EPFL

Sterile Neutrino Dark Matter and Magnetic Fields

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Based on works with

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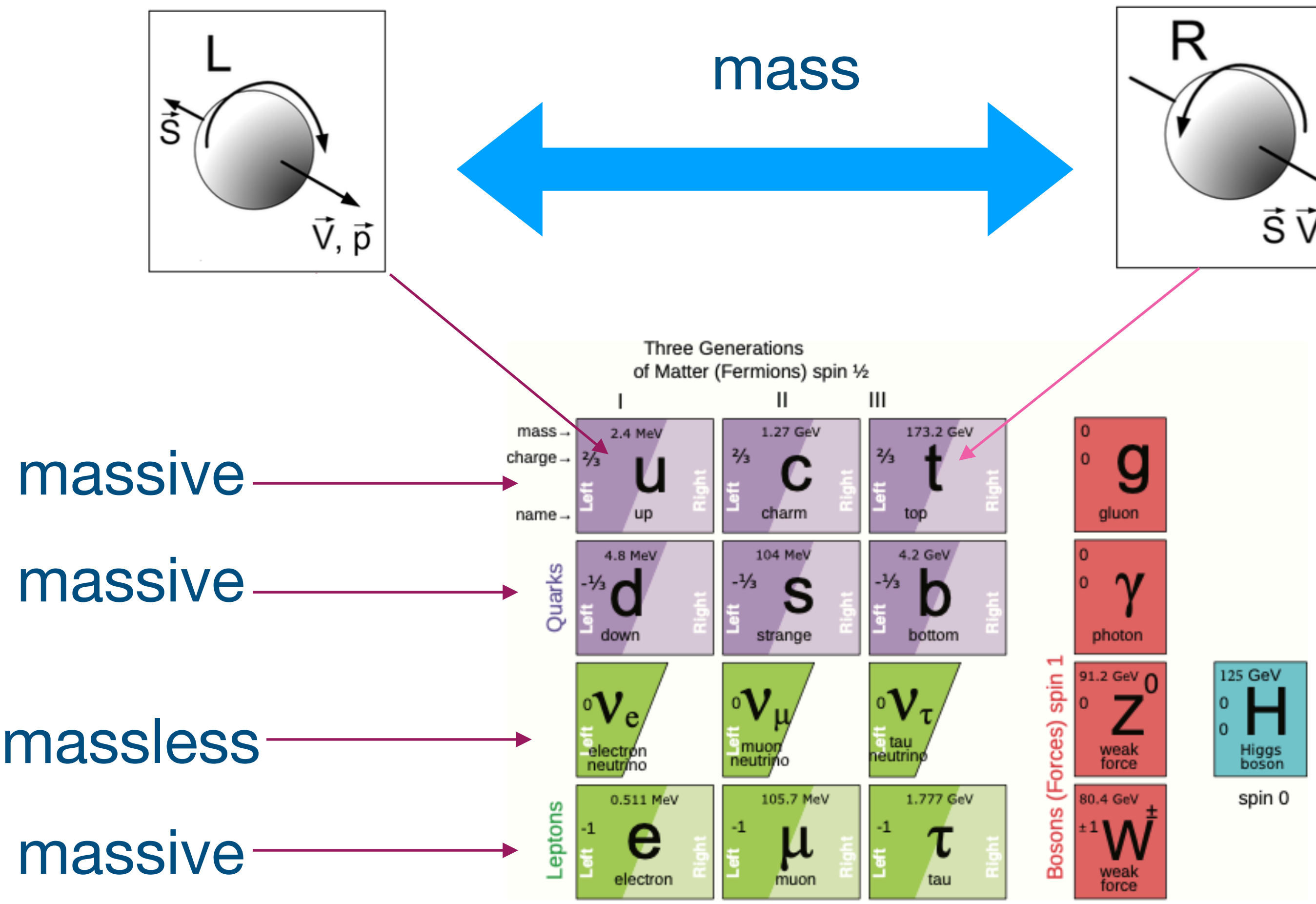
Outline

- Anomalous fermion number non-conservation (a short reminder)
- (Hyper)-magnetic fields as a storage for lepton/baryon/chiral asymmetry
- Transition from (hyper) magnetic to magnetic fields in the Early Universe
- Sterile neutrino DM and magnetic fields
- Conclusions

Anomalous fermion number non-conservation

Chirality in the Standard Model of elementary particles



mass

massive

massive

massless

massive

Left-right asymmetric content

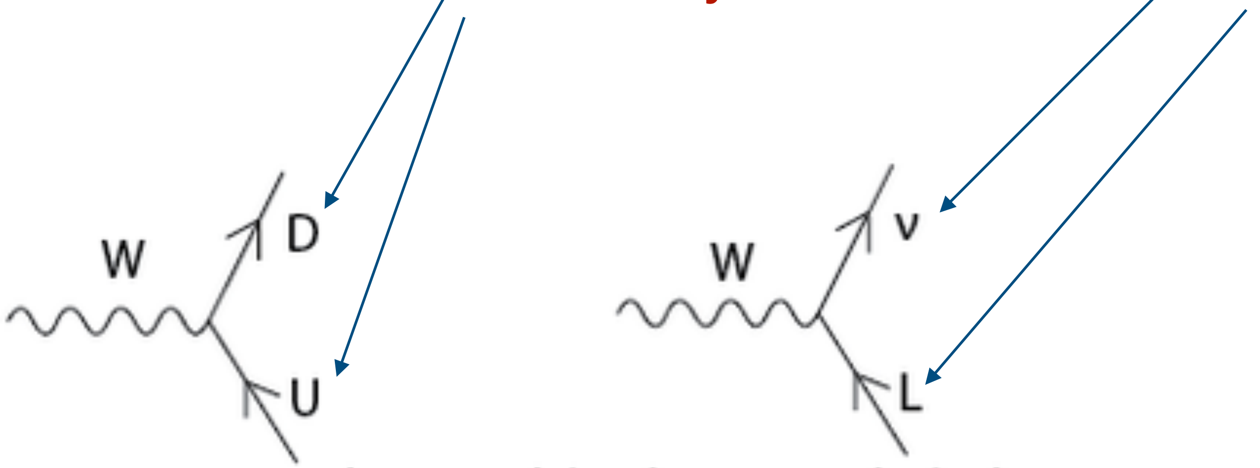
Chirality in the Standard Model of elementary particles

Left- and right- handed fermions interact with the weak bosons differently, leading to parity breaking!

perturbative effect

left-handed only

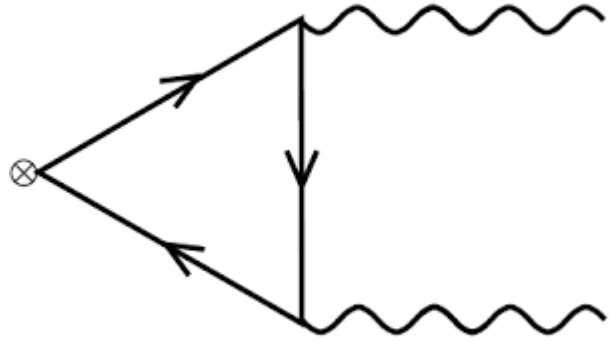
left-handed only



U is a up-type quark; D is a down-type quark.

L is a lepton and ν is the corresponding neutrino.

Anomalous fermion number non-conservation: anomaly



Fermionic current

SU(2) and U(1) gauge fields

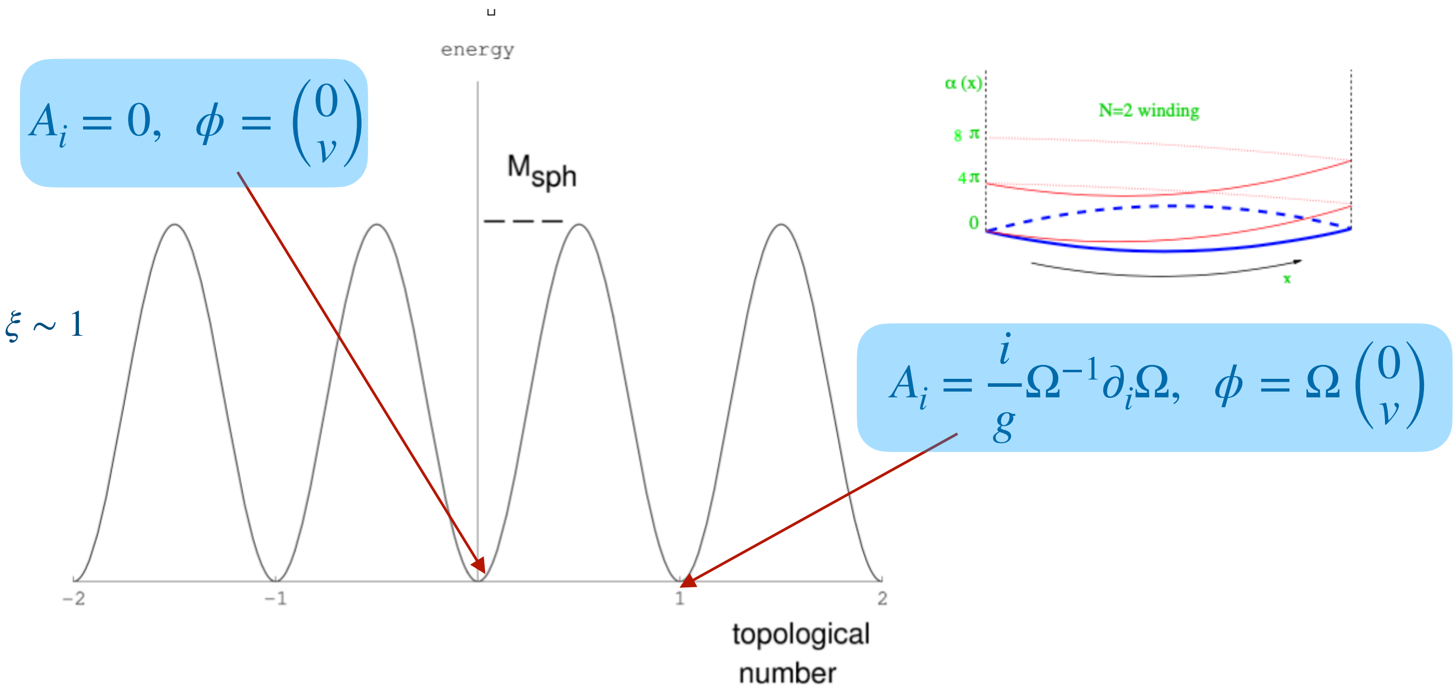
Non-Abelian SU(2) Abelian U(1)

$$\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = 3\left(\frac{g^2}{16\pi^2} Tr F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{g'^2}{32\pi^2} Y_{\mu\nu}\tilde{Y}^{\mu\nu}\right)$$

SU(2) and U(1) gauge fields can be converted into fermions,
and fermions can be converted into gauge fields

Anomalous fermion number non-conservation: SU(2)

$$M_{sph} = \xi \frac{M_W}{\alpha_W} \sim 10\ \mathrm{TeV},\ \xi \sim 1$$



$$A_i = 0,\quad \phi = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$A_i = \frac{i}{g}\Omega^{-1}\partial_i\Omega,\quad \phi = \Omega\begin{pmatrix} 0 \\ v \end{pmatrix}$$

Change in topology: space $S_3 \rightarrow S_3$ SU(2) group.

$$\int_{t_1}^{t_2} d^4x \frac{g^2}{16\pi^2} TrF_{\mu\nu}\tilde{F}^{\mu\nu} = N_{CS}(t_2) - N_{CS}(t_1),\quad N_{CS} \neq 0 \text{ for vacuum SU(2) configurations}$$

Fermion number goes into vacuum.

Anomalous fermion number non-conservation: the SU(2) rate

Non-Abelian SU(2)

$$\Gamma \sim \begin{cases} \exp(-\frac{4\pi}{\alpha_W}) \sim 10^{-160}, & T = 0 \\ \exp\left(-\frac{M_{sph}}{T}\right), & T < T_c \\ (\alpha_W)^5 T^4, & T > T_c \end{cases}$$

Quantum-mechanical tunnelling through barrier

Thermal over-barrier fluctuations

Thermal fluctuations

$T_c \simeq 160$ GeV is the temperature of the electroweak crossover:
above it the SU(2) symmetry is “restored”.
Baryogenesis happens above 130 GeV - freezing temperature of sphalerons.

(Hyper)-magnetic fields as a storage for lepton/baryon/chiral asymmetry

High temperature phase of the electroweak theory

Anomalous fermion number non-conservation: $U(1)_Y$

The vacuum configurations of the Abelian U(1) group have no topological number, and

$$\int_{t_1}^{t_2} d^4x \frac{g'^2}{32\pi^2} Y_{\mu\nu}\tilde{Y}^{\mu\nu} = 0$$

for vacuum to vacuum transitions. This is not the case if the hypermagnetic field $\vec{B}$ is non-zero,

$$N_{CS} = \frac{g'^2}{8\pi^2}\vec{A}_Y\vec{B}_Y,$$

where $\vec{A}_Y$ is the hypercharge field vector-potential,

$$\int_{t_1}^{t_2} d^4x \frac{g'^2}{32\pi^2} Y_{\mu\nu}\tilde{Y}^{\mu\nu} = N_{CS}^Y(t_2) - N_{CS}^Y(t_1) \neq 0$$

Anomalous fermion number non-conservation: the $U(1)_Y$ rate

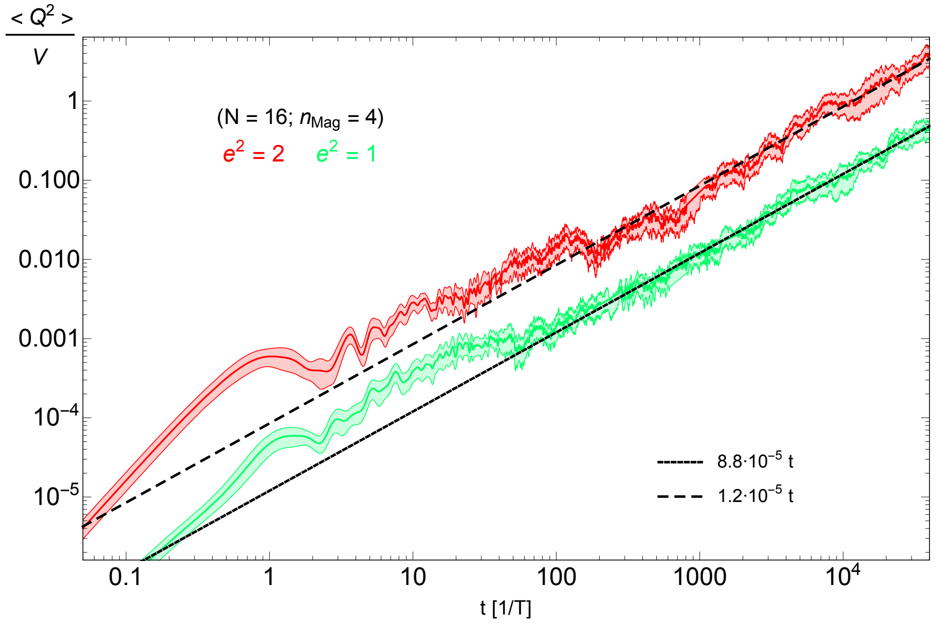
- MHD equations, temperature above the EW crossover:

$$\Gamma_Y \simeq \frac{3g'^4}{4\pi^4 \sigma_Y T^2} B_Y^2 \propto \alpha_Y^3 B_Y^2$$

σ_Y - hypercharge conductivity of the plasma

- Non-perturbative lattice simulation give a result with a factor ~10 times larger (Figueroa,Florio, MS)

- If there is hypercharge magnetic field, it can be a “storage” for baryon and lepton numbers - Chern-Simons condensate



Diffusion of the Chern-Simons number

$$Q \propto \int d^4x F\tilde{F}$$

Instabilities

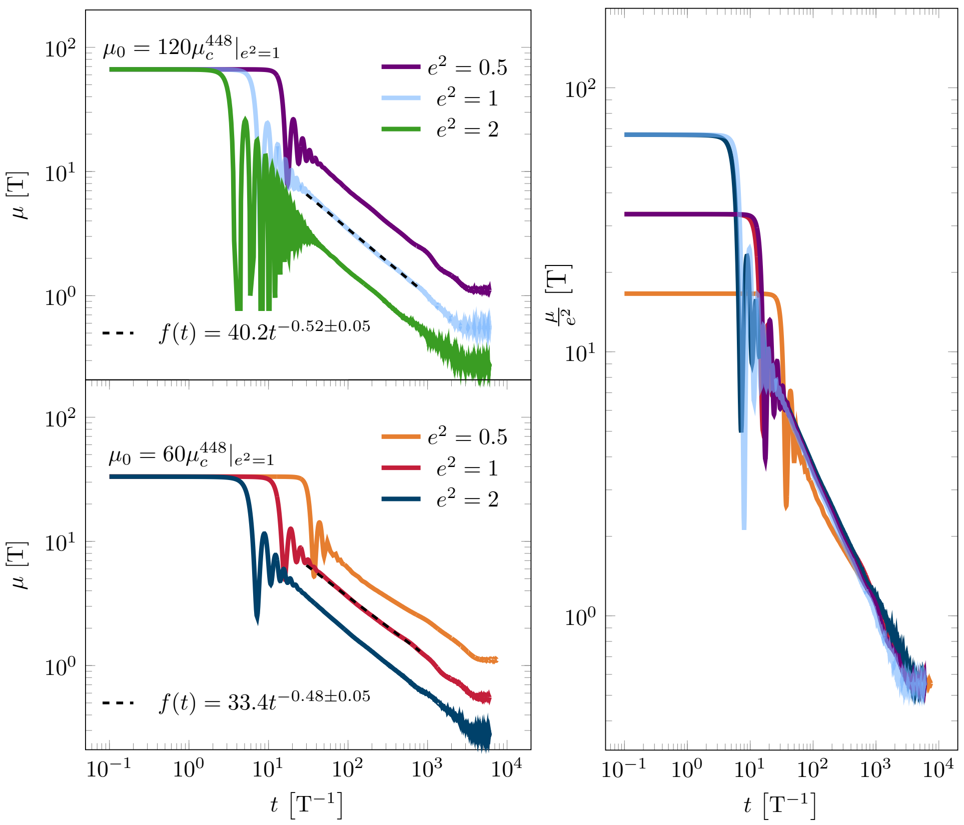
If asymmetries in baryon or lepton numbers are present, the hypermagnetic field is generated even if it was zero before.

Effective action:

$S_{CS} \simeq \mu N_{CS}$, μ - fermionic chemical potential

The infrared hypercharge modes are unstable for $k < k_{\text{inst}} \equiv \dfrac{\alpha_Y}{\pi}\mu$

The instability eats up the asymmetry and converts it into abelian fields



Behaviour of the chemical potential

Low temperature phase of the electroweak theory

At small temperatures we are in the Higgs phase of the Standard Model:

Just change hypercharge to charge, hypermagnetic to magnetic, baryon and lepton number to chirality, (almost) everything remains in force.

Below the electroweak cross-over we have massless photon γ. Anomaly moves to another sector of the theory: chiral charge of the electron distribution,

$$\partial_\mu J_5^\mu = \frac{2\alpha}{\pi} \vec{E}_\gamma \vec{B}_\gamma + m_e \bar{\Psi}_e \gamma_5 \Psi_e$$

$J_5^0 =$ density of left-handed electrons - density of right-handed electrons. At $T >$ few GeV the electron mass can be neglected: magnetic helicity can go to electron chiral charge and back. Modifications of magnetohydrodynamics equations, change of the evolution of magnetic fields (Giovannini, MS; Boyarsky, Fröhlich, Ruchayskiy; Dam Thanh Son, Yamamoto;…, Rogachevskii, Ruchayskiy, Boyarsky, Fröhlich, Kleeorin, Brandenburg, Schober; applications to astrophysics)

Transition from (hyper) magnetic to magnetic fields in the Early Universe

From hypermagnetic to magnetic

Non-Abelian SU(2)   Abelian U(1)

Anomaly equation: $\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = 3\left(\frac{g^2}{16\pi^2} TrF_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{g'^2}{32\pi^2} Y_{\mu\nu}\tilde{Y}^{\mu\nu}\right)$

- Temperatures above the electroweak cross-over, $T > T_c = 160$ GeV: massless hyper-charge photon Y_μ. Anomaly equation contains the piece $\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = -\frac{3g'^2}{16\pi^2}\vec{E}_Y \vec{B}_Y + \cdot\cdot$, where $\vec{E}_Y$ and $\vec{B}_Y$ are hyper-electric and hyper-magnetic fields.

- Temperatures below the electroweak cross-over, $T < T_c = 160$ GeV: massless photon. In terms of low-energy fields W, Z and γ: $\partial_\mu J_B^\mu = \partial_\mu J_L^\mu \propto Z_{\mu\nu}\tilde{\gamma}^{\mu\nu} + \mathcal{O}(W^2, Z^2)$

No term $\propto \gamma_{\mu\nu}\tilde{\gamma}^{\mu\nu} \propto \vec{E}_\gamma \vec{B}_\gamma$!

Storage for baryon and lepton numbers - the CS condensate - disappears

The puzzle

How does this happen and when?

- No perturbative corrections to the CS term beyond one-loop (Coleman&Hill, for our problem Laine&MS): absence of coupling of anomaly to the massless photon is an exact perturbative result. Order parameter for the electroweak phase transition?
- Contradicts to the lattice studies of the SM at high temperatures: no phase transition, smooth crossover for Higgs masses above 80 GeV

Answer is unknown, but is important for cosmological applications

Cosmological implications of U(1) anomaly

- The change of hyper-magnetic helicity leads to production/destruction of baryon number

- Baryon/lepton asymmetries change the evolution of hyper-magnetic fields: the state with $B_Y = 0$ is unstable if the Universe is charge asymmetric

Several ideas (Joyce, MS; Kamada, Long; Vachaspati, Vilenkin,Kamada, Domcke…):

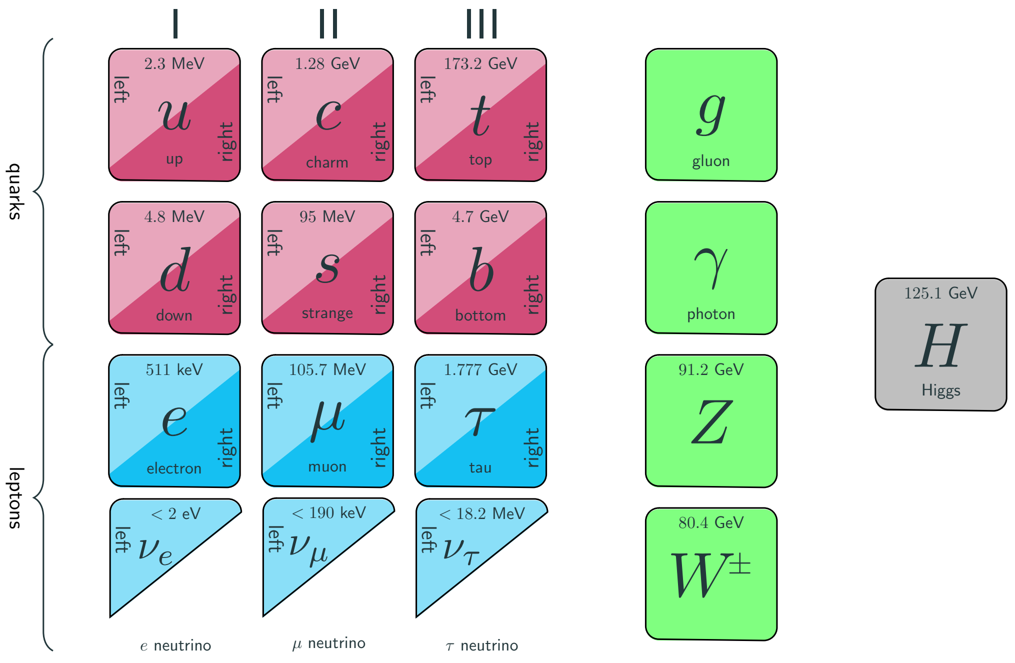
- preexisting (parity breaking inflation?) hyper-magnetic fields may lead to baryogenesis

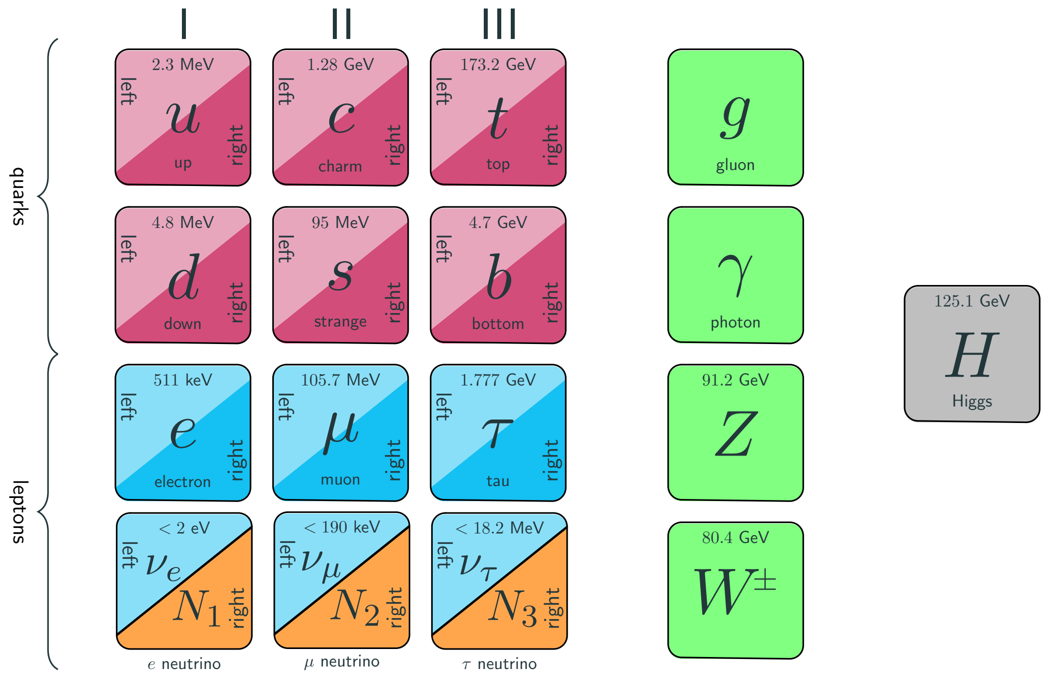
- charge asymmetries in the early Universe may lead to generation of primordial hyper-magnetic fields

Sterile neutrino Dark Matter and magnetic fields

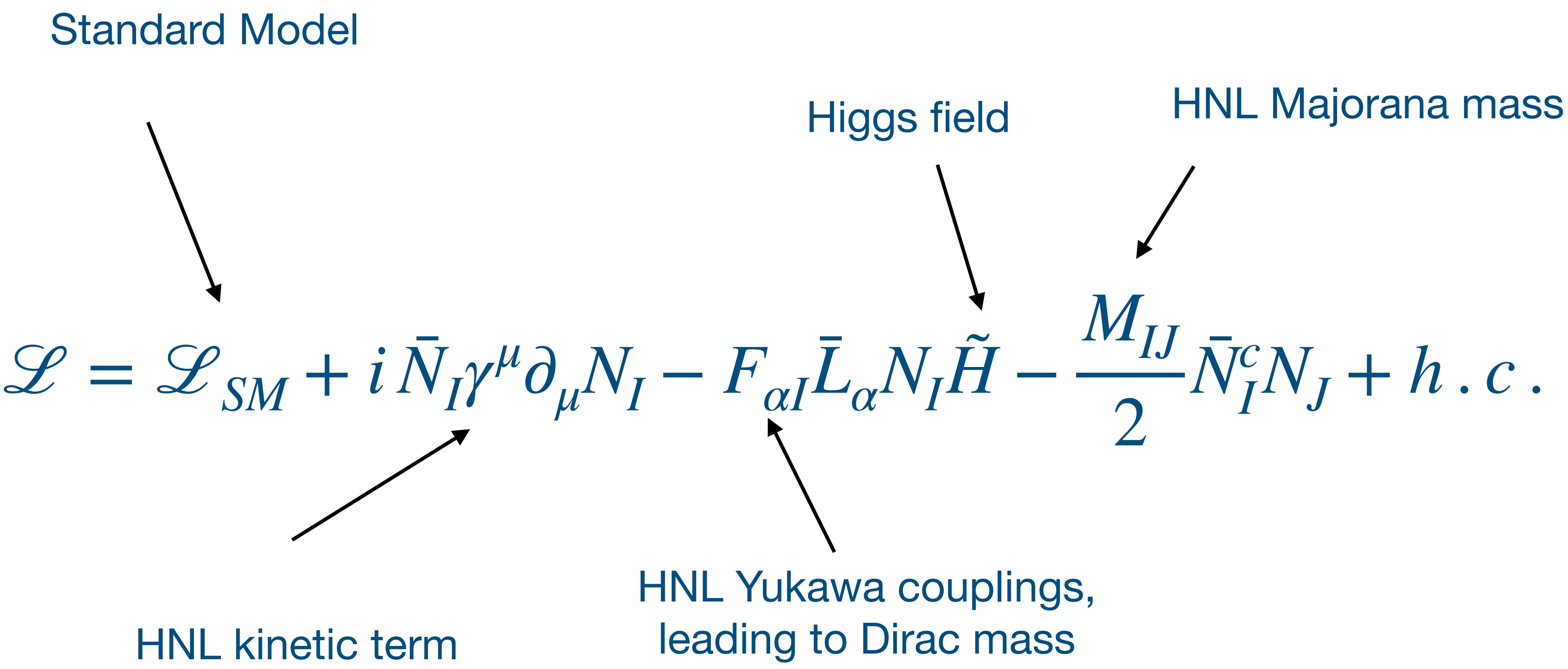
νMSM as a minimal model of new physics

The simplest theory of new physics which can explain all experimental drawbacks of the Standard Model (neutrino masses and oscillations, dark matter, baryon asymmetry of the Universe, incorporating cosmological Higgs inflation leading to the observable universe) is at extension of the SM by 3 right-handed neutrinos (or heavy neutral leptons - HNLs) : the minimal type I see-saw model or νMSM.





Most general renormalisable see-saw Lagrangian with Majorana neutrinos:



Standard Model

Higgs field

HNL Majorana mass

$$\mathscr{L} = \mathscr{L}_{SM} + i\bar{N}_I\gamma^\mu\partial_\mu N_I - F_{\alpha I}\bar{L}_\alpha N_I\tilde{H} - \frac{M_{IJ}}{2}\bar{N}^c_I N_J + h.c.$$

HNL kinetic term

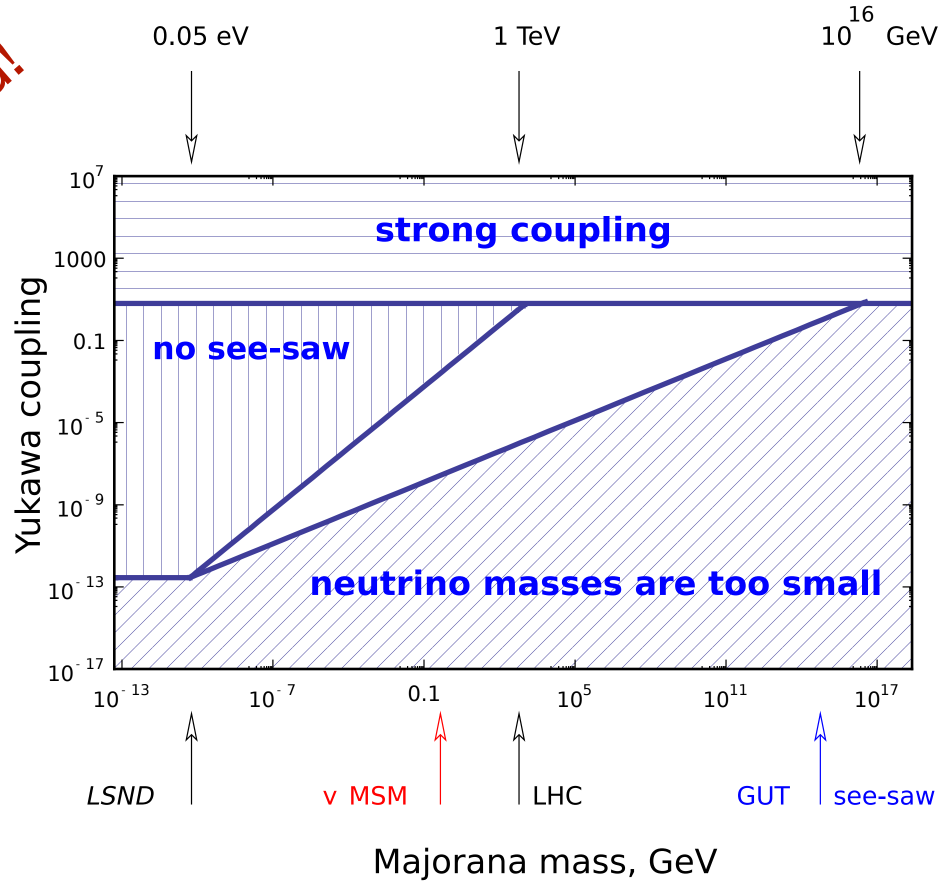
HNL Yukawa couplings,
leading to Dirac mass

Neutrino masses and Yukawa couplings from Neutrino physics

$$Y^2 = Trace[F^\dagger F]$$

Scale F as x, and M as x^2, low energy neutrino physics is not changed!

$$m_\nu \propto \frac{F^2 v^2}{M}$$



HNL roles in the νMSM

N_1- Dark Matter particle (Dodelson, Widrow; Shi, Fuller; Dolgov, Hansen;….)

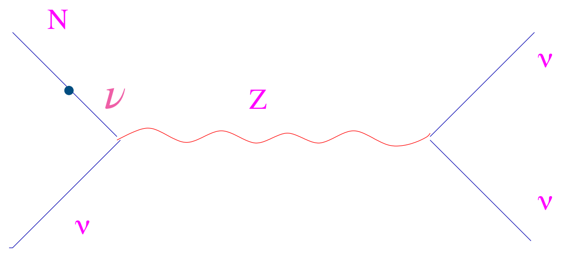
$N_{2,3}$ - responsible for neutrino masses and baryogenesis (See-saw team - Minkowski and others; Fukugita, Yanagida, …; baryogenesis: Akhmedov, Rubakov, Smirnov; Asaka, MS,…)

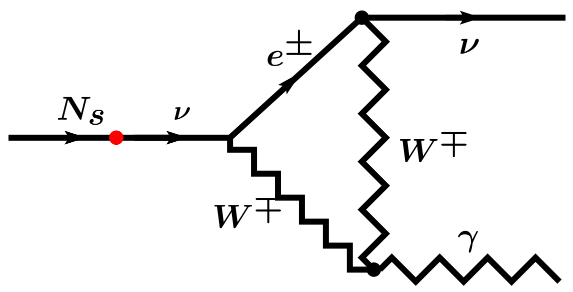
Constraints on DM sterile neutrino N₁

- **Stability.** N₁ must have a lifetime larger than that of the Universe. Main decay mode $N_1 \to 3\nu$ is not observable.

- **X-rays.** N₁ decays radiatively, $N_1 \to \gamma\nu$, producing a narrow line $E_\gamma = M_1/2$ which can be detected by X-ray telescopes (such as Chandra or XMM-Newton).

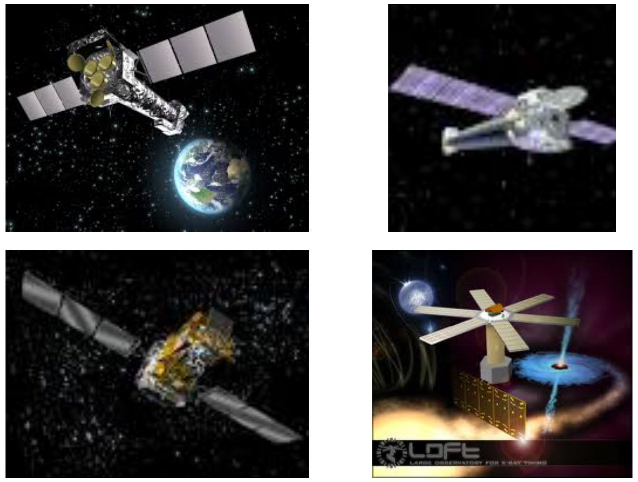
$$\theta = m_D/M_M$$



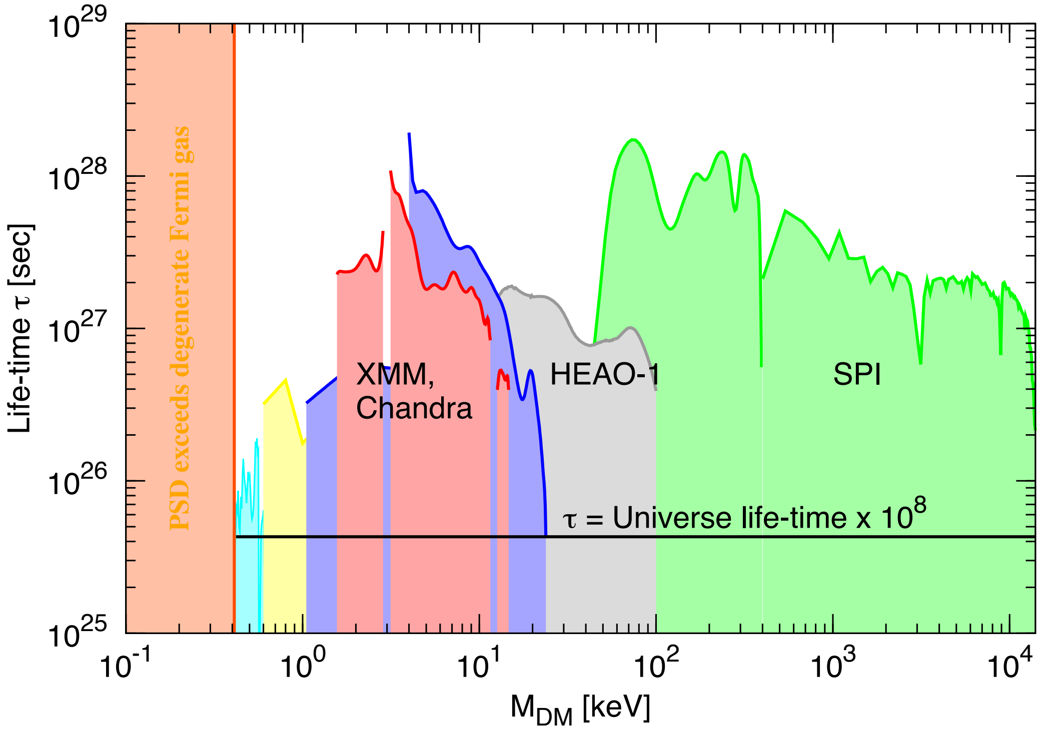


$$\Gamma_{\rm rad} = \frac{9\alpha G_F^2}{256 \cdot 4\pi^4} \sin^2(2\theta) M_s^5$$

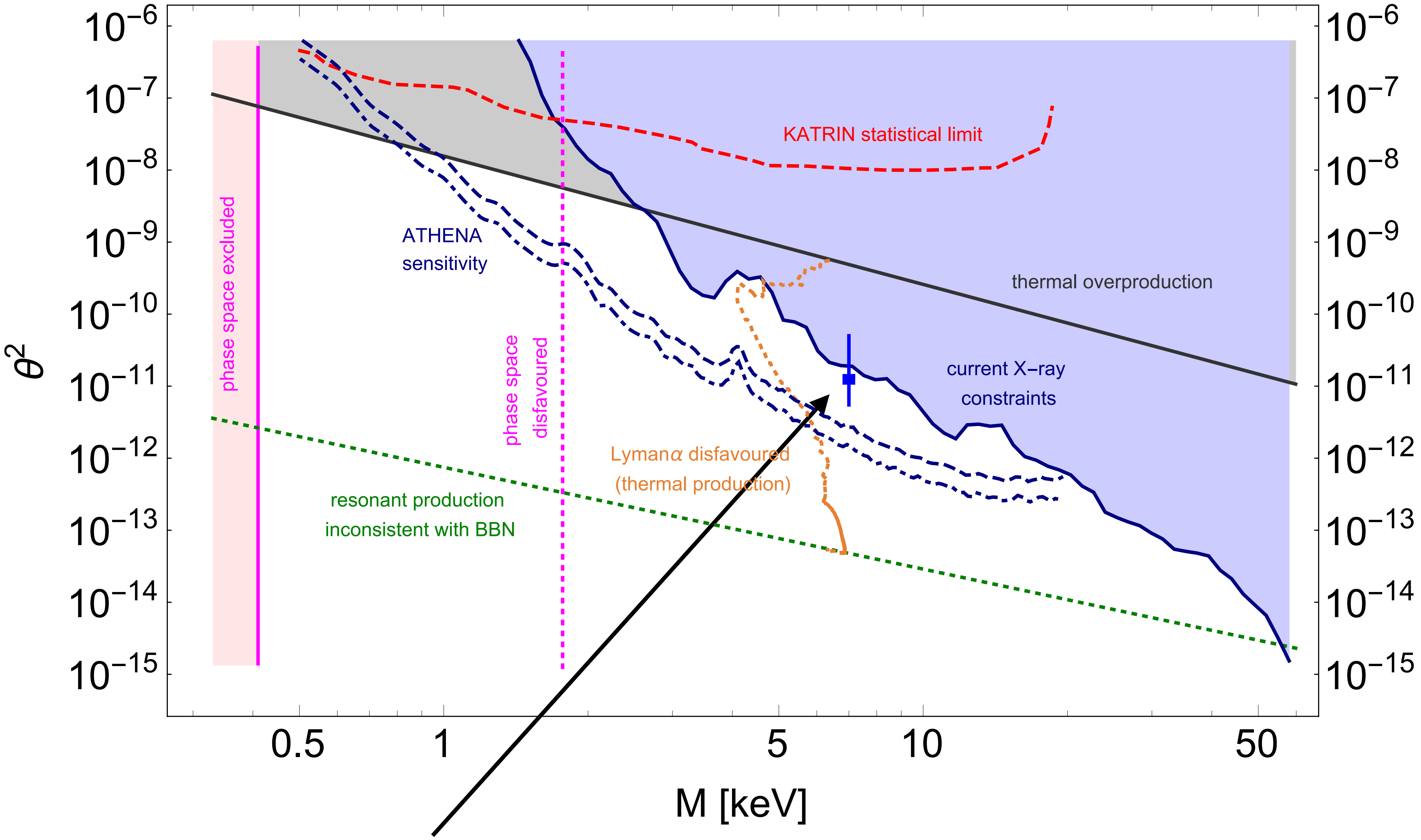
Lifetime constraints



Available X-ray satellites:
Suzaku, XMM-Newton, Chandra, INTEGRAL, NuStar

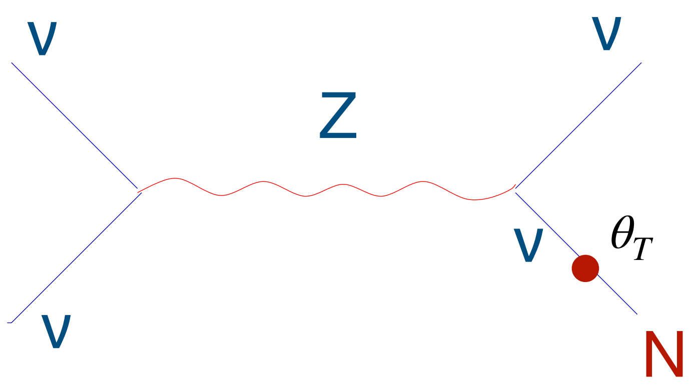


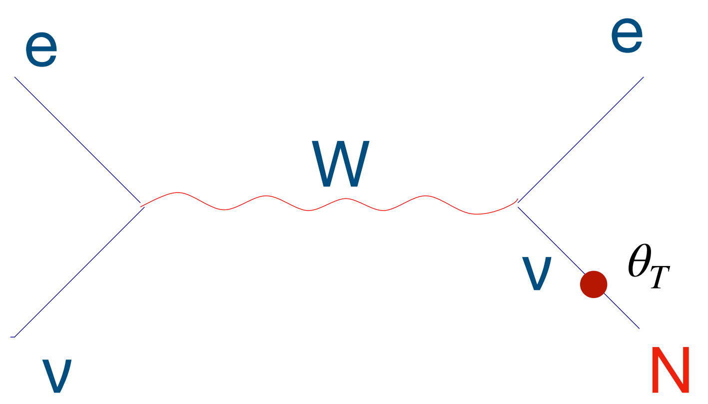
X-ray and structure formation constraints



Possible detection (?), still controversial, to be resolved in 2023-2024
Bulbul et al; Boyarsky et al

DM sterile neutrino production at low temperatures





$$\Gamma_N \simeq \Gamma_\nu \theta_T^2, \quad \theta_T \simeq \frac{\theta_0 M_N^2}{M_N^2 - (E_\nu^2 - \vec{p}_\nu^{\,2})}$$

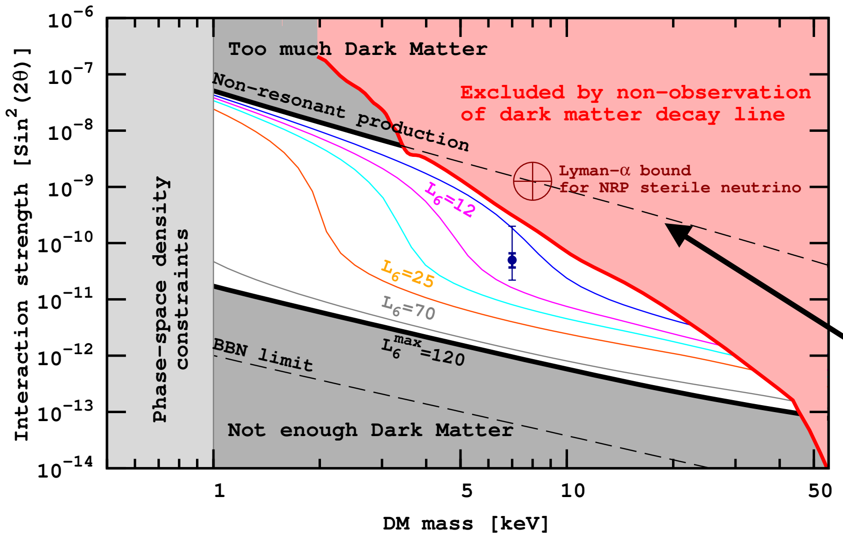
Temperature dependent mixing angle

The temperature of production of DM sterile neutrinos: the QCD epoch

$$T \simeq 250 \left(\frac{M_1}{7\ \mathrm{keV}} \right)^{1/3} \ \mathrm{MeV}$$

Dodelson, Widrow; Shi, Fuller; Dolgov, Hansen; Abazajian, Fuller, Patel; … Asaka, Laine, MS;…

Non-resonant production



Relation between the lifetime and abundance.

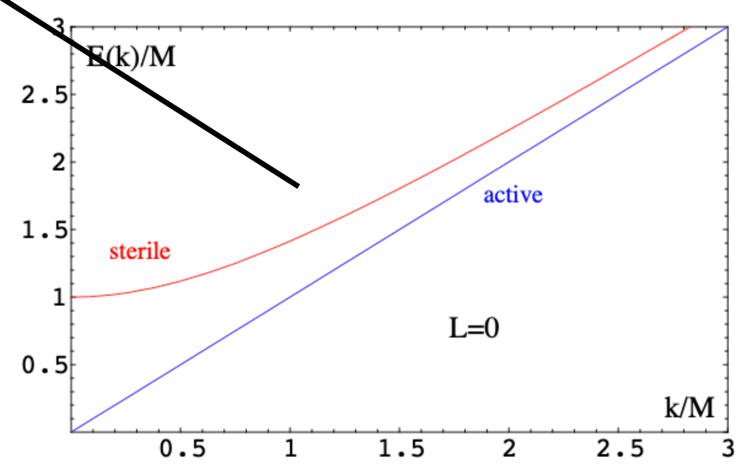
Momentum of sterile neutrino $\simeq 0.85 p_T$

$$\theta_1^2 \to \theta_M^2(T) \simeq \frac{\theta_1^2}{\left(1+\frac{2p}{M_1^2}\left(b(p,T) \pm \cancel{c(T)}\right)\right)^2+\theta_1^2}$$

$$b(p,T) = \frac{16G_F^2}{\pi\alpha_W}p(2+\cos^2\theta_W)\frac{7\pi^2T^4}{360}$$

$$\cancel{c(T) = 3\sqrt{2}G_F\left(1+\sin^2\theta_W\right)(n_{\nu_e}-n_{\bar{\nu}_e})}$$

DW mechanism



Transitions $\nu \longrightarrow N_1$

Dodelson-Widrow

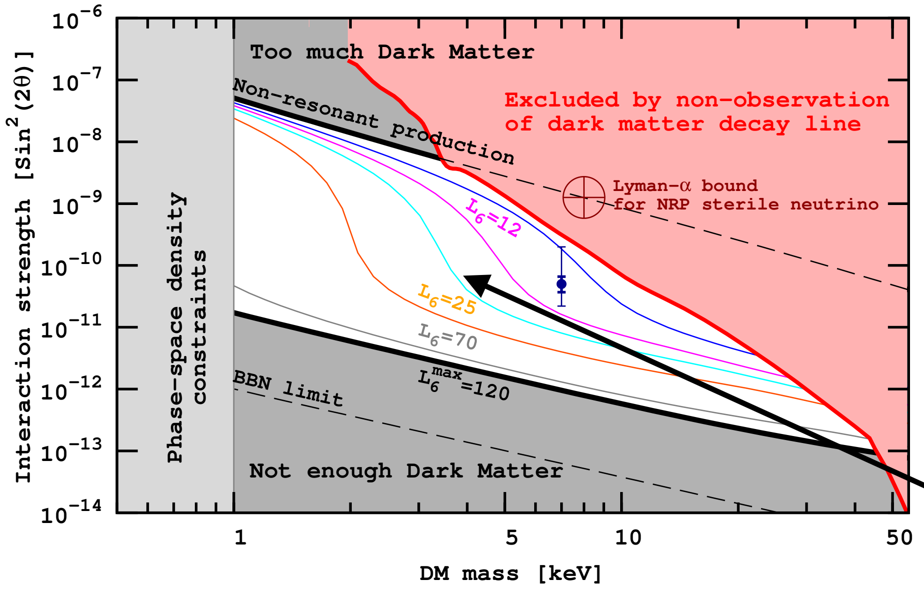
Resonant production

L_6 - lepton asymmetry in units 10^{-6}

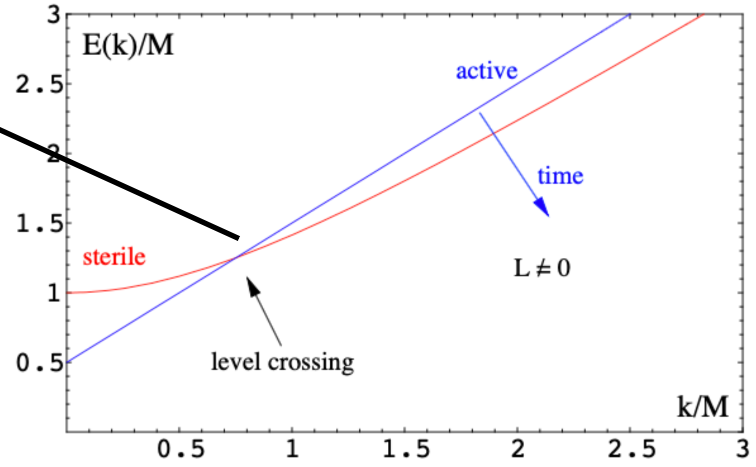
$$\theta_1^2 \to \theta_M^2(T) \simeq \frac{\theta_1^2}{\left(1+\frac{2p}{M_1^2}\left(b(p,T)\pm c(T)\right)\right)^2+\theta_1^2}$$

$$b(p,T) = \frac{16G_F^2}{\pi\alpha_W}p(2+\cos^2\theta_W)\frac{7\pi^2T^4}{360}$$

$$c(T) = 3\sqrt{2}G_F\left(1+\sin^2\theta_W\right)(n_{\nu_e}-n_{\bar{\nu}_e})$$



SF mechanism



Resonant transitions

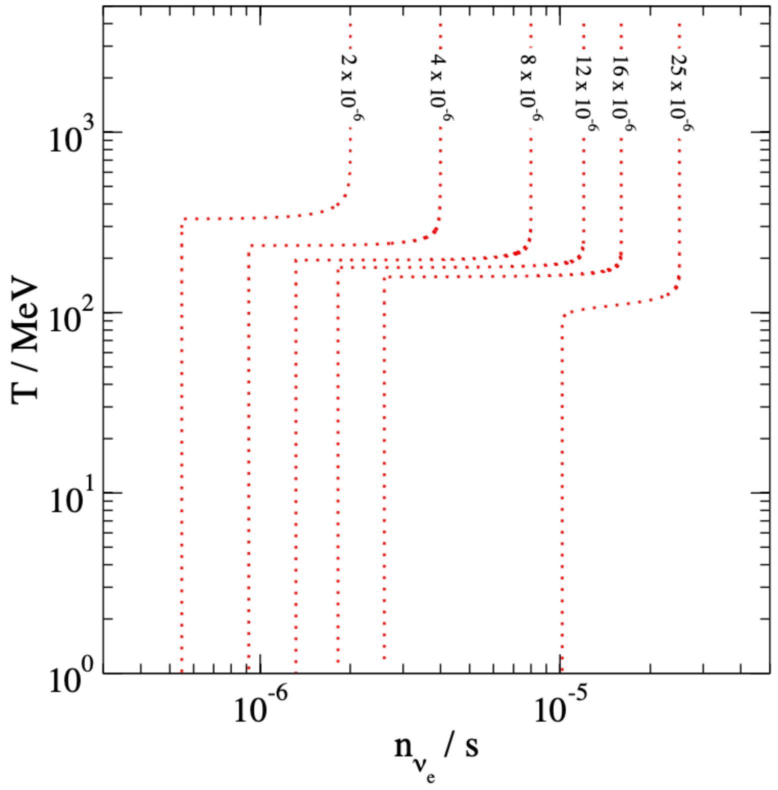
Shi-Fuller

Relation between the lifetime abundance, and lepton asymmetry.

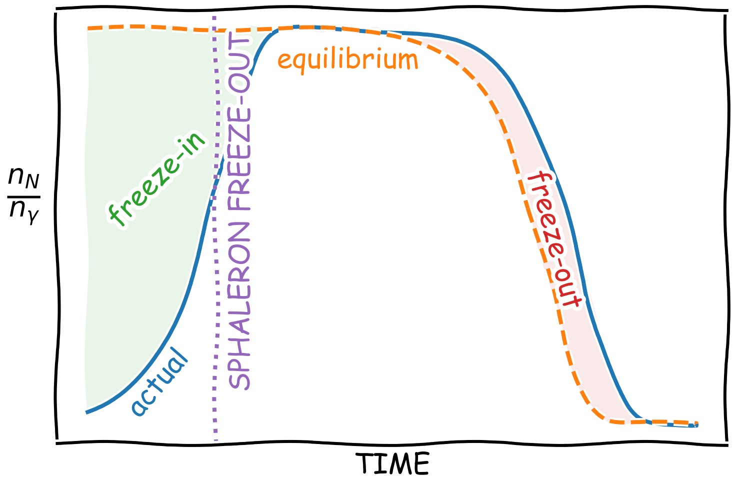
Momentum of sterile neutrino $\simeq 0.3p_T$

Asymmetry transfer to DM population

Large fraction of lepton asymmetry is converted into DM abundance of sterile neutrinos:



Low scale leptogenesis



Low scale leptogenesis

Creation of baryon asymmetry is a complicated process involving creation of HNLs in the early universe and their coherent CP-violating oscillations, interaction of HNLs with SM fermions, sphaleron processes with lepton and baryon number non-conservation. One need to deal with resummations, hard thermal loops, Landau-Pomeranchuk-Migdal effect, etc.

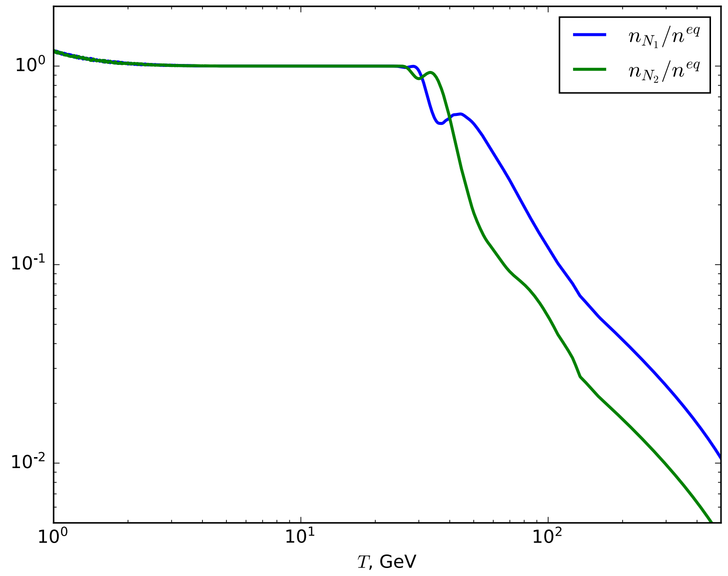
Initial idea: Akhmedov, Rubakov, Smirnov ‘98

Formulation of kinetic theory and demonstration that NuMSM can explain simultaneously neutrino masses, dark matter, and baryon asymmetry of the Universe: Asaka, M.S. ’05

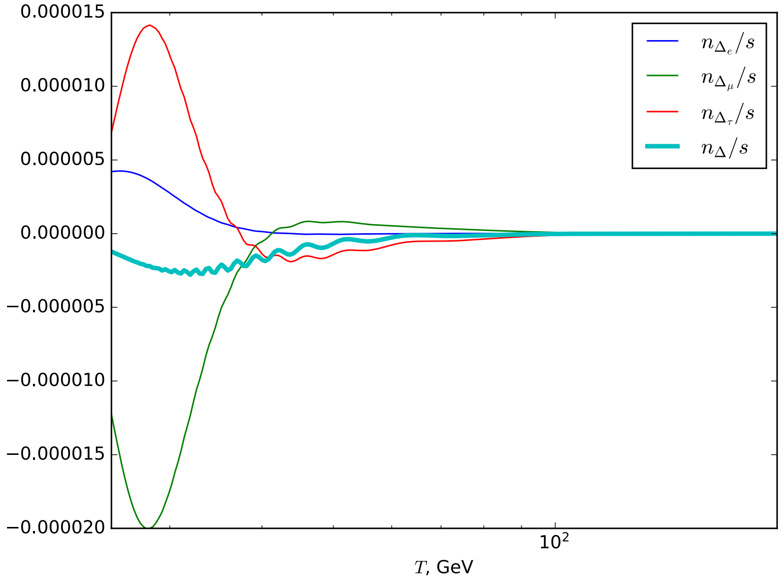
Analysis of baryon asymmetry generation in the NuMSM: Asaka, M.S., Canetti, Drewes, Frossard; Abada, Arcadia, Domcke, Lucente; Hernández, Kekic, J. López-Pavón, Racker, J. Salvado; Drewes, Garbrech, Guetera, Klariç; Hambye, Teresi; Eijima, Timiryasov; Ghiglieri, Laine,…

Time evolution

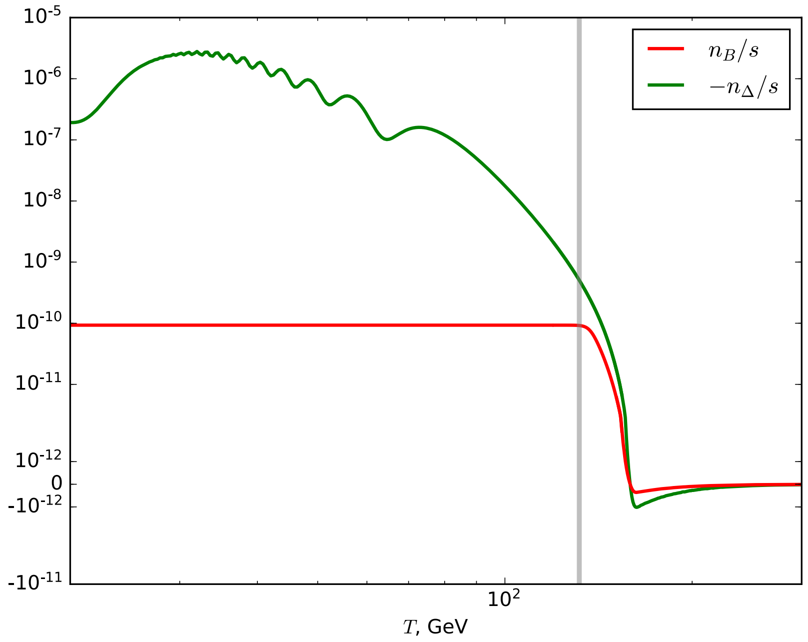
HNL densities



Lepton asymmetries

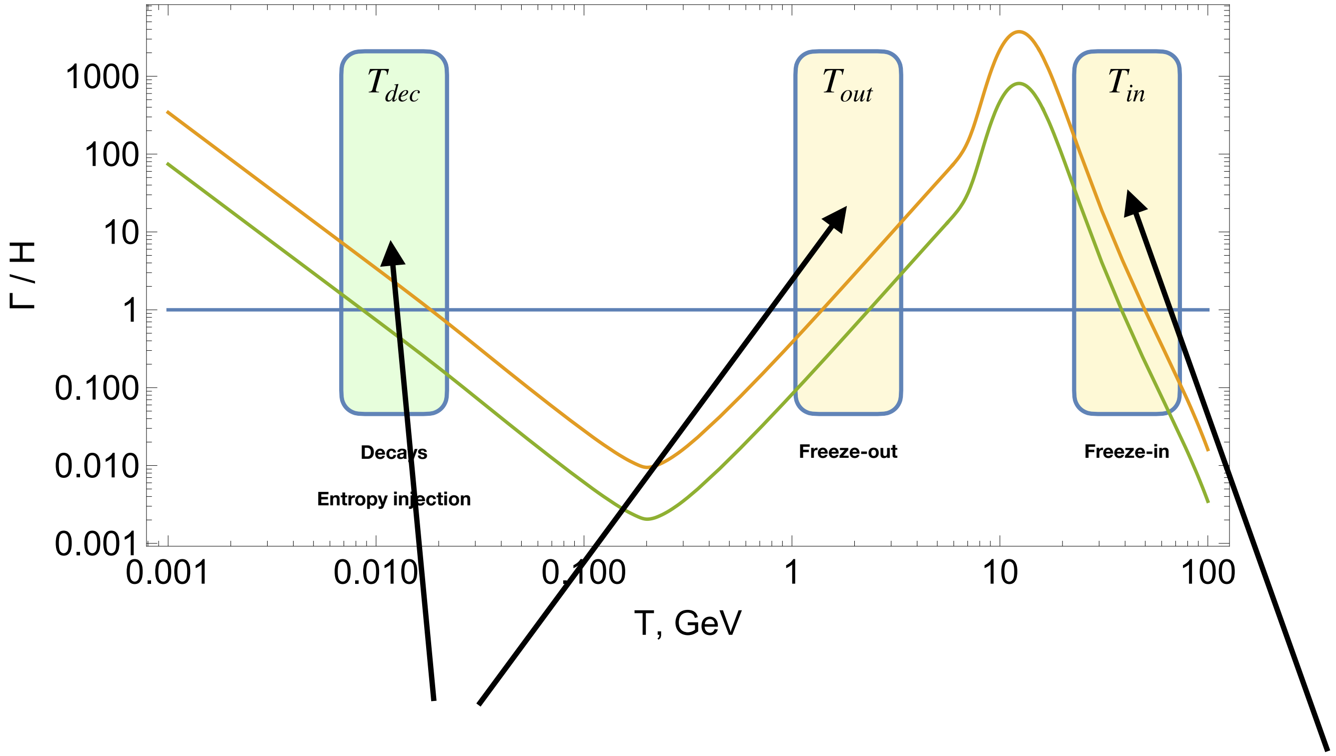


Baryon asymmetry and total lepton asymmetry



Lepton asymmetry can be much larger than the baryon as it is generated below the sphaleron freeze-out

Leptogenesis at few GeV



Relation between baryogenesis and DM

Freeze out leptogenesis can ensure 100% of DM, but very strong degeneracy between N2,3 is required

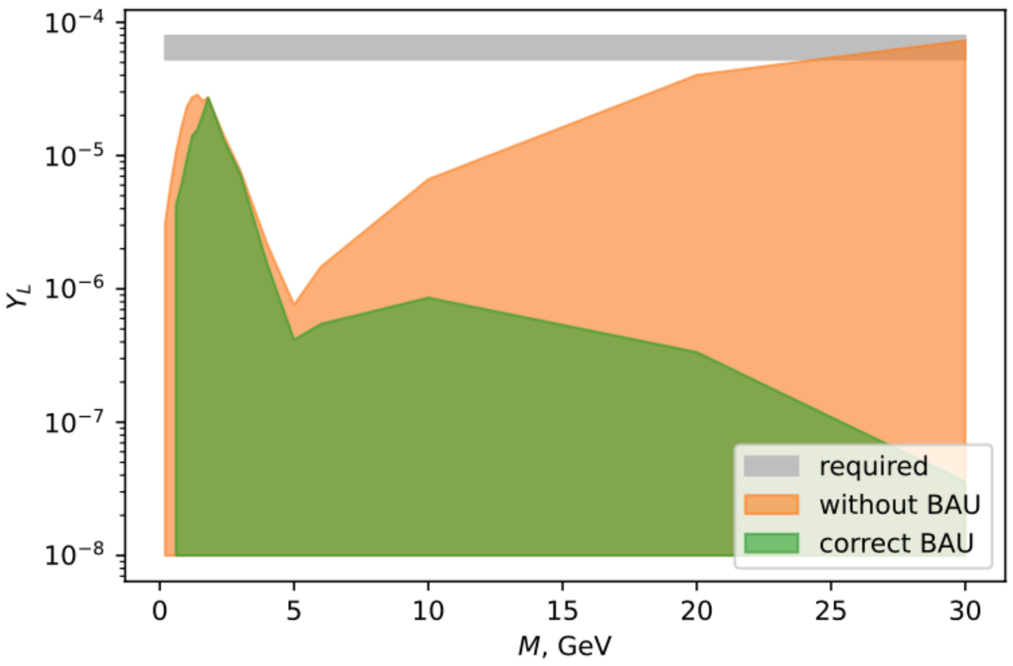
Freeze in leptogenesis can ensure ~50% of DM

MS; Canetti, Drewes, Frossard, MS; Eijima, Timiryasov, MS; Laine, Ghiglieri

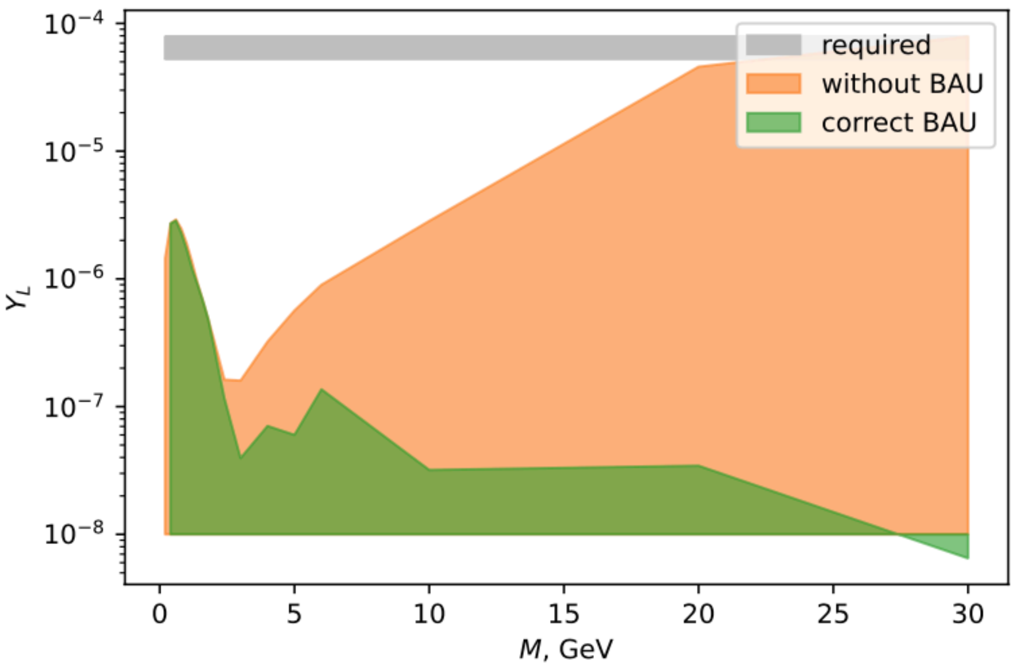
Asymmetry created by HNLs

Eijima, MS, Timiryasov

Normal ordering of neutrino masses

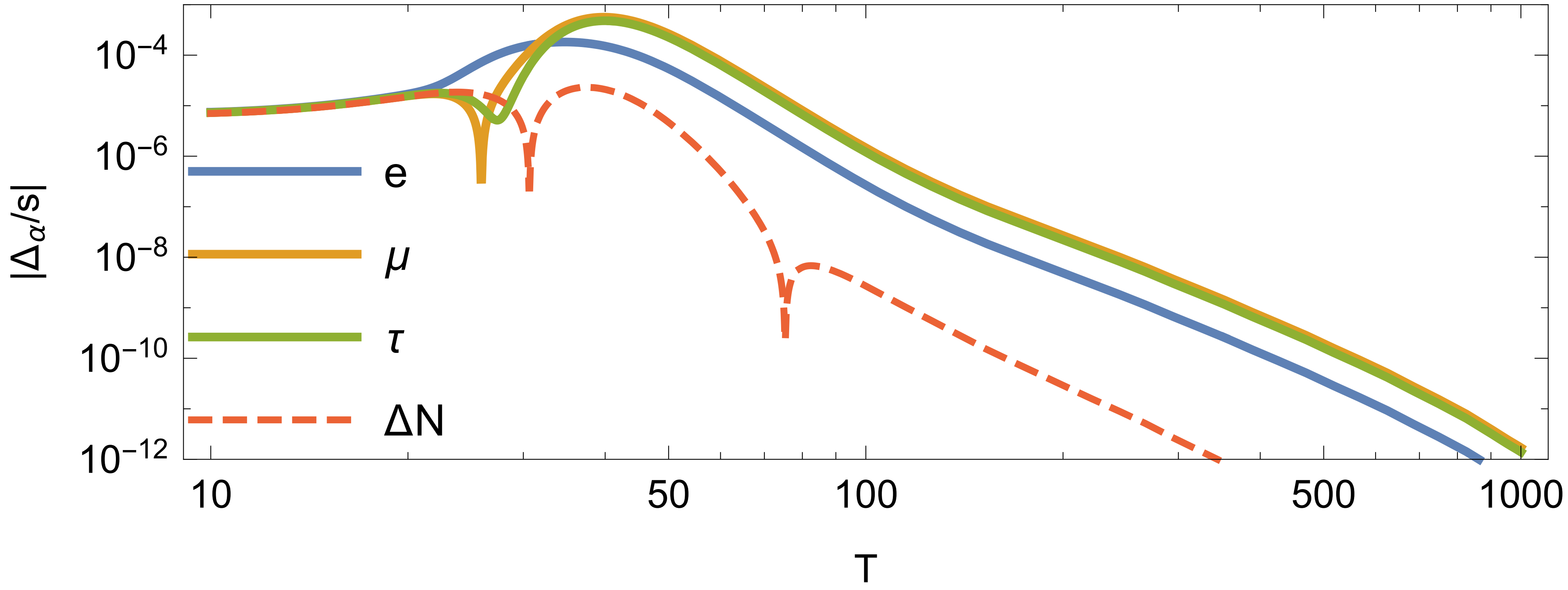


Inverted ordering of neutrino masses



Maximal lepton asymmetries which can be generated at the freeze-in of the HNLs. Not enough for 100% sterile neutrino DM.

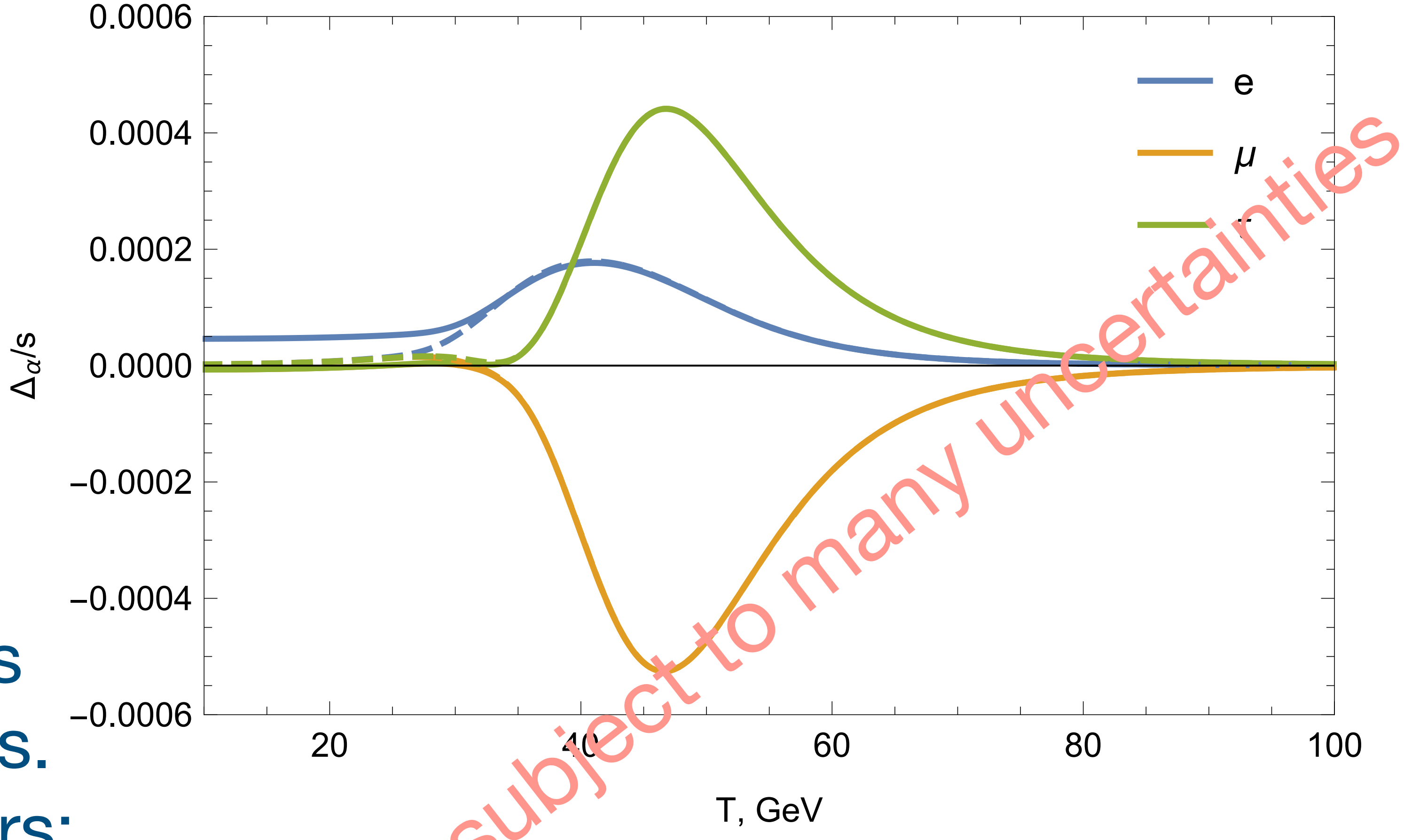
Lepton asymmetry and magnetic fields

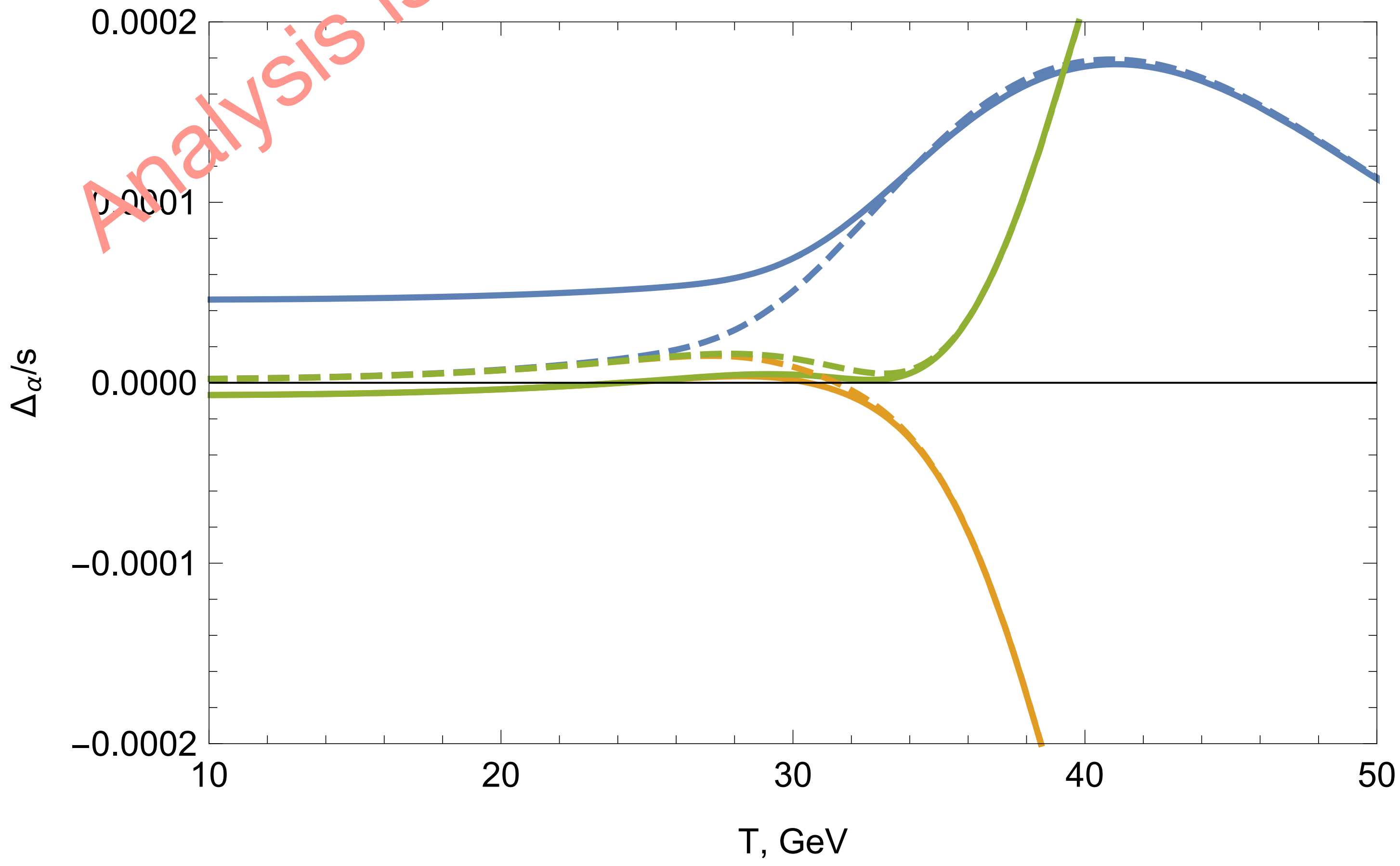


- Without accounting for U(1) fields: production of temporary large lepton asymmetries, which are diluted later by HNL interactions

- With U(1) fields: generation of magnetic fields due to instability, hiding the asymmetry in CS condensate (immune to the washout due to HNL interactions). Decay of CS condensate gives back the asymmetry

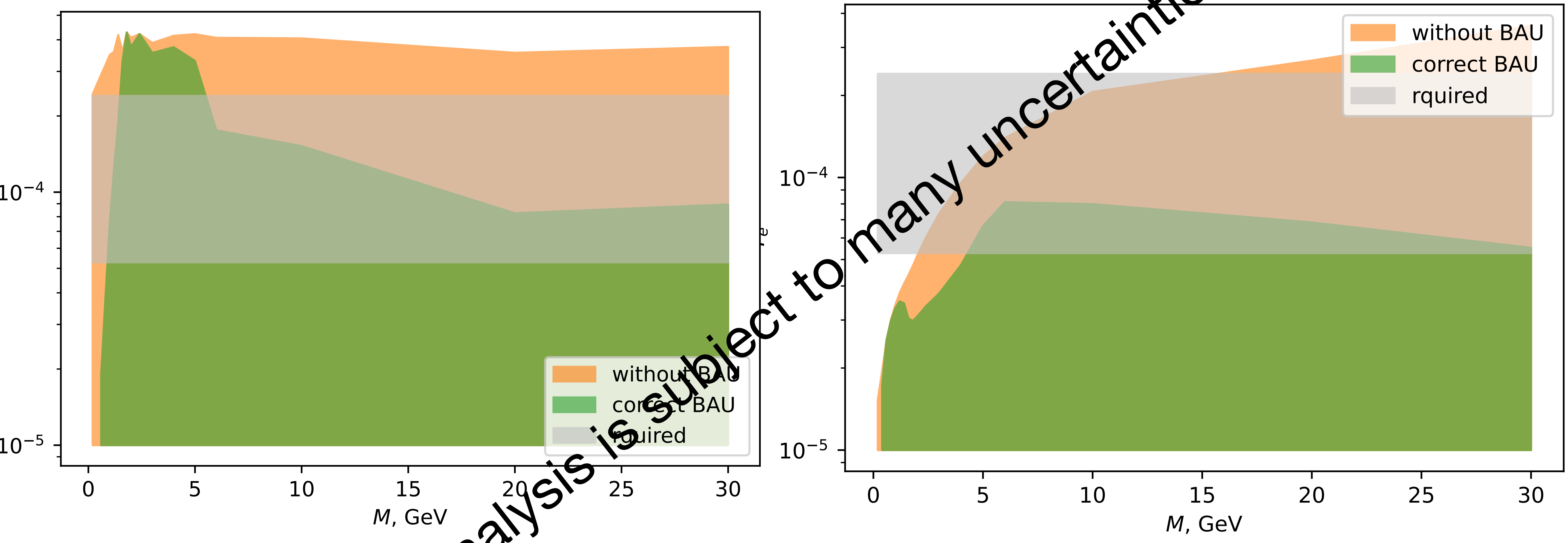
Evolution of lepton asymmetries with and without magnetic fields. Only electron asymmetry matters: others are destroyed by the chirality flip transitions





Analysis is subject to many uncertainties

Asymmetry created by HNLs



Normal

Inverted

Analysis is subject to many uncertainties

Maximal lepton asymmetries which can be generated at the freeze-in of the HNLs. May be enough for 100% sterile neutrino DM.

Conclusions

- Not only the SU(2) electroweak anomaly, but also the U(1) anomaly may be important for cosmology
- It may lead to the (temporary) existence of the (hyper) magnetic fields, and to CS condensate hiding the baryon and lepton asymmetries
- New prospects for baryogenesis
- New prospects for sterile neutrino DM productions
- Theoretical challenge - how to reconcile the evidence for the absence of EW phase transition with CS non-renormalisation theorems