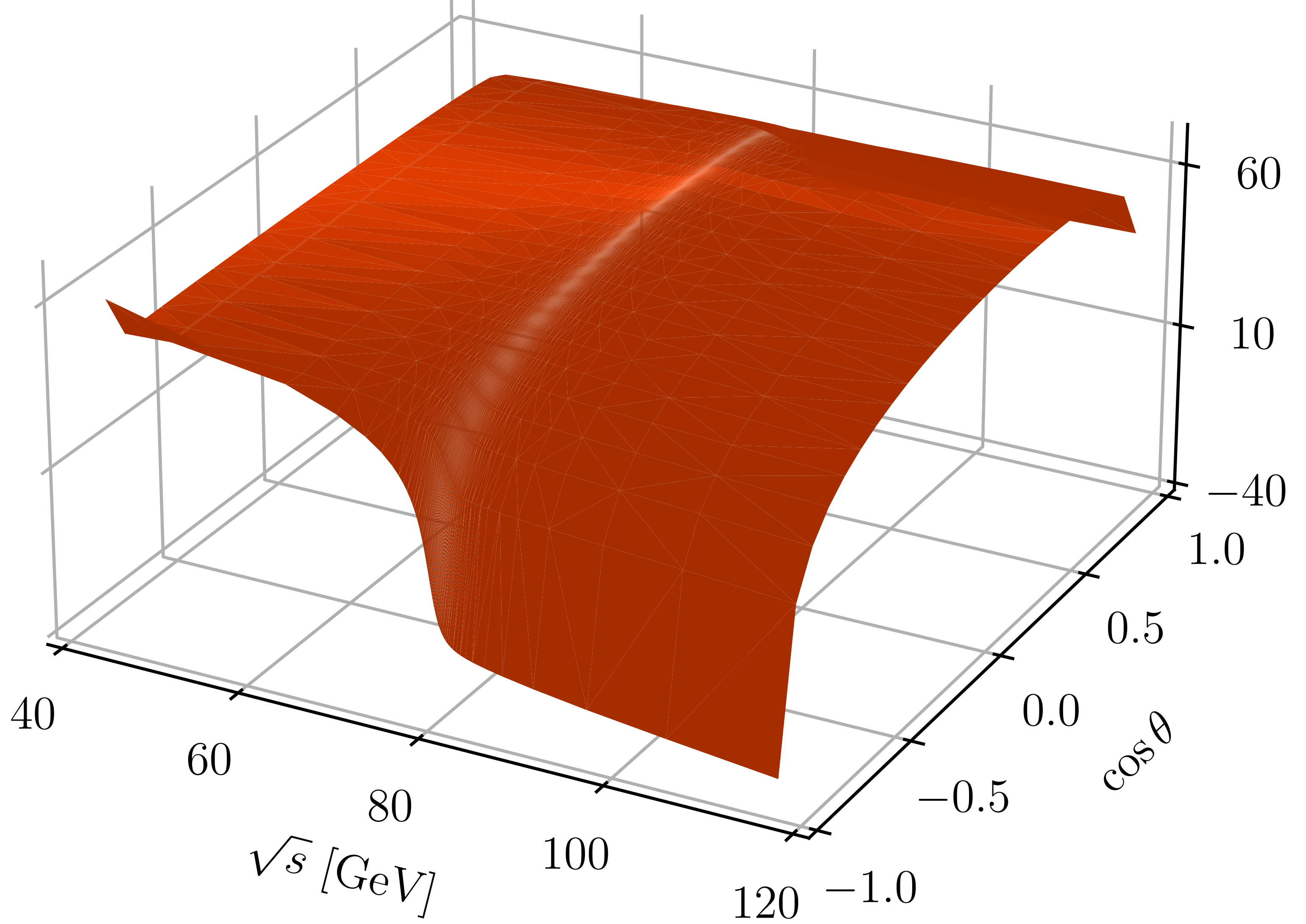


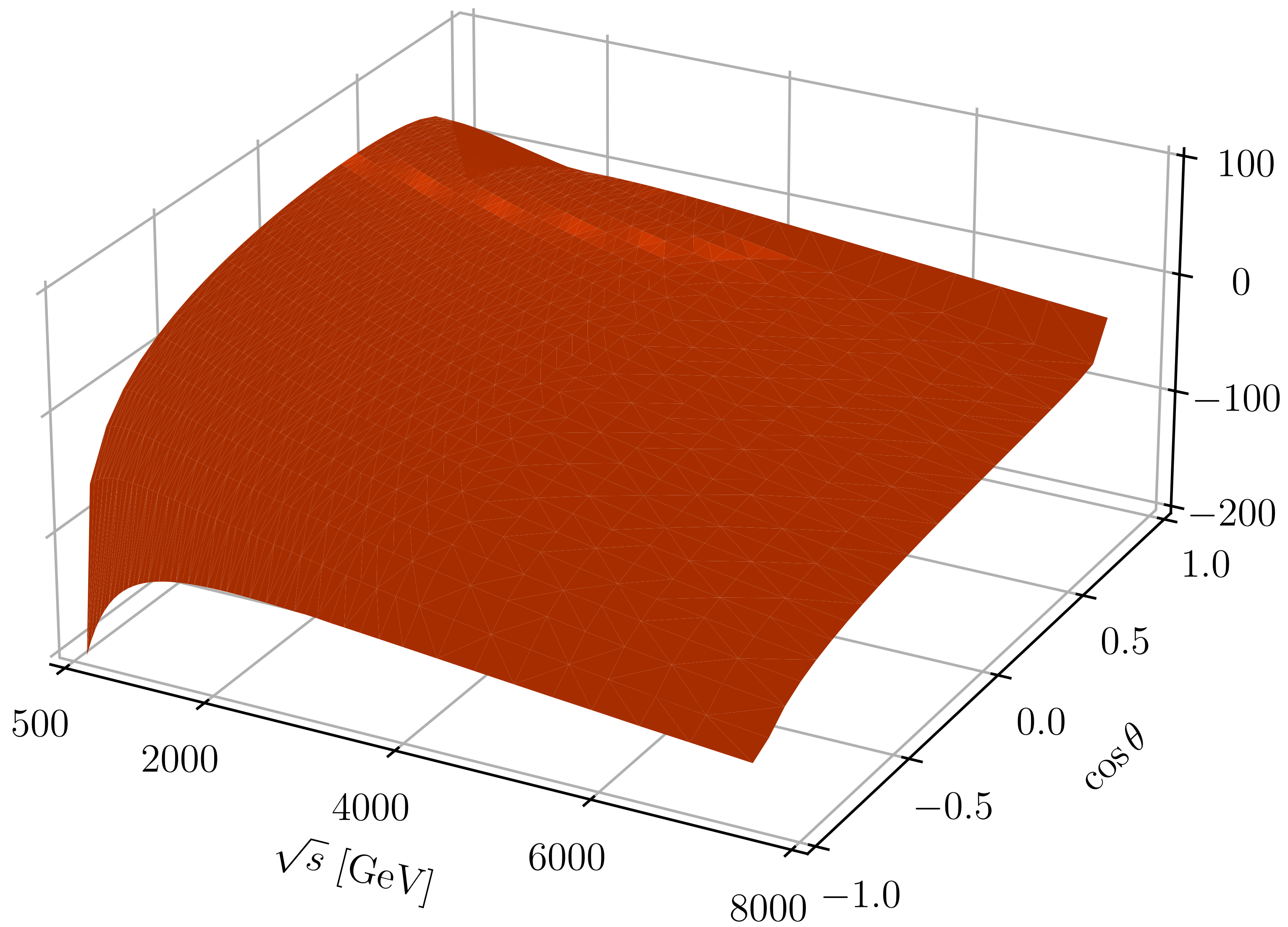


in units $\frac{\alpha}{\pi}\frac{\alpha_s}{\pi}\sigma_0$

- Expected large effects at large transverse masses, analogously to the NC DY case
- Improved theoretical stability in PDFs determination at (sub)percent level
- Relevance in the discussion of the W resonance region, when matching fixed-order and QCD-QED resummation → m_W fit

Finite 2-loop exact QCD-EW virtual corrections to Charged-Current Drell-Yan





in units $\frac{\alpha}{\pi}\frac{\alpha_s}{\pi}\sigma_0$

- Expected large effects at large transverse masses, analogously to the NC DY case
- Improved theoretical stability in PDFs determination at (sub)percent level
- Relevance in the discussion of the W resonance region, when matching fixed-order and QCD-QED resummation → m_W fit

- In the evaluation of the corrections to CC DY we have not optimised the choice of the Master Integrals
  → the diff.eqs. systems are not triangular (like in the NC DY case) but they are generic coupled systems

  SeaSyde is able to handle such systems, achieving a relative precision of 10^{-14} at every phase-space point

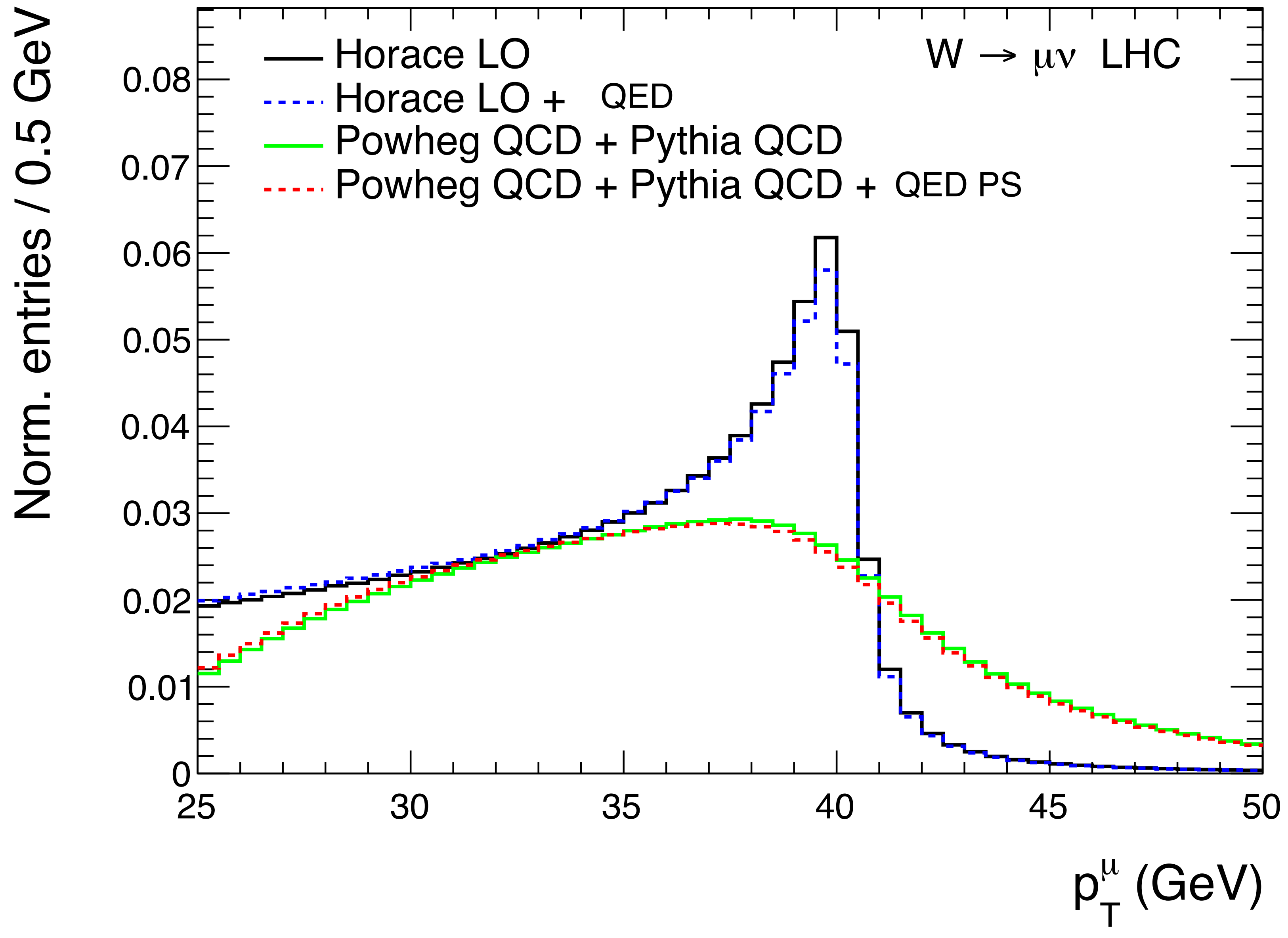
  Potential limitations: the size of the diff.eqs. system can lead to long evaluation time
  Computing the full CC DY grid for LHC applications (3250 points in (s, t)) requires 3 weeks on one 26-core machine

Towards a tool matching QCD+QED resummation with NNLO QCD-EW fixed order

- The exact NNLO QCD-EW corrections yield large effects at large transverse/invariant masses → BSM searches
- Relevance in the discussion of the boson resonance region, when matching fixed-order and QCD-QED resummation → m_W fit

Towards a tool matching QCD+QED resummation with NNLO QCD-EW fixed order

- The exact NNLO QCD-EW corrections yield large effects at large transverse/invariant masses → BSM searches
- Relevance in the discussion of the boson resonance region, when matching fixed-order and QCD-QED resummation → m_W fit



POWHEG simulation NLO QCD+EW +QCDPS + QEDPS

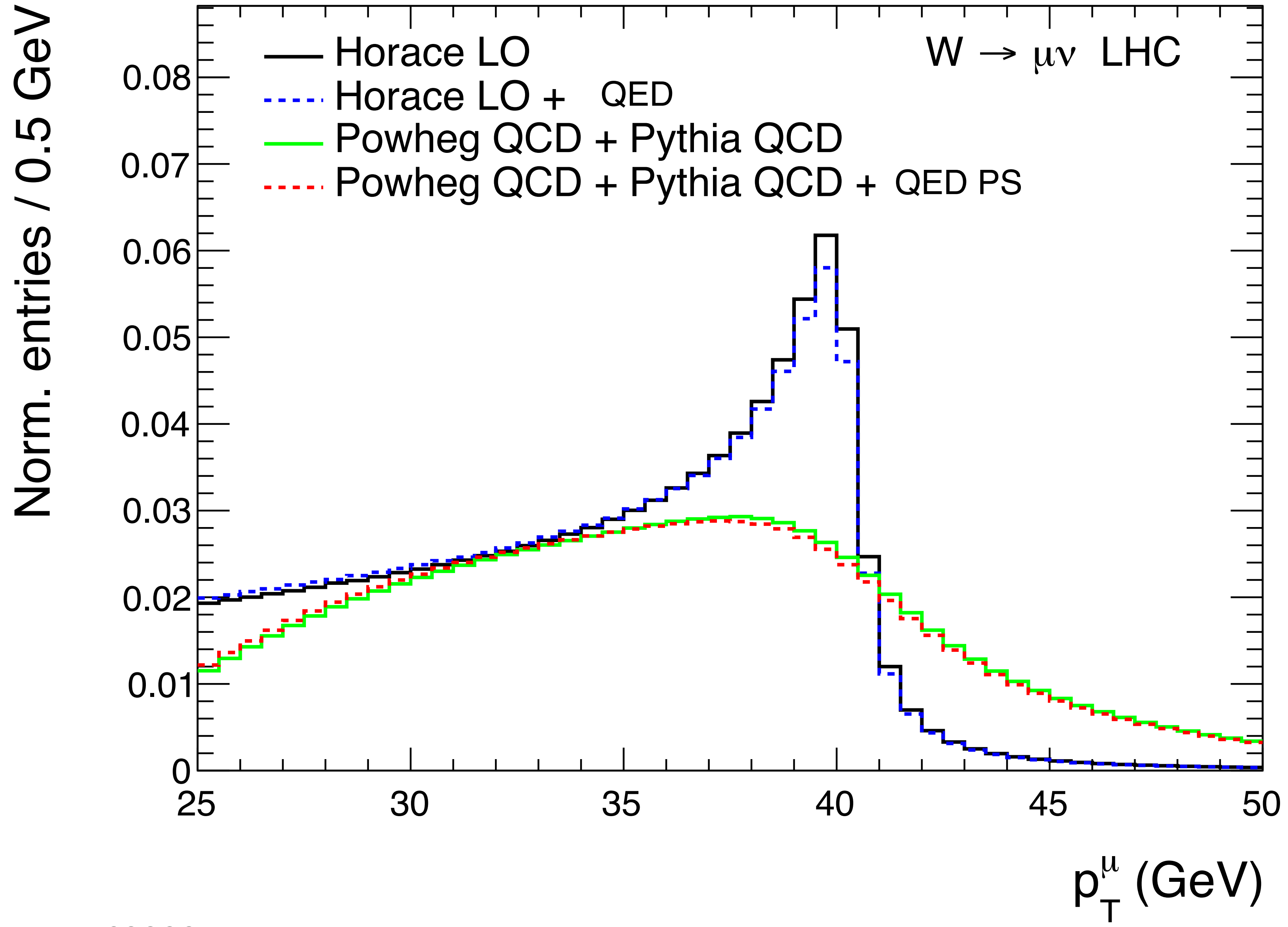
$pp \to W^+$, $\sqrt{s} = 14$ TeV			M_W shifts (MeV)			
Templates accuracy: NLO-QCD+QCD$_{PS}$			$W^+ \to \mu^+\nu$		$W^+ \to e^+\nu$(dres)	
Pseudodata accuracy		QED FSR	M_T	p_T^ℓ	M_T	p_T^ℓ
1	NLO-QCD+(QCD+QED)$_{PS}$	Pythia	-95.2±0.6	-400±3	-38.0±0.6	-149±2
2	NLO-QCD+(QCD+QED)$_{PS}$	Photos	-88.0±0.6	-368±2	-38.4±0.6	-150±3
3	NLO-(QCD+EW)+(QCD+QED)$_{PS}$two-rad	Pythia	-89.0±0.6	-371±3	-38.8±0.6	-157±3
4	NLO-(QCD+EW)+(QCD+QED)$_{PS}$two-rad	Photos	-88.6±0.6	-370±3	-39.2±0.6	-159±2

Huge impact of QED and mixed QCD-QED corrections in the m_W determination

What is the theoretical uncertainty on this estimated shift ?

Towards a tool matching QCD+QED resummation with NNLO QCD-EW fixed order

- The exact NNLO QCD-EW corrections yield large effects at large transverse/invariant masses → BSM searches
- Relevance in the discussion of the boson resonance region, when matching fixed-order and QCD-QED resummation → m_W fit

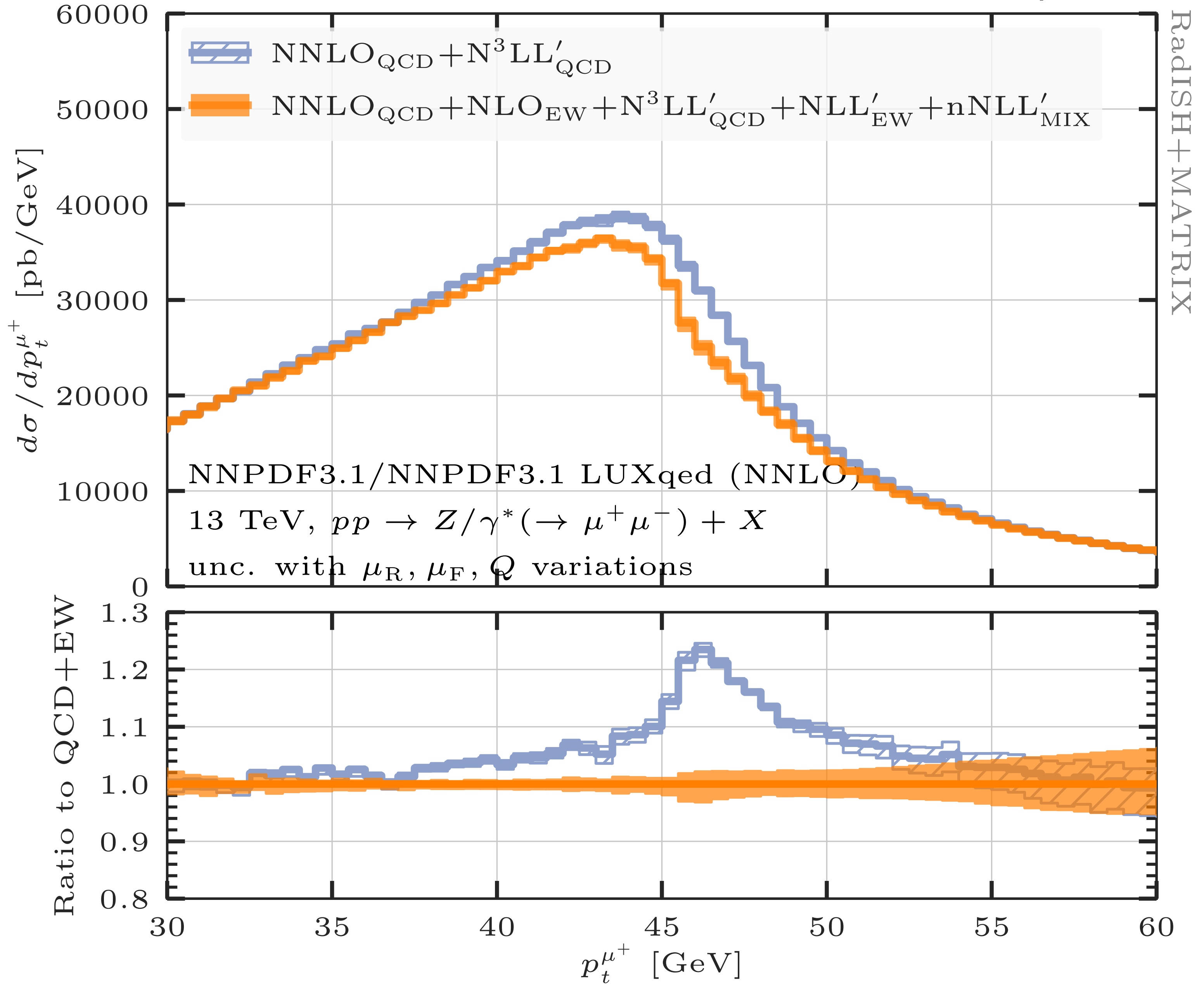


POWHEG simulation NLO QCD+EW +QCDPS + QEDPS

	$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: NLO-QCD+QCD$_{\rm PS}$ Pseudodata accuracy	QED FSR	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^ℓ	 $W^+ \to e^+\nu$(dres) M_T	 p_T^ℓ
1	NLO-QCD+(QCD+QED)$_{\rm PS}$	Pythia	-95.2±0.6	-400±3	-38.0±0.6	-149±2
2	NLO-QCD+(QCD+QED)$_{\rm PS}$	Photos	-88.0±0.6	-368±2	-38.4±0.6	-150±3
3	NLO-(QCD+EW)+(QCD+QED)$_{\rm PS}$two-rad	Pythia	-89.0±0.6	-371±3	-38.8±0.6	-157±3
4	NLO-(QCD+EW)+(QCD+QED)$_{\rm PS}$two-rad	Photos	-88.6±0.6	-370±3	-39.2±0.6	-159±2

Huge impact of QED and mixed QCD-QED corrections in the m_W determination

What is the theoretical uncertainty on this estimated shift ?



L.Buonocore, L.Rottoli, P.Torrielli, arXiv:2404.15112

Matching in full QCD-EW SM at N3LL'-QCD + NLL'-EW + nNLL'-mixed accuracy including QED effects from all charged legs (see P.Torrielli's talk)

Matching with the exact NNLO QCD-EW will be needed to reach full NNLL-mixed

→ Reliable estimate of the reduced residual theoretical uncertainties

Concluding remarks

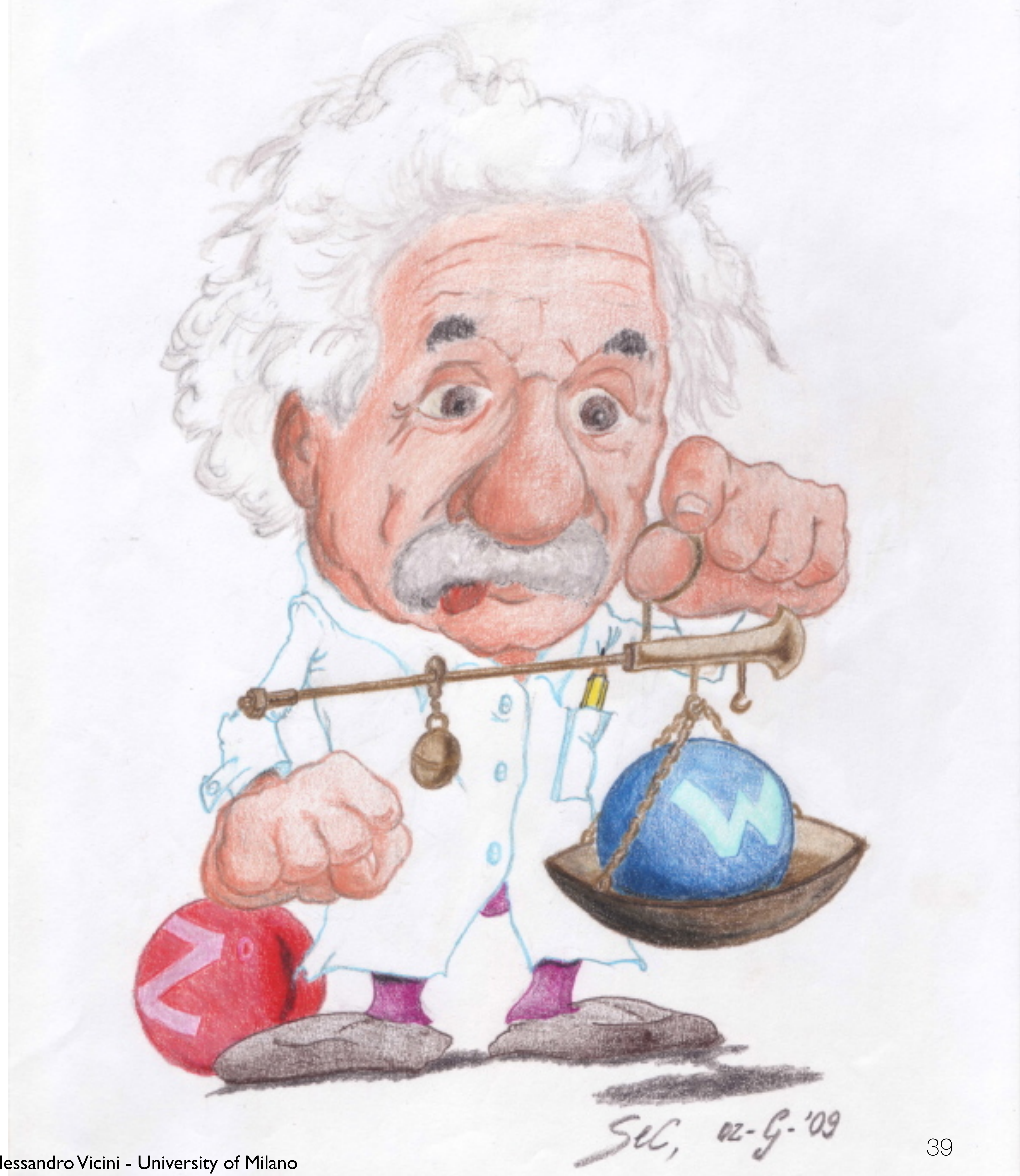
Conclusions

The precision tests of the Standard Model at the LHC are an active research field
They require the development of advanced computational techniques to evaluate complex 2-loop amplitude

The semi-automatic evaluation, with arbitrary numerical precision
of the exact mixed QCD-EW corrections to the NC- and CC-DY processes
opens the way to a new class of calculations

The cross section evaluation requires a non-trivial infrastructure
to consistently include all the real and virtual sets of corrections (e.g. Matrix)

The matching of these fixed-order results with a joined QCD-QED all orders resummation
will allow a robust estimate of the theoretical uncertainties affecting the W-mass determination



Thank you

back-up

Computational framework of NNLO QCD-EW corrections to NC DY

The complete calculation has been included in the Munich/Matrix framework

- fully automatic generation and bookkeeping of all the double-real and real-virtual contributions based on an interface with OpenLoops and Recola/Collier
- the 2-loop virtual corrections are separately computed and provided in fast-evaluation format

In this specific framework, main compatibility requirement to include the double-virtual corrections:
the q_T-subtraction formalism to handle the IR singularities (Catani, Grazzini, 2007)

Upon inclusion of the appropriate scheme-dependent subtraction term,
the double virtual corrections can be used with any other simulation code

The q_T-subtraction and the residual cut-off dependency

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

When $q_T/Q > r_{cut}$ the double-real and the real-virtual contributions, subtracted with CS dipoles, are finite

$d\sigma_{CT}^{(1,1)}$ is obtained by expanding to fixed order the q_T resummation formula

The q_T-subtraction and the residual cut-off dependency

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

When $q_T/Q > r_{cut}$ the double-real and the real-virtual contributions, subtracted with CS dipoles, are finite

$d\sigma_{CT}^{(1,1)}$ is obtained by expanding to fixed order the q_T resummation formula

Logarithmic sensitivity on r_{cut} in the double unresolved limit $\int d\sigma_R^{(1,1)} \sim \sum_{i=1}^{4} c_i \ln^i r_{cut} + c_0 + \mathcal{O}(r_{cut}^m)$

The counterterm removes the IR sensitivity to the cutoff variable $\int \left(d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right) \sim c_0 + \mathcal{O}(r_{cut}^m)$

→ we need small values of the cutoff

→ explicit numerical tests to quantify the bias induced by the cutoff choice (cfr. Buonocore, Kallweit, Rottoli, Wiesemann, arXiv:2111.13661 Camarda, Cieri, Ferrera, arXiv:2111.14509)

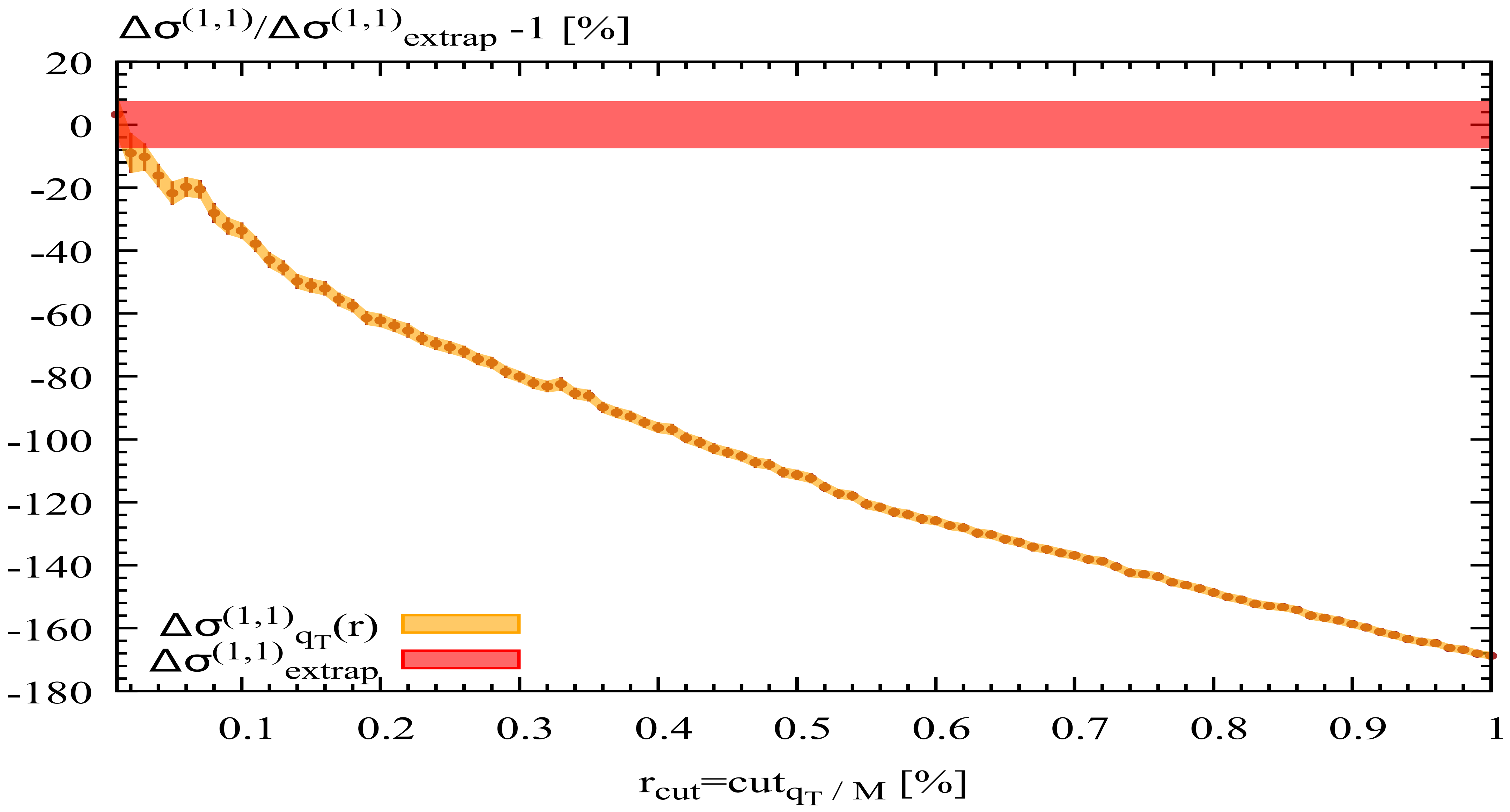
we can fit the r_{cut} dependence and extrapolate in the $r_{cut} \to 0$ limit

Dependence on r_{cut} of the NNLO QCD-EW corrections to NC DY

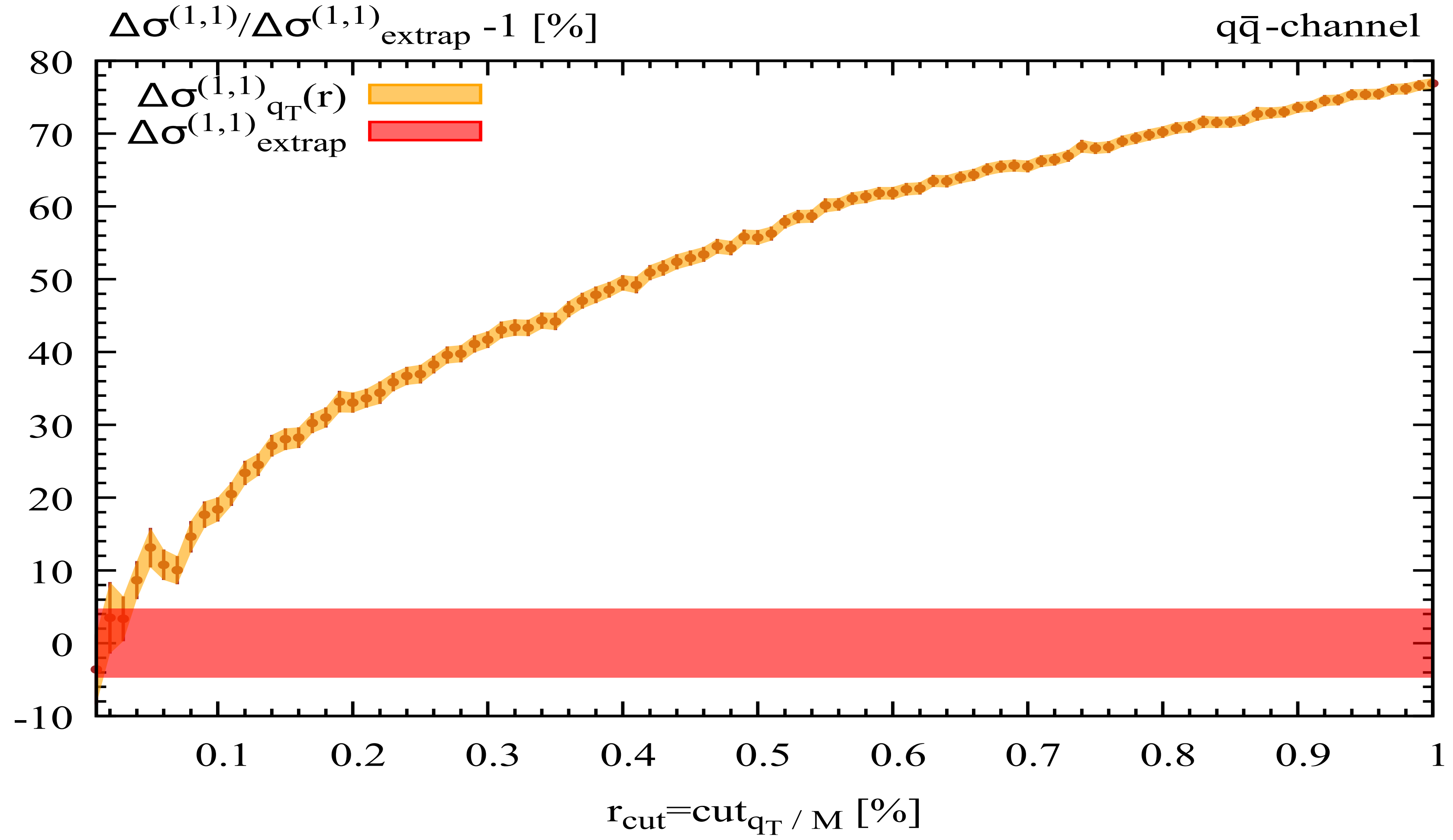
courtesy of S.Kallweit

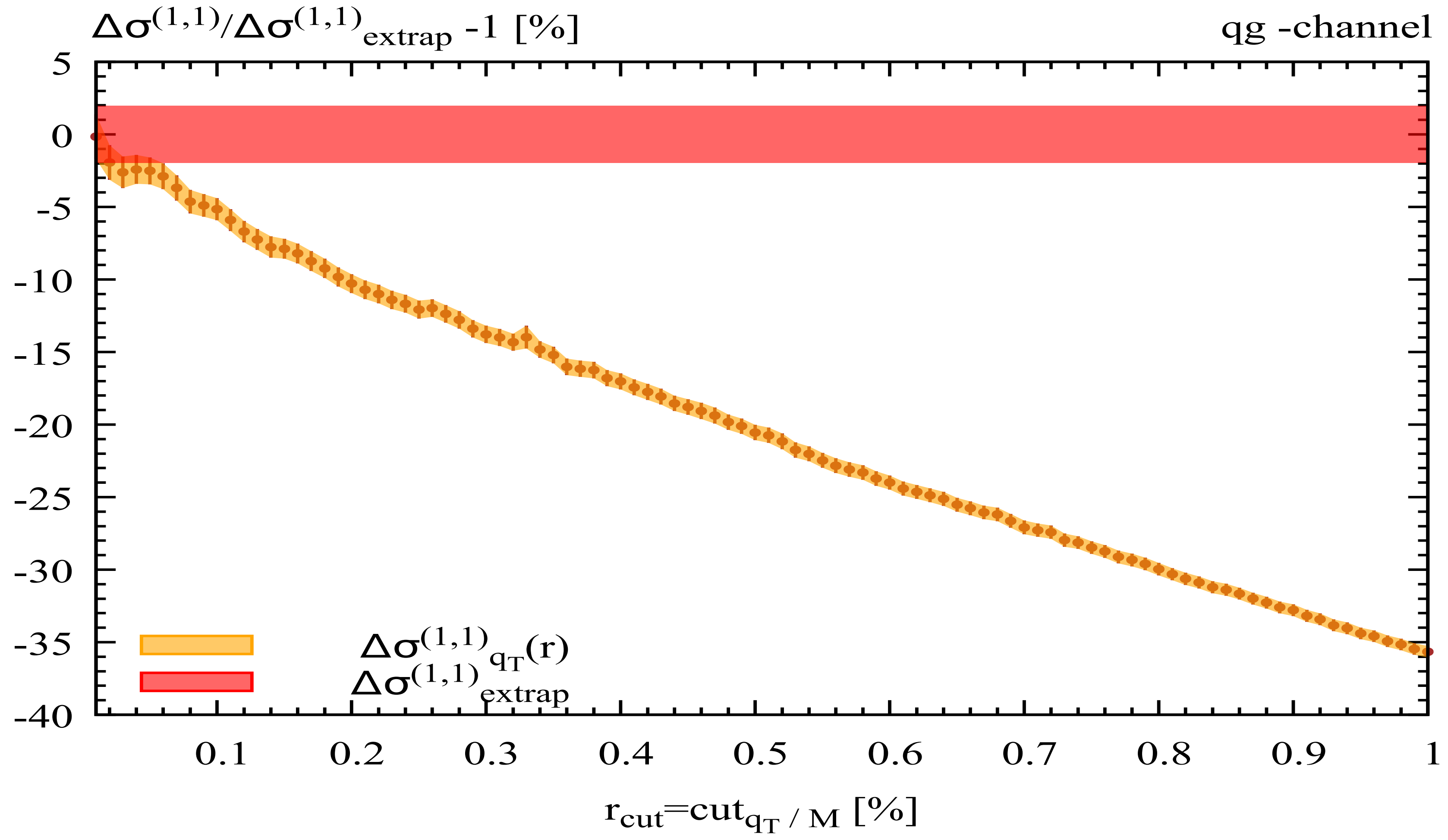
Symmetric-cut scenario

$p_{T,\ell^\pm} > 25$ GeV $\quad y_{\ell^\pm} < 2.5 \quad m_{\ell\ell} > 50$ GeV



Splitting into partonic channels





- **large power corrections in $r_{\rm cut}$ for mixed corrections**
  - explained by overall small size of corrections, and in parts also by cancellation between partonic channels
- **by far less dramatic dependence at level of cross sections**
  - better than permille precision at inclusive level

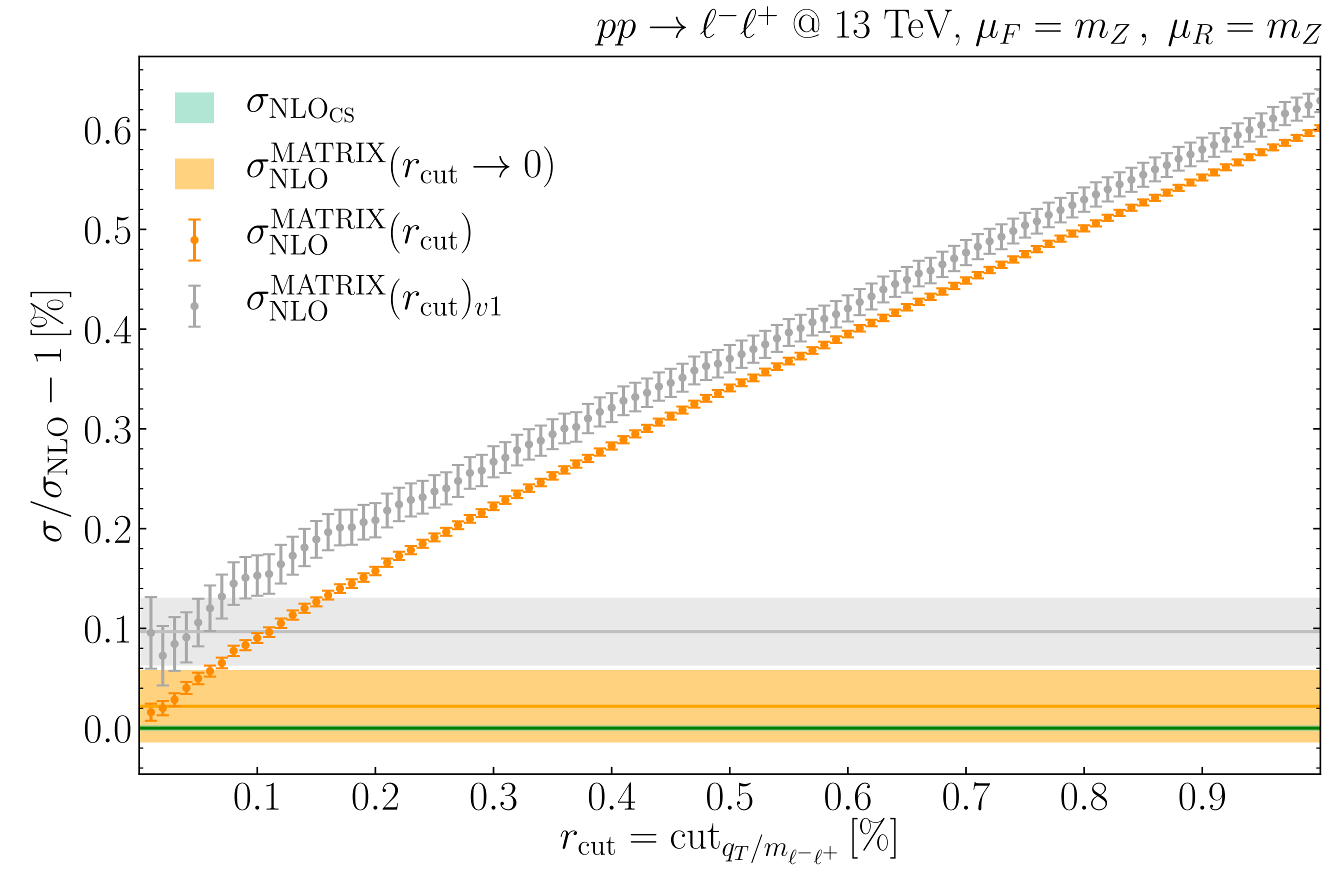
The q_T-subtraction and the residual cut-off dependency in different acceptance setups

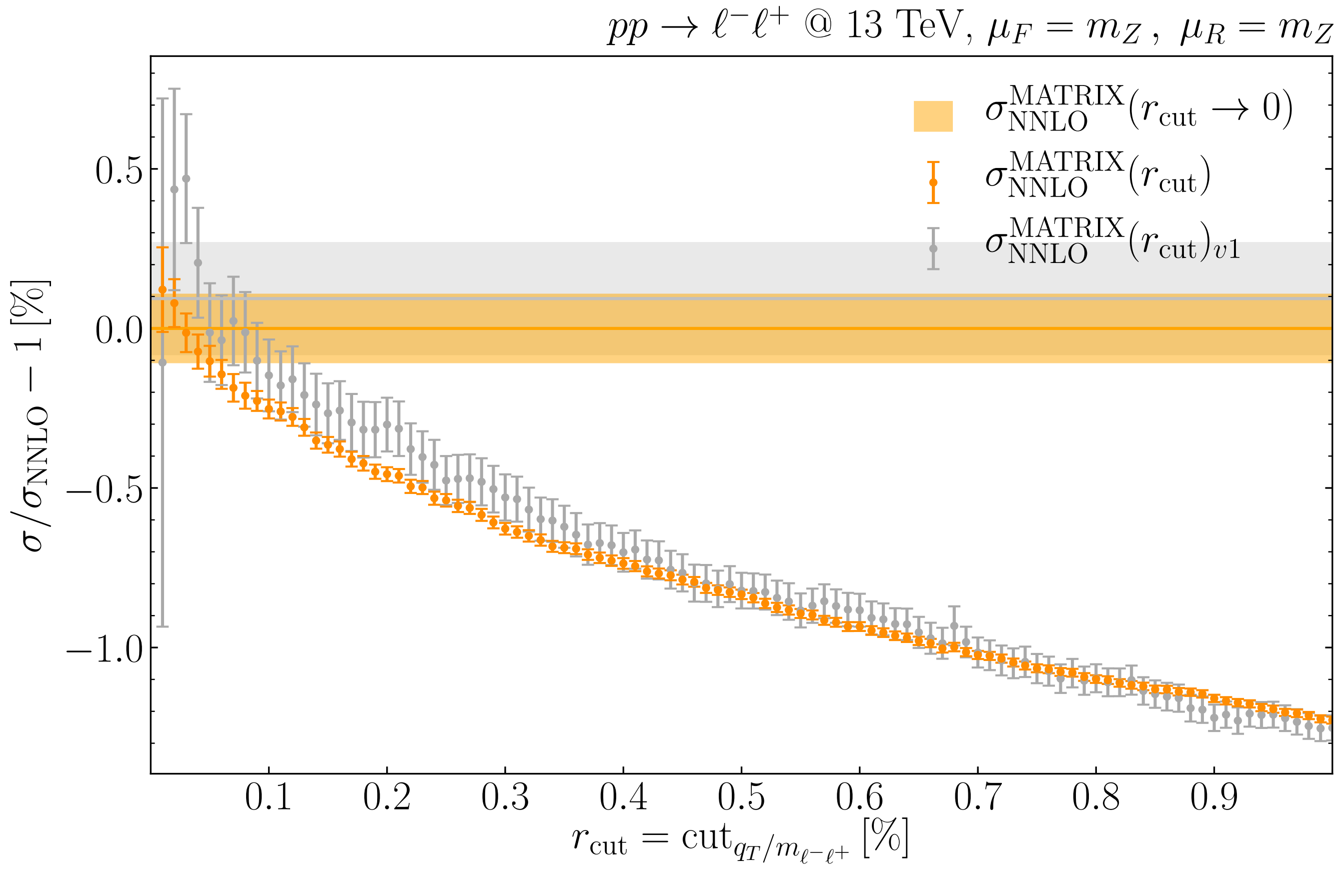
courtesy of S.Kallweit

(cfr. Buonocore, Kallweit, Rottoli, Wiesemann, 2111.13661)

Symmetric cuts

- $p_{\mathrm{T},\ell^\pm} > 25$ GeV

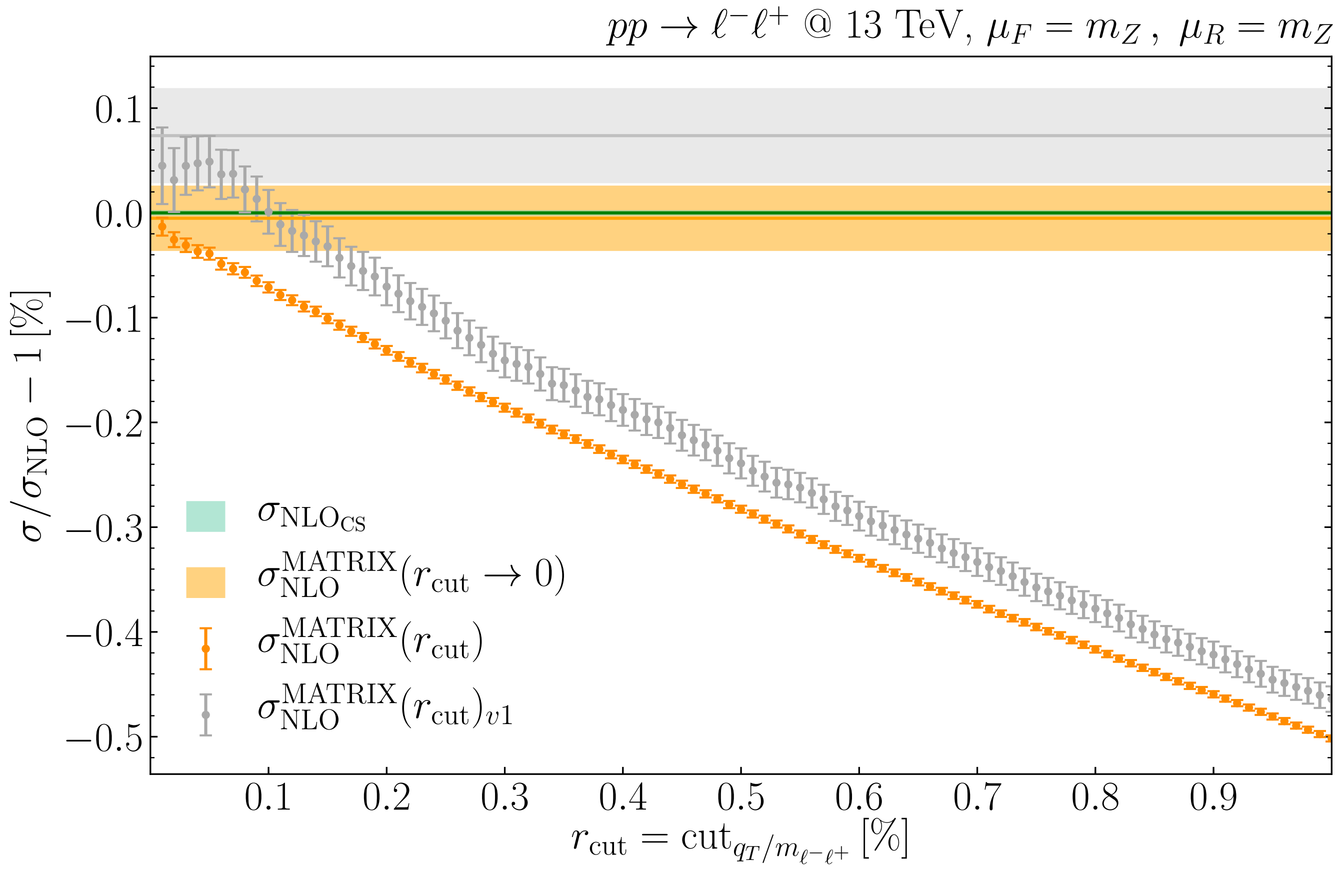


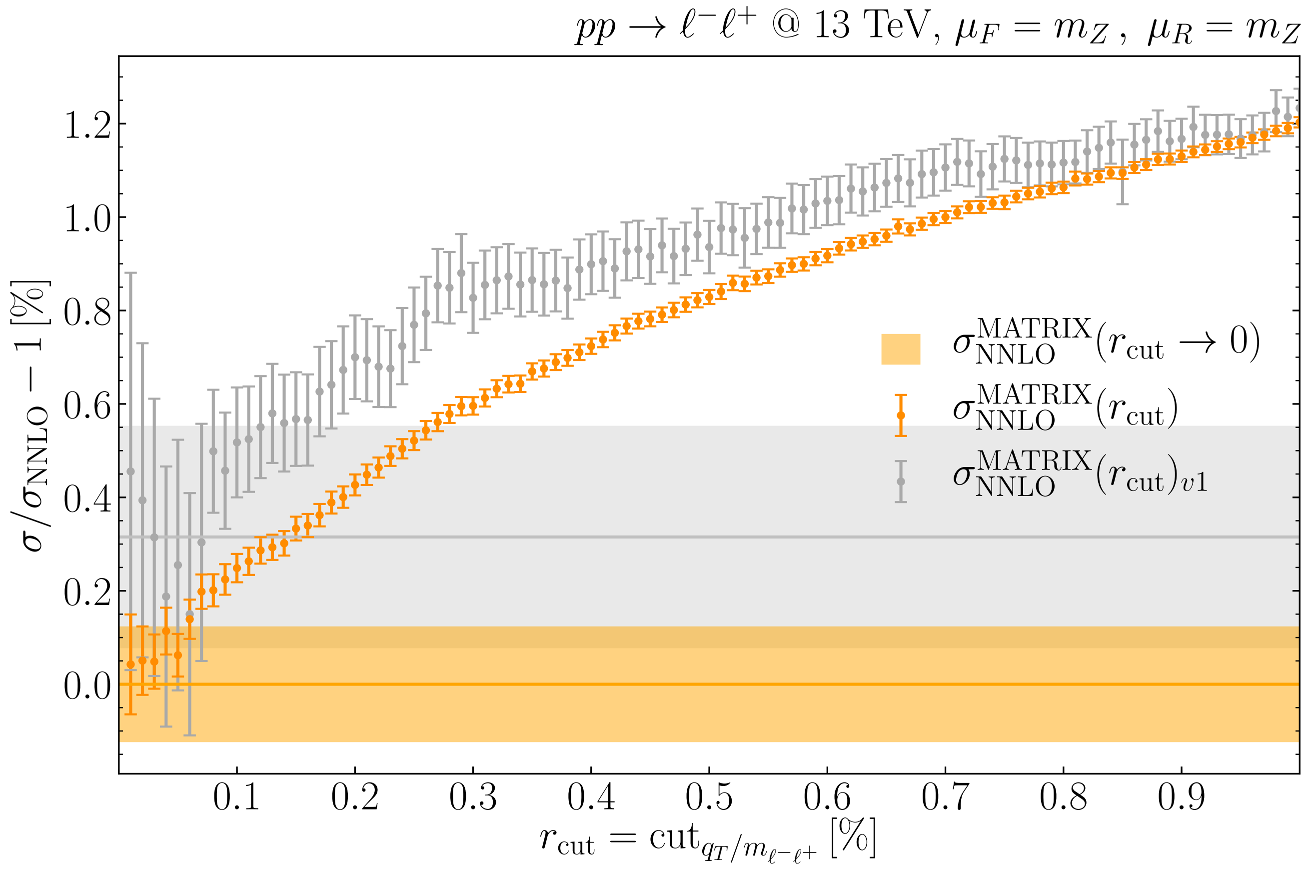


➥ large power corrections in r_{cut}

Asymmetric cuts on ℓ_1 and ℓ_2

- $p_{\mathrm{T},\ell_1} > 25$ GeV $p_{\mathrm{T},\ell_2} > 20$ GeV

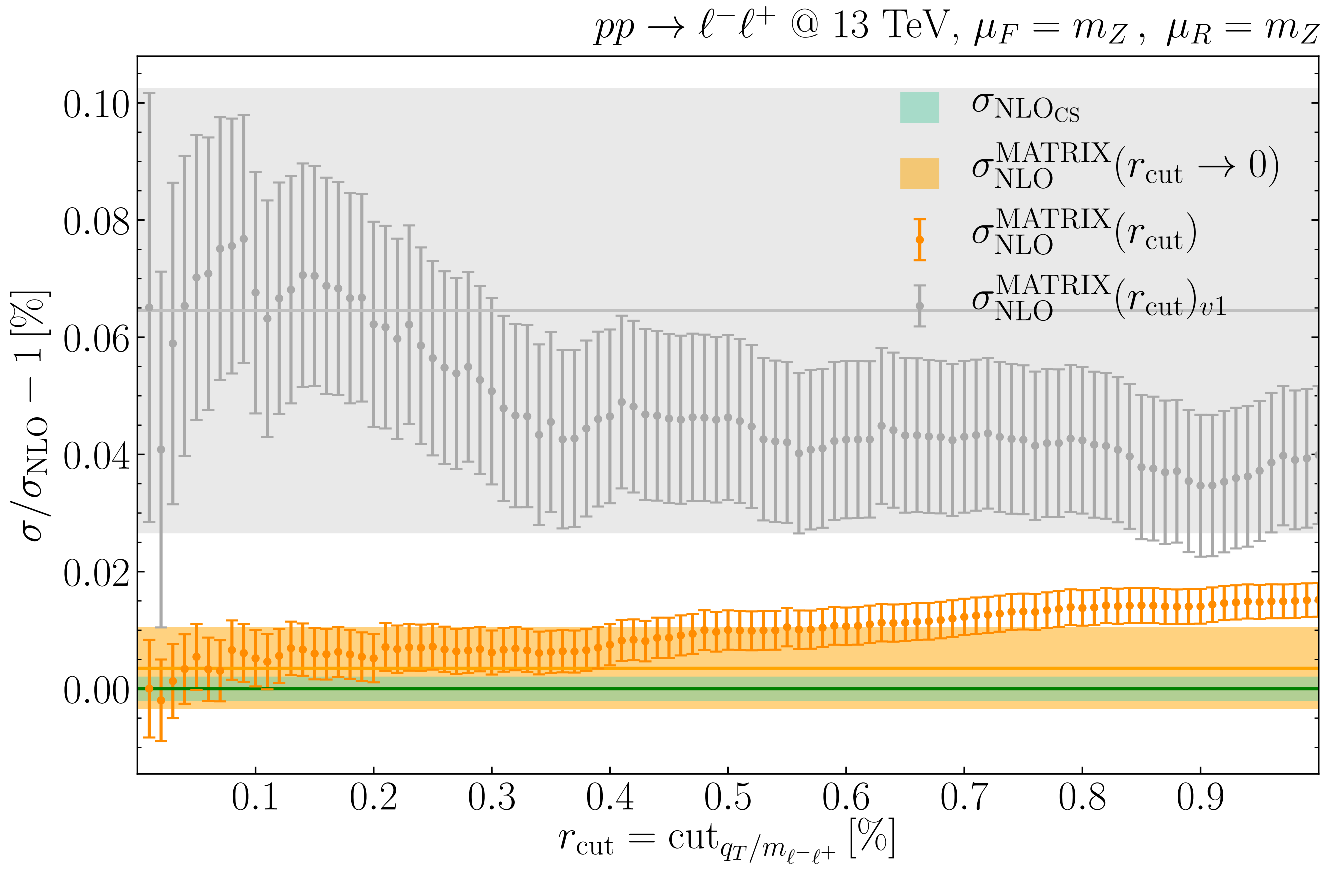


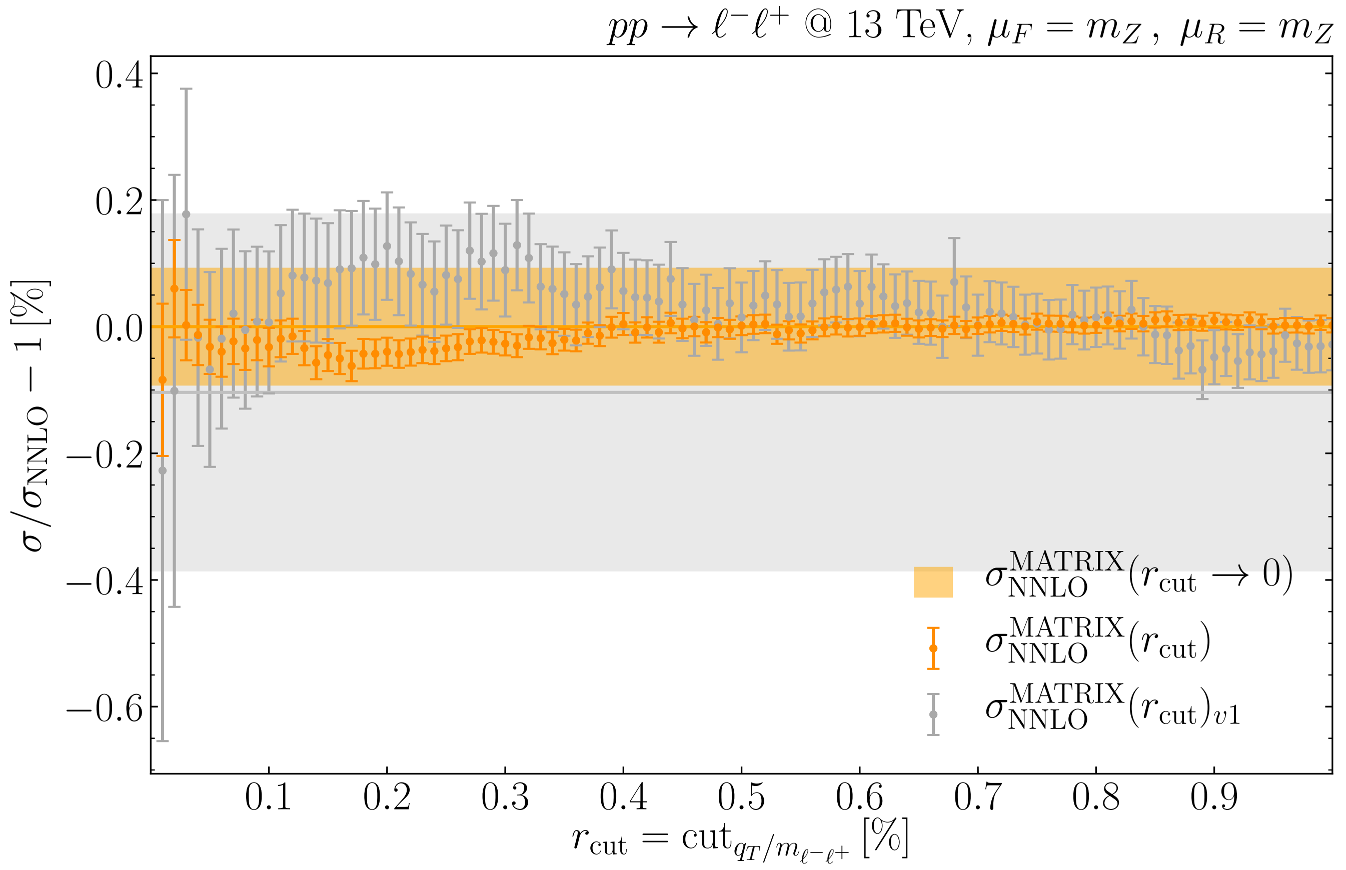


➥ large power corrections in r_{cut}

Asymmetric cuts on ℓ^+ and ℓ^-

- $p_{\mathrm{T},\ell^+} > 25$ GeV $p_{\mathrm{T},\ell^-} > 20$ GeV



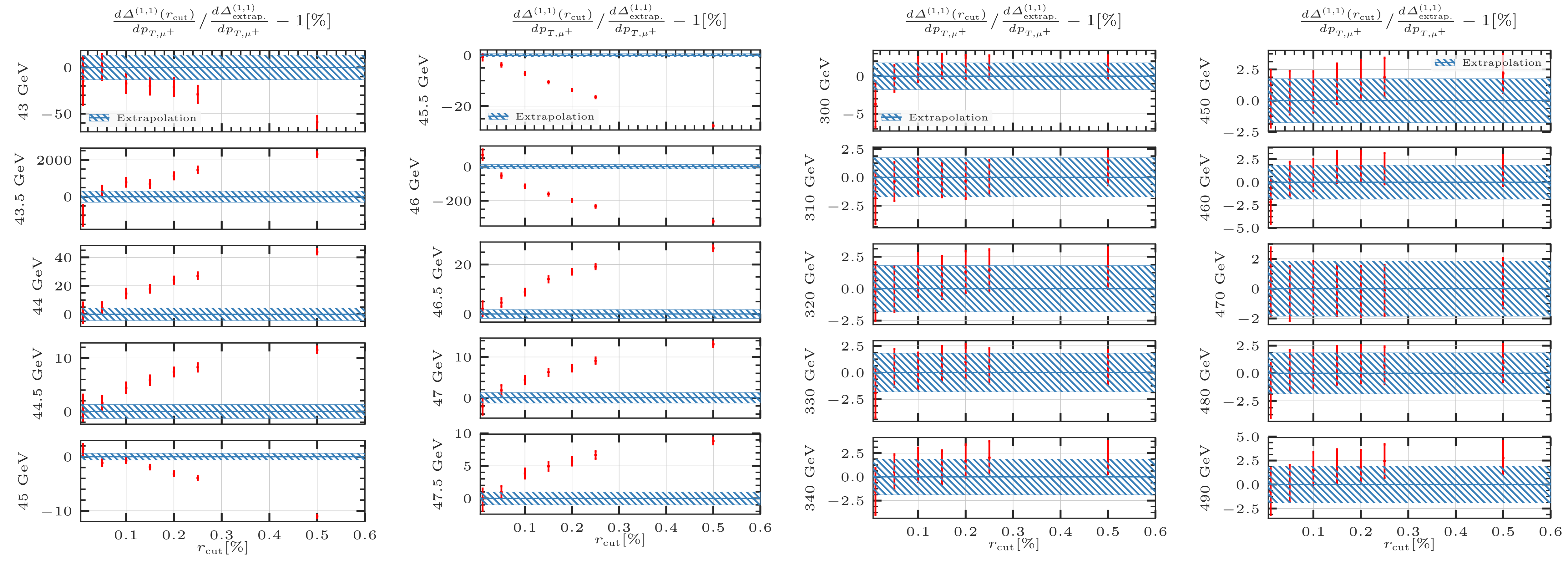


➥ no significant dependence on r_{cut}

Differential sensitivity to r_{cut}

Binwise r_{cut} dependence of the mixed NNLO QCD–EW corrections for NC Drell–Yan

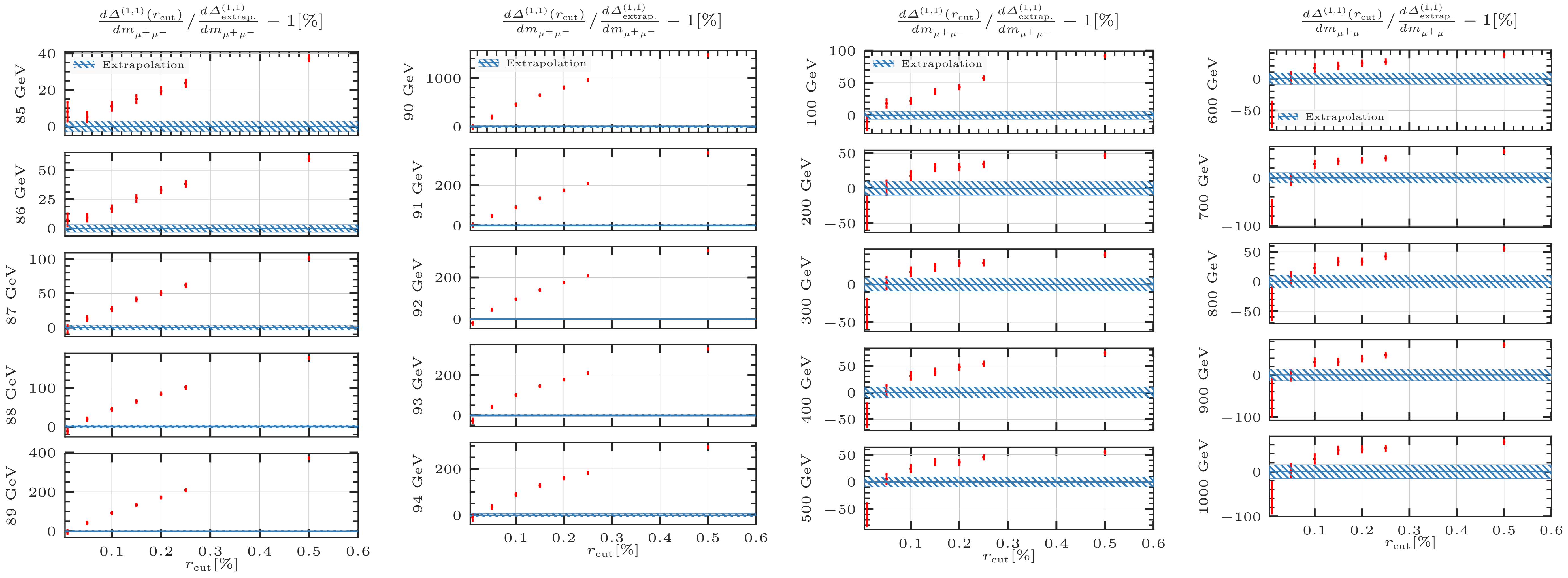
Differential distribution in p_{T,μ^+}: peak (left panels) and tail (right panels) regions



- ➥ large r_{cut} dependence in particular around the peak of the distribution, and typically precision of $\lesssim 3\%$ on the relative mixed QCD–EW corrections (artificially large where corrections are basically zero)

Binwise r_{cut} dependence of the mixed NNLO QCD–EW corrections for NC Drell–Yan

Differential distribution in $m_{\mu^+\mu^-}$: peak (left panels) and tail (right panels) regions



- ➥ quite large r_{cut} dependence throughout, and lower numerical precision of $\lesssim 10\%$ on the relative mixed QCD–EW corrections (but still permille-level precision at the level of cross sections)

The hard-virtual coefficient

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

$$\mathscr{H}^{(1,1)} = H^{(1,1)}\, C_1\, C_2$$

The process independent collinear functions C_1, C_2 are known up to N3LO

The process dependent hard function H is defined
upon subtraction of the universal IR contributions

The hard-virtual coefficient

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathcal{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

$$\mathcal{H}^{(1,1)} = H^{(1,1)} C_1 C_2$$

The process independent collinear functions C_1, C_2 are known up to N3LO

The process dependent hard function H is defined upon subtraction of the universal IR contributions

$$2\mathrm{Re}\langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(1,1)} \rangle = \sum_{k=-4}^{0} \varepsilon^k f_i(s,t,m)$$

after UV renormalisation the poles are only of IR origin

$$|\mathcal{M}_{fin}\rangle \equiv (1 - I) |\mathcal{M}\rangle \qquad H \propto \langle \mathcal{M}_0 | \mathcal{M}_{fin} \rangle$$

$$H^{(1,0)} = \frac{2\mathrm{Re}\langle \mathcal{M}^{(0,0)} | \mathcal{M}_{fin}^{(1,0)} \rangle}{|\mathcal{M}^{(0,0)}|^2}, \qquad H^{(0,1)} = \frac{2\mathrm{Re}\langle \mathcal{M}^{(0,0)} | \mathcal{M}_{fin}^{(0,1)} \rangle}{|\mathcal{M}^{(0,0)}|^2}, \qquad H^{(1,1)} = \frac{2\mathrm{Re}\langle \mathcal{M}^{(0,0)} | \mathcal{M}_{fin}^{(1,1)} \rangle}{|\mathcal{M}^{(0,0)}|^2}$$

NLO-QCD NLO-EW NNLO QCD-EW

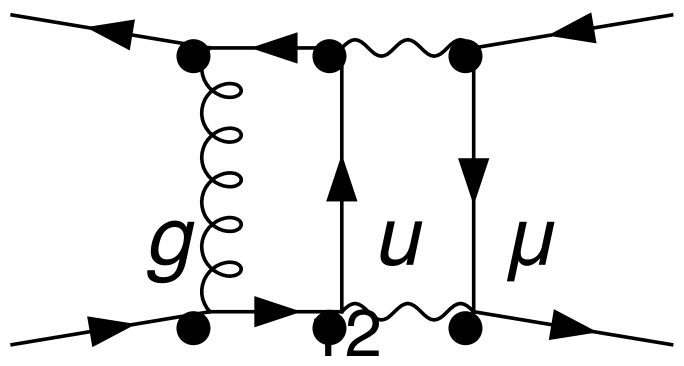
The double virtual amplitude: regularisation of the IR divergences

The evaluation of the amplitudes is done in $n = 4 - 2\varepsilon$ dimensions

In the q_T-subtraction formalism, the final state leptons are massive, yielding mass singular logarithms
→ also the 2-loop virtual corrections should be evaluated with massive leptons

We start with a fully massive final state 2-loop amplitude
We retain only collinear singular terms ($\sim \log(m_l^2/M_Z^2)$) and discard those suppressed by a power of m_l^2/M_Z^2

Among the 2-loop boxes



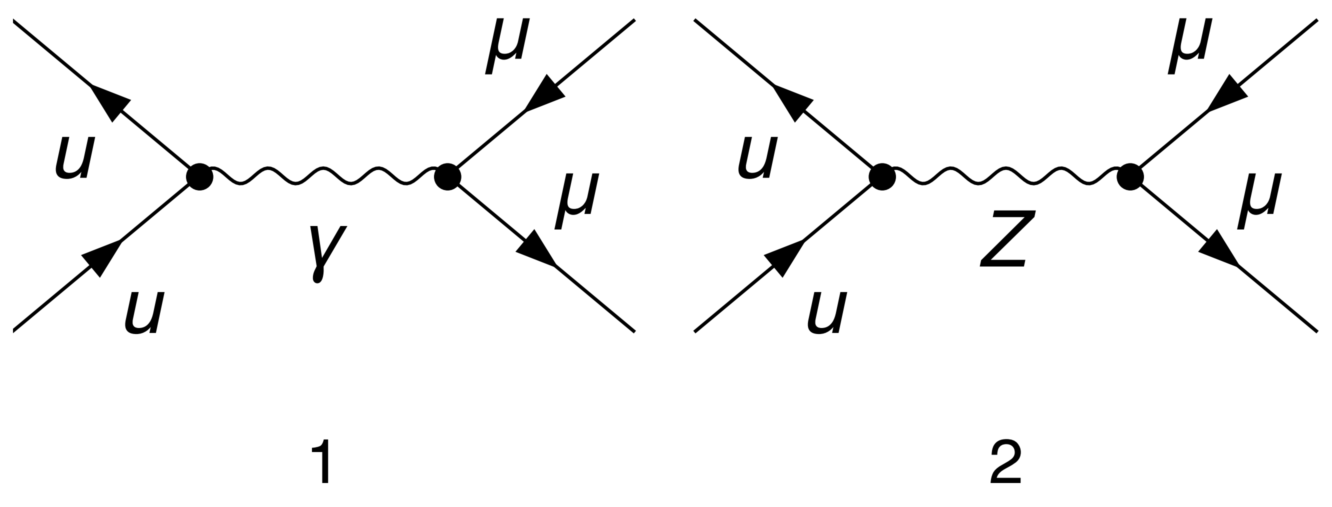
WW and ZZ boxes do not develop collinear singularities
→ evaluated with Master Integrals with massless external lines

$\gamma\gamma$ and γZ boxes individually develop collinear singularities, but in the sum they exactly cancel

→ explicit check in the $\gamma\gamma$ case, based on the massive MIs known from $t\bar{t}$ production
in the γZ check that the residual singularity is the soft divergence

The double virtual amplitude: generation of the amplitude

$\mathcal{M}^{(0,0)}(q\bar{q} \to l\bar{l}) =$



$\mathcal{M}^{(1,1)}(q\bar{q} \to l\bar{l}) =$ O(1000) self-energies + O(300) vertex corrections +O(130) box corrections + 1loop x 1loop (before discarding all those vanishing for colour conservation, e.g. no fermonic triangles)

Two independent calculations based on QGraf and FeynArts in the EW Background Field Gauge

The BFG choice guarantees the validity of EW Ward identities for the initial state vertex → additional technical checks

- UV finiteness when combining 2-loop vertex and quark WF in the full EW SM → that combination has only IR poles
- UV renormalisation is confined to the gauge-boson propagators sector, where IR divergences are absent

The 1-loop check of the gauge-parameter independence identifies those subsets of diagrams yielding the cancellation.

The 2-loop calculation is organised splitting the total amplitude in the combination of different subsets, according to their EW charges (# of Ws, Zs, γs)

The double virtual amplitude: UV renormalization

G.Degrassi, AV, hep-ph/0307122, S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229 S.Dittmaier, arXiv:2101.05154

Complex mass scheme

$$\mu_{W0}^2 = \mu_W^2 + \delta\mu_W^2, \quad \mu_{Z0}^2 = \mu_Z^2 + \delta\mu_Z^2, \quad e_0 = e + \delta e$$

$$\frac{\delta s^2}{s^2} = \frac{c^2}{s^2}\left(\frac{\delta\mu_Z^2}{\mu_Z^2} - \frac{\delta\mu_W^2}{\mu_W^2}\right)$$

the mass counterterms are defined at the complex pole of the propagator

the weak mixing angle is complex valued $c^2 \equiv \mu_W^2/\mu_Z^2$

BFG EW Ward identity → cancellation of the UV divergences combining vertex and fermion WF corrections

The bare couplings of Z and photon to fermions in the (G_μ, μ_W, μ_Z) input scheme are given by

$$\frac{g_0}{c_0} = \sqrt{4\sqrt{2}G_\mu\mu_Z^2}\left[1 - \frac{1}{2}\Delta r + \frac{1}{2}\left(2\frac{\delta e}{e} + \frac{s^2 - c^2}{c^2}\frac{\delta s^2}{s^2}\right)\right] \equiv \sqrt{4\sqrt{2}G_\mu\mu_Z^2}\left(1 + \delta g_Z^{G_\mu}\right)$$

$$g_0 s_0 = \sqrt{4\sqrt{2}G_\mu\mu_W^2 s^2}\left[1 + \tfrac{1}{2}\left(-\Delta r + 2\tfrac{\delta e}{e}\right)\right] \equiv e_{ren}^{G_\mu}\left(1 + \delta g_A^{G_\mu}\right)$$

Gauge boson renormalised propagators

$$\Sigma_{R,T}^{AA}(q^2) = \Sigma_T^{AA}(q^2) + 2\,q^2\,\delta g_A$$

$$\Sigma_{R,T}^{ZZ}(q^2) = \Sigma_T^{ZZ}(q^2) - \delta\mu_Z^2 + 2\,(q^2 - \mu_Z^2)\,\delta g_Z$$

$$\Sigma_{R,T}^{AZ}(q^2) = \Sigma_T^{AZ}(q^2) - q^2\,\frac{\delta s^2}{sc}$$

$$\Sigma_{R,T}^{ZA}(q^2) = \Sigma_T^{ZA}(q^2) - q^2\,\frac{\delta s^2}{sc},$$

After the UV renormalisation, the singular structure is entirely due to IR soft and/or collinear singularities

The double virtual amplitude: γ_5 treatment

The absence of a consistent definition of γ_5 in $n = 4 - 2\varepsilon$ dimensions yields a practical problem

The trace of Dirac matrices and γ_5 is a polynomial in ε
The UV or IR divergences of Feynman integrals appear as poles $1/\varepsilon$

$$\mathrm{Tr}(\gamma_\alpha \ldots \gamma_\mu \gamma_5) \times \int d^n k \frac{1}{[k^2 - m_0^2][(k+q_1)^2 - m_1^2][(k+q_2)^2 - m_2^2]} \sim (a_0 + a_1 \varepsilon + \ldots) \times \left(\frac{c_{-2}}{\varepsilon^2} + \frac{c_{-1}}{\varepsilon} + c_0 + \ldots \right)$$

If a_1 is evaluated in a non-consistent way,
then poles might not cancel and the finite part of the xsec might have a spurious contribution

The double virtual amplitude: γ_5 treatment

The absence of a consistent definition of γ_5 in $n = 4 - 2\varepsilon$ dimensions yields a practical problem

The trace of Dirac matrices and γ_5 is a polynomial in ε
The UV or IR divergences of Feynman integrals appear as poles $1/\varepsilon$

$$Tr(\gamma_\alpha \ldots \gamma_\mu \gamma_5) \times \int d^n k \frac{1}{[k^2 - m_0^2][(k+q_1)^2 - m_1^2][(k+q_2)^2 - m_2^2]} \sim (a_0 + a_1 \varepsilon + \ldots) \times \left(\frac{c_{-2}}{\varepsilon^2} + \frac{c_{-1}}{\varepsilon} + c_0 + \ldots \right)$$

If a_1 is evaluated in a non-consistent way,
then poles might not cancel and the finite part of the xsec might have a spurious contribution

- 't Hooft-Veltman treat γ_5 (anti)commuting in (4) $n-4$ dimensions preserving the cyclicity of the traces (one counterterm is needed)
- Kreimer treats γ_5 anticommuting in n dimensions, abandoning the cyclicity of the traces (→ need of a starting point)

- Heller, von Manteuffel, Schabinger verified that the IR-subtracted squared matrix element are identical in the two approaches

- we adopted the naive anticommuting prescription (Kreimer); we use $\gamma_5 = \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$ to compute traces with one γ_5
  - we computed the 2-loop amplitude and, independently, the IR subtraction term; both depend on the prescription chosen
  - the cancellation of all the lowest order poles is checked (and non trivial)
  - absence of fermionic triangles because of colour conservation

Differential equations and IBPs

- Not all the Feynman integrals in one amplitude are independent
  → exploit Integration-by-parts (IBP) and Lorentz identities to reduce to a basis of independent Master Integrals

$$\int \frac{d^n k_1}{(2\pi)^n} \int \frac{d^n k_2}{(2\pi)^n} \frac{\partial}{\partial k_1^\mu} \frac{(k_1^\mu, k_2^\mu, p_r^\mu)}{[k_1^2 - m_0^2]^{\alpha_0} [(k_1+p_1)^2 - m_1^2]^{\alpha_1} \dots [(k_1+k_2+p_j)^2 - m_j^2]^{\alpha_j} \dots [(k_2+p_l)^2 - m_l^2]^{\alpha_l}} = 0$$

$$\int \frac{d^n k_1}{(2\pi)^n} \int \frac{d^n k_2}{(2\pi)^n} \frac{\partial}{\partial k_2^\mu} \frac{(k_1^\mu, k_2^\mu, p_r^\mu)}{[k_1^2 - m_0^2]^{\alpha_0} [(k_1+p_1)^2 - m_1^2]^{\alpha_1} \dots [(k_1+k_2+p_j)^2 - m_j^2]^{\alpha_j} \dots [(k_2+p_l)^2 - m_l^2]^{\alpha_l}} = 0$$

- Henn's conjecture (2013): if a change of basis exists which leads to $d\vec{J}(\vec{s};\varepsilon) = \varepsilon \tilde{\mathbf{A}}(\vec{s}) \cdot \vec{J}(\vec{s};\varepsilon)$
  then the solution is expressed in terms of iterated integrals (Chen integral representation)
  depending only on the results at previous orders in the ε expansion

Differential equations and IBPs

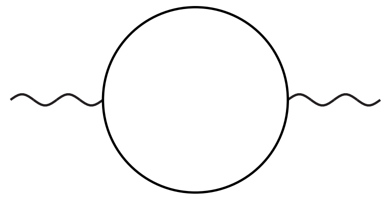
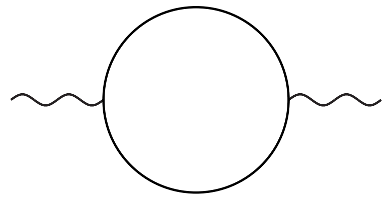
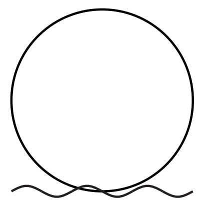
- Not all the Feynman integrals in one amplitude are independent
  → exploit Integration-by-parts (IBP) and Lorentz identities to reduce to a basis of independent Master Integrals

$$\int \frac{d^n k_1}{(2\pi)^n} \int \frac{d^n k_2}{(2\pi)^n} \frac{\partial}{\partial k_1^\mu} \frac{(k_1^\mu,\, k_2^\mu,\, p_r^\mu)}{[k_1^2 - m_0^2]^{\alpha_0}\, [(k_1+p_1)^2 - m_1^2]^{\alpha_1} \,\ldots\, [(k_1+k_2+p_j)^2 - m_j^2]^{\alpha_j} \,\ldots\, [(k_2+p_l)^2 - m_l^2]^{\alpha_l}} = 0$$

$$\int \frac{d^n k_1}{(2\pi)^n} \int \frac{d^n k_2}{(2\pi)^n} \frac{\partial}{\partial k_2^\mu} \frac{(k_1^\mu,\, k_2^\mu,\, p_r^\mu)}{[k_1^2 - m_0^2]^{\alpha_0}\, [(k_1+p_1)^2 - m_1^2]^{\alpha_1} \,\ldots\, [(k_1+k_2+p_j)^2 - m_j^2]^{\alpha_j} \,\ldots\, [(k_2+p_l)^2 - m_l^2]^{\alpha_l}} = 0$$

- The independent Master Integrals (MIs) satisfy a system of first-order linear differential equations
  with respect to each of the kinematical invariants / internal masses

When considering the complete set of MIs, the system can be cast in homogeneous form: $d\vec{I}(\vec{s};\varepsilon) = \mathbf{A}(\vec{s};\varepsilon) \cdot \vec{I}(\vec{s};\varepsilon)$

$\dfrac{d}{dk^2}$  $+ \dfrac{1}{2}\left[\dfrac{1}{k^2} - \dfrac{(D-3)}{(k^2+4m^2)}\right]$  $= -\dfrac{(D-2)}{4m^2}\left[\dfrac{1}{k^2} - \dfrac{1}{(k^2+4m^2)}\right]$ 

- Henn's conjecture (2013): if a change of basis exists which leads to $d\vec{J}(\vec{s};\varepsilon) = \varepsilon \tilde{\mathbf{A}}(\vec{s}) \cdot \vec{J}(\vec{s};\varepsilon)$
  then the solution is expressed in terms of iterated integrals (Chen integral representation)
  depending only on the results at previous orders in the ε expansion

A Simple Example

$$\begin{cases} f'(x) + \dfrac{1}{x^2-4x+5} f(x) = \dfrac{1}{x+2} \\ f(0) = 1 \end{cases}$$

$$f_{hom}(x) = x^r \sum_{k=0}^{\infty} c_k x^k$$

$$f'_{hom}(x) = \sum_{k=0}^{\infty} (k+r)\, c_k\, x^{(k+r-1)}$$

$$\begin{cases} r c_0 = 0 \\ \frac{1}{5} c_0 + c_1 (r+1) = 0 \\ \frac{4}{25} c_0 + \frac{1}{5} c_1 + c_2 (2+r) = 0 \\ \ldots \end{cases}$$

$$f_{hom}(x) = 5 - x - \frac{3}{10} x^2 + \frac{11}{150} x^3 + \ldots$$

Expanded around $x' = 0$

$$f_{part}(x) = f_{hom}(x) \int_0^x dx' \, \overbrace{\frac{1}{(x'+2)} f_{hom}^{-1}(x')}$$

$$= \frac{1}{2} x - \frac{7}{40} x^2 + \frac{2}{75} x^3 + \ldots$$

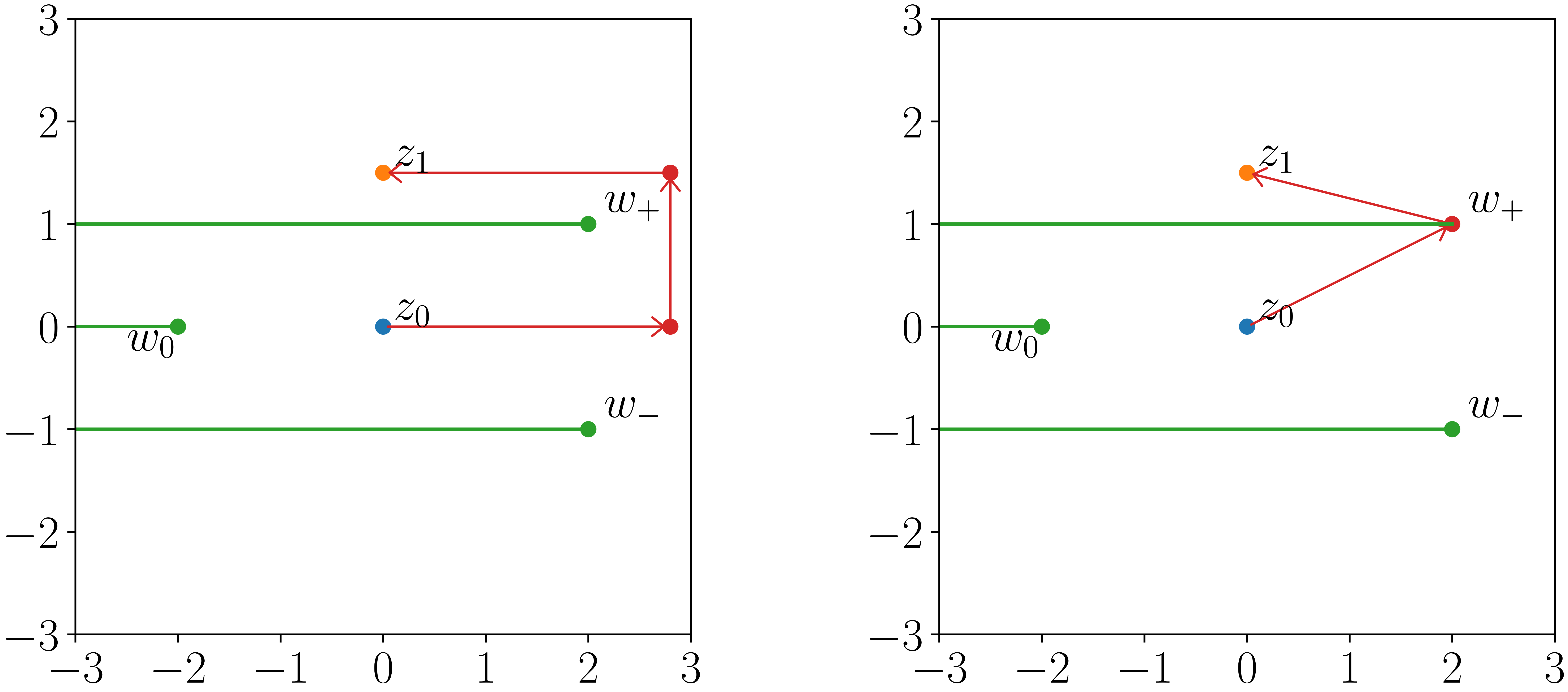
$$f(x) = f_{part}(x) + C f_{hom}(x)$$

$$f(0) = 1 \to C = \frac{1}{5}$$

Evaluation of the Master Integrals by series expansions

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

- **Taylor expansion**: **avoids** the singularities;
- **Logarithmic expansion**: uses the singularities as **expansion points**.
- Logarithmic expansion has larger convergence radius but requires longer evaluation time. **We use Taylor expansion as default**.

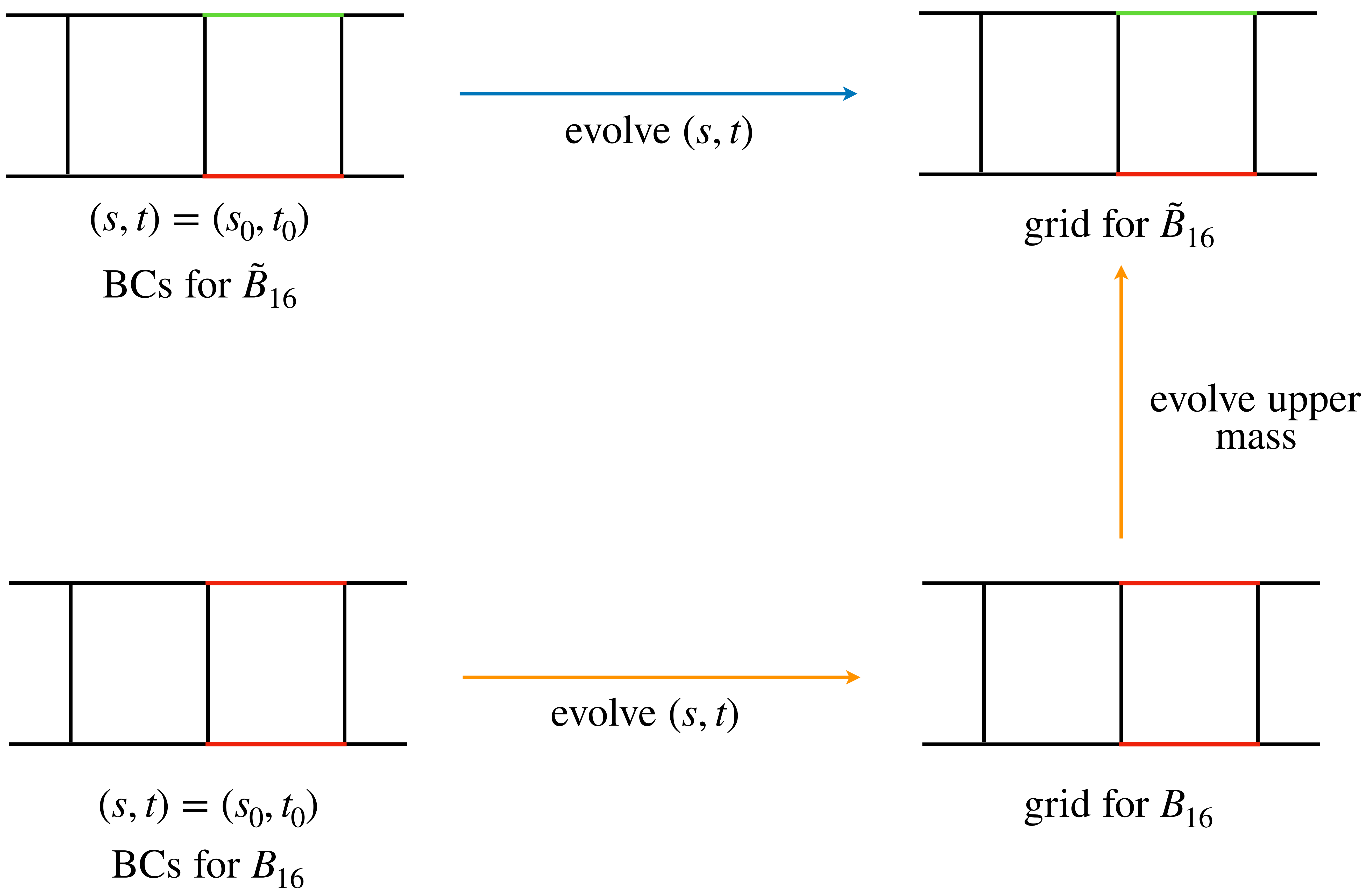


Exploiting the flexibility of the Differential Equations approach

The CC-DY Master Integrals can be evaluated with two different approaches:

- compute the BCs with AMFlow and then solve the differential equations in the invariants *s* and *t*

- use the results of the NC DY process as BCs (two equal internal masses, arbitrary *s* and *t*) then solve the differential equation in the mass parameter from (m_Z, m_Z) to (m_W, m_Z)



Perfect agreement of the two approaches

Estimate of the residual uncertainties: total cross section

The impact of the NNLO QCD-EW corrections is twofold:

- more accurate predictions (additional higher orders)
- reduced uncertainties (scale, inputs, matching)

Ongoing phenomenological studies for full NC DY

Estimate of the residual uncertainties: total cross section

The impact of the NNLO QCD-EW corrections is twofold: more accurate predictions (additional higher orders)
reduced uncertainties (scale, inputs, matching)

Ongoing phenomenological studies for full NC DY

A representative example from the results for the on-shell Z production total cross section

R.Bonciani, F.Buccioni, N.Rana, AV, arXiv:2007.06518, arXiv:2111.12694

→ dependence on the EW input-scheme choice

comparison of (G_μ, M_W, M_Z) and $(\alpha(0), M_W, M_Z)$ (very conservative choice that maximises the spread of the results)

order	G_μ	$\alpha(0)$	$\delta(G_\mu-\alpha(0))$ (%)
NNLO-QCD	55787	53884	3.53
NNLO-QCD+NLO-EW	55501	55015	0.88
NNLO-QCD+NLO-EW+ NNLO QCD-EW	55469	55340	0.23

Estimate of the residual uncertainties: total cross section

The impact of the NNLO QCD-EW corrections is twofold: more accurate predictions (additional higher orders)
reduced uncertainties (scale, inputs, matching)

Ongoing phenomenological studies for full NC DY

A representative example from the results for the on-shell Z production total cross section
R.Bonciani, F.Buccioni, N.Rana, AV, arXiv:2007.06518, arXiv:2111.12694

→ dependence on the EW input-scheme choice

comparison of (G_μ, M_W, M_Z) and $(\alpha(0), M_W, M_Z)$ (very conservative choice that maximises the spread of the results)

order	G_μ	$\alpha(0)$	$\delta(G_\mu - \alpha(0))$ (%)
NNLO-QCD	55787	53884	3.53
NNLO-QCD+NLO-EW	55501	55015	0.88
NNLO-QCD+NLO-EW+ NNLO QCD-EW	55469	55340	0.23

the LO + NLO-EW result would suffer of only 0.55% spread;
the NLO-QCD and NNLO-QCD corrections are only LO-EW and reintroduce a dependence (→0.88%)
which is reduced by the NNLO QCD-EW (→0.23%)

Estimate of the residual uncertainties: total cross section

The impact of the NNLO QCD-EW corrections is twofold: more accurate predictions (additional higher orders)
reduced uncertainties (scale, inputs, matching)

Ongoing phenomenological studies for full NC DY

A representative example from the results for the on-shell Z production total cross section
R.Bonciani, F.Buccioni, N.Rana, AV, arXiv:2007.06518, arXiv:2111.12694

→ dependence on the EW input-scheme choice

comparison of (G_μ, M_W, M_Z) and $(\alpha(0), M_W, M_Z)$ (very conservative choice that maximises the spread of the results)

order	G_μ	$\alpha(0)$	$\delta(G_\mu-\alpha(0))$ (%)
NNLO-QCD	55787	53884	3.53
NNLO-QCD+NLO-EW	55501	55015	0.88
NNLO-QCD+NLO-EW+ NNLO QCD-EW	55469	55340	0.23

the LO + NLO-EW result would suffer of only 0.55% spread;
the NLO-QCD and NNLO-QCD corrections are only LO-EW and reintroduce a dependence (→0.88%)
which is reduced by the NNLO QCD-EW (→0.23%)

The availability of N3LO-QCD and NNLO QCD-EW results can bring the study of EW gauge bosons in the per mille arena !!!
Is the full NNLO-EW calculation negligible at this level ?

W-boson mass prediction

The renormalisation of the SM and a framework for precision tests

- The Standard Model is a renormalizable gauge theory based on $SU(3) \times SU(2)_L \times U(1)_Y$
- The EW gauge sector of the SM lagrangian is assigned specifying (g, g', v, λ) in terms of 4 measurable inputs
- More observables can be computed and expressed in terms of the input parameters, including the available radiative corrections, at any order in perturbation theory
- The validity of the SM can be tested comparing these predictions with the corresponding experimental results

- The input choice $(g, g', v, \lambda) \leftrightarrow (\alpha, G_\mu, m_Z, m_H)$ minimises the parametric uncertainty of the predictions

$$
\begin{aligned}
\alpha(0) &= 1/137.035999139(31) \\
G_\mu &= 1.1663787(6) \times 10^{-5}\ \mathrm{GeV}^{-2} \\
m_Z &= 91.1876(21)\ \mathrm{GeV}/c^2 \\
m_H &= 125.09(24)\ \mathrm{GeV}/c^2
\end{aligned}
$$

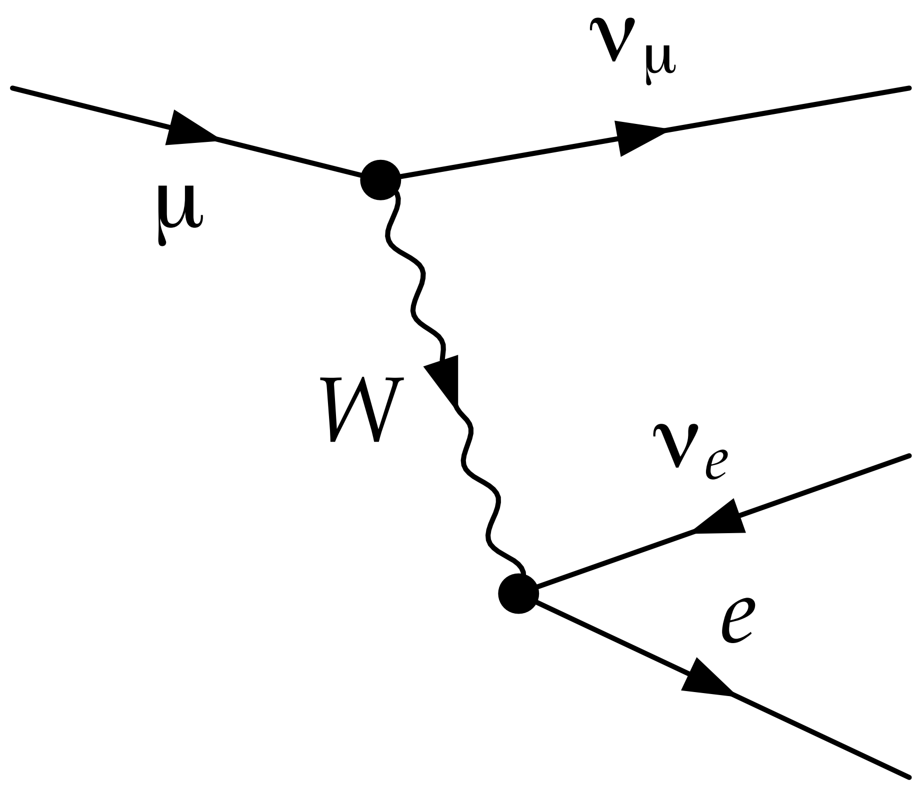
- with these inputs, m_W and the weak mixing angle are predictions of the SM, to be tested against the experimental data

The W boson mass: theoretical prediction

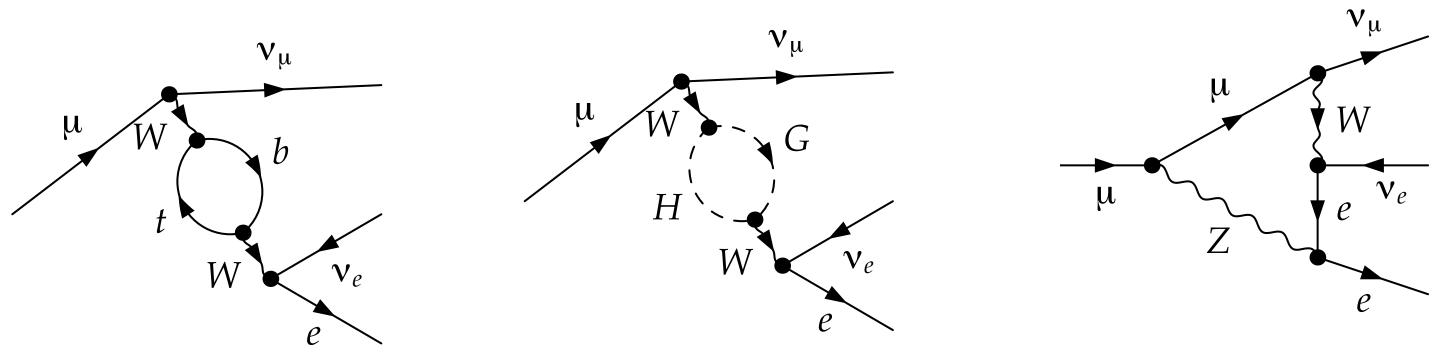
$$\mathcal{L}_{SM} = \mathcal{L}_{SM}(\alpha,\, G_\mu,\, m_Z; m_H; m_f; CKM)$$

$\rightarrow$ we can compute m_W

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8 m_W^2}\,(1+\Delta r)$$



+



+ ....

$$\Delta r = \Delta r(\alpha, G_\mu, m_Z, m_H; m_f; CKM)$$

$$m_W^2 = \frac{m_Z^2}{2}\left(1+\sqrt{1-\frac{4\pi\alpha}{G_\mu\sqrt{2}m_Z^2}(1+\Delta r)}\right)$$

The W boson mass: theoretical prediction

<u>on-shell scheme:</u> dominant contributions to Δr

$$\Delta r = \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta r_{\rm rem}$$

$$\Delta\alpha = \Pi^{\gamma}_{\rm ferm}(M_Z^2) - \Pi^{\gamma}_{\rm ferm}(0) \quad \to \quad \alpha(M_Z) = \frac{\alpha}{1-\Delta\alpha}$$

$$\Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2} = 3\frac{G_F m_t^2}{8\pi^2\sqrt{2}} \quad \text{[one-loop]} \quad \sim \frac{m_t^2}{v^2} \sim \alpha_t$$

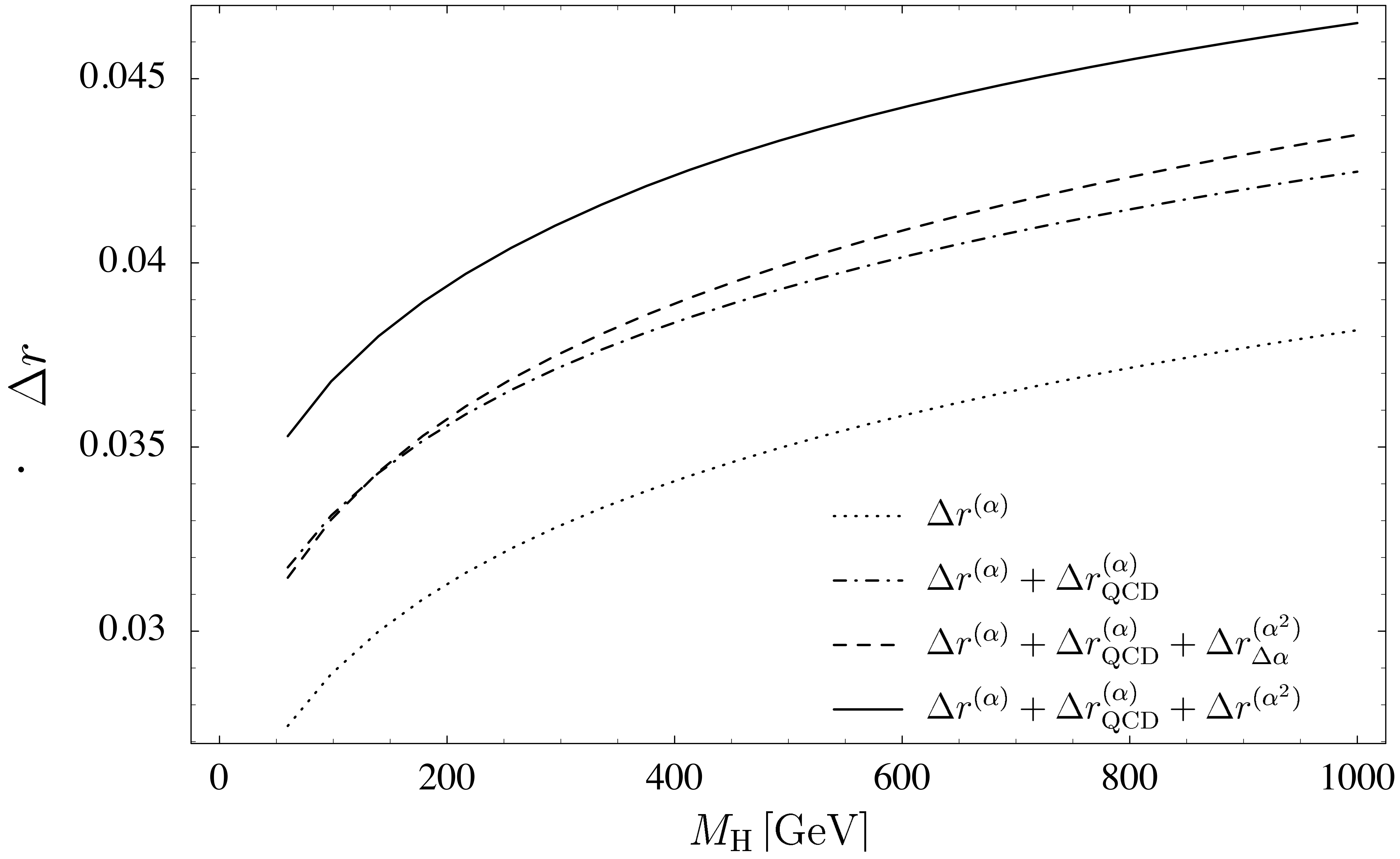
beyond one-loop order: $\sim \alpha^2,\ \alpha\alpha_t,\ \alpha_t^2,\ \alpha^2\alpha_t,\ \alpha\alpha_t^2,\ \alpha_t^3, \ldots$

reducible higher order terms from $\Delta\alpha$ and $\Delta\rho$ via

$$1 + \Delta r \ \to\ \frac{1}{(1-\Delta\alpha)\left(1+\frac{c_w^2}{s_w^2}\Delta\rho\right) + \cdots}$$

$$\rho = 1 + \Delta\rho \ \to\ \frac{1}{1-\Delta\rho}$$

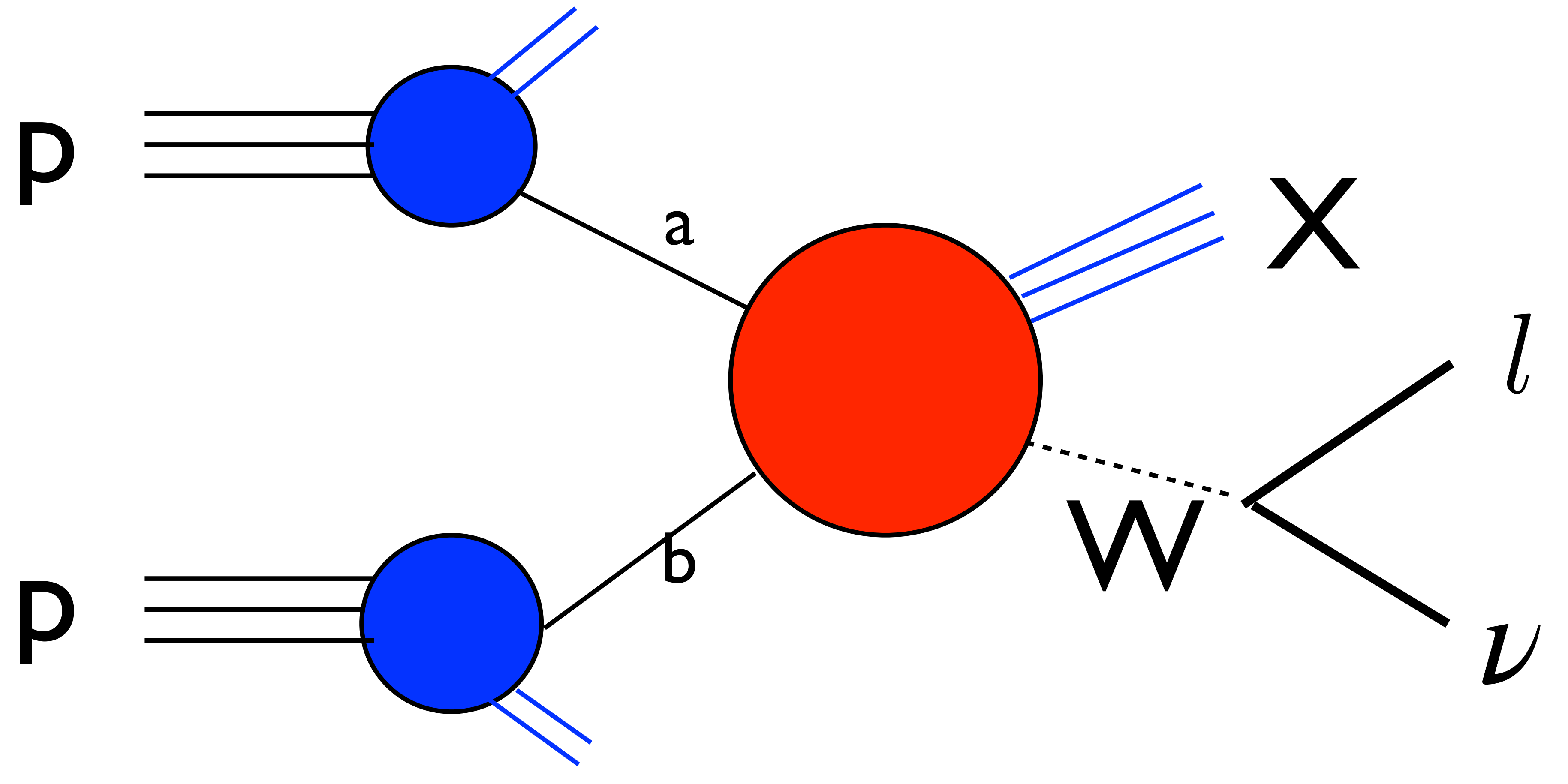
effects of higher-order terms on Δr



(Consoli, Hollik, Jegerlehner)

W-boson mass determination

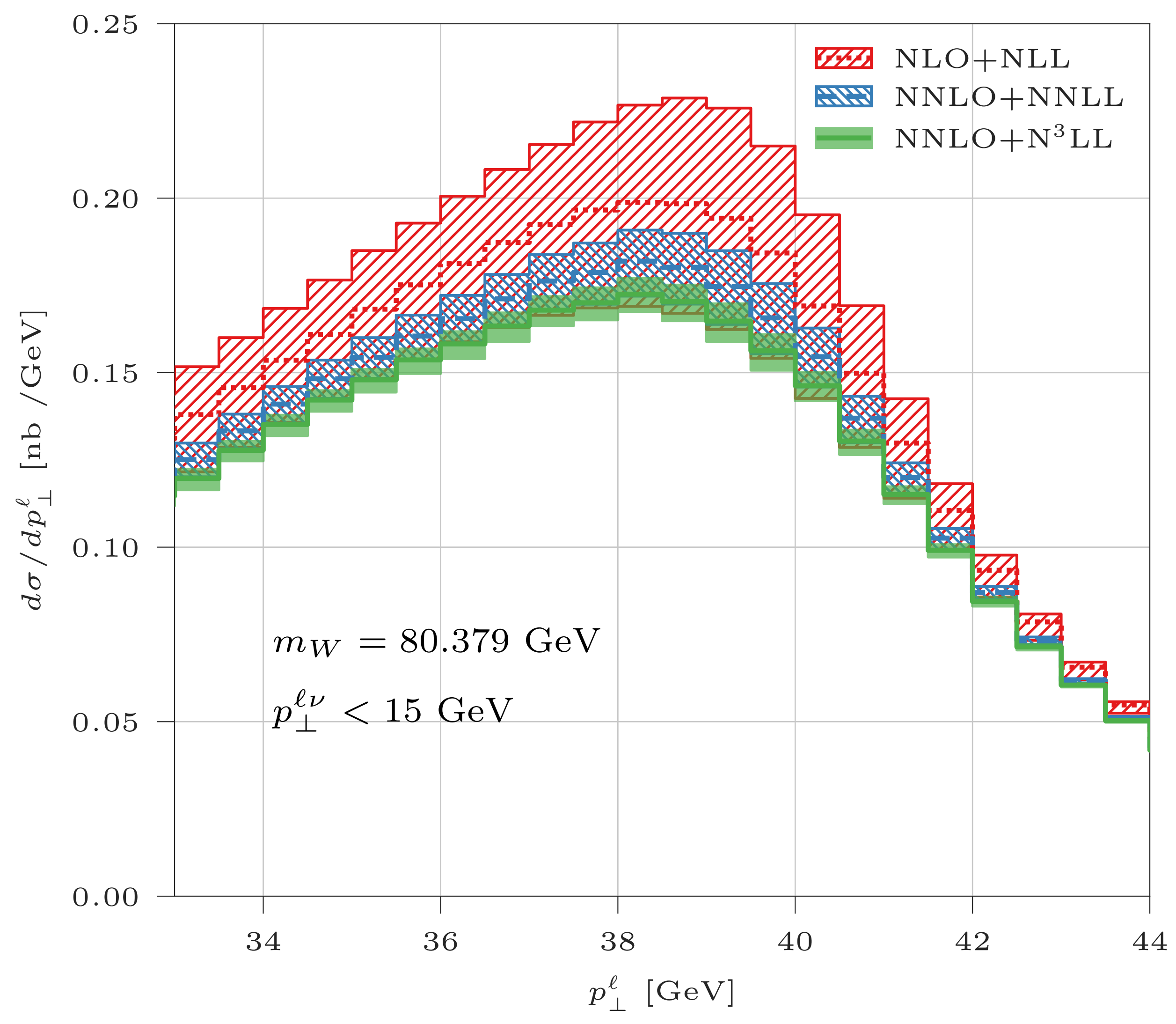
m_W determination at hadron colliders



- In charged-current DY,
  it is NOT possible to reconstruct the lepton-neutrino invariant mass
  Full reconstruction is possible (but not easy) only in the transverse plane

- A generic observable has a linear response to an m_W variation
  With a goal for the relative error of 10^{-4}, the problem seems to be unsolvable

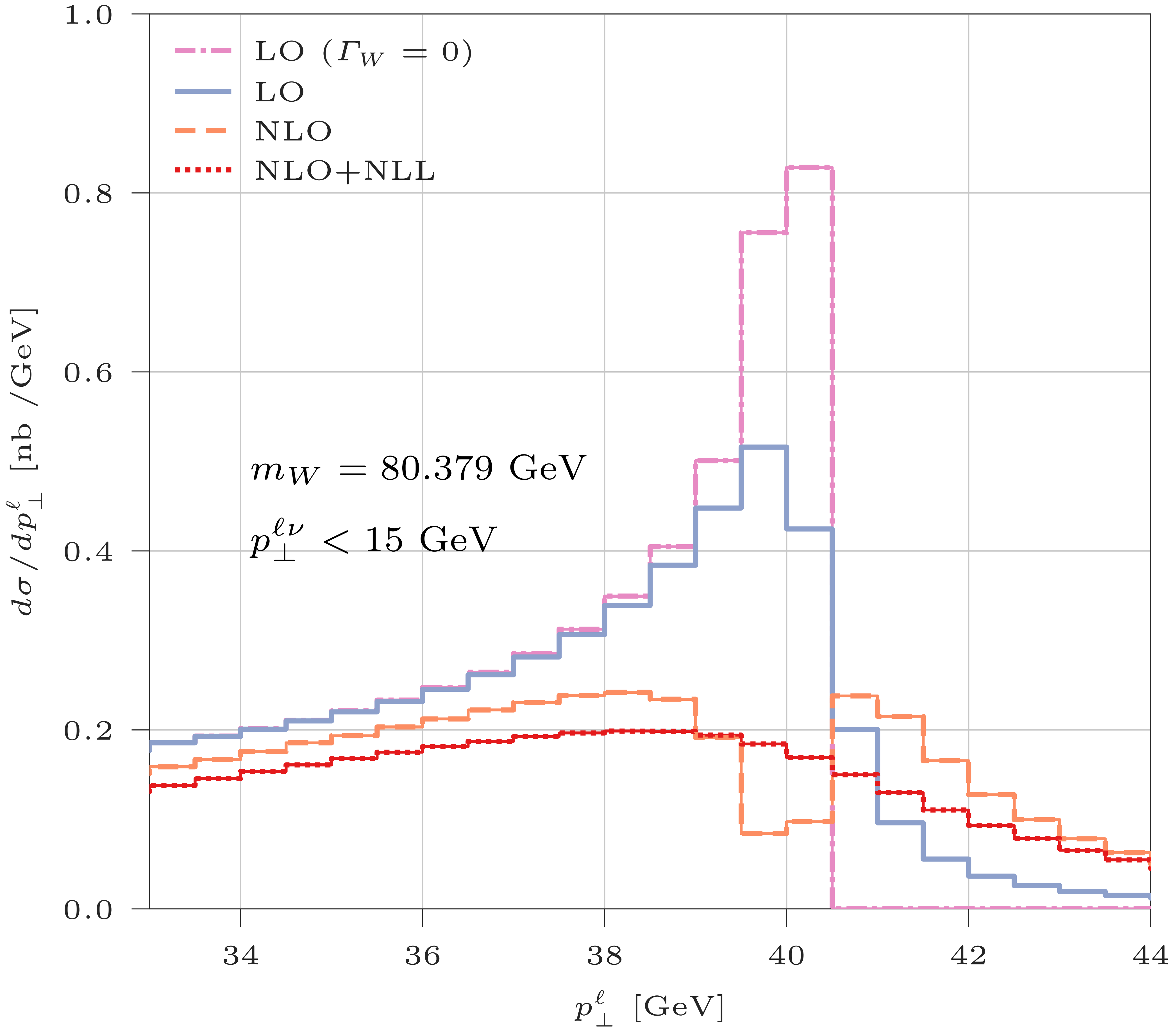
- m_W extracted from the study of the shape of the $p_\perp^l, M_\perp$ and $E_\perp^{miss}$ distributions in CC-DY
  thanks to the jacobian peak that enhances the sensitivity to m_W



$$\frac{d}{dp_\perp^2} \to \frac{2}{s}\frac{1}{\sqrt{1-4p_\perp^2/s}}\frac{d}{d\cos\theta} \sim \frac{d}{dp_\perp^2} \to \frac{2}{s}\frac{1}{\sqrt{1-4p_\perp^2/m_W^2}}\frac{d}{d\cos\theta}$$

→ enhanced sensitivity at the 10^{-3} level ($p_\perp^l$ distribution)
or even at the 10^{-2} level ($M_\perp$ distribution)

The lepton transverse momentum distribution in charged-current Drell-Yan



The lepton transverse momentum distribution has a jacobian peak induced by the factor $1/\sqrt{1-\frac{s}{4p_\perp^2}}$.

When studying the W resonance region, the peak appears at $p_\perp \sim \frac{m_W}{2}$

Kinematical end point at $\frac{m_W}{2}$ at LO

The decay width allows to populate the upper tail of the distribution

Sensitivity to soft radiation → double peak at NLO-QCD

The QCD-ISR next-to-leading-log resummation broadens the distribution and cures the sensitivity to soft radiation at the jacobian peak.

In the $p_\perp^\ell$ spectrum the sensitivity to m_W and important QCD features are closely intertwined

m_W determination at hadron colliders: template fitting

Given one experimental kinematical distribution

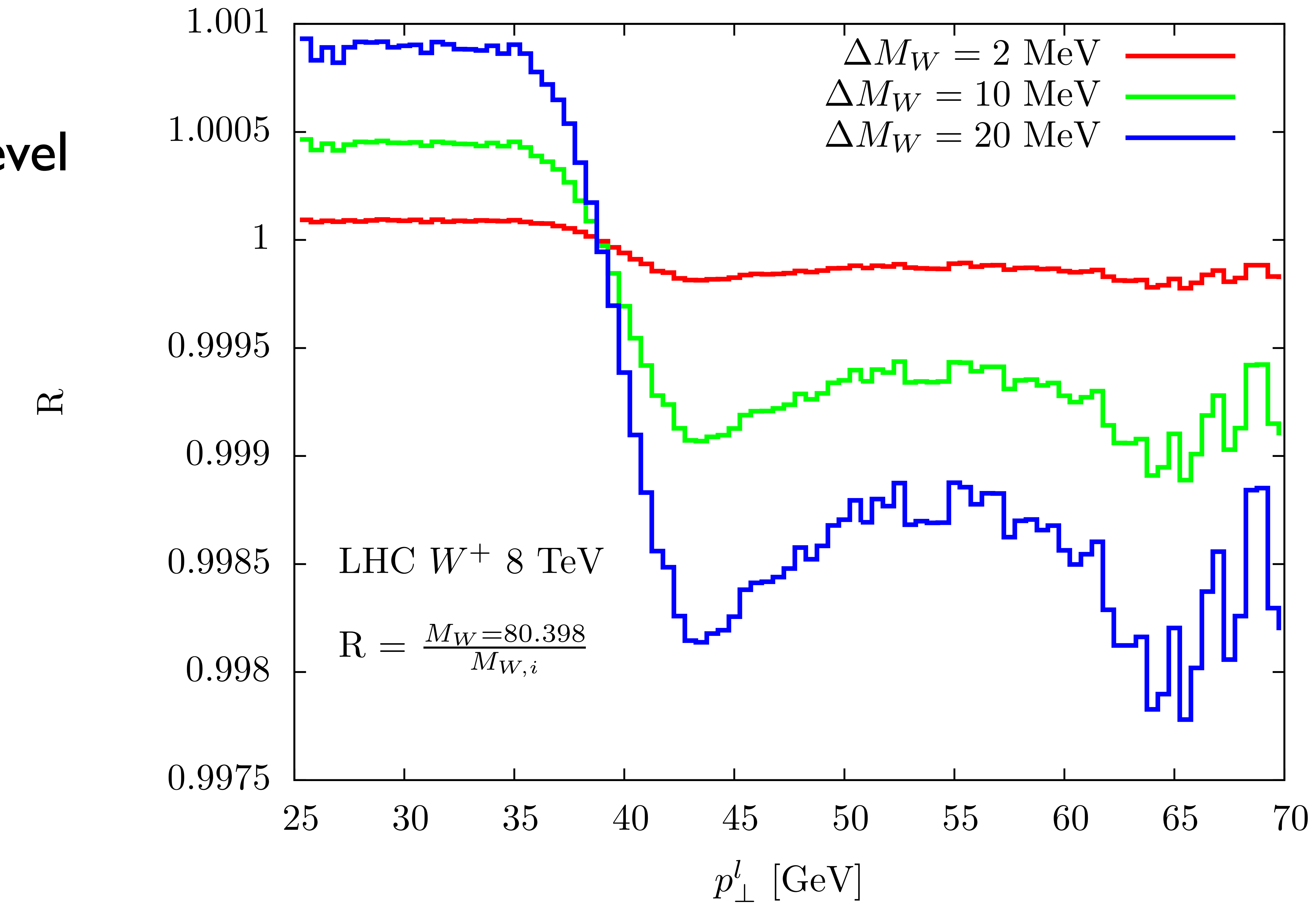
- we compute the corresponding theoretical distribution for several hypotheses of one Lagrangian input parameters (e.g. m_W)
- we compute, for each $m_W^{(k)}$ hypothesis, a χ^2_k defined in a certain interval around the jacobian peak (fitting window)
- we look for the minimum of the χ^2 distribution

The m_W value associated to the position of the minimum of the χ^2 distribution is the experimental result

A determination at the 10^{-4} level requires
a control over the shape of the distributions at the per mille level

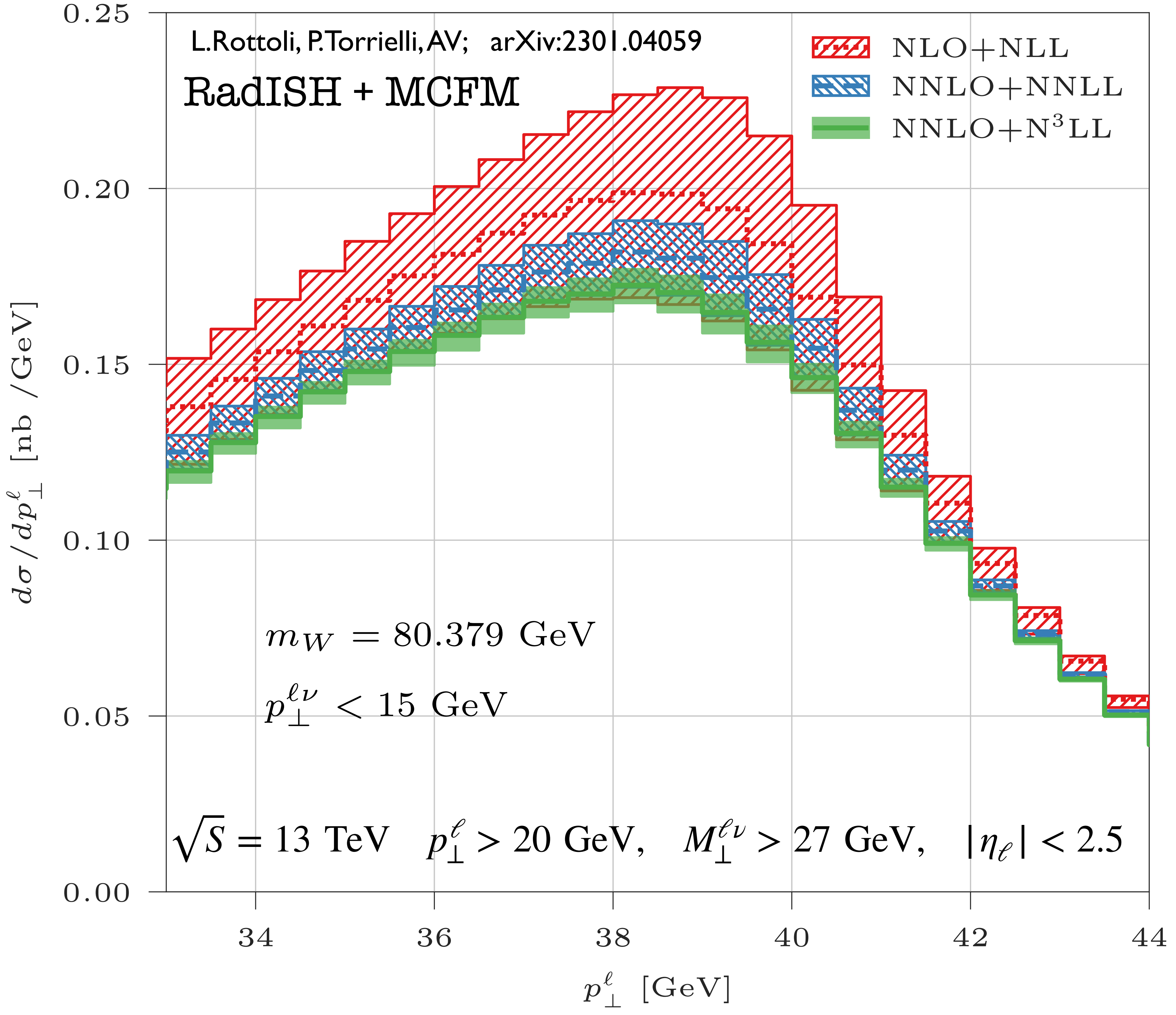
The theoretical uncertainties of the templates
contribute to the theoretical systematic error on m_W

- higher-order QCD
- non-perturbative QCD
- PDF uncertainties
- heavy quarks corrections
- EW corrections



Template fitting: description of the single lepton transverse momentum distribution

The template fitting procedure is acceptable if the data are described by the theoretical distribution with high quality



Scale variation of the NNLO+N3LL prediction for ptlep provides a set of equally good templates
but the width of the uncertainty band is at the few percent level
a factor 10 larger than the naive estimate would require !

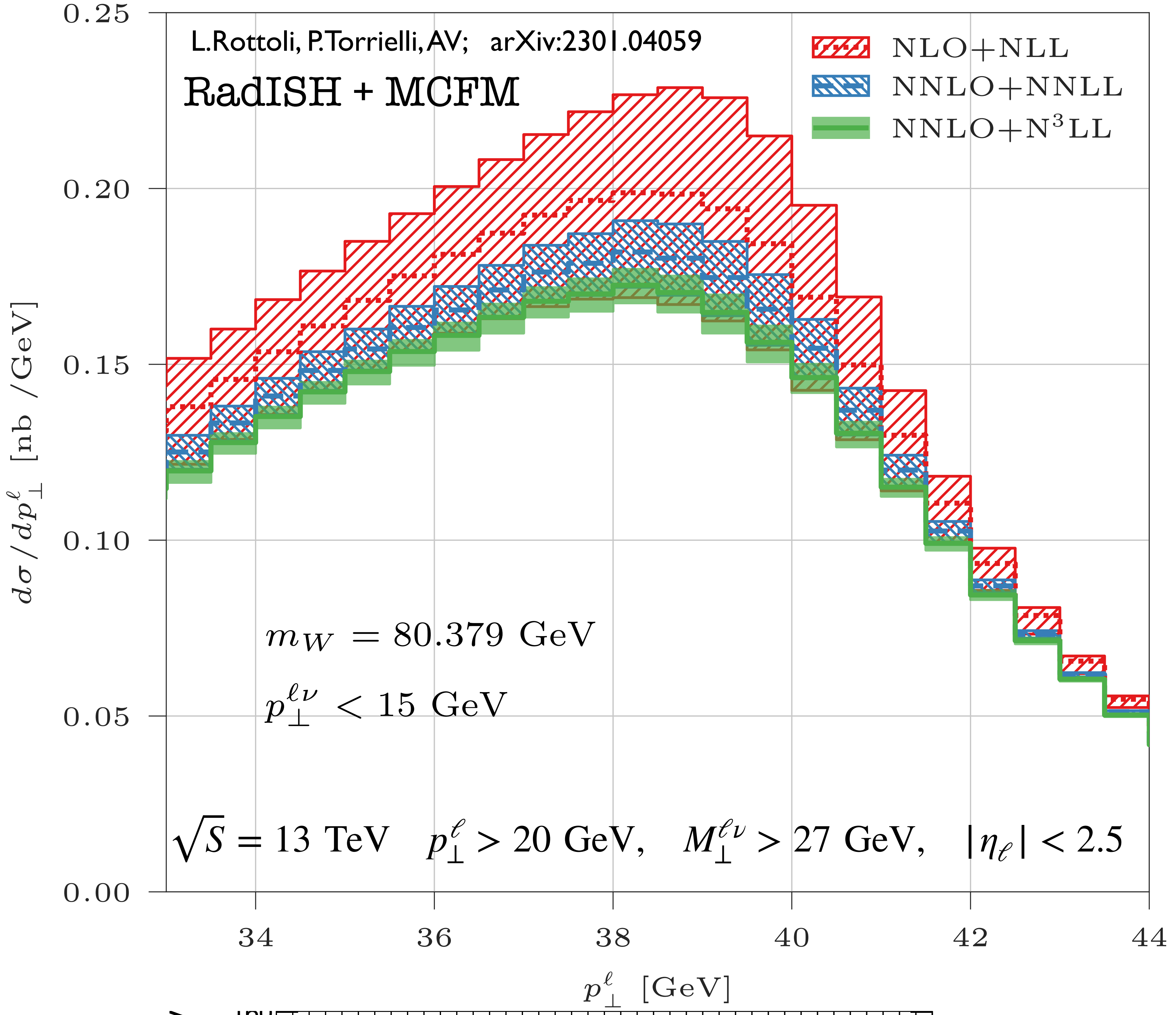
→ data driven approach
a Monte Carlo event generator is tuned to the data in NCDY ($p_\perp^Z$)
for one QCD scale choice

↓

the same parameters are then used to prepare the CCDY templates

Template fitting: description of the single lepton transverse momentum distribution

The template fitting procedure is acceptable if the data are described by the theoretical distribution with high quality

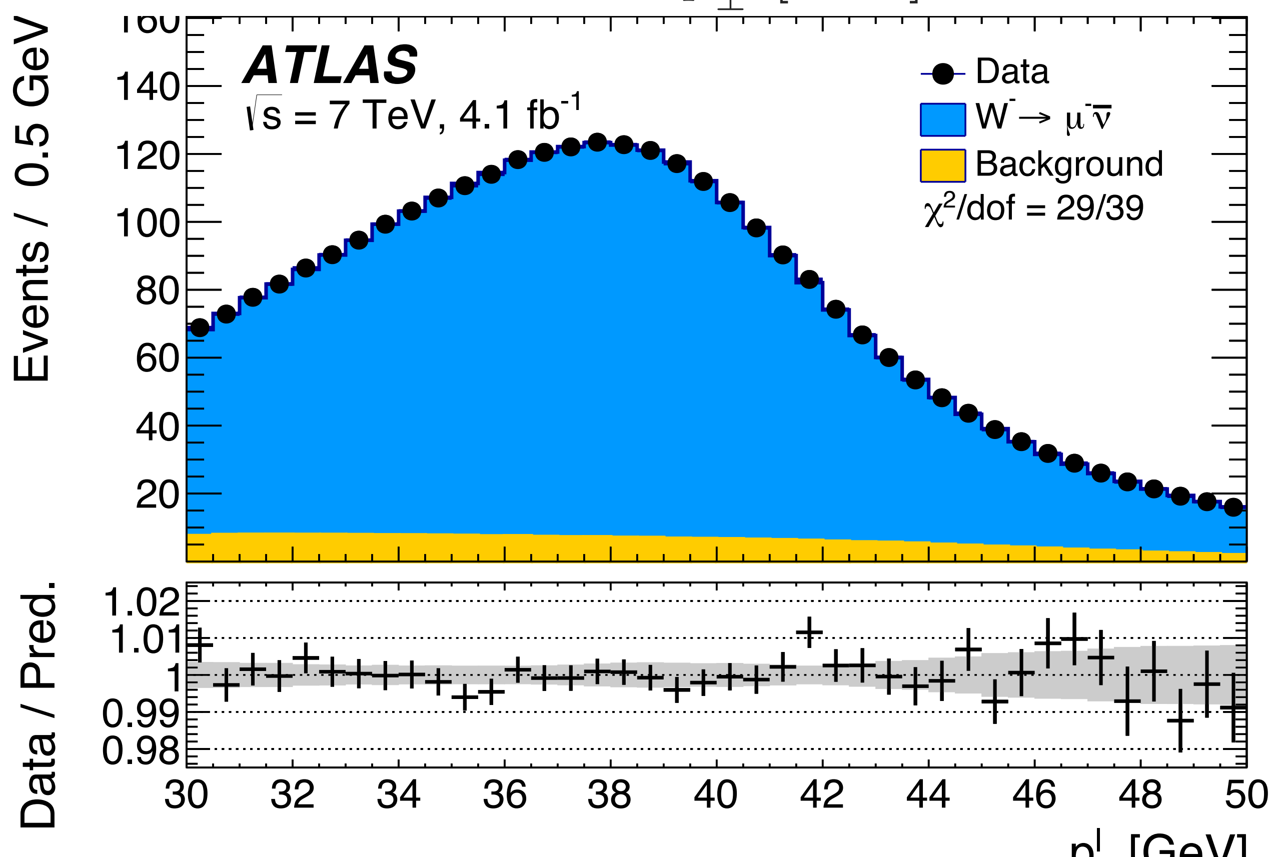


Scale variation of the NNLO+N3LL prediction for ptlep
provides a set of equally good templates
but the width of the uncertainty band is at the few percent level
a factor 10 larger than the naive estimate would require !

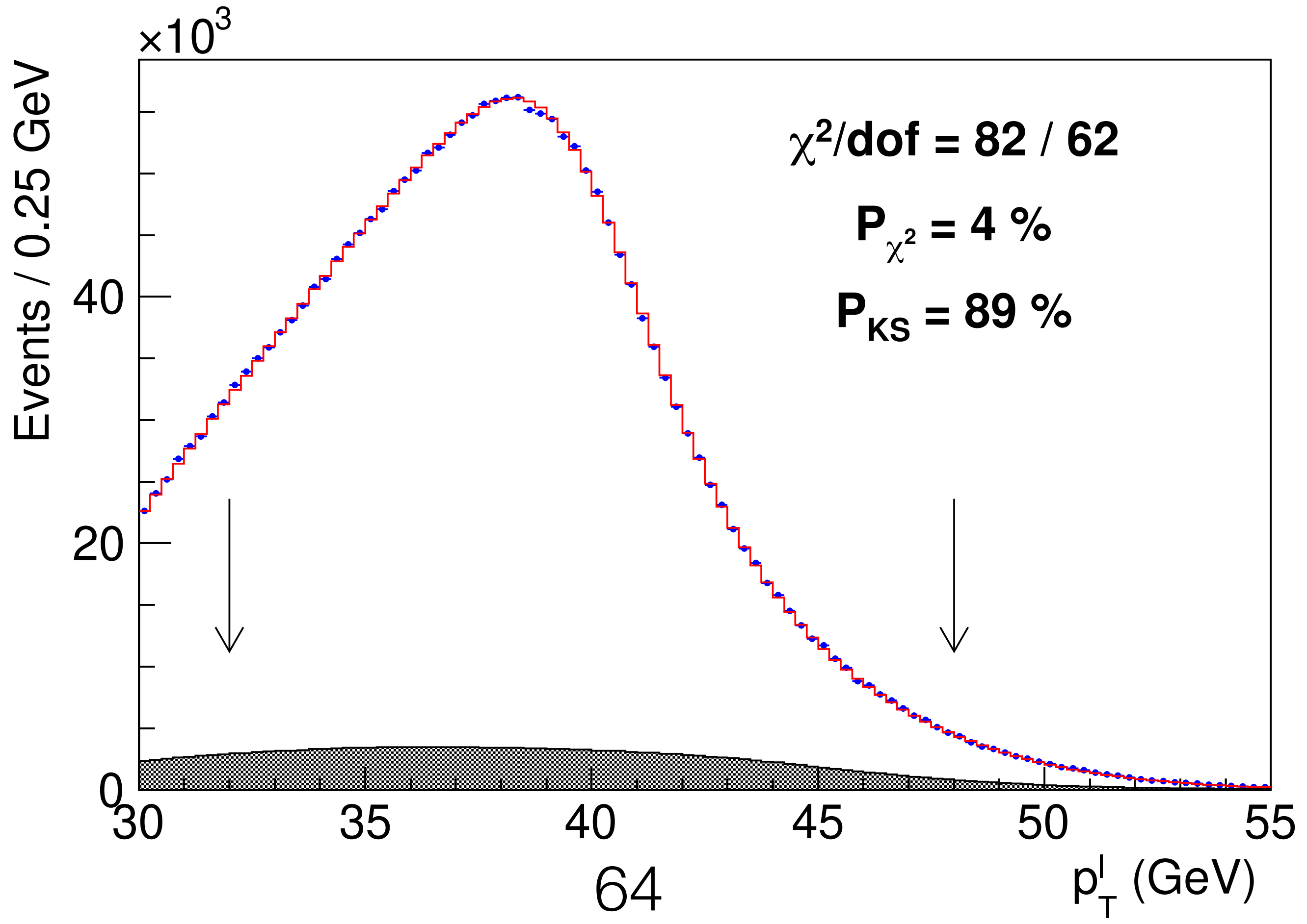
→ data driven approach
a Monte Carlo event generator is tuned to the data in NCDY ($p_\perp^Z$)
for one QCD scale choice

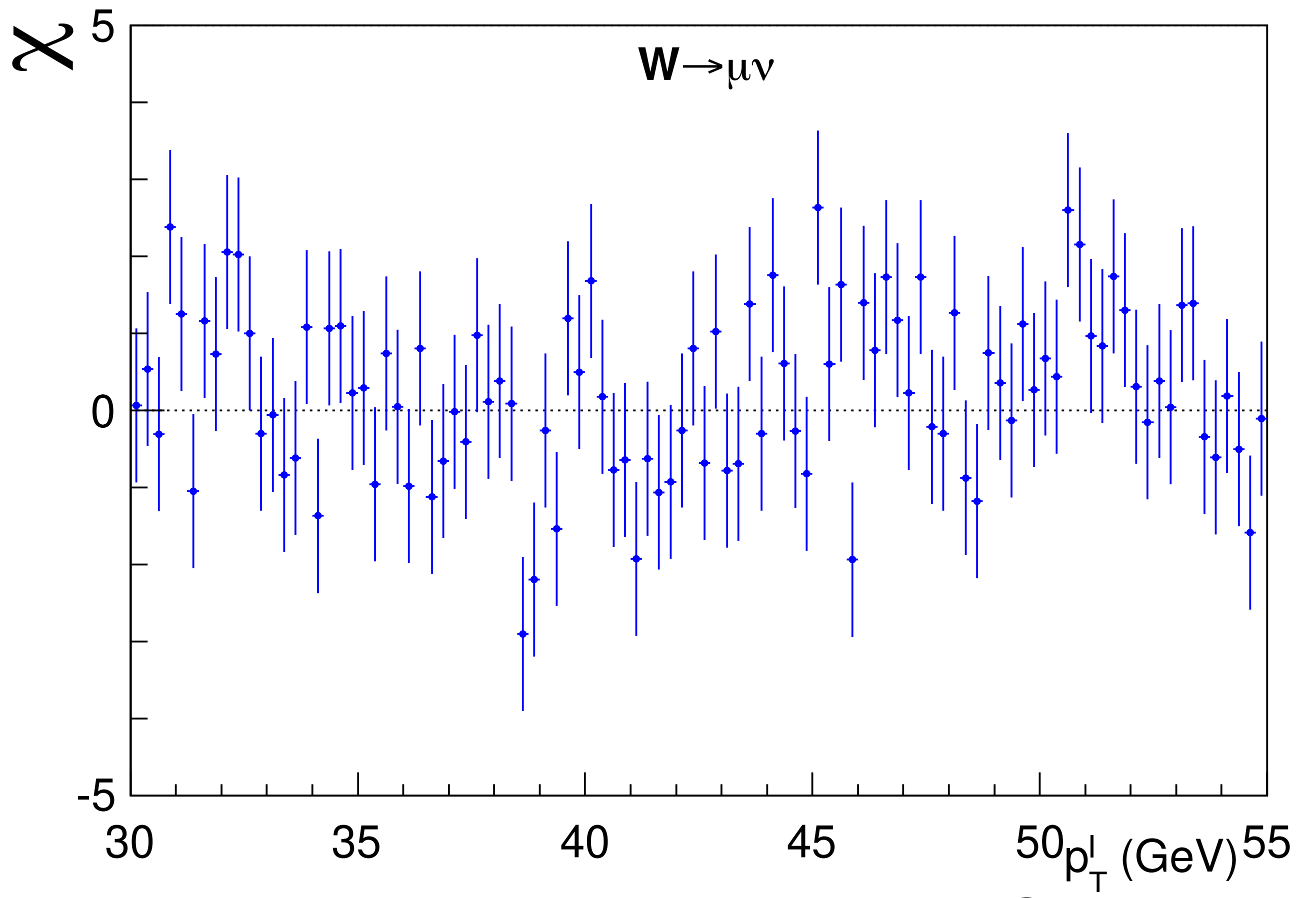
↓

the same parameters are then used to prepare the CCDY templates



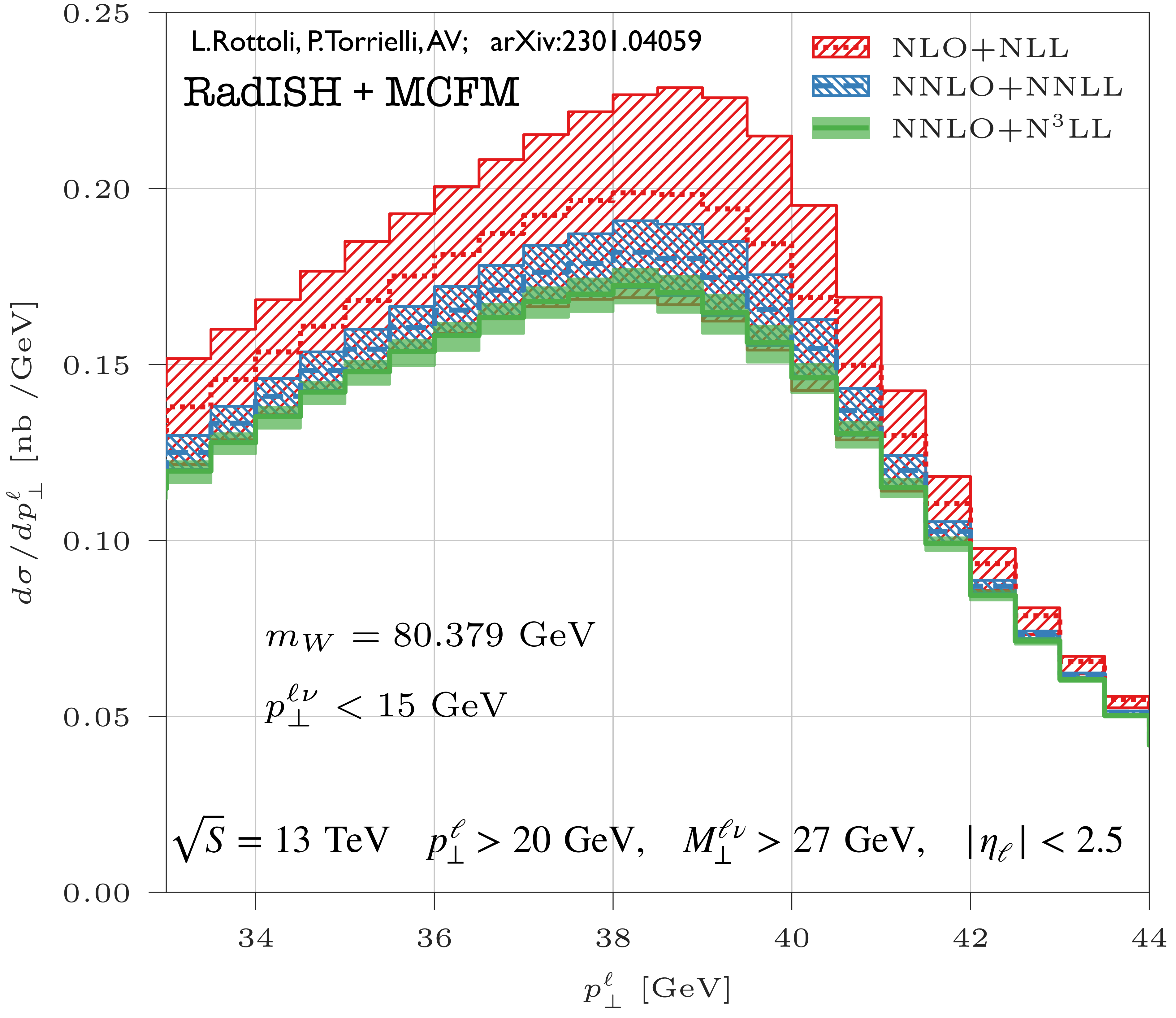
CDF collaboration, Scince 376, 170-176 (2022)





Template fitting: description of the single lepton transverse momentum distribution

The template fitting procedure is acceptable if the data are described by the theoretical distribution with high quality



Scale variation of the NNLO+N3LL prediction for ptlep provides a set of equally good templates but the width of the uncertainty band is at the few percent level a factor 10 larger than the naive estimate would require !

→ data driven approach

a Monte Carlo event generator is tuned to the data in NCDY ($p_\perp^Z$) for one QCD scale choice

↓

the same parameters are then used to prepare the CCDY templates

A data driven approach improves the accuracy of the model (i.e. its ability to describe the data)

does not improve the precision of the model (the intrinsic ambiguities in the model formulation)

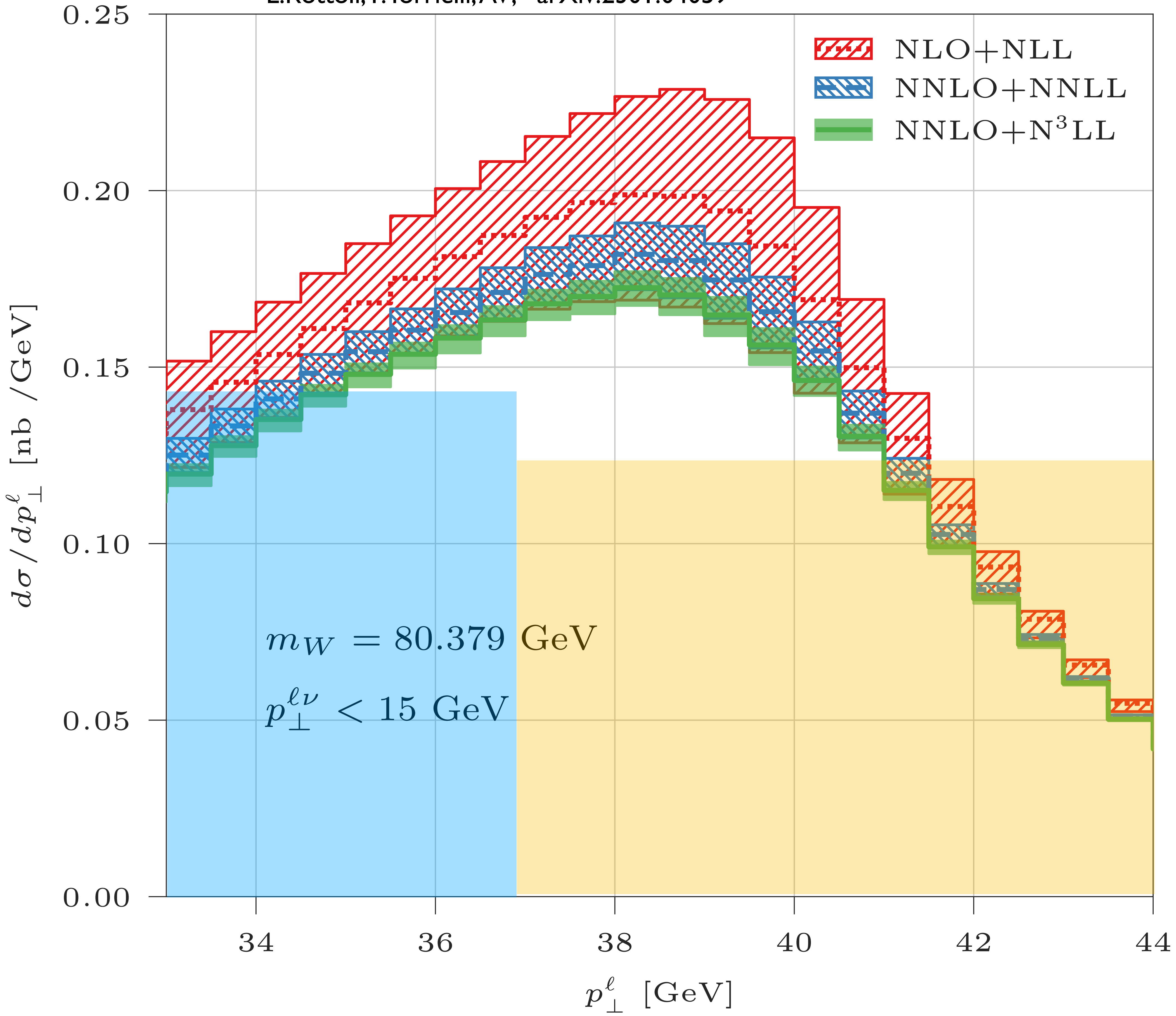
What are the limitations of the transfer of information from NCDY to CCDY ?

MW from a jacobian asymmetry

L.Rottoli, P.Torrielli, AV, arXiv:2301.04059

The jacobian asymmetry $\mathcal{A}_{p_\perp^\ell}$

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



$$L_{p_\perp^\ell} \equiv \int_{p_\perp^{\ell,\min}}^{p_\perp^{\ell,\mathrm{mid}}} dp_\perp^\ell \frac{d\sigma}{dp_\perp^\ell}, \qquad U_{p_\perp^\ell} \equiv \int_{p_\perp^{\ell,\mathrm{mid}}}^{p_\perp^{\ell,\max}} dp_\perp^\ell \frac{d\sigma}{dp_\perp^\ell}$$

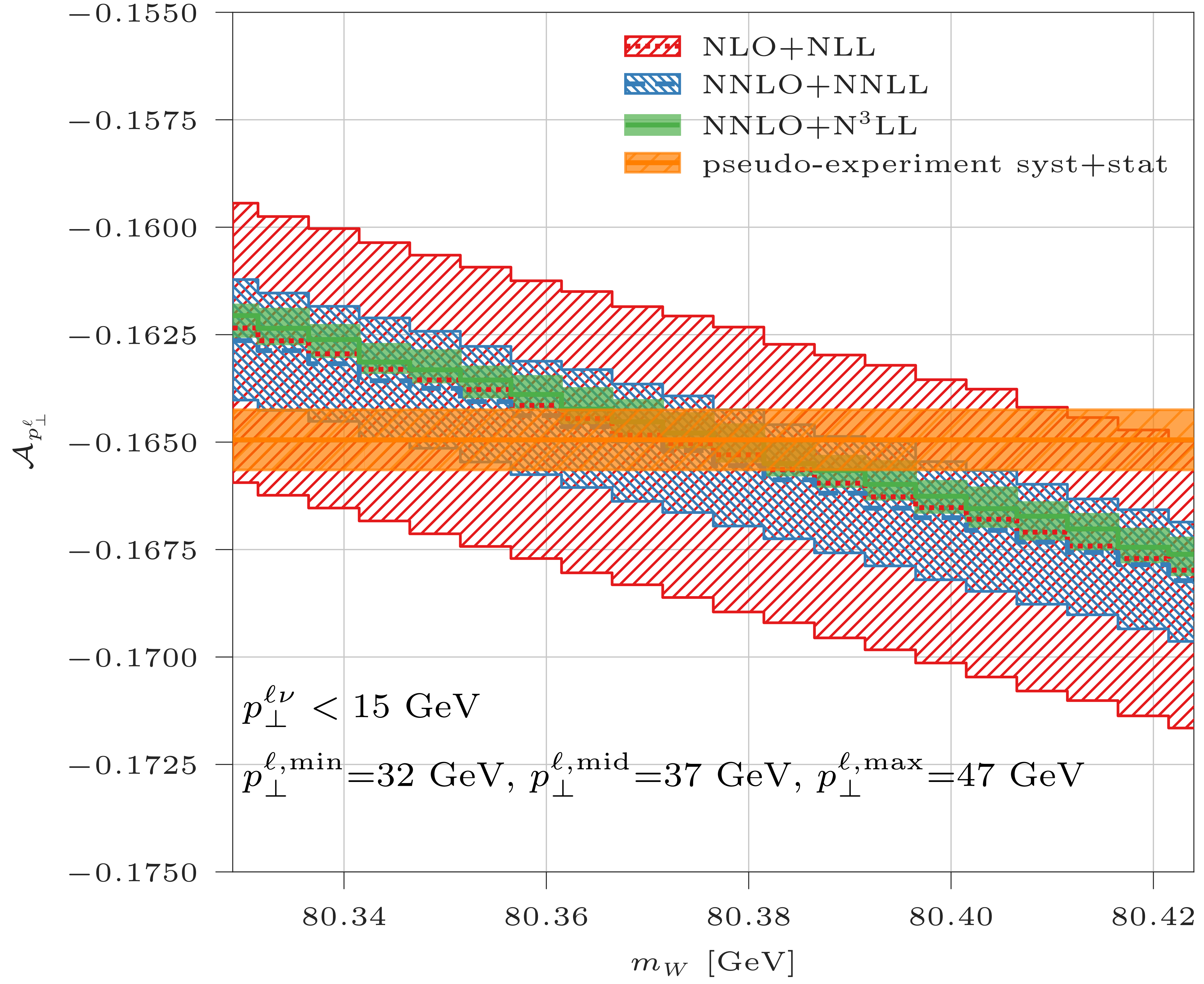
$$\mathcal{A}_{p_\perp^\ell}(p_\perp^{\ell,\min}, p_\perp^{\ell,\mathrm{mid}}, p_\perp^{\ell,\max}) \equiv \frac{L_{p_\perp^\ell} - U_{p_\perp^\ell}}{L_{p_\perp^\ell} + U_{p_\perp^\ell}}$$

The asymmetry is an observable (i.e. it is measurable via counting): its value is one single scalar number
It depends only on the edges of the two defining bins

Increasing m_W shifts the position of the peak to the right $\rightarrow$ Events migrate from the blue to the orange bin
$\rightarrow$ The asymmetry decreases

The jacobian asymmetry $\mathscr{A}_{p_\perp^\ell}$ as a function of m_W

L.Rottoli, P.Torrielli, AV;   arXiv:2301.04059



The asymmetry $\mathscr{A}_{p_\perp}$ has a linear dependence on m_W,
stemming from the linear dependence on the end-point position

The slope of the asymmetry expresses the sensitivity to m_W ,
in a given setup $(p_\perp^{\ell,min}, p_\perp^{\ell,mid}, p_\perp^{\ell,max})$

The slope is the same with every QCD approximation
(factorization of QCD effects, perturbative and non-perturbative)

The "large" size of the two bins $\mathcal{O}(5-10)$ GeV leads to
- small statistical errors
- excellent stability of the QCD results (inclusive quantity)
- ease to unfold the data to particle level   (m_W combination)

The experimental value and the theoretical predictions can be directly compared (m_W from the intersection of two lines)

The main systematics on the two fiducial cross sections is related to the lepton momentum scale resolution

Compatibility and combination of world W-boson mass determinations

LHC-TeV MW working group, arXiv:2308.09417

CDF, Science 376 (2022) 170; D0, PRL 103 (2009) 141801 and PRD 89 (2014) 012005; ATLAS, EPJC 78 (2018) 110; LHCb, JHEP 01 (2022) 036; LEP, Phys Rept 532 (2013) 119

Input Measurements for combination

- CDF – $p\bar{p}$ collisions @ $\sqrt{s}$ = 1.96 TeV; fit variables are p_T^l , p_T^ν and m_T.
- D0 – two separate measurements using $p\bar{p}$ collisions @ $\sqrt{s}$ = 1.96 TeV; fit variables are p_T^e, m_T and p_T^ν.
- ATLAS – pp collisions @ $\sqrt{s}$ = 7 TeV; central region at LHC; fit variables are p_T^l and m_T. [*Original analysis used following agreement to use published results*]
- LHCb – pp collisions @ $\sqrt{s}$ = 13 TeV; forward region at LHC; fit variable is q/p_T^μ.
- LEP – legacy combination from LEP experiments.

Experiment	Event requirements	Fit ranges
CDF	$30 < p_T^\ell < 55$ GeV $\lvert\eta_\ell\rvert < 1$ $30 < E_T^{miss} < 55$ GeV $65 < m_T < 90$ GeV $u_T < 15$ GeV	$32 < p_T^\ell < 48$ GeV $32 < E_T^{miss} < 48$ GeV $60 < m_T < 100$ GeV
D0	$p_T^e > 25$ GeV $\lvert\eta_\ell\rvert < 1.05$ $E_T^{miss} > 25$ GeV $m_T > 50$ GeV $u_T < 15$ GeV	$32 < p_T^e < 48$ GeV $65 < m_T < 90$ GeV
ATLAS	$p_T^\ell > 30$ GeV $\lvert\eta_\ell\rvert < 2.4$ $E_T^{miss} > 30$ GeV $m_T > 60$ GeV $u_T < 30$ GeV	$32 < p_T^\ell < 45$ GeV $66 < m_T < 99$ GeV
LHCb	$p_T^\mu > 24$ GeV $2.2 < \eta_\mu < 4.4$	$28 < p_T^\mu < 52$ GeV

QCD challenges

The measurements span two decades → remarkable theoretical progress

The analyses are based on different PDF sets and event generators, with different theoretical content

- D0: RESBOS CP (N2LO, N2LL) with CTEQ66 PDFs (NLO)
- CDF: RESBOS C (NLO, N2LL) with CTEQ6M PDFs (NLO) [CDF publication applied a correction to reproduce Resbos2 + NNPDF3.1]
- ATLAS: POWHEG + Pythia8 (NLO+PS) with DYTurbo for Angular Distribution (N2LO) with CT10 PDFs (NNLO)
- LHCb: POWHEG + Pythia8 (NLO+PS) with DYTurbo for Angular Distribution (N2LO) with averaged result from MSHT20, NNPDF31 and CT18 PDFs (NLO)

The combination study seeks to "update" the measurements to a common QCD framework before their compatibility is assessed and, eventually, the results are combined

$$m_W^{update} = m_W^{ref} + \delta m_W^{PDF} + \delta m_W^{pol} + \delta m_W^{other}$$

- m_W^{ref}: Published value
- δm_W^{PDF}: Update to common PDF
- δm_W^{pol}: Common W polarisation
- δm_W^{other}: Additional (small) updates

The LHCb measurement has been "repeated", using the same code framework but different PDF sets
Effect of updates on other measurements estimated with two simulated samples from two models

Compatibility of PDF sets with Drell-Yan data

Measurement	NNPDF3.1	NNPDF4.0	MMHT14	MSHT20	CT14	CT18	ABMP16
CDF y_Z	24 / 28	28 / 28	30 / 28	32 / 28	29 / 28	27 / 28	31 / 28
CDF A_W	11 / 13	14 / 13	12 / 13	28 / 13	12 / 13	11 / 13	21 / 13
D0 y_Z	22 / 28	23 / 28	23 / 28	24 / 28	22 / 28	22 / 28	22 / 28
D0 $W \to e\nu$ A_ℓ	22 / 13	23 / 13	52 / 13	42 / 13	21 / 13	19 / 13	26 / 13
D0 $W \to \mu\nu$ A_ℓ	12 / 10	12 / 10	11 / 10	11 / 10	11 / 10	12 / 10	11 / 10
ATLAS peak CC y_Z	13 / 12	13 / 12	58 / 12	17 / 12	12 / 12	11 / 12	18 / 12
ATLAS W^- y_ℓ	12 / 11	12 / 11	33 / 11	16 / 11	13 / 11	10 / 11	14 / 11
ATLAS W^+ y_ℓ	9 / 11	9 / 11	15 / 11	12 / 11	9 / 11	9 / 11	10 / 11
Correlated χ^2	75	62	210	88	81	41	83
Total χ^2 / d.o.f.	200 / 126	196 / 126	444 / 126	270 / 126	210 / 126	162 / 126	236 / 126
$p(\chi^2, n)$	0.003%	0.007%	$< 10^{-10}$	$< 10^{-10}$	0.0004%	1.5%	10^{-8}

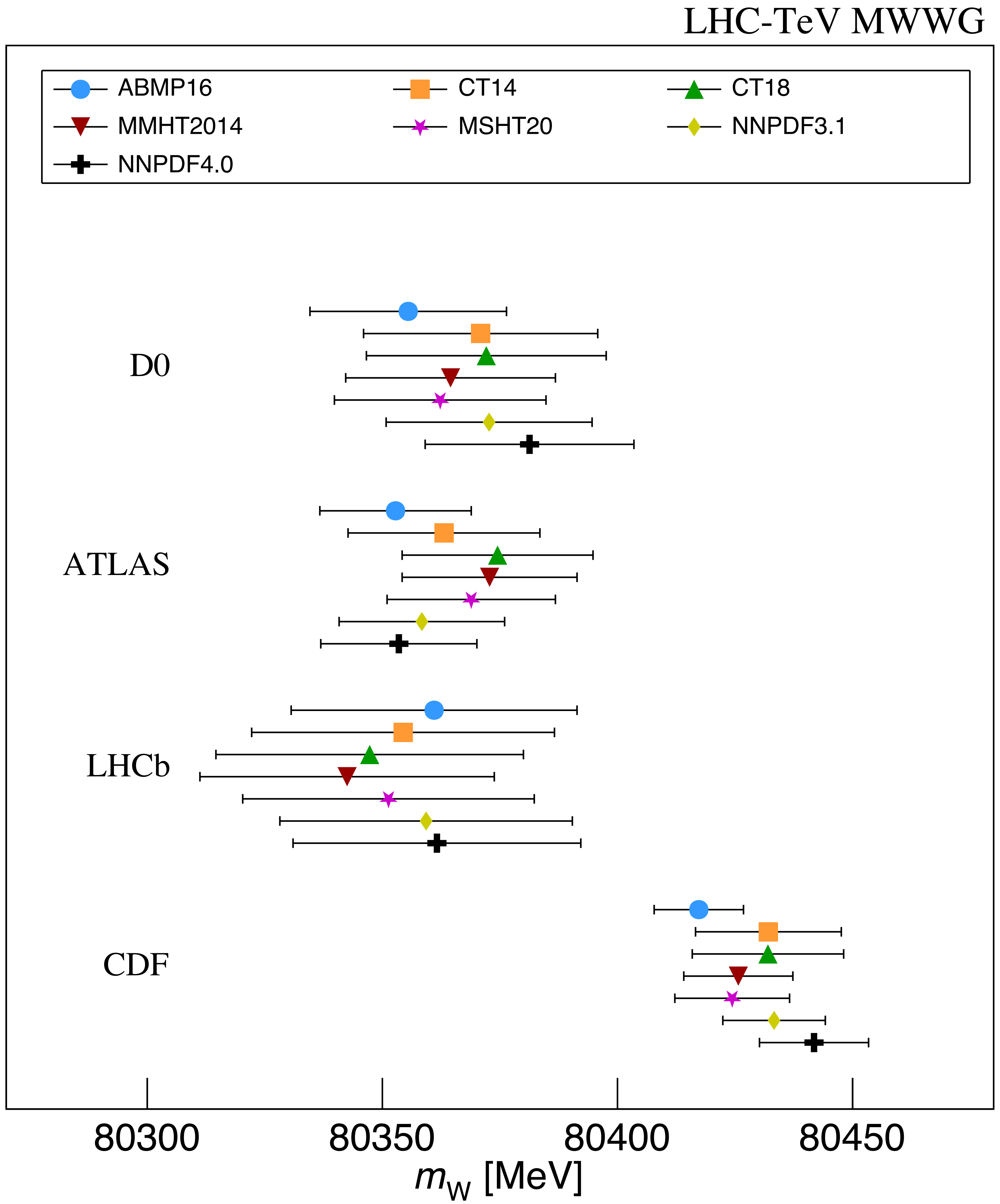
No PDF set provides a good description of the full Tevatron+LHC dataset

Best description given by CT18 (which has larger uncertainties)

CT18 therefore taken as the default PDF set

Combination

Input measurements with updates applied



All experiments (4 d.o.f.)				
PDF set	m_W	σ_{PDF}	χ^2	$p(\chi^2, n)$
ABMP16	80392.7 ± 7.5	3.2	29	0.0008%
CT14	80393.0 ± 10.9	7.1	16	0.3%
CT18	80394.6 ± 11.5	7.7	15	0.5%
MMHT2014	80398.0 ± 9.2	5.8	17	0.2%
MSHT20	80395.1 ± 9.3	5.8	16	0.3%
NNPDF3.1	80403.0 ± 8.7	5.3	23	0.1%
NNPDF4.0	80403.1 ± 8.9	5.3	28	0.001%

No combination of all measurements provides a good χ^2 probability

the full combination, including CDF, is disfavoured

MW combinations (cfr arXiv:2308.09417 for all the preparatory steps of the combination)

Combinations with CDF excluded have good compatibility: $m_W = 80369.2 \pm 13.3$ MeV (CT18)

the χ^2 probability is 91%

relative weights: 42% (ATLAS), 23% (D0), 18% (LHCb), 16% (LEP)

The inclusion of CDF brings the χ^2 probability below 0.5%

Combination of the different m_W determinations

Results combined using BLUE

Validation by reproducing internal experimental combinations

The CDF measurement contains an *a posteriori* shift $\delta m_W \sim 3 \text{ MeV}$
accounting for (CTEQ6M→NNPDF3.1, mass modelling, polarisation effects) removed before the combination

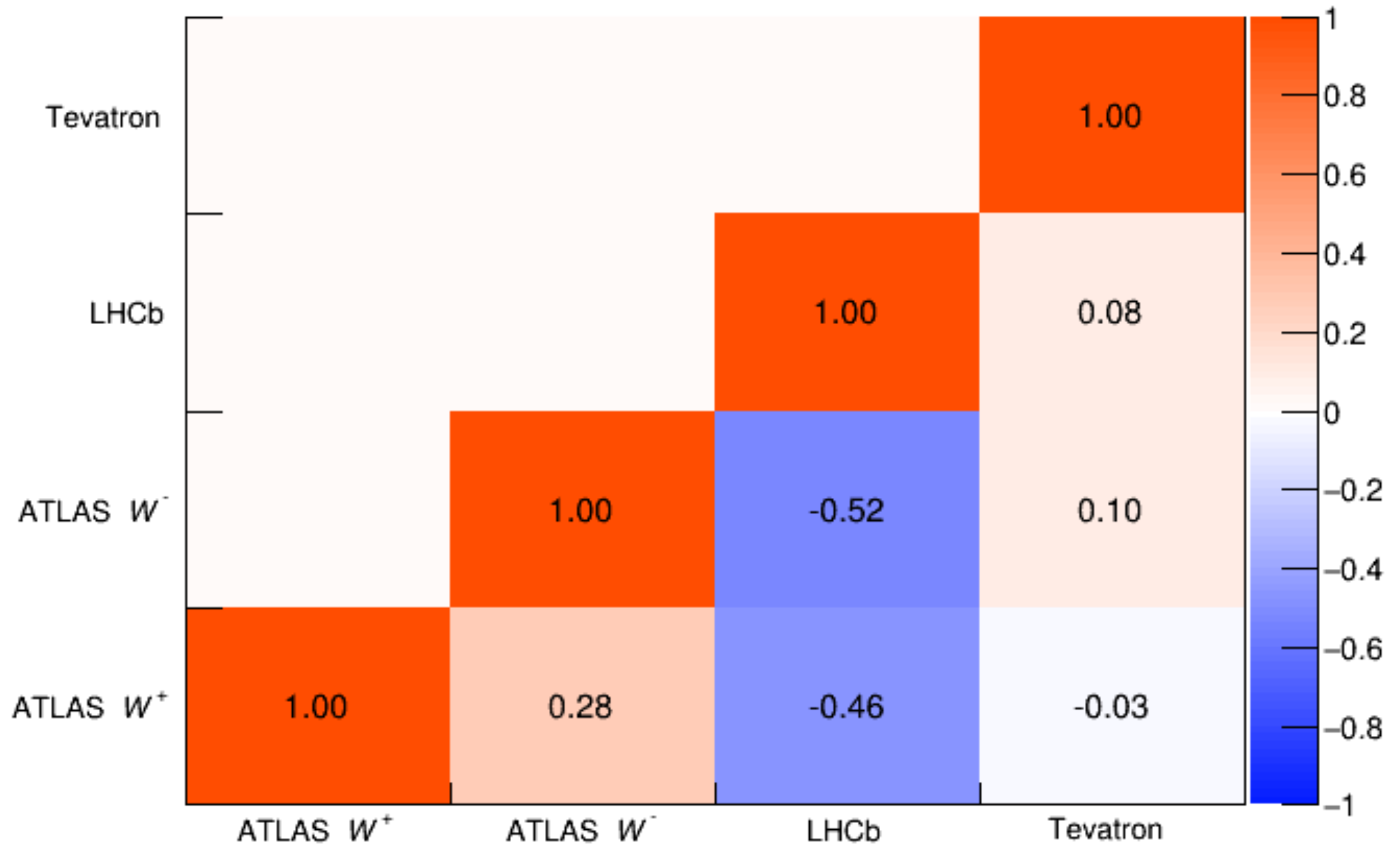
PDF correlations in the combination

Correlations needed in the combination

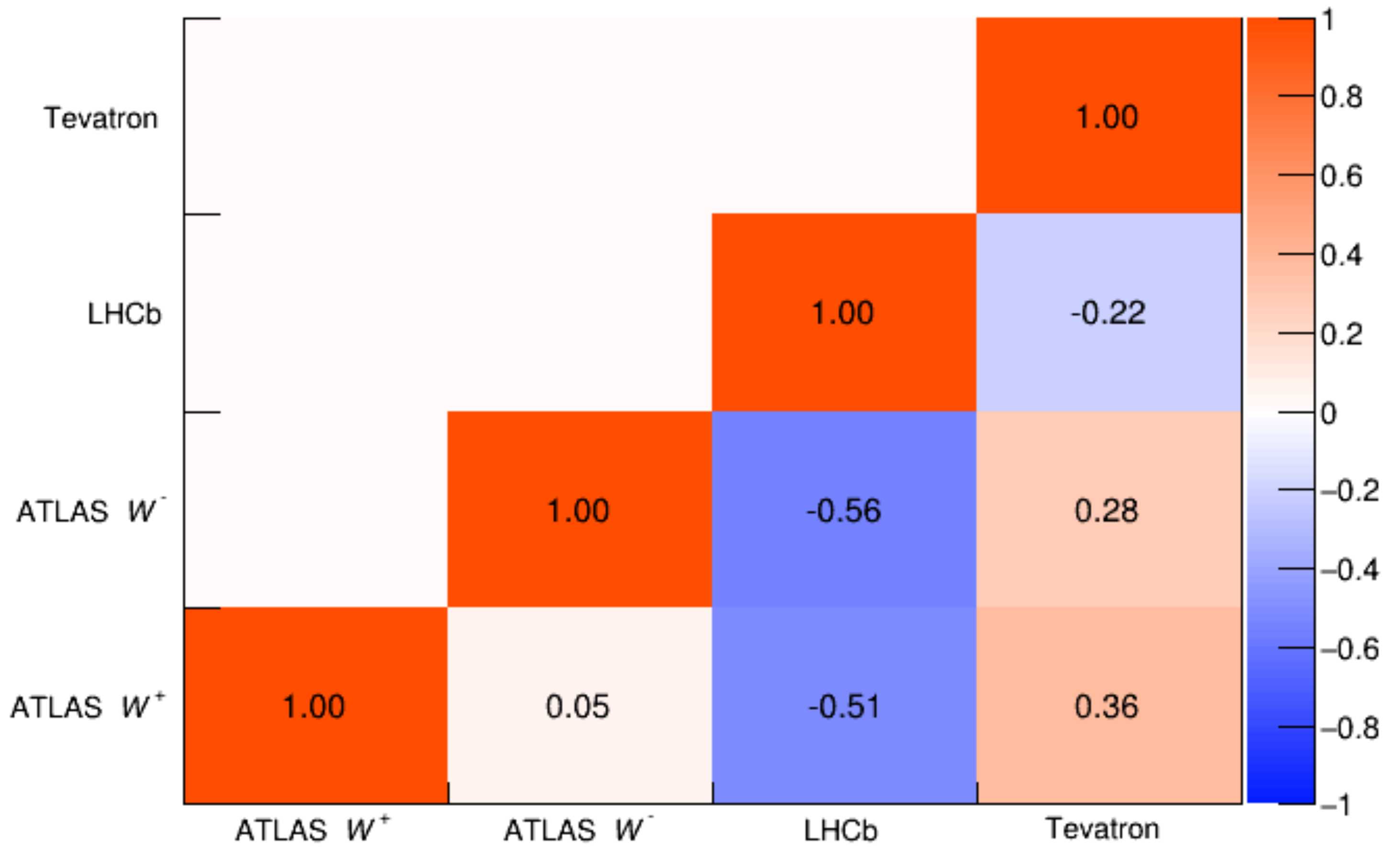
Significantly different correlations between the various PDF sets

PDF anti-correlations between experiments leads to more stable results and reduced PDF dependence
cfr. G.Bozzi, L.Citelli, AV, M.Vesterinen, arXiv:1501.05587, arXiv:1508.06954

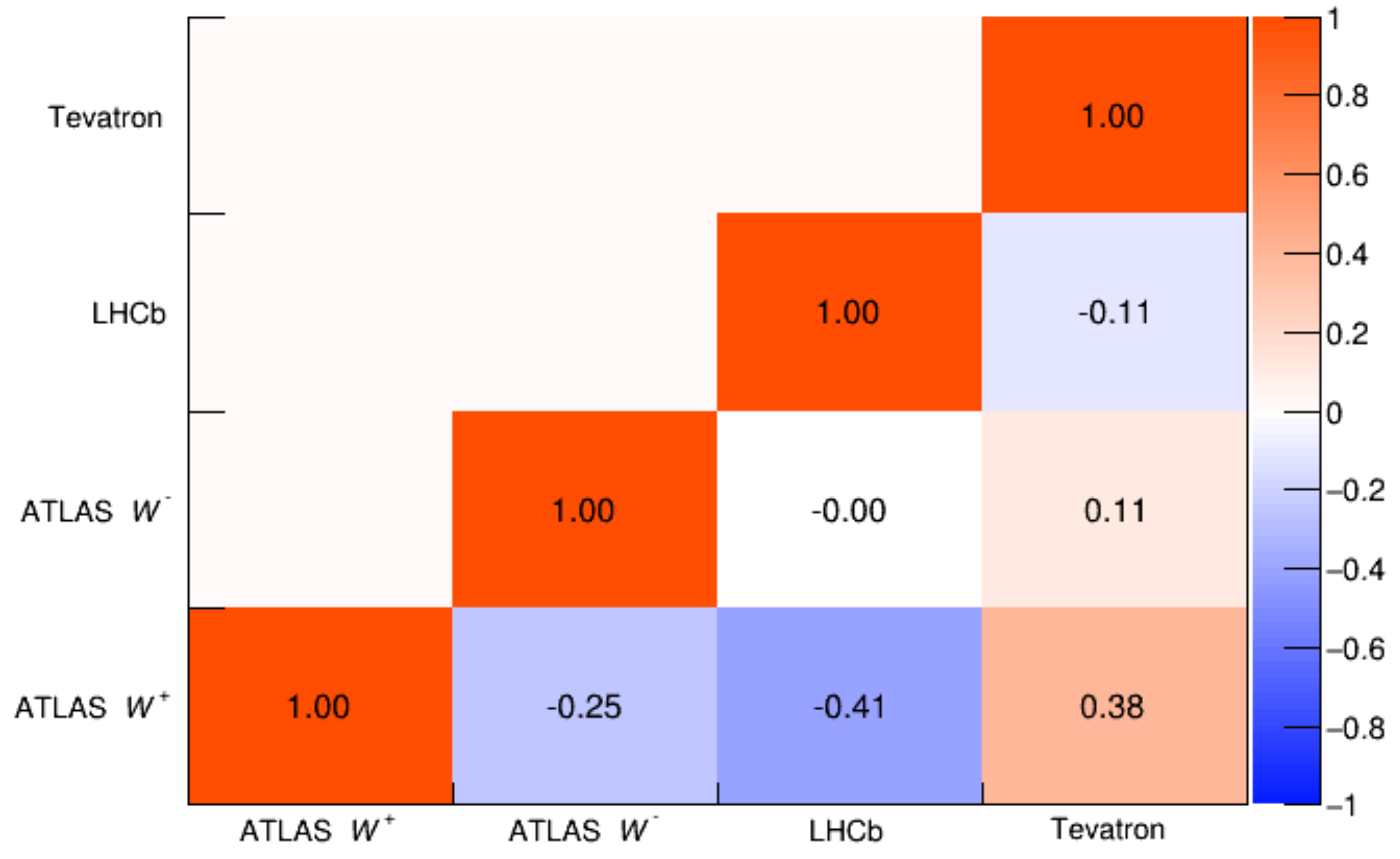
CT18



MSHT20



NNPDF4.0



Conclusions about the m_W combination effort

Extensive effort to provide a common treatment of PDF and pQCD modelling for the m_W determination at hadron colliders

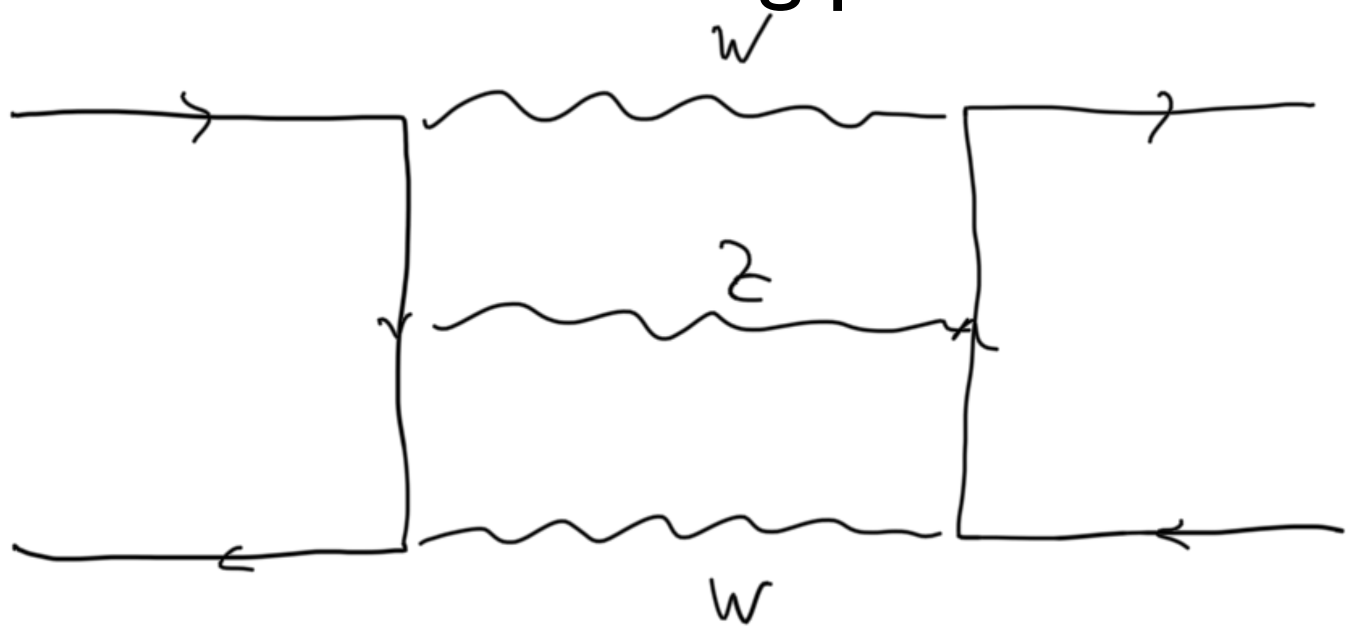
The updated treatment is unable to solve the tension between the existing measurements

The full combination $m_W = 80394.6 \pm 11.5\ \mathrm{MeV}$ (CT18) is disfavoured due to low χ^2 probability (0.5%)

The combination with CDF excluded $m_W = 80369.2 \pm 13.3\ \mathrm{MeV}$ (CT18) has good χ^2 probability (91%)

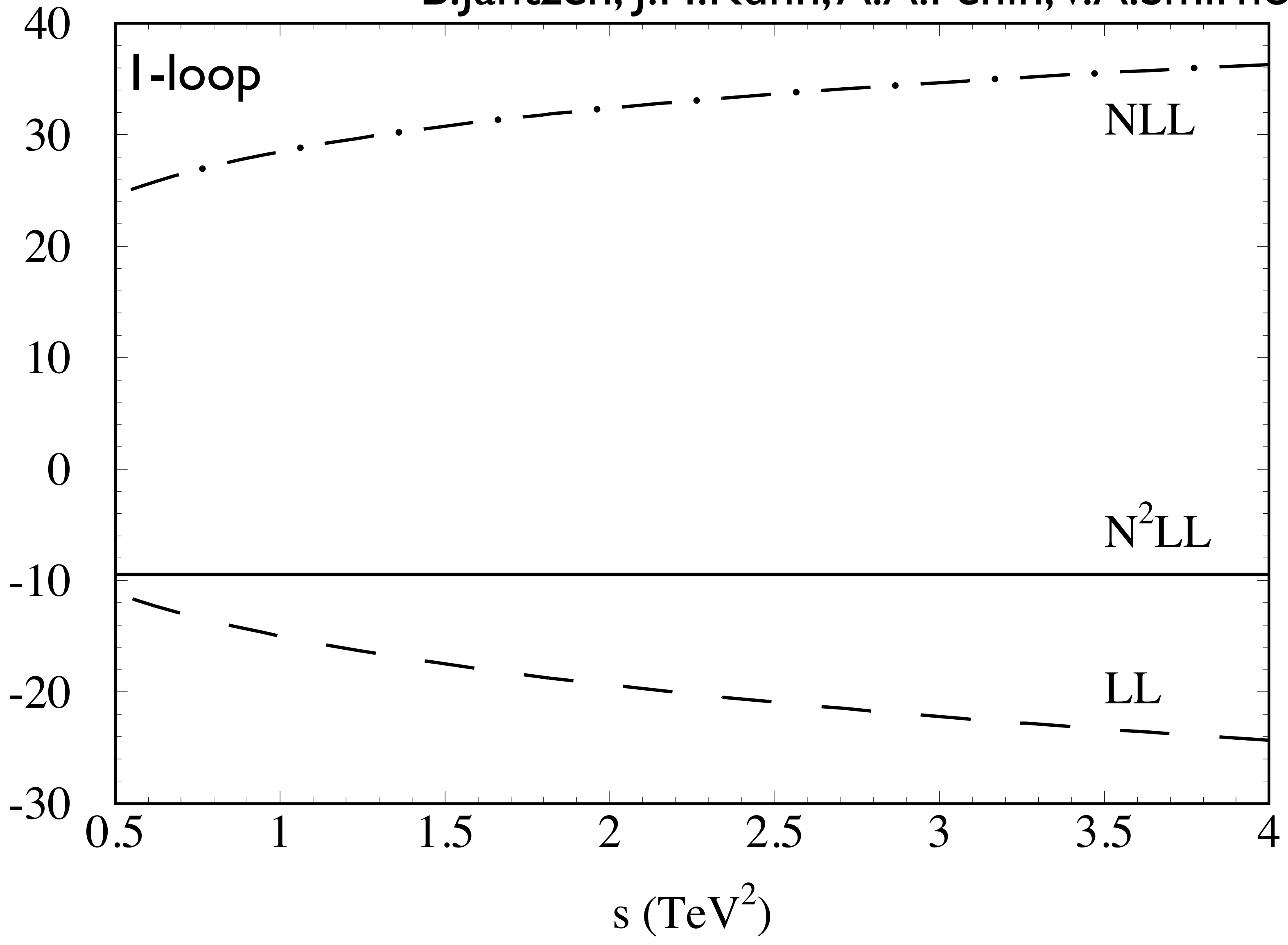
Need for a full NNLO-EW calculation to reduce the uncertainties to sub-percent level

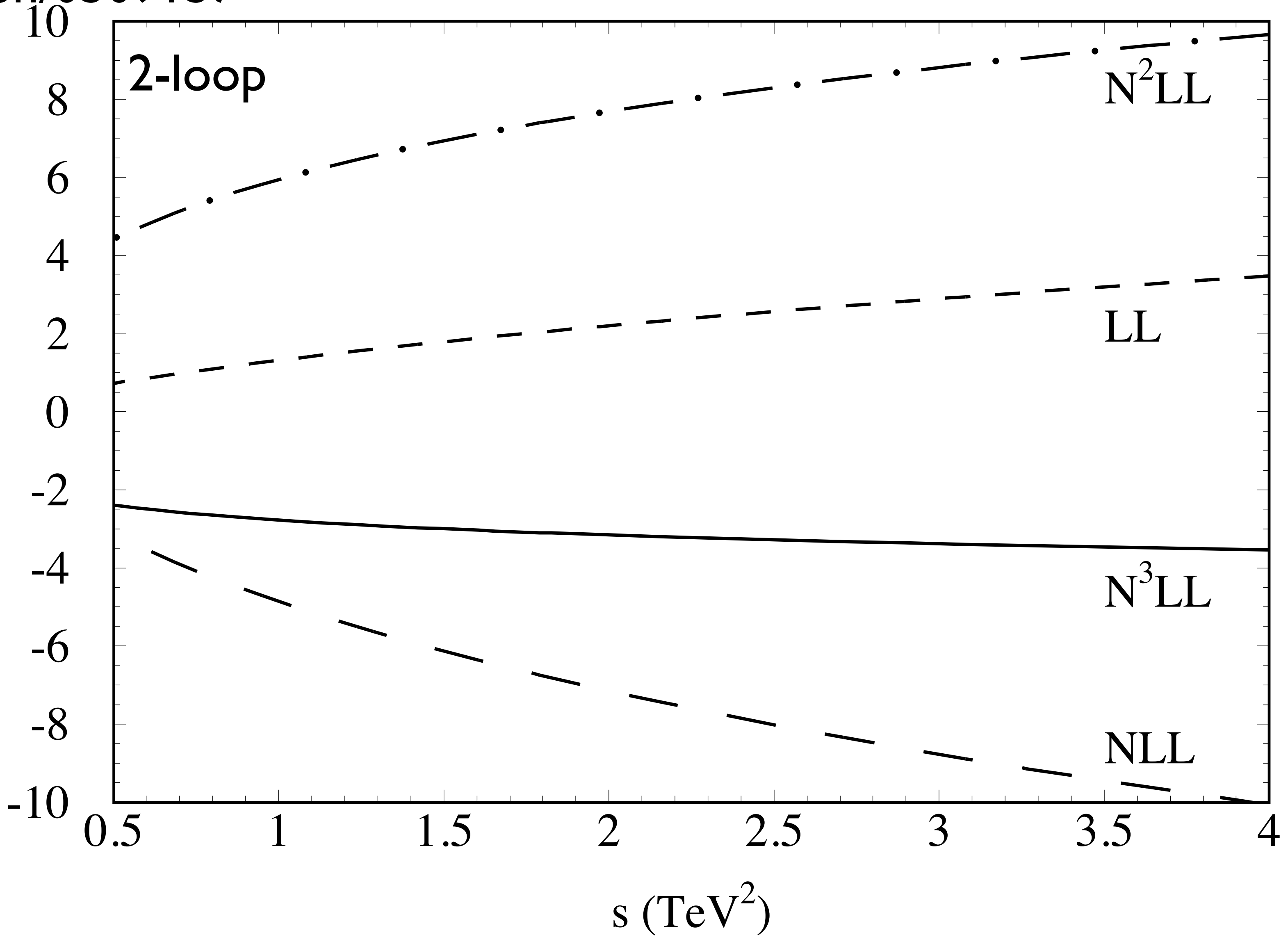
The NNLO-EW corrections to scattering processes are still today one of the frontiers in QFT



The NNLO-EW corrections could modify in a non-trivial way the large-mass/momentum tails of the distributions
Large logarithmic corrections (EW Sudakov logs) appear in the virtual corrections
At two-loop level, we have up to the fourth power of $\log(s/m_V^2)$,

B.Jantzen, J.H.Kühn, A.A.Penin, V.A.Smirnov, hep-ph/0509157



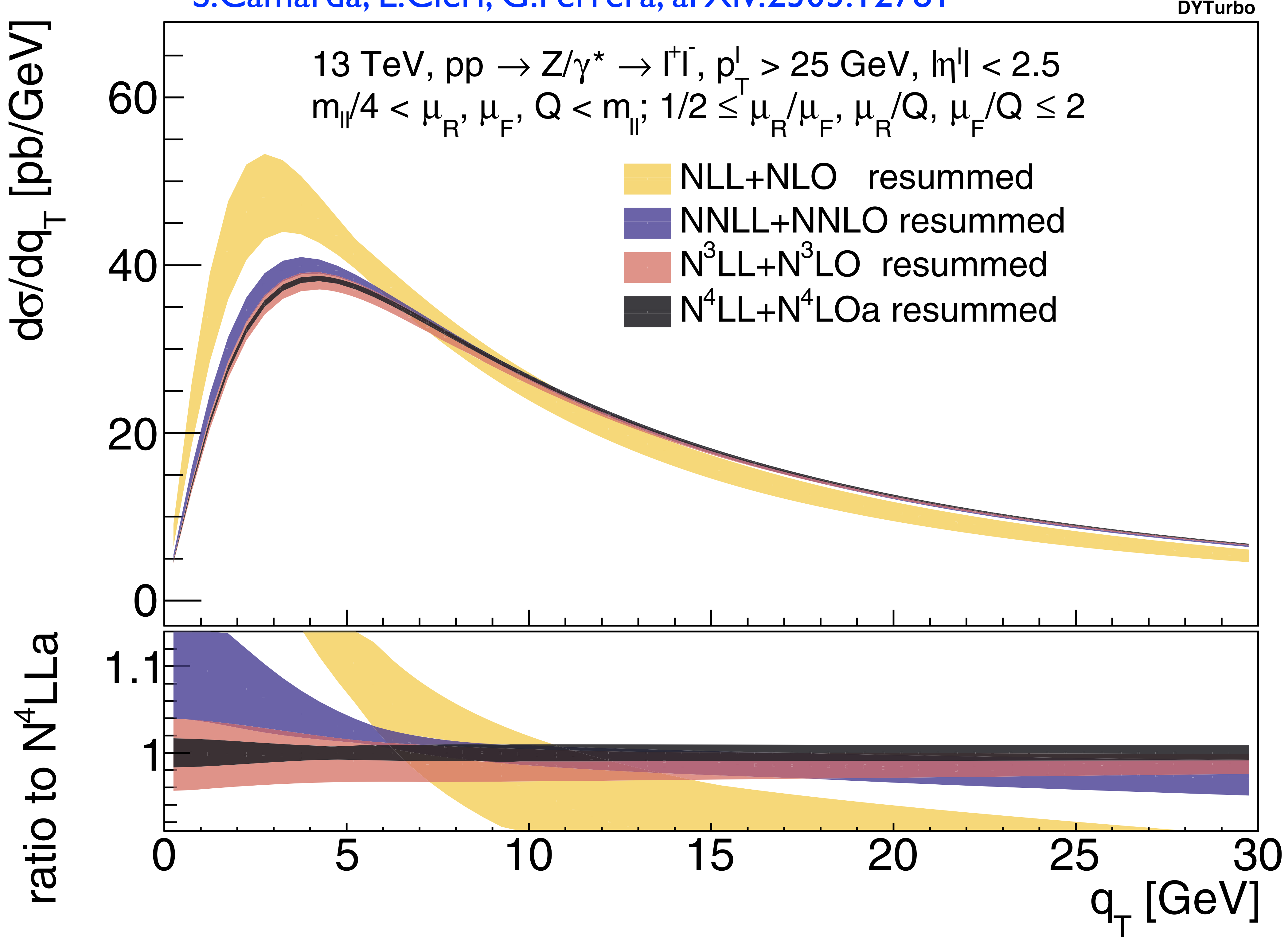


corrections to $e^+e^- \to q\bar{q}$
due to EW Sudakov logs

urgently needed to match sub-percent precision in the TeV region

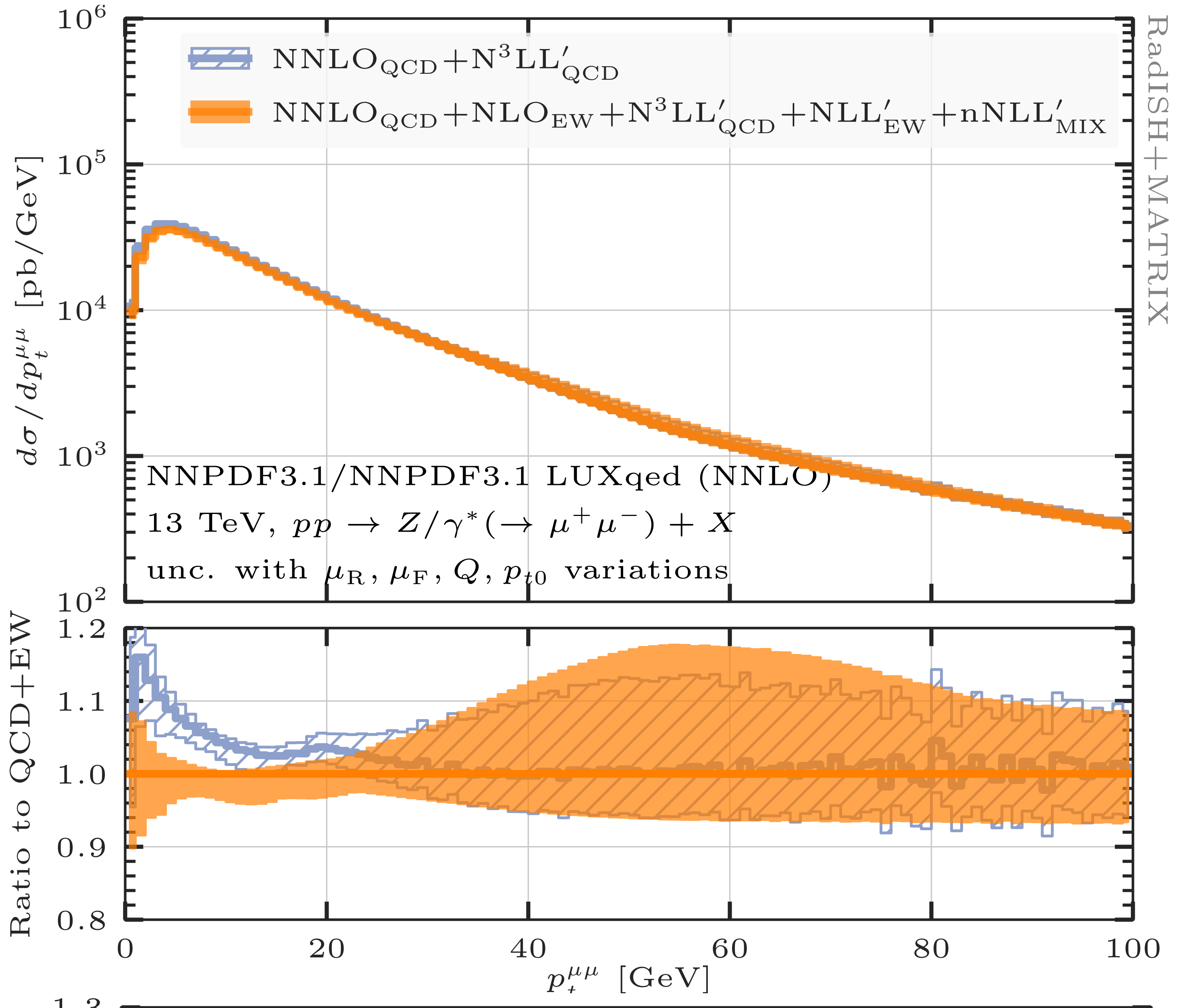
Beyond fixed order: Drell-Yan cross sections resumming large logarithmic corrections

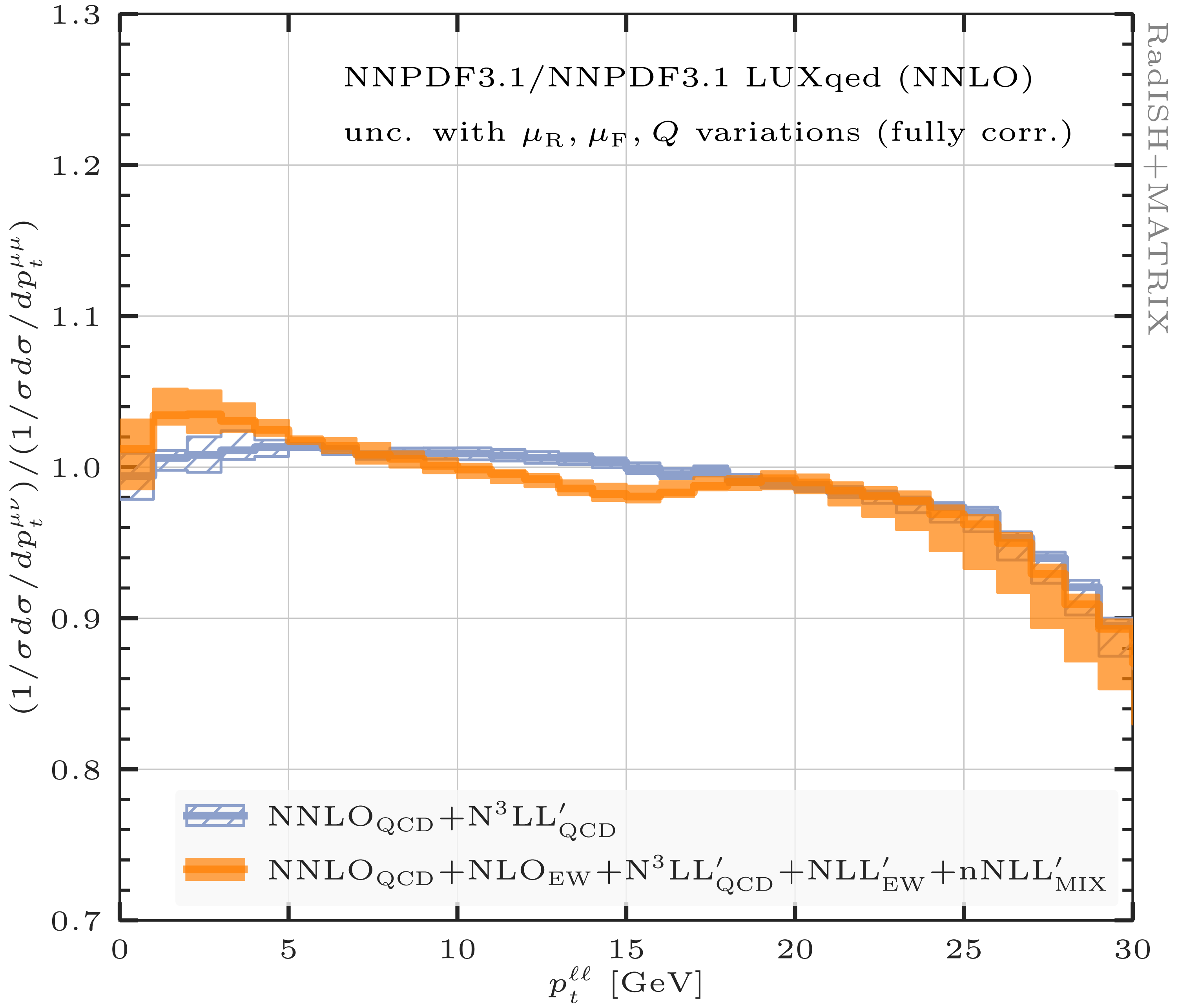
S.Camarda, L.Cieri, G.Ferrera, arXiv:2303.12781



Matching in pure QCD
at approximated N4LL+N4LO accuracy

L.Buonocore, L.Rottoli, P.Torrielli, arXiv:2404.15112





Matching in full QCD-EW SM at N3LL'-QCD + NLL'-EW + nNLL'-mixed accuracy