Scenarios of inflationary magnetogenesis

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Cosmological magnetic fields

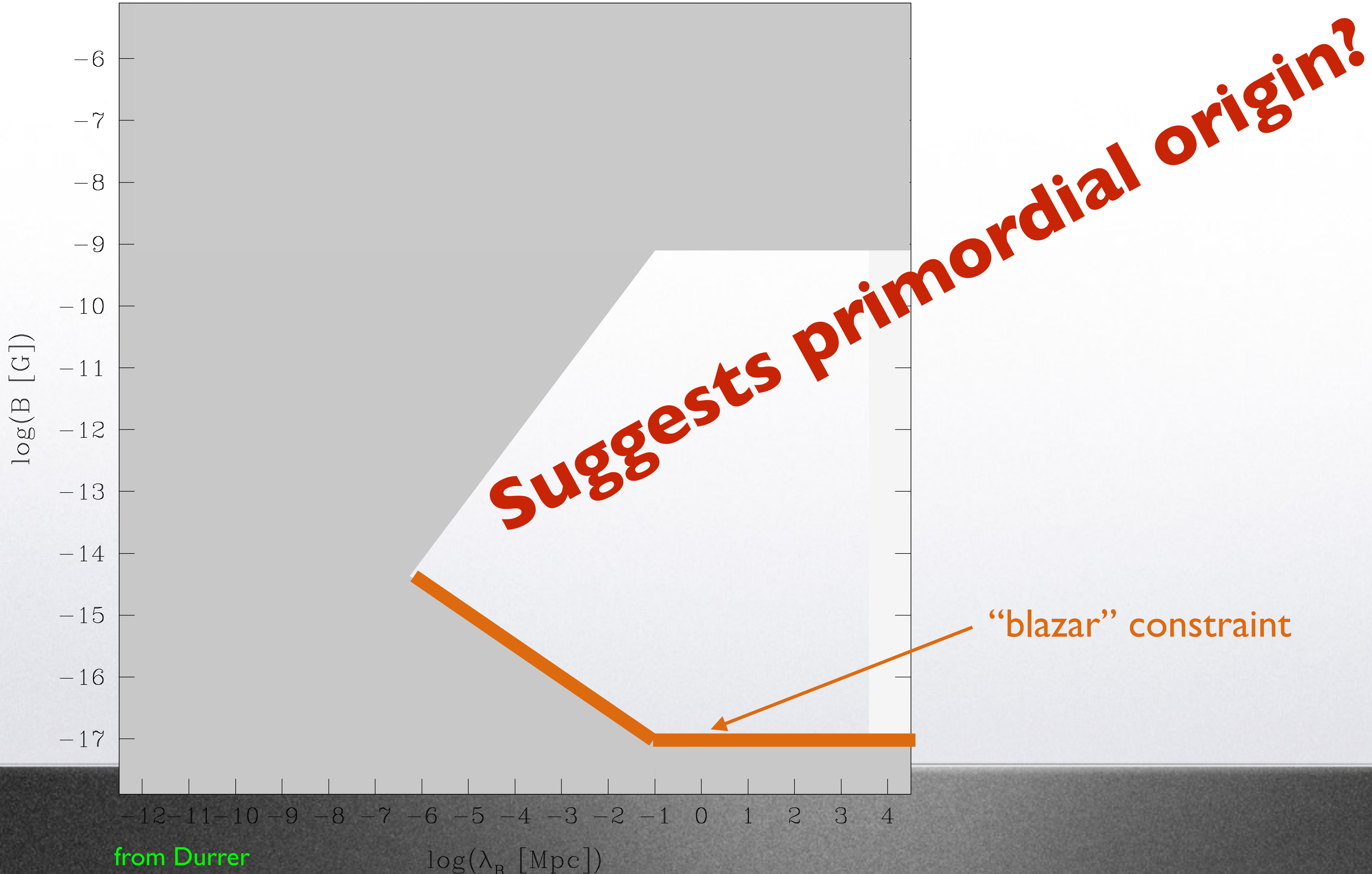
Observed with a number of techniques

- In the Galaxy (~kpc), solid evidence of $B \cong \mu G$.

  [From dynamo amplification
  of primordial $10^{-21 \div -23}$ G @ Mpc scale]

- At cosmological scales (~1 Mpc), blazars: $B \cong 10^{-17}$ G
  [x$(L/1\ \mathrm{Mpc})^{1/2}$ for $L<1$ Mpc]

Constraints on cosmological magnetic fields



from Durrer and Neronov 13

Inflationary magnetogenesis

Pro: possible to create large coherence lengths

Con: must modify standard model

$$\mathcal{S}_{\rm Maxwell} = \int d^4\mathbf{x}\sqrt{g}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) = \int d^4\mathbf{x}\left(\frac{1}{2}A_i'\,A_i' - \frac{1}{2}\partial_a A_i\,\partial_a A_i\right)$$

where

$$g_{\mu\nu} = a^2(\tau)\,\left(-d\tau^2 + d\mathbf{x}^2\right)$$ conformally flat Universe

$$A_0 = 0,\ \partial_i\,A_i = 0$$ Coulomb gauge (assumed throughout)

⇒Maxwell on conformally flat space-time = free theory on Minkowski

⇒no effects from inflation

Inflationary magnetogenesis

Turner, Widrow 87
Calzetta, Kandus Mazzitelli 97

One idea: light charged scalar ϕ
with mass $m<H$
gets fluctuations during inflation

Electric currents

Magnetic field

…but unfortunately…

Inflationary magnetogenesis

…but unfortunately…

Giovannini, Shaposhnikov 00

Very red spectrum of magnetic field
(currents are slow at large scale)

$$B \propto \frac{H^{5/2}}{m^{3/2}\, M_P} \frac{1}{\ell^2}$$

(e.g., $B=10^{-45}$ G @ 1 Mpc
for $H=10^{12}$ GeV, $m=100$ GeV)

☹️

Inflationary magnetogenesis

Actually, there **is** a standard mechanism of amplification:

Maroto 00

Metric perturbations during inflation break conformal invariance!

…but perturbations freeze at super-Hubble scales

Blue spectrum again, and very weak fields

☹

Inflationary magnetogenesis

Let us try to modify the gauge-invariant Lagrangian for electromagnetism, then!

scalar inflaton

Ratra:

$$\mathcal{S}_{\rm Ratra} = \int d^4\mathbf{x}\sqrt{g}\left(-\frac{f(\phi)^2}{4}\,F_{\mu\nu}\,F^{\mu\nu}\right)$$

Ratra 92

pseudoscalar inflaton

Axion:

$$\mathcal{S}_{\rm Axion} = \int d^4\mathbf{x}\sqrt{g}\left(-\frac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu} + \frac{\phi}{4\,f}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu}\right)$$

Turner Widrow 87,
also Carroll Field Garretson 92

Ratra magnetogenesis

$$\mathcal{S}_{\rm Ratra} = \int d^4\mathbf{x}\sqrt{g}\left(-\frac{f(\phi)^2}{4}\,F_{\mu\nu}\,F^{\mu\nu}\right) = \int d^4\mathbf{x}\,f(\phi)^2\,\left(\frac{1}{2}A_i'\,A_i' - \frac{1}{2}\partial_a A_i\,\partial_a A_i\right)$$

f(ϕ) through *ϕ(τ)* gives *f(τ)* modeled as

$$f(\tau) = (-H\,\tau)^{-n}$$

n<0 to avoid strong coupling (charge of electron ~f^{-1})

Demozzi et al 09

Canonically normalized field

$$\tilde{A}_i = f\,A_i$$

obeys

$$\tilde{A}_i'' + \left(k^2 - \frac{n\,(n+1)}{\tau^2}\right)\tilde{A}_i = 0$$

amplification at large scales

will assume inflation with H constant and $a=1$ at end of inflation

Ratra magnetogenesis

At end of inflation $B(\ell) \simeq H^2 \left(\frac{H^{-1}}{\ell}\right)^{n+3}$ (scale invariant for n=-3)

$n=-3$, $H \sim 10^{12}$ GeV ⇒ **$B \sim 10^{-12}$ G** at all scales Good!

…however…

electric field: $E(\ell) \simeq H^2 \left(\frac{H^{-1}}{\ell}\right)^{n+2}$ (IR-divergent for n=-3!)

Backreaction from electric energy avoided for $n>-2$

⇒ $B<10^{-32}$ G at 1 Mpc 😒

Demozzi et al 09

Ratra magnetogenesis:

ways out?

Ferreira, Jain, Sloth 13, 14

Difficult…

Assume:

- Ratra active only after *1 Mpc* scales leave the horizon
- $n=-2+\ldots$
- Low scale inflation $\varrho^{1/4} \sim 10\ MeV$

$$B \sim 10^{-15}\ G\ @\ 1\ Mpc$$

Axion magnetogenesis

$$\mathcal{S}_{\rm Axion} = \int d^4\mathbf{x}\sqrt{g}\left(-\frac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu} + \frac{\phi}{4\,f}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu}\right)$$

by parts

$$= \frac{\dot{\phi}}{2\,f}\,\epsilon_{ijk}\,A_i\,\partial_j\,A_k$$

sign depends on momentum: not defined!

Convenient to decompose photon in helicity modes

$$\mathbf{A}(\mathbf{x},\tau) = \sum_{\lambda=\pm}\int\frac{d^3\mathbf{k}}{(2\,\pi)^{3/2}}\left[a_{\mathbf{k}}^{\lambda}\,A_{\lambda}^{\mathbf{k}}(\tau)\;\mathbf{e}^{\lambda}(\mathbf{k})\,e^{i\mathbf{k}\,\mathbf{x}} + a_{\mathbf{k}}^{\lambda\,\dagger}\,A_{\lambda}^{*\,\mathbf{k}}(\tau)\;\mathbf{e}^{\lambda\,*}(\mathbf{k})\,e^{-i\mathbf{k}\,\mathbf{x}}\right]$$

Axion magnetogenesis

Equation for mode functions:

$$A_\lambda'' + \left(\mathbf{k}^2 + \lambda\,\frac{\phi'}{f}\,|\mathbf{k}|\right)\,A_\lambda = 0$$

for λ=-, the "mass term" is negative and large for *~1* Hubble time

Exponential amplification of *left handed modes only*!

parity violation!

$$A_L \propto \exp\left\{\frac{\pi}{2}\,\frac{\dot{\phi}}{f\,H}\right\}$$

Axion magnetogenesis

Magnetic spectrum at end of inflation

$$B(\ell) \simeq H^2\, e^{\pi\xi} \left(\frac{H^{-1}}{\ell}\right)^2$$

$$\xi \equiv \frac{\dot{\phi}}{2\, f\, H}$$

😊 overall amplitude tunable

😞 very blue spectrum

Carroll Field Garretson 92

If ξ chosen to saturate no backreaction condition $(B^2 < H^2\, M_P{}^2)$ then B too small @ Mpc scales

…however…

Evolving the field in the cosmic plasma

The magnetic field produced has *maximal helicity* (generated by parity-violating background)

$$\mathcal{H} \equiv \int_V d^3x \, \mathbf{B} \cdot \mathbf{A}$$

and helicity is (almost) conserved for large conductivities

$$\frac{d\mathcal{H}}{dt} = -\frac{1}{4\pi\sigma} \int_V d^3x \mathbf{B} \cdot (\nabla \times \mathbf{B}) \cong 0$$

Dissipative processes suppress power at small scales

In order to conserve helicity,
power has to go to larger scales:

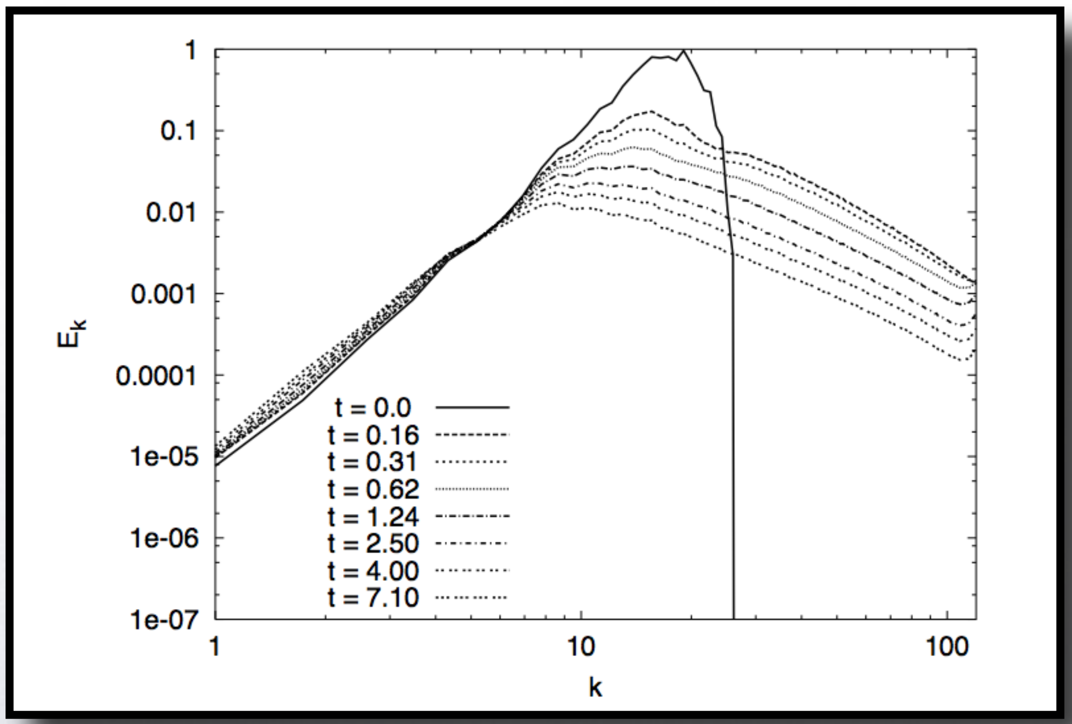
Inverse cascade

Son 99
Field and Carroll 00
Vachaspati 01,
Sigl 02...

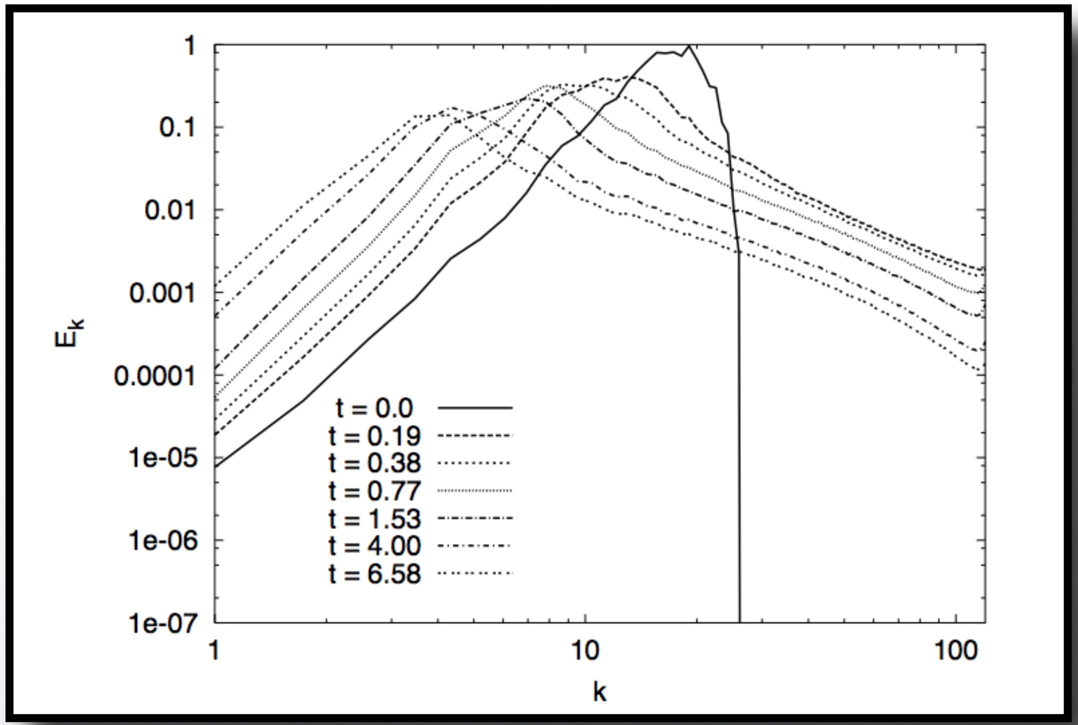
Numerical solutions

Evolution of the comoving magnetic field:

without helicity



with helicity



From Jedamzik and Banerjee 2004

Scalings:

- ☛ Coherence length $\propto \tau^{2/3}$
- ☛ Magnetic field strength $\propto \tau^{-1/3}$

$$\Rightarrow \quad B_0^2\, L_0 = B_{\rm rh}^2\, L_{\rm rh} \left(\frac{a_{\rm rh}}{a_0}\right)^3$$

- ☛ Spectral index for scales>coherence length:

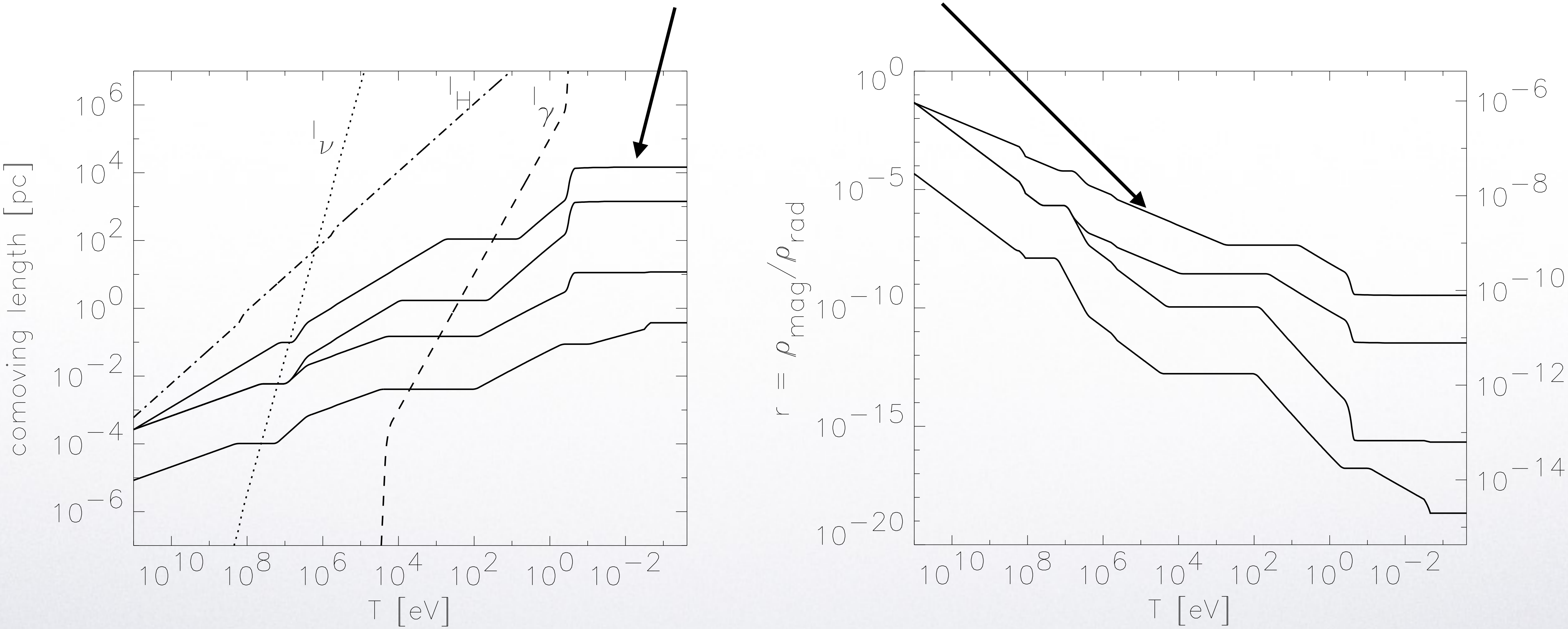
constant

(property of *self-similarity*)

In practice the story is more complicated…

From Jedamzik and Banerjee 2004

evolution of maximally helical field



In practice the story is more complicated…

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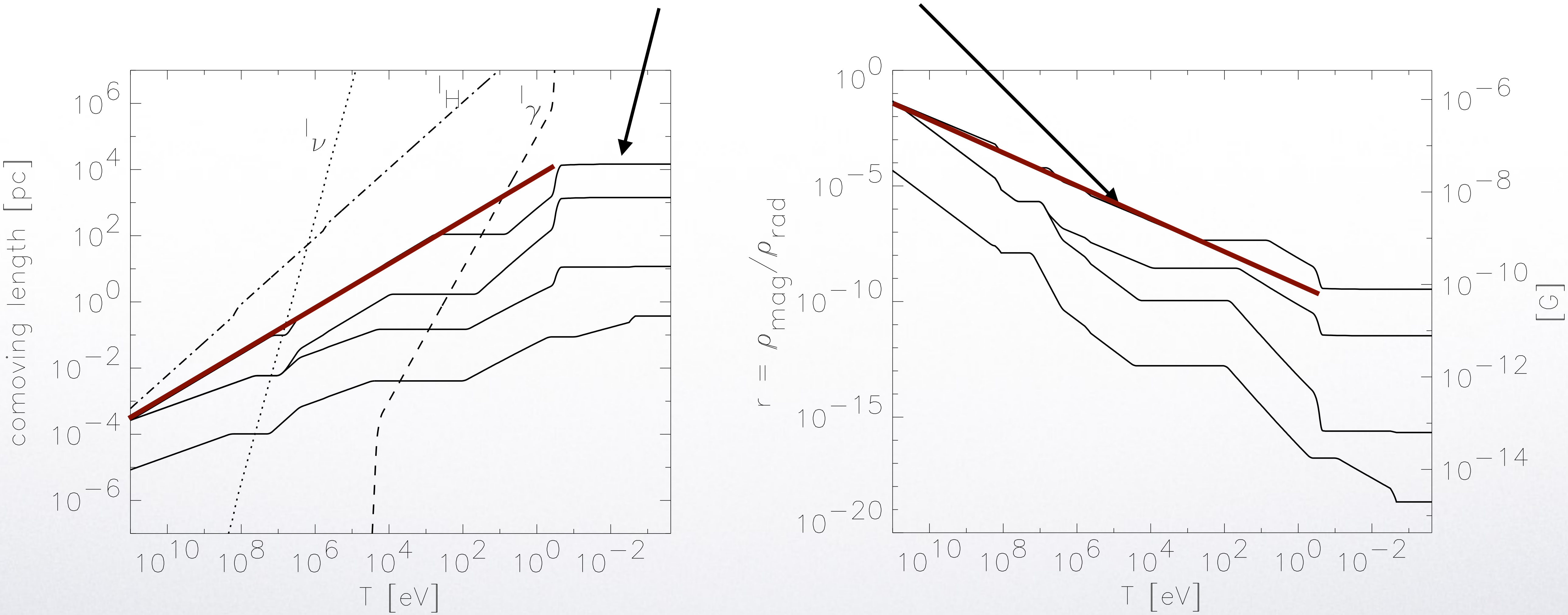


…but the final result is simple:

(assuming instantaneous reheating)

Coherence length grows: $$L = L_0 \frac{T_{RH}}{T_{rec}} \left(1 + \frac{B_0}{T_{RH}^2} \frac{1}{L_0 H_{RH}} \frac{T_{RH}}{T_{rec}}\right)^{2/3}$$

(physical)

Magnetic field decreases: $$B = B_0 \frac{T_{rec}^2}{T_{RH}^2} \left(1 + \frac{B_0}{T_{RH}^2} \frac{1}{L_0 H_{RH}} \frac{T_{RH}}{T_{rec}}\right)^{-1/3}$$

with $T_{rec}=0.3\ eV$, temperature at recombination

simplifies to

$$\frac{B}{L} = \frac{T_{rec}^4}{M_P} \simeq 10^{-8} \frac{\mathrm{G}}{\mathrm{Mpc}} \qquad B^2 L = B_{RH}^2 L_{RH} \left(\frac{T_{rec}}{T_{RH}}\right)^3$$

(assuming instantaneous reheating)

Coherence length grows:

$$L = L_0 \frac{T_{RH}}{T_{rec}} \left(1 + \frac{B_0}{T_{RH}^2} \frac{1}{L_0 H_{RH}} \frac{T_{RH}}{T_{rec}}\right)^{2/3}$$

(physical)

Magnetic field decreases:

$$B = B_0 \frac{T_{rec}^2}{T_{RH}^2} \left(1 + \frac{B_0}{T_{RH}^2} \frac{1}{L_0 H_{RH}} \frac{T_{RH}}{T_{rec}}\right)^{-1/3}$$

with $T_{rec}=0.3\ eV$, temperature at recombination

Anber, LS 2006

Can obtain $10^{-17}\ G$ @ $1\ Mpc$ with

$\xi \sim 16$ ⇒ scale of inflation $\rho^{1/4} \sim 10^{10}\ GeV$

…but unfortunately…

Constraints from nongaussianities

The produced electromagnetic modes
infect the inflaton perturbations through the coupling $\phi F\widetilde{F}$,
contributing to its three-point function

Barnaby Peloso 10

NONGAUSSIANITIES

$$f_{NL}^{\mathrm{equil}} \simeq 8.9\,\times 10^4\,\frac{H^6}{\epsilon^3\,M_P^6}\,\frac{e^{6\,\pi\,\xi}}{\xi^9}$$

Planck constrains $|f_{NL}{}^{equil}|<50$

$\xi<2.2$

Axion model
ruled out

How to find a way out?

Problems	Ways out
Ratra does not control amplitude Axion does not control spectral index	Use a hybrid of the two models
Too much f_{NL}	ϕ is not the inflaton, use instead a rolling spectator σ

The Lagrangian

$$\mathcal{L} = f(\tau)^2 \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\gamma}{8} \epsilon_{\mu\nu\rho\lambda} F^{\mu\nu} F^{\rho\lambda} \right)$$

$f(\tau)$ from $f(\sigma)$ through $\sigma(\tau)$
modeled as $f(\tau) = (-H\,\tau)^{-n}$

$n<0$ to avoid strong coupling
$\gamma \equiv -\xi/n$ an $O(10)$ constant

equation of motion:

Canonically normalized field

$$\tilde{A}''_\sigma + \left(k^2 + 2\,\xi\,\sigma\,\frac{k}{\tau} - \frac{n\,(n+1)}{\tau^2} \right) \tilde{A}_\sigma = 0$$

helicity dependent,
dominates at intermediate times,
exponential amplification

helicity independent,
dominates at late times,
determines spectral index

A model

Supergravity Lagrangian for *U(1)* gauge field

$$\mathcal{L} = -\frac{1}{4}\,\mathrm{Re}\,\{f\}\;F_{\mu\nu}\,F^{\mu\nu} - \frac{1}{4}\,\mathrm{Im}\,\{f\}\;F_{\mu\nu}\,\tilde{F}^{\mu\nu}$$

f=gauge kinetic function, assume $f(X,\,Y) = X\,Y$

with everything but *Re{X}* stabilized to

$$\mathrm{Re}\{Y\} = Y_0\,, \quad \mathrm{Im}\{Y\} = \gamma\,Y_0\,, \quad \mathrm{Im}\{X\} = 0$$

then: $$\mathcal{L} = X_R\,Y_0\left(-\frac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu} - \frac{\gamma}{4}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu}\right)$$ 👍

Mode functions

$$\tilde{A}_+(k,\,\tau) = \frac{1}{\sqrt{2\,k}}\,\left(G_{-n-1}(\xi,\,-k\,\tau) + i\,F_{-n-1}(\xi,\,-k\,\tau)\right)$$

$\tilde{A}_- \sim 0$

Coulomb wave functions

At large scales

$$\tilde{A}_\sigma(k,\,\tau) \simeq \sqrt{-\frac{\tau}{2\,\pi}}\,e^{\pi\xi}\,\Gamma\left(|2n+1|\right)\,|2\,\xi\,k\,\tau|^{-|n+1/2|}$$

Exponential amplification

Arbitrary spectral index

Subsequent evolution:

Assume instantaneous reheating

Assume inverse cascade until recombination

$$B_0 \simeq 10^{-8}\,\mathrm{G}\,\left(\frac{L_0}{\mathrm{Mpc}}\right) \qquad B_0^2\,L_0 = B_{\mathrm{rh}}^2\,L_{\mathrm{rh}}\,\left(\frac{a_{\mathrm{rh}}}{a_0}\right)^3$$

with

$$B_{\mathrm{rh}}^2 = H^4\,\frac{e^{2\,\pi\,\xi}}{\xi^5}\,\frac{\Gamma(4-2\,n)\,\Gamma(6+2\,n)}{2^8\times 3^2\times 5\times 7\times \pi^3}$$

$$L_{\mathrm{rh}} = \frac{18\,\pi}{(3-2\,n)\,(5+2\,n)}\,\frac{\xi}{H}$$

Constraints on parameter space

n<0 to avoid strong coupling
n>-2 to avoid IR divergence of electric field

will focus on ***-2<n<0***

First constraint: overproduction of GWs by magnetic field during inflation?

Primordial gravitational waves

Tensor components of the metric

$$g_{\mu\nu}(\mathbf{x},\,t)\,dx^\mu\,dx^\nu = -dt^2 + a^2(t)\,\left(\delta_{ij} + h_{ij}(\mathbf{x},\,t)\right)\,dx^i\,dx^j$$

$$\sum_{ij}\delta^{ij}\,h_{ij} = \sum_i \partial_i\,h_{ij} = 0$$

the tensor mode has two components (=helicity ±2)
so we can decompose it, in momentum space,
into left handed and right handed modes

$$h_{ij}(\mathbf{k},\,t) = h_L(\mathbf{k},\,t)\,\epsilon^L_{ij}(\mathbf{k}) + h_R(\mathbf{k},\,t)\,\epsilon^R_{ij}(\mathbf{k})$$

Generation of (parity violating) gravitational waves by *U(1)* gauge field during inflation

The energy of the electromagnetic field sources gravitational waves:

(note: this is an operator equation)

$$\ddot{h}_\lambda + 3\frac{\dot{a}}{a}\dot{h}_\lambda + \frac{\mathrm{k}^2}{a^2}h_\lambda = \frac{2}{M_p^2}\Pi_\lambda^{ij}\,T_{ij}^{\mathrm{EM}}$$

$\equiv T_\lambda$

Projector on helicity-λ components

Spatial components of gauge field stress-energy tensor

RHS is known, so obtain h_λ with retarded propagator

The amplitude of the helicity-λ gravitational waves

If $G_k(t,t')$ is retarded propagator
for operator $d^2/dt^2+3\,H\,d/dt+k^2/a^2$, then

$$h_\lambda(\mathbf{k},\,t) = \frac{2}{M_P^2}\int dt'\,G_k\,(t,\,t')\;T_\lambda(\mathbf{k},\,t)$$

and from this we obtain the amplitude

$$\langle h_\lambda(\mathbf{k},\,t)h_\lambda(\mathbf{q},\,t)\rangle = \frac{4}{M_P^4}\int dt' G_k(t,\,t')\int dt'' G_q(t,\,t'')\langle T_\lambda(\mathbf{k},\,t')T_\lambda(\mathbf{q},\,t'')\rangle$$

where $\langle T_\lambda(\mathbf{k},\,t')\,T_\lambda(\mathbf{q},\,t'')\rangle$ is quartic in the gauge field A
and can be computed in terms of the functions $A_\lambda{}^{\mathbf{k}}(t)$

The amplitude of the helicity-λ gravitational waves

Denoting $$\langle h_\lambda(\mathbf{x},\, t)\, h_\lambda(\mathbf{y},\, t)\rangle = \int \frac{d^3\mathbf{k}}{(2\,\pi)^3}\,\frac{\mathcal{P}_\lambda(\mathbf{k})}{k^3}\,e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}$$

$$\mathcal{P}_\pm(\mathbf{k}) = \frac{H^2}{\pi^2\,M_P^2}\left(1 + f_\pm(n)\,\frac{H^2}{M_P^2}\,\frac{e^{4\pi\xi}}{\xi^6}\right)$$

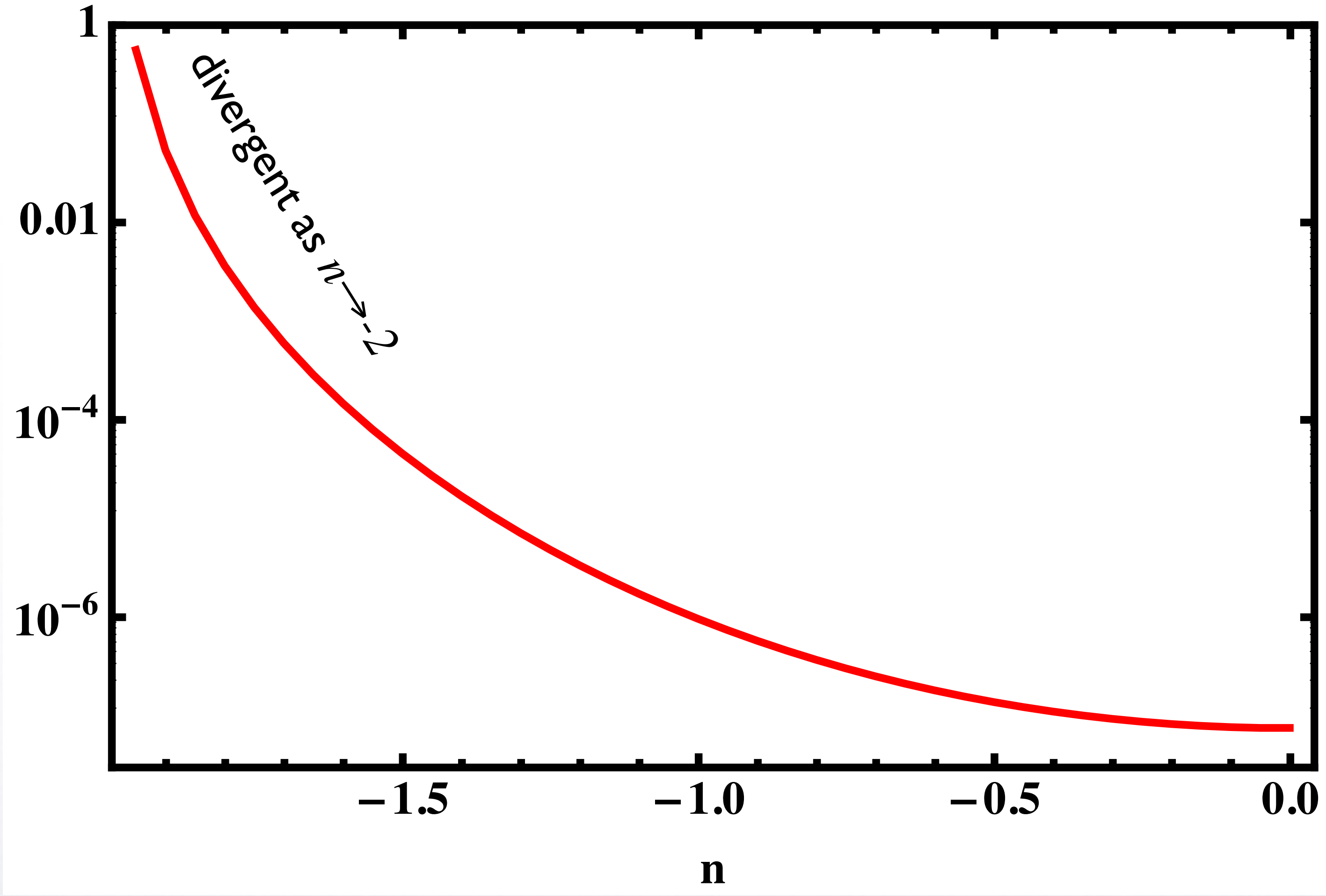
standard part

parity-violation

induced by gauge fields

The amplitude of the helicity-λ gravitational waves

$f_+(n)$



$[f_-(n) << f_+(n)]$

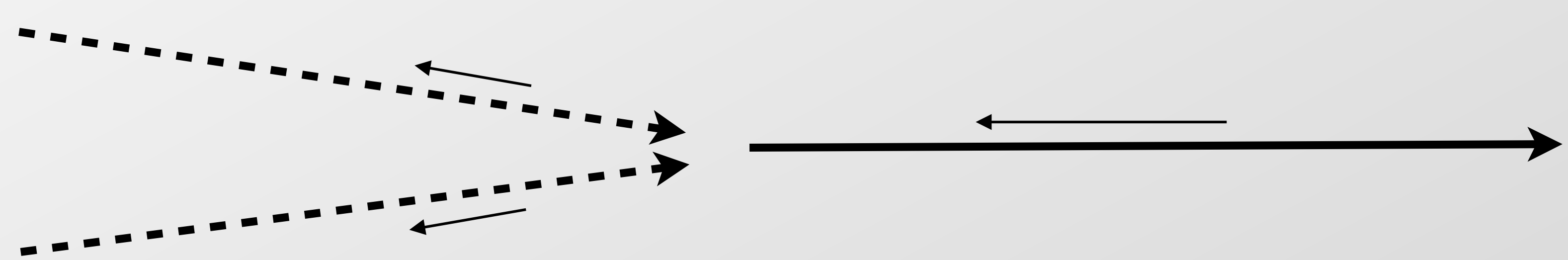
Parity violating gravitational waves

Sorbo 10

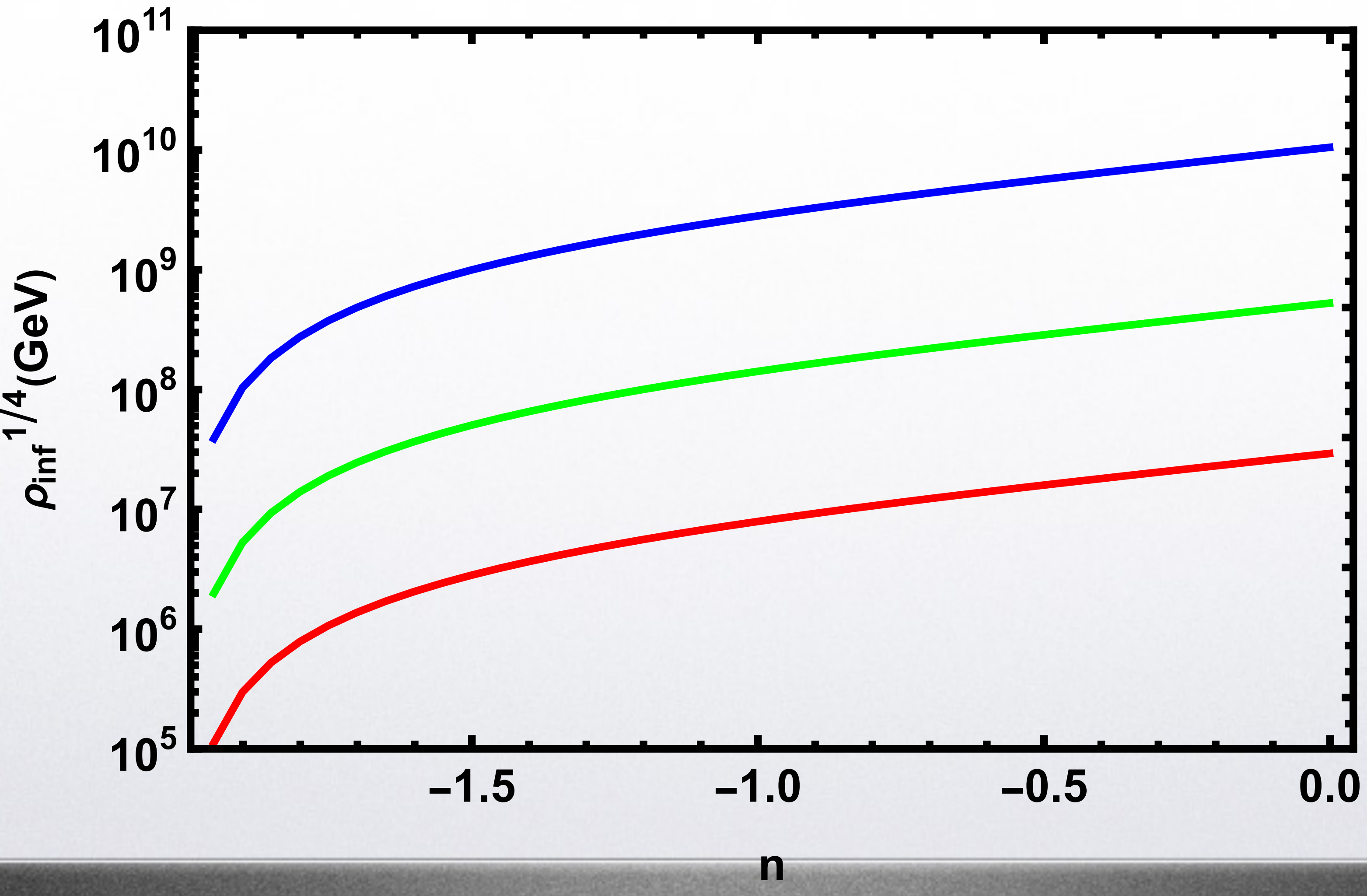
A_+ and A_- have different amplitudes

$$\langle h_+ h_+ \rangle \neq \langle h_- h_- \rangle$$

Physics: in the limit of small transverse momentum two LH photons cannot create a RH graviton



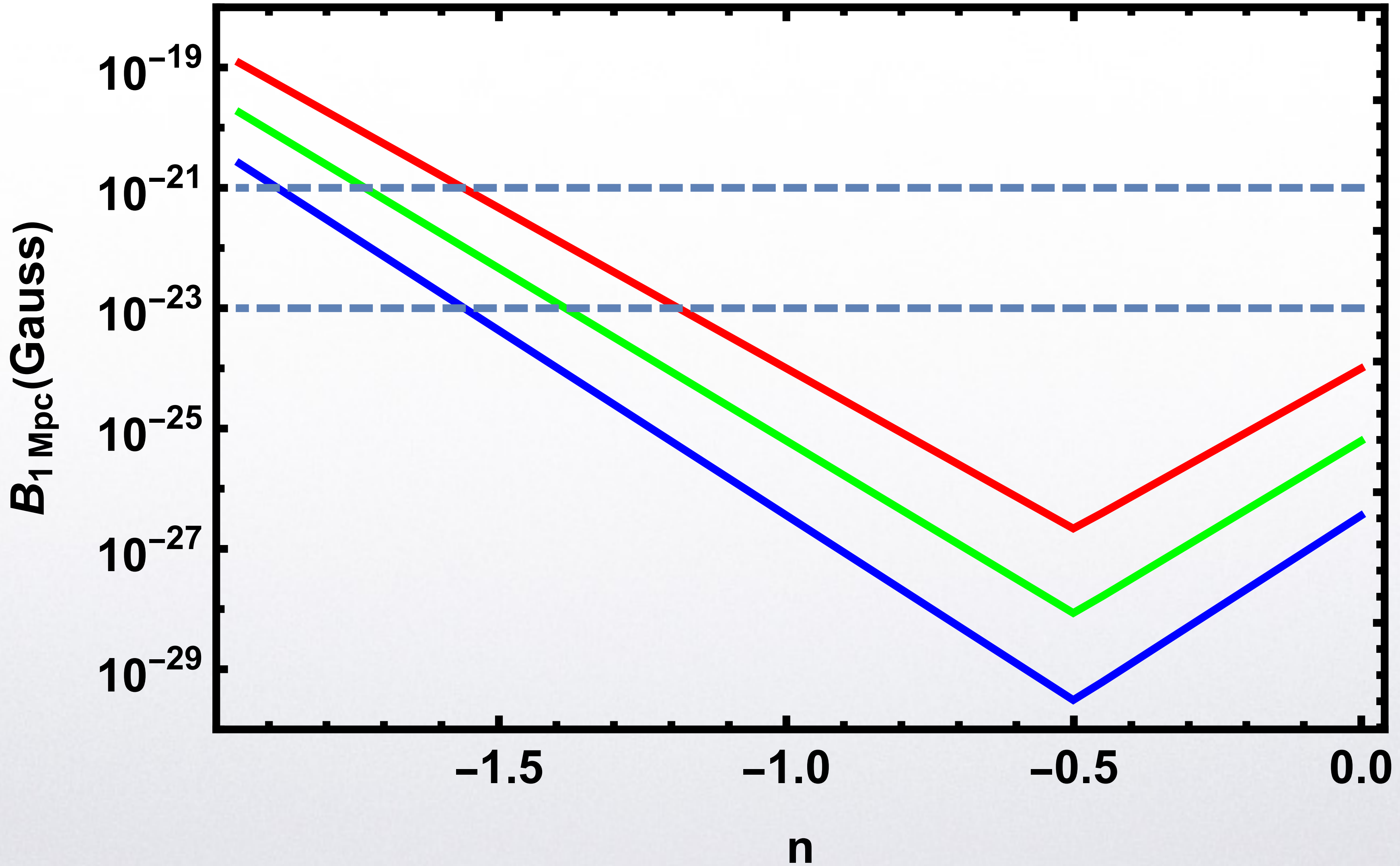
Imposing $r<0.035$: upper bound on $\varrho_{\rm inf}$ as function of n for $B=10^{-16}$, 2.5×10^{-17}, 6×10^{-18} $(1\ Mpc/L)^{1/2}$ G



How about *galactic* magnetic fields?

Intensity of B at $1\ Mpc$ scales for

$B = 10^{-16}, 2.5x10^{-17}, 6x10^{-18}\ (1\ Mpc/L)^{1/2}\ G$



Dynamo optimistic and pessimistic lower bound

A second constraint!

Ferreira, Sloth 14

Isocurvature perturbations are partially converted into curvature perturbations during inflation

$$\sigma \Longrightarrow A_\mu \Longrightarrow \delta\sigma \Longrightarrow \delta\phi$$

⬇

Nongaussian component in curvature perturbations, strongly constrained by Planck!

A second constraint!

Caprini, Guzzetti, Sorbo 17

$$\delta\varphi''_{\rm flat} + 2\,\mathcal{H}\,\delta\varphi'_{\rm flat} + \left(k^2 + a^2\,V_{\varphi\varphi}\right)\,\delta\varphi_{\rm flat} - \left(\frac{a^2\,\varphi'^2}{\mathcal{H}}\right)'\,\frac{\delta\varphi_{\rm flat}}{M_{pl}^2 a^2} - \left(\frac{a^2\,\varphi'\,\sigma'}{\mathcal{H}}\right)'\,\frac{\delta\sigma_{\rm flat}}{M_{pl}^2\,a^2} =$$
$$= 2\,\varphi_0'\,S^{(3)} + \frac{\varphi_0'}{\mathcal{H}}\,S'^{(3)} + \frac{\varphi_0'}{\mathcal{H}}\,S^{(2)}$$

$$S^{(2)} = -\frac{a^2}{2\,M_{pl}^2}\,\rho_{\rm em}(\mathbf{k}) = -\frac{I^2\,a^2}{4\,M_{pl}^2}\,[E_i * E_i + B_i * B_i]$$
$$S^{(3)} = \frac{a}{2\,M_{pl}^2}\,\frac{i\,\hat{k}_j}{k}\,q_{{\rm em}j}(\mathbf{k}) = \frac{I^2\,a^2}{2\,M_{pl}^2}\,\frac{i\,\hat{k}_j}{k}\,\epsilon_{jlm}[E_l * B_m]\,,$$

…computing and computing and computing…

A second constraint!

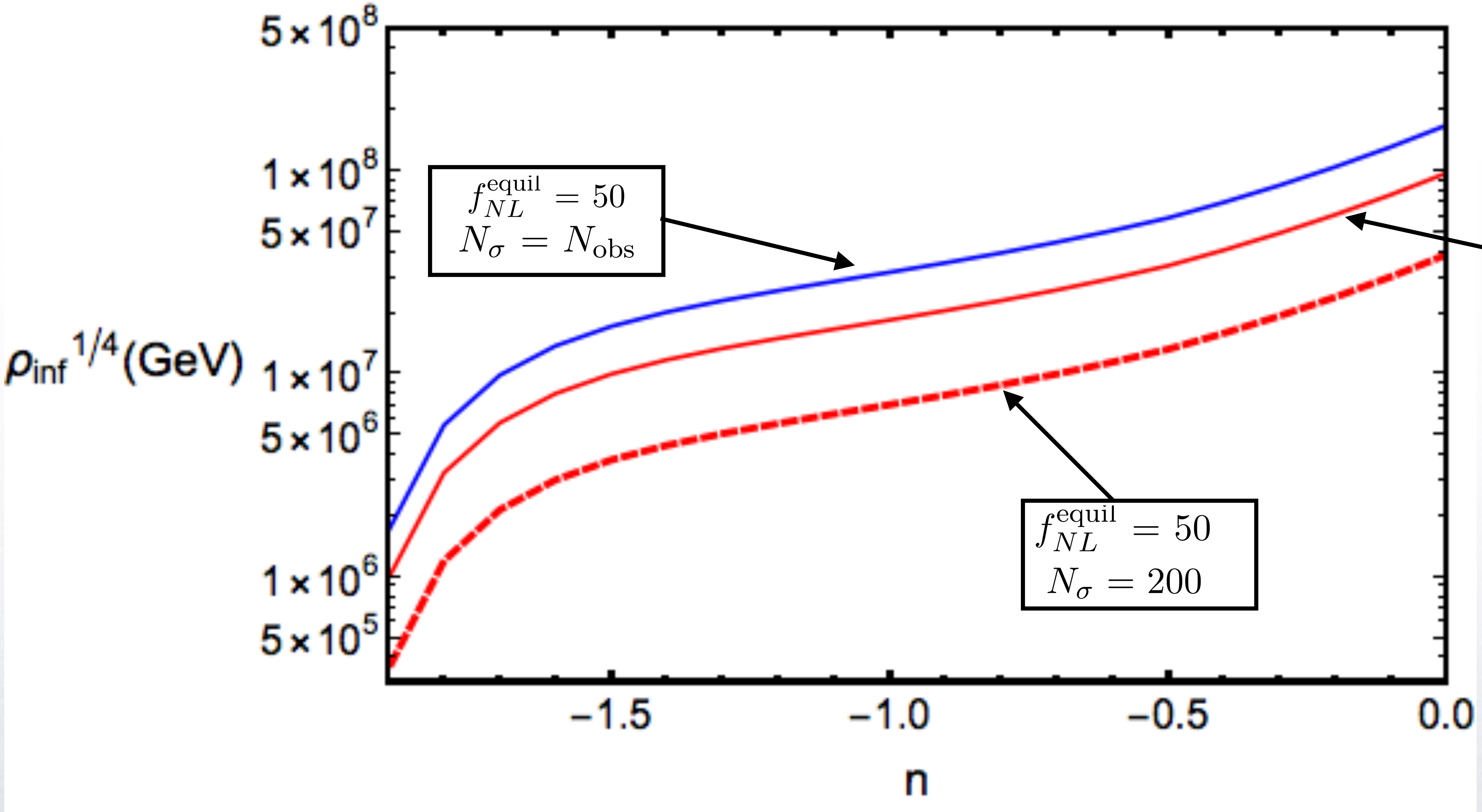
$$\langle \mathcal{R}(\mathbf{k}_1)\,\mathcal{R}(\mathbf{k}_2)\,\mathcal{R}(\mathbf{k}_3)\rangle = \frac{3}{10}(2\pi)^{5/2} f_{\rm NL}^{\rm equil}\mathcal{P}_{\mathcal{R}}^2\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{\sum k_i^3}{\Pi k_i^3}$$

$$f_{\rm NL}^{\rm equil} = \frac{10}{9}\frac{1}{(2\pi)^{5/2}}\frac{1}{\mathcal{P}_{\mathcal{R}}^2}\frac{H^6}{M_{pl}^6}\frac{e^{6\pi\xi}}{\xi^9}N_\sigma^3\, f(n)$$

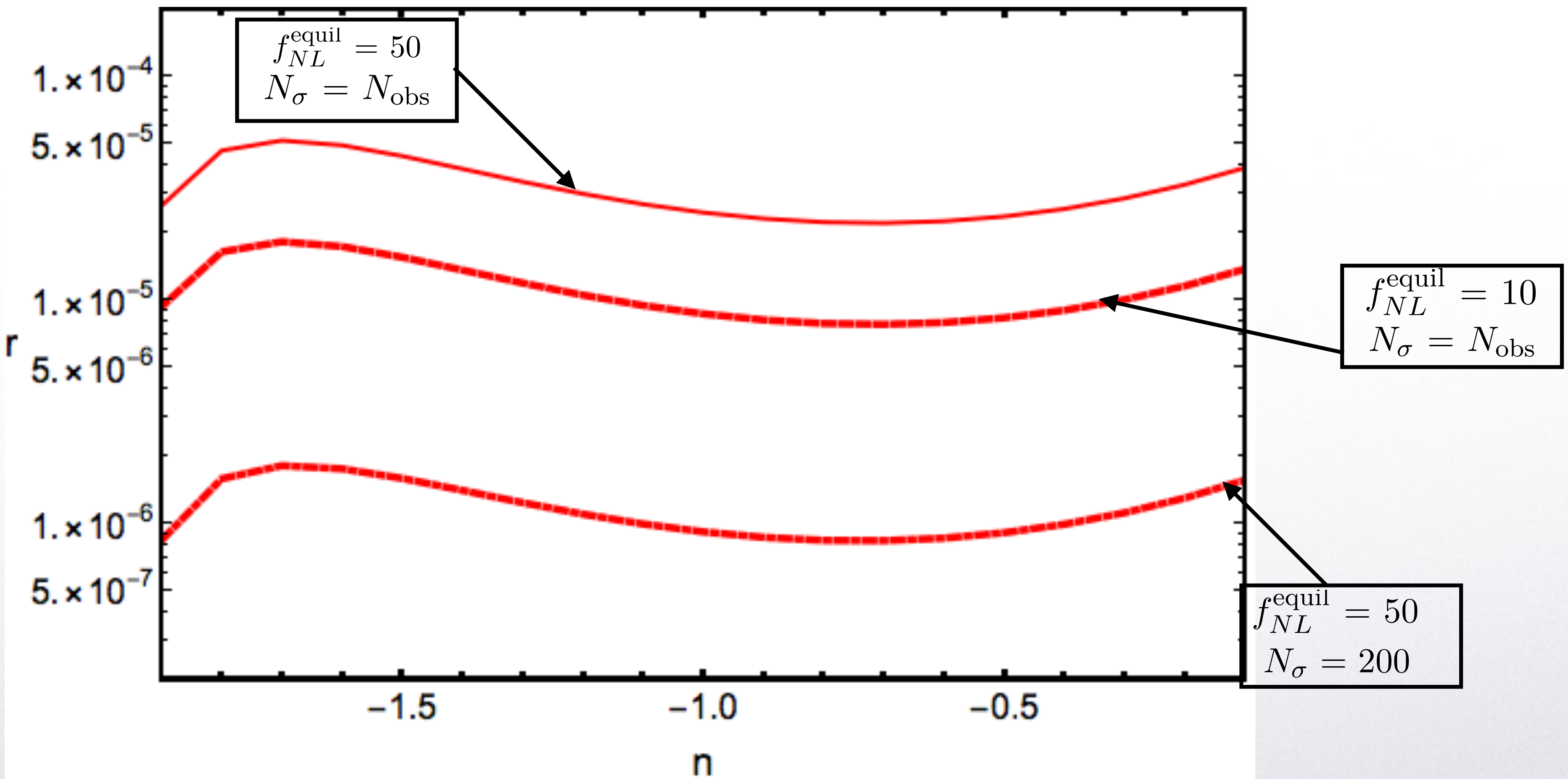
$$f(n) \simeq 1.7\times 10^{-5}\,(2+n)^{4.8}$$

Number of efolds of slow-rolling of σ

⟹a limit from f_{NL} on inflationary energy scale (assume $B=10^{-17}\,(1\ Mpc/L)^{1/2}\ G$)



…and an induced limit on r…

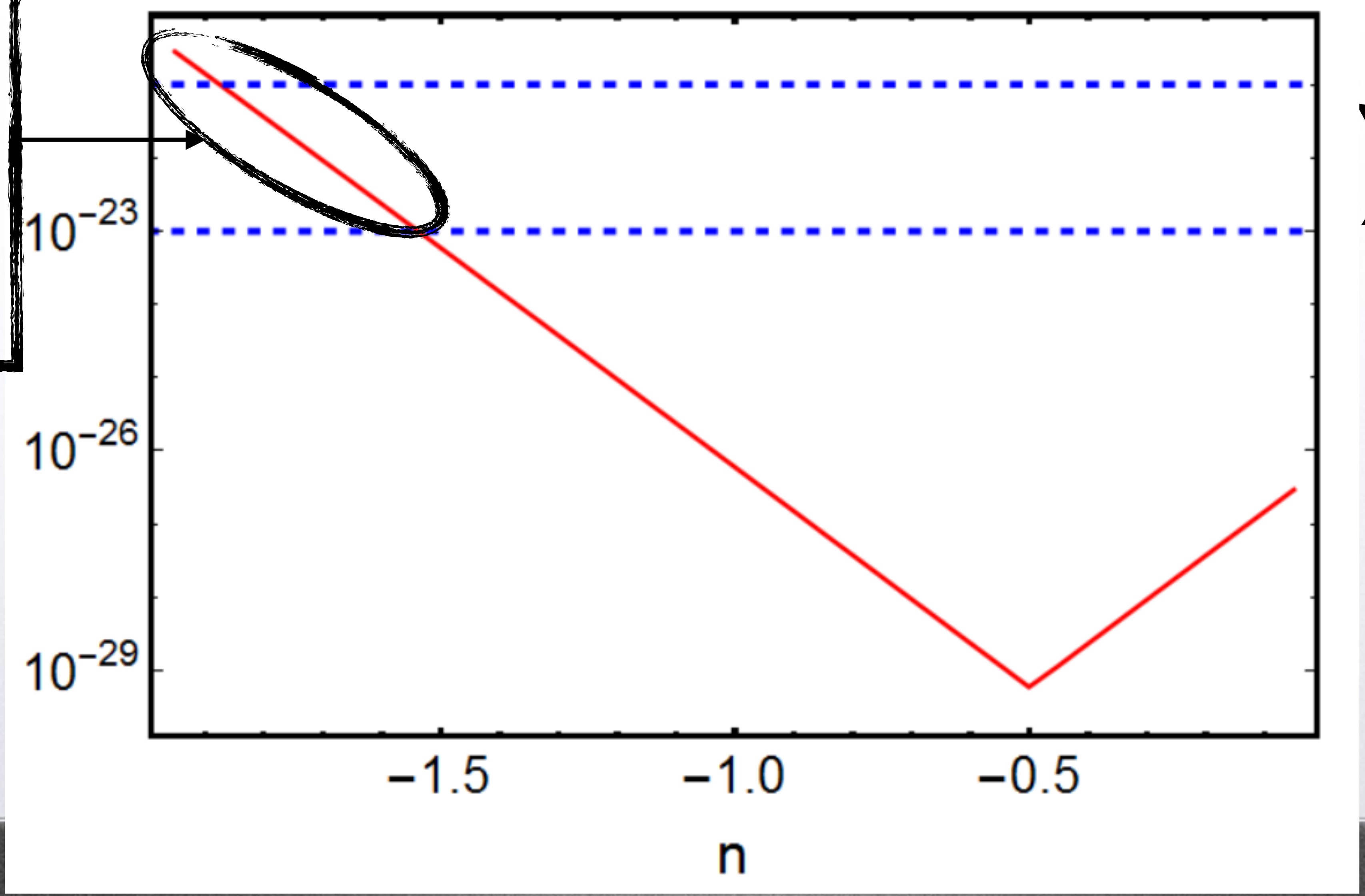


How about *galactic* magnetic fields?

Intensity of B at $1\ Mpc$ scales for

$$B=10^{-17}\,(1\ Mpc/L)^{1/2}\ G$$

Works with all constraints if optimistic lower bound for dynamo is valid!



Dynamo optimistic and pessimistic lower bound

Comments

- Despite the details, an order of magnitude estimate!
- Magnetic fields would be helical (detectable signature?)
- $B<<nG$ at cosmological scales: no effects in CMB
- Another signature: chiral GWs (hard to see?)

Conclusions

- Inflationary magnetogenesis notoriously difficult problem
- Presented a (not-so-)simple model consistent with observations