

Backreaction from magnetic fields generated during inflation

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- In observational cosmology we try to constrain the history of the Universe by the observation of relics. The best example of this is the CMB which represents not only a relic of the time of recombination, $t \simeq 3 \times 10^5$ years after the big bang, but probably also of a much earlier moment, $t \lesssim 10^{-35}$ sec, when something like inflation took place and generated correlations on very large scales.

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- But there are other very interesting events which may have left observable traces, relics, in the universe. Most notably confinement at $t \simeq 10^{-4}$ sec
- or the electroweak transition at $t \simeq 10^{-10}$ sec which may have led to the observed baryon asymmetry in the Universe.
- It has been proposed that confinement and, especially the electroweak (phase) transition but also inflation might generate primordial magnetic fields which represent seeds for the magnetic fields observed in galaxies, clusters, filaments and even voids.

In order to generate micro Gauss fields in galaxies and clusters today via a dynamo mechanism, at least 10^{-20} Gauss are needed all over the Universe, even in voids. The Neronov & Vovk result indicates that fields of more than 10^{-16} Gauss might actually be present also in voids. How did they get there?

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For these reasons I believe it is very difficult to generate the observed ubiquitous magnetic fields at late times. But can we do it in the early universe?

There are three main ideas how magnetic fields may have formed in the early Universe:

- **Second order perturbations:** To generate magnetic fields in the cosmic fluid one needs vorticity and a charge and current density. Vorticity can be obtained only in **second order perturbation theory** (at first order vector perturbations decay) and currents require charge separation which is obtained only at **second order in the tight coupling** limit. Estimates have shown that typical fields do not exceed 10^{-29} Gauss ([Fenu et al. 2010](#)). This is far too small to be consistent with the [Neronov-Vovk \(2010\)](#) bound or with the minimal amplitude needed for dynamo amplification.

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- **Phase transitions:** First order phase transitions are violent events which proceed via bubble nucleation. Charge separation and turbulent fluid motion can lead to the generation of magnetic fields.
- **Inflation:** The electromagnetic field is conformally coupled to gravity and is therefore usually not generated during inflation. However, if conformal symmetry is explicitly broken or if the electromagnetic field is coupled to the inflation, it can also be generated during inflation.

The initial spectrum

We assume that the process leading to a magnetic field is statistically homogeneous and isotropic. A magnetic field spectrum generated by such a process is of the form

$$\langle B_i(\eta, \mathbf{k}) B_j^*(\eta, \mathbf{q}) \rangle = \frac{(2\pi)^3}{2} \delta(\mathbf{k} - \mathbf{q}) \left\{ (\delta_{ij} - \hat{k}_i \hat{k}_j) P_S(k, \eta) - i \epsilon_{ijn} \hat{k}_n P_A(k, \eta) \right\}$$

The Dirac- δ is due to statistical homogeneity and the requirement $\nabla \cdot \mathbf{B}$ dictates the tensor structure. Note that the pre-factor of P_S is even under parity while the one of P_A is odd under parity.

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$k^3 P_A \propto |B_+|^2 - |B_-|^2$ determines the helicity of the magnetic field. Its integral is the helicity density while $k^3 P_S \propto |B_+|^2 + |B_-|^2$ determines the energy density in magnetic fields.

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On smaller scales the magnetic field evolves in the plasma. Below a damping scale it is damped by the viscosity of the cosmic plasma, $P_S = P_A = 0$ for $k > k_d(t)$. Here $k_d(t)$ is a time-dependent damping scale.

If the magnetic field is generated e.g. during a phase transition, its correlation length is finite. It is typically of the size of the largest bubbles when they coalesce and the phase transition terminates. This is a fraction of the Hubble scale at the transition. On scales larger than the Hubble scale, correlations vanish by causality.

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Hence the correlation function is a function of compact support; and therefore its Fourier transform is analytic. Usually this signifies white noise (flat) on large scales but since we have to additional condition $\nabla \cdot \mathbf{B} = 0$, for magnetic fields we must require $P_S \propto k^2$ on large scales. Correspondingly n_H must be odd and the physical requirement $|P_A| \leq P_S$ then implies $P_A \propto k^3$ (RD & Caprini, 2003).

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For magnetic fields generated during inflation, the correlation length is arbitrary and no such causality arguments apply.

Magnetic fields from inflation

The fact that magnetic fields from phase transitions tend to have rather small coherence scales motivates to study what can be obtained from inflation. There, scales which were initially inside the horizon grow very large and the annoying 'causality constraint' does not apply.

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Let us first discuss a simple case where we couple the inflation to the electromagnetic field (Ratra '92).

$$S = \int d^4x \sqrt{-g} \left[R + \frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{I_1(\phi)}{4} F^2 \right]$$

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With this modification in the action, the modified evolution equation for the 'renormalized' electromagnetic potential $\mathcal{A} = a I_1(\phi) A$ in Fourier space becomes (in Coulomb gauge)

$$\ddot{\mathcal{A}} + \left(k^2 - \frac{\ddot{I}_1}{I_1} \right) \mathcal{A} = 0$$

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This is a wave equation with a time-dependent mass term. We know how to calculate the generation of its modes out of the vacuum.

For example if $l_1 \propto a^\gamma$ is a simple power law, we can compute the resulting magnetic fields spectrum to

$$k^3 P_B \propto k^{n_B} \quad \text{with} \quad n_B = \begin{cases} 4 - 2\gamma & \text{if } \gamma \geq -1/2 \\ 6 + 2\gamma & \text{if } \gamma \leq -1/2 \end{cases}$$

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Since there is no infrared cutoff, the spectral index should not be negative otherwise $\frac{d\rho_B}{d\log(k)} \propto k^3 P_S \propto k^{n_B}$ or $\frac{d\rho_E}{d\log(k)} \propto k^3 P_E \propto k^{n_E}$ diverges in the ir. This limits

$$-2 \lesssim \gamma \lesssim 2.$$

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For this result we have normalized $l_1 = 1$ at the end of inflation. Since l_1 is growing rapidly during inflation this means that $l_1 \ll 1$ for most of the time during inflation. But since charged particles couple to the canonically normalized field $l_1 F_{\mu\nu}$, their charge during inflation has the renormalized value $e_N = e/l_1 \gg e$.

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Hence **during inflation the electron charge was much larger than 1. In this regime we cannot trust perturbation theory** and our calculation does actually not apply (Demozzi et al. '09).

Generation of helical magnetic fields during inflation

Especially the strong coupling problem motivated people to study what can be gained from a helical coupling of the inflaton to the magnetic field. The idea was that this does not mix with the electron charge, and because of the **inverse cascade** invoked by helicity conservation in the evolution of the field after inflation, a bluer spectral index might be admissible.

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We start with the action

$$S = \int d^4x \sqrt{-g} \left[R + \frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{1}{4}F^2 + \frac{I_2(\phi)}{4}F \cdot \tilde{F} \right]$$

where $\tilde{F}^{\mu\nu} = \frac{1}{2}\eta^{\mu\nu\alpha\beta}F_{\alpha\beta}$ is the dual of the electromagnetic field tensor.

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In this case the evolution equation for the two helicity modes of the vector potential, $\mathcal{A}_\pm = aA_\pm$ (in Coulomb gauge) becomes

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where $\tilde{F}^{\mu\nu} = \frac{1}{2} \eta^{\mu\nu\alpha\beta} F_{\alpha\beta}$ is the dual of the electromagnetic field tensor.

[Anber Sorbo \(2010\)](#)

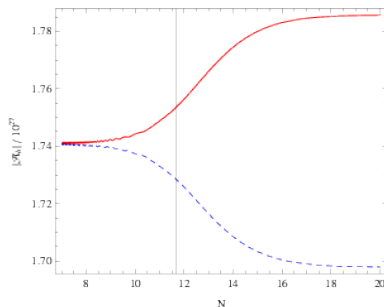
In this case the evolution equation for the two helicity modes of the vector potential, $\mathcal{A}_\pm = aA_\pm$ (in Coulomb gauge) becomes

$$\ddot{\mathcal{A}}_\pm + \left[k^2 \pm k I_2 \right] \mathcal{A}_\pm = 0.$$

Again, a wave equation with time-dependent mass term. There are two main differences to the non-helical case: Now one of the helicity modes is amplified while the other is reduced depending on the sign of I_2 , and the mass-term is proportional to k .

Generation of helical magnetic fields during inflation

Both differences are very important: the first leads to helicity and the second is the cause of the short duration of the amplification phase



RD, Hollenstein & Jain, 2010

Because the duration during which the mass term is relevant is always just the Hubble exit time $k \sim \mathcal{H}$, before the \ddot{A} term is much larger and later the $k^2 \mathcal{A}$ term dominates, the vacuum fluctuation of one mode are always amplified while those of the other mode are suppressed. And this by the same, k -independent factor.

Generation of helical magnetic fields during inflation

Since the vacuum fluctuations of the vector potential behave like k^{-1} , this yields

$$P_S \propto P_A \propto k,$$

$$\frac{d\rho_B}{d\log(k)} \propto k^4, \quad n_B = 4.$$

Despite the inverse cascade rather low scale inflation is needed to satisfy $\frac{d\rho_B}{d\log(k_*)} < \rho_r$ and obtain a significant correlation scales and amplitude after the inverse cascade.

(RD, Hollenstein & Jain, '10)

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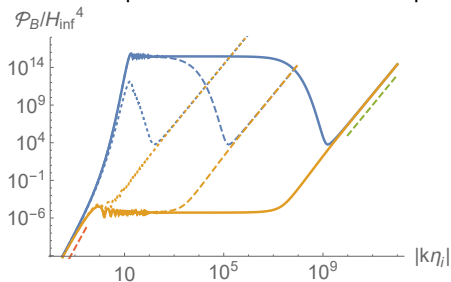
Furthermore, the back reaction on the perturbations still leads to non-Gaussianities (Barnaby, Namba & Peloso, '11).

Another interesting mechanism to generate helical magnetic fields is via an initial non-vanishing chirality, $\mu_5 = \mu_L - \mu_R$. At $T \gtrsim 10\text{MeV}$, due to the electroweak anomaly, only the sum $H + \lambda\mu_5$ is conserved where H is the magnetic helicity and λ is a phenomenological parameter. Hence chirality can generate helical magnetic fields and vice versa

(A. Boyarsky, J. Fröhlich & O. Ruchayskiy, '12; Schober, Fujita & Durrer, '20).

Generation of helical magnetic fields during & after inflation by a 'spectator field'

- Considered coupling of the electromagnetic field to a second, scalar field, subdominant during inflation, a **spectator field** (Caprini & Sorbo, '15) which continues to role after inflation during reheating (RD & Fujita, '19).
- In this way can generate magnetic fields which are helical, have a scale invariant spectrum and achieve an amplitude of about 10^{-16} GeV today.



(RD & Fujita, 2019)

- As soon as many charged particles are produced , magnetogenesis terminates.
- In this model, back-reaction and strong coupling remain under control if we choose a sufficiently low inflation scale, $H_{\text{inf}} \lesssim 10^5 \text{ GeV}$.

Higgs-Starobinsky inflation

Recently we (RD, Sobol & Vilchinskii, 2022) have also considered the generation of gauge fields during Higgs-Starobinsky inflation.

$$S = S_{\text{HS}} + S_{\text{GF}}$$

$$S_{\text{HS}}[g_{\mu\nu}, h] = \int d^4x \sqrt{-g} \left[-\frac{M_p^2}{2} \left(1 + \frac{\xi_h h^2}{M_p^2} \right) R + \frac{\xi_s}{4} R^2 + \frac{1}{2} g^{\mu\nu} \partial_\mu h \partial_\nu h - \frac{\lambda}{4} h^4 \right]$$

$$S_{\text{GF}} = \int d^4x \sqrt{-g} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{(-R)^n}{2M_p^{2n}} \left(\kappa_1 F_{\mu\nu} F^{\mu\nu} + \kappa_2 F_{\mu\nu} \tilde{F}^{\mu\nu} \right) \right]$$

Even though S_{HS} looks like a 3 parameter Lagrangian, the inflationary dynamics in Einstein frame is single field inflation with an inflaton ϕ and only depends on

$$\xi = \frac{\xi_h^2}{\lambda} + \xi_s$$

To obtain the correct amplitude one needs

$$A_s = \frac{N_*^2}{72\pi^2 \xi} \simeq 2 \times 10^{-9} \text{ hence with } N_* \simeq 50 \quad \xi \simeq 2 \times 10^9$$

In Einstein frame the gauge field coupling becomes

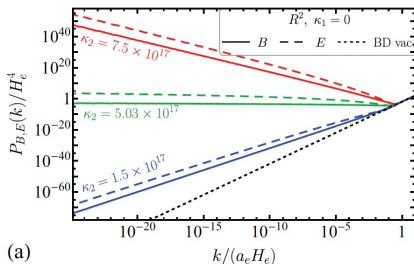
$$S_{\text{GF}} = -\frac{1}{4} \int d^4x \sqrt{-\bar{g}} \left[l_1 F_{\mu\nu} F^{\mu\nu} + l_2 F_{\mu\nu} \tilde{F}^{\mu\nu} \right]$$

with

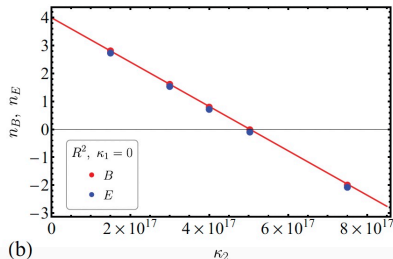
$$l_j = \delta_{1j} + 2\kappa_j \left(\frac{12\pi^2 A_s}{N_*^2} \right)^n \left(e^{\sqrt{\frac{2}{3}} \frac{\phi}{M_p}} - 1 \right)^n$$

Higgs-Starobinsky inflation : Results

- The relevant coupling constants are not the κ_j but $\kappa_j c_0^n$ with $c_0 = \frac{12\pi^2 A_S}{N_*^2} \simeq 10^{-10}$.
To avoid strong coupling and to be in the perturbative regime we just have to request $|\kappa_j c_0^n| \ll 1$.
- for $n = 1$ we always obtain blue spectra, $n_B = 4$ (see [Hollenstein et al. 2011](#)).
- for $n = 2$ the spectral index depends on c_0 , $n_B = 4 - 256\pi\kappa_2 c_0^2$.
- for $n > 2$ the spectral index is scale dependent but always red on very large scales, i.e. require an ir cutoff.



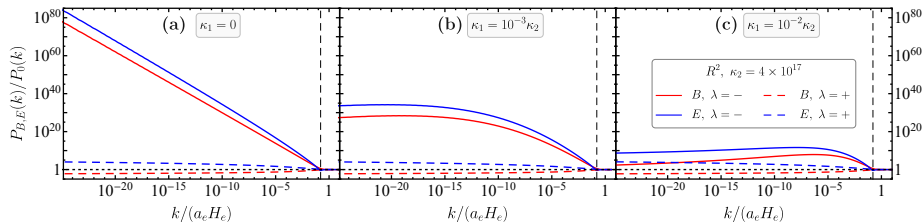
(a)



(b)

RD, Sobol & Vilchinskii (2022)

Higgs-Starobinsky inflation : Results

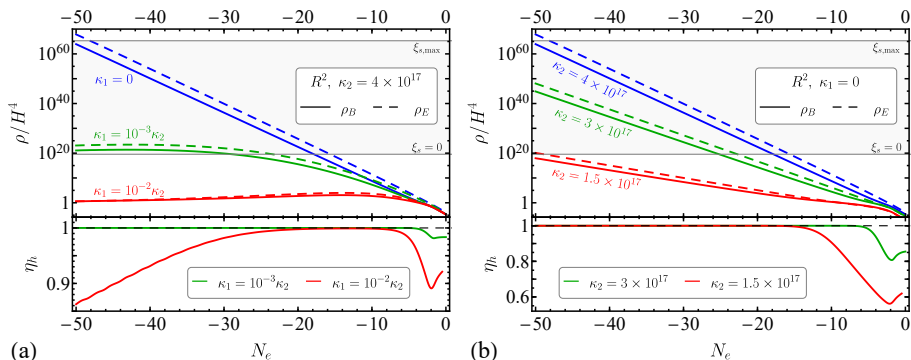


RD, Sobol & Vilchinskii (2022)

For $n = 2$ and $\kappa_2 = 4 \times 10^{17}$:

We can obtain red, scale invariant and blue spectra, depending on the value of κ_1 .

Higgs-Starobinsky inflation : Results



RD, Sobol & Vilchinskii (2022)

Especially at early times, the gauge field energy density is high and backreaction might be important!

Backreaction in Higgs-Starobinsky inflation : A non-perturbative Lagrangian

In slow roll inflation the inflaton field, the scalar curvature and the Ricci tensor are simple related since we are in nearly de-Sitter expansion. Therefore, models with different gauge-field couplings are similar. We want a coupling which does not require higher orders once the gauge field becomes large:

To study backreaction we (RD, Sobol & Vilchinskii, 2023) chose Starobinsky inflation with

$$\Delta\mathcal{L} = \frac{\xi_s}{4} \left[1 + \frac{\kappa_1}{M_{\text{P}}^4} F_{\mu\nu} F^{\mu\nu} + \frac{\kappa_2}{M_{\text{P}}^4} F_{\mu\nu} \tilde{F}^{\mu\nu} \right]^{-1} R^2 = \frac{\xi_s}{4} \frac{1}{\Delta(F_{\mu\nu})} R^2$$

with the free dimensionless parameters ξ_s , κ_1 and κ_2 .

In the Einstein frame this becomes

$$S = \int d^4x \sqrt{-g} \left[-\frac{M_{\text{P}}^2}{2} R + \frac{3M_{\text{P}}^2}{4\Psi^2} \partial_\mu \Psi \partial^\mu \Psi - \frac{M_{\text{P}}^4}{4\xi_s} \frac{(1-\Psi)^2}{\Psi^2} \Delta(F_{\mu\nu}) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

$$\Psi = \exp\left(\sqrt{\frac{2}{3}} \frac{\phi}{M_{\text{P}}}\right)$$

Backreaction in Higgs-Starobinsky inflation : The gradient expansion

To take into account backreaction we want to consider the effects of the gauge field on the inflaton and on the expansion of the Universe.

Backreaction in Higgs-Starobinsky inflation : The gradient expansion

To take into account backreaction we want to consider the effects of the gauge field on the inflaton and on the expansion of the Universe.

For this we need to determine the quantum average of terms quadratic in the gauge field, $I_1 \langle \mathbf{E}^2 + \mathbf{B}^2 \rangle$, $I_2 \langle \mathbf{E} \cdot \mathbf{B} \rangle$ where $\langle \dots \rangle$ denote the vacuum expectation value.

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To this aim we apply the gradient expansion: we introduce the following quadratic expressions

$$\mathcal{E}^{(n)} \equiv \frac{l_1}{a^n} \langle \mathbf{E} \cdot \text{rot}^n \mathbf{E} \rangle,$$

$$\mathcal{G}^{(n)} \equiv -\frac{l_1}{2a^n} \langle \mathbf{E} \cdot \text{rot}^n \mathbf{B} + \text{rot}^n \mathbf{B} \cdot \mathbf{E} \rangle,$$

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Maxwell's eqn. result in a hierarchy of linear equations for these quantities which, in our case, contain time dependent source terms which come from the boundary of the k-integrals due to renormalisation (only modes with sufficiently large wave numbers are excited) which we can calculate analytically. We then solve the hierarchy with cutoff at $n=150$ numerically.

$\mathcal{E}^{(0)}$ and $\mathcal{B}^{(0)}$ determine the gauge field energy density while $\mathcal{G}^{(0)}$ enters the axial coupling to the inflaton.

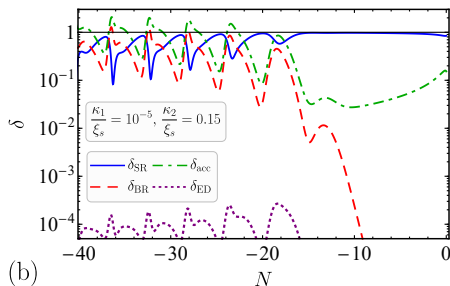
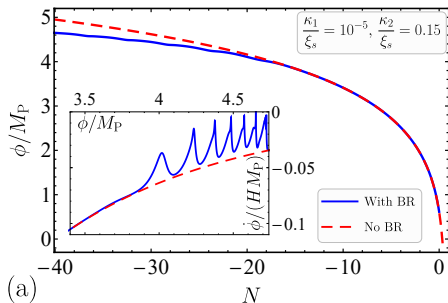
Backreaction in Higgs-Starobinsky inflation : Results

We introduce the dimensionless quantities

$$\delta_{\text{BR}} = \left| \frac{l'_1(\phi)(\mathcal{E}^{(0)} - \mathcal{B}^{(0)}) - 2l'_2(\phi)\mathcal{G}^{(0)}}{2l_1(\phi)V'(\phi)} \right|$$

$$\delta_{\text{acc}} = \left| \frac{\ddot{\phi}}{V'(\phi)} \right|, \quad \delta_{\text{SR}} = \left| \frac{3H\dot{\phi}}{V'(\phi)} \right|,$$

$$\delta_{\text{ED}} = \frac{\rho_{\text{GF}}}{\rho_{\text{inf}}} = \frac{\frac{1}{2}(\mathcal{E}^{(0)} + \mathcal{B}^{(0)})}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}.$$

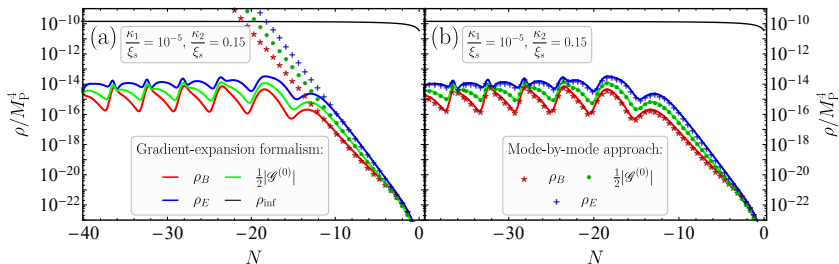


RD, Sobol & Vilchinskii (2022)

Similar results had been found before [V. Domcke et al. \(2020\)](#)

Backreaction in Higgs-Starobinsky inflation : Results

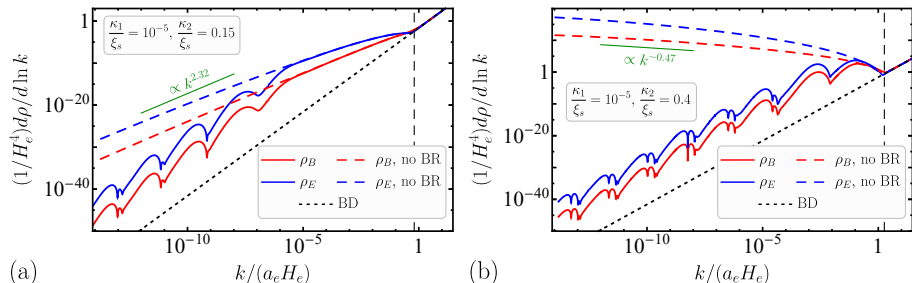
The effect of backreaction on the gauge field energy density:



RD, Sobol & Vilchinskii (2022)

Backreaction in Higgs-Starobinsky inflation : Results

The effect of backreaction on the gauge field power spectrum :



RD, Sobol & Vilchinskii (2022)

Magnetic fields are very small at the end of inflation and have a strong blue tilt, $\sim k^4$.

But: We have not considered the effect on fluctuations of the inflaton field and on metric perturbations.

A full numerical lattice calculation has recently found (Figueroa, Lizerraga, Uria & Urrestilla, 2023) that the 'oscillations' may be spurious (washed out by couplings of higher k modes).

Backreaction in a perturbed Friedman universe

(RD, R. von Eckardstein, D. Garg, K. Schmitz, O. Sobol & S. Vilchinskii,
arXiv:2404.19694)

We consider a fully general Lagrangian

$$S[g_{\mu\nu}, \phi, A_\mu] = \int d^4x \sqrt{-g} \left[-\frac{M_p^2}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \frac{1}{4} l_1(\phi) (F_{\mu\nu} F^{\mu\nu}) - \frac{1}{4} l_2(\phi) (F_{\mu\nu} \tilde{F}^{\mu\nu}) \right]$$

In a perturbed Friedmann Universe

$$g_{\mu\nu} dx^\mu dx^\nu = (1 + 2\Psi) a^2 d\eta^2 - (1 - 2\Phi) a^2 \delta_{ij} dx^i dx^j$$

$$\phi = \langle \phi \rangle(\eta) + \delta\varphi = \phi_c + \delta\varphi.$$

We consider only scalar perturbations but include also gauge field perturbations which are generated via $l_{1,2}$.

$$F_\nu = -\frac{a^2 l_1(\phi_c)}{2M_p^2} \frac{\partial_i}{\Delta} \varepsilon_{ijk} [(E_j B_k) - \langle (E_j B_k) \rangle]$$

$$F_\rho, \quad F_\pi \quad \dots$$

$$\begin{aligned}\zeta &= \Phi + \frac{\mathcal{H}}{\mathcal{H}^2 - \mathcal{H}'} (\Phi' + \mathcal{H}\Psi) \\ &= \Phi + \frac{\mathcal{H}}{\mathcal{H}^2 - \mathcal{H}'} \left(\frac{\phi'_c}{2M_{\text{p}}^2} \delta\varphi + F_v \right)\end{aligned}$$

$$\zeta'' + p\zeta' + q\zeta = \mathcal{S}(\eta, k, F_v, F_\pi, F'_\pi, E \cdot B)$$

If backreaction can be neglected,

$$q = k^2, \quad p = 2 \frac{\mathcal{H}^2 - \mathcal{H}'}{\mathcal{H}} + \frac{(\mathcal{H}^2 - \mathcal{H}')'}{\mathcal{H}^2 - \mathcal{H}'} = 2 \frac{z'}{z}$$

where z is the Mukhanov–Sasaki variable,

$$z = \frac{a\phi'_c}{\mathcal{H}}$$

Otherwise there are corrections containing $\mathcal{E}^{(0)}$, $\mathcal{B}^{(0)}$ and $\mathcal{G}^{(0)}$

$$\zeta(\eta, k) = \zeta_{\text{in}}(\eta, k) + \zeta_S(\eta, k)$$

Inflaton gauge fields.

$$\zeta_S(\eta, k) = \int_{-\infty}^{\eta} G(\eta, \eta', k) S(\eta', k, F_\nu, F_\pi, F'_\pi, E \cdot B) d\eta'$$

This term is new and highly non-Gaussian. If close to scale invariance, its spectrum and amplitude are strongly constrained by CMB observations. If blue it might lead to the formation of primordial black holes.

$$\langle \zeta_{\text{in}} \zeta_S \rangle = 0 \quad \langle \zeta_S(k_1) \zeta_S(k_2) \zeta_S(k_3) \rangle \neq 0$$

- An application to axion inflation where backreaction on the background evolution is neglected is presented in the talk by [Deepen Garg](#).
- Including backreaction, effect on ζ_{in} .
- Transition from inflation to radiation may lead in addition to the well known 'passive' and 'compensated' modes to a dangerous 'constant' mode (see [Bonvin, Caprini & RD, \(2011\)](#)).
- Can there be any close to scale invariant solutions with negligible backreaction ?
- Study blue spectra, primordial black holes?

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- Magnetic fields generated during inflation are severely affected by **backreaction**, even if their energy density remains small. It remains to be seen how this backreaction affects the scalar metric perturbations generated during inflation in 'realistic' models.
- **Discovering large scale magnetic fields especially in voids** where they are not processed further by galaxy formation processes would be a very strong sign of their primordial nature.