PHYSICS AT THE TERA SCALE
Helmholtz Alliance

Evolution with Subleading Color Contributions

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DESY

in collaboration with Dave Soper

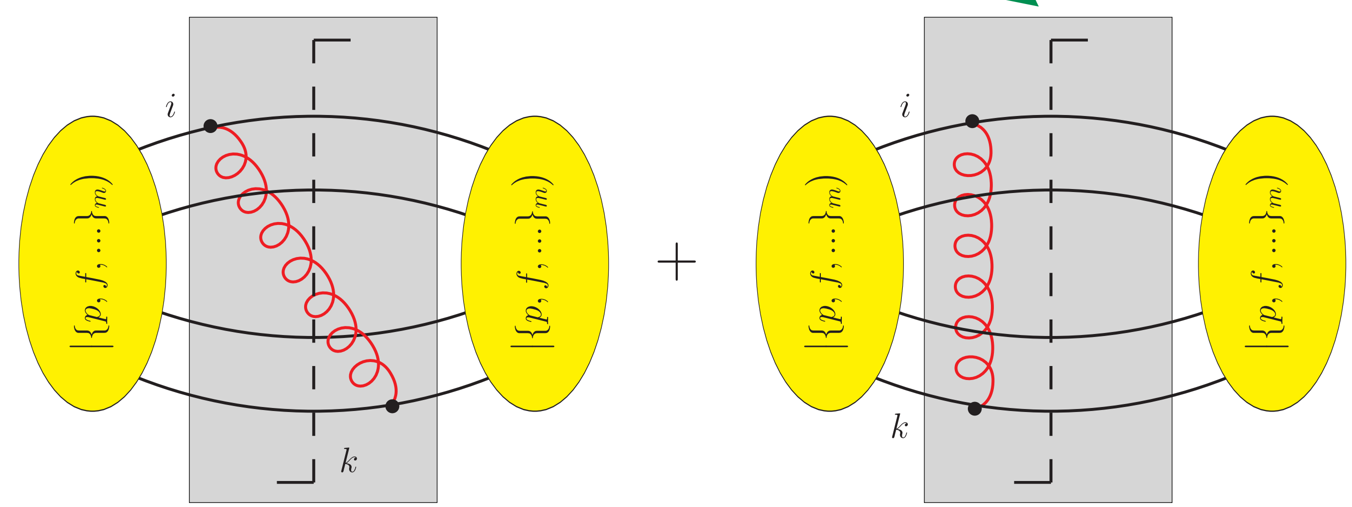
Shower Evolution

Using the factorization properties of the QCD the approximated order by order calculation can be organized according to

$$\mathcal{U}(t,t') = 1 + \int_{t'}^{t} d\tau\, \mathcal{U}(t,\tau)\big[\mathcal{H}_I(\tau) - \mathcal{V}(\tau)\big]$$

From the unitary condition:

$$\big(1\big|\mathcal{V}(t) = \big(1\big|\mathcal{H}_\mathrm{I}(t)$$



resolved radiations unresolved radiation

The shower form of the solution is

$$\mathcal{U}(t,t') = \mathcal{N}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}(t,\tau)\, \mathcal{H}_I(\tau)\, \mathcal{N}(\tau,t')$$

and the Sudakov operator is

$$\mathcal{N}(t,t') = \mathbb{T}\exp\left(-\int_{t'}^{t} d\tau\, \mathcal{V}(\tau)\right)$$

Full Splitting Operator

Very general splitting operator (*no spin correlation*) is

$$\big(\{\hat{p},\hat{f},\hat{c}',\hat{c}\}_{m+1}\big|\mathcal{H}(t)\big|\{p,f,c',c\}_m\big)$$

$$= \sum_{l=\mathrm{a,b},1,\dots,m} \delta\Big(t - T_l\big(\{\hat{p},\hat{f}\}_{m+1}\big)\Big)\,\big(\{\hat{p},\hat{f}\}_{m+1}\big|\mathcal{P}_l\big|\{p,f\}_m\big)\,\frac{m+1}{2}$$

$$\times \frac{n_\mathrm{c}(a)n_\mathrm{c}(b)\,\eta_\mathrm{a}\eta_\mathrm{b}}{n_\mathrm{c}(\hat{a})n_\mathrm{c}(\hat{b})\,\hat{\eta}_\mathrm{a}\hat{\eta}_\mathrm{b}}\,\frac{f_{\hat{a}/A}(\hat{\eta}_\mathrm{a},\mu_F^2)f_{\hat{b}/B}(\hat{\eta}_\mathrm{b},\mu_F^2)}{f_{a/A}(\eta_\mathrm{a},\mu_F^2)f_{b/B}(\eta_\mathrm{b},\mu_F^2)}\sum_k \Psi_{lk}(\{\hat{f},\hat{p}\}_{m+1})$$

$$\times \sum_{\beta=L,R}(-1)^{1+\delta_{lk}}\big(\{\hat{c}',\hat{c}\}_{m+1}\big|\mathcal{G}_\beta(l,k)\big|\{c',c\}_m\big)$$

P. Skands and D. Soper inveted a name for this kind of parton shower:
Partitioned dipole shower

Splitting kernel is

$$\Psi_{lk} = \frac{\alpha_\mathrm{s}}{2\pi}\,\frac{1}{\hat{p}_l\cdot\hat{p}_{m+1}}\left[A_{lk}\,\frac{2\hat{p}_l\cdot\hat{p}_k}{\hat{p}_k\cdot\hat{p}_{m+1}} + H_{ll}^{\mathrm{coll}}(\{\hat{f},\hat{p}\}_{m+1})\right]$$

Important:

$$A_{lk} + A_{kl} = 1$$

Color operator for gluon emission is

$$\big(\{\hat{c}',\hat{c}\}_{m+1}\big|\mathcal{G}_R(l,k)\big|\{c',c\}_m\big)$$

$$= {}_D\big\langle\{\hat{c}\}_{m+1}\big|t_l^\dagger\big|\{c\}_m\big\rangle\,\big\langle\{c'\}_m\big|t_k\big|\{\hat{c}'\}_{m+1}\big\rangle_D\ .$$

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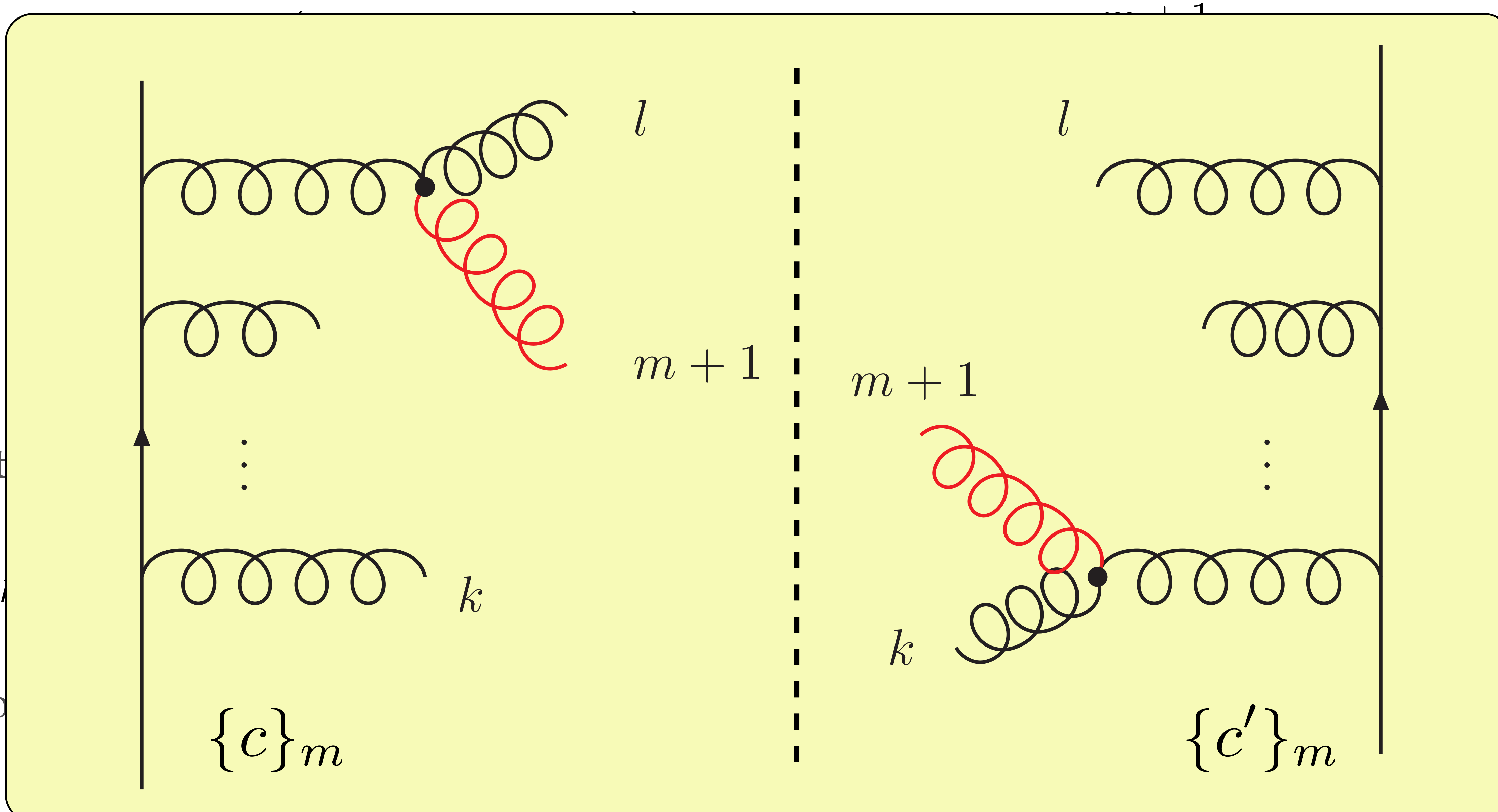

$$= {}_D\!\left\langle\{\hat{c}\}_{m+1}\middle|t_l^\dagger\middle|\{c\}_m\right\rangle\left\langle\{c'\}_m\middle|t_k\middle|\{\hat{c}'\}_{m+1}\right\rangle_D \ .$$

Angular Ordered Shower

What would happen if we used angular ordering?

$$t_{\angle} = T_l\big(\{\hat{p},\hat{f}\}_{m+1}\big) = \log 2 - \log\frac{\hat{p}_l\cdot\hat{p}_{m+1}\,\hat{Q}^2}{\hat{p}_l\cdot\hat{Q}\,\hat{p}_{m+1}\cdot\hat{Q}} = \log\frac{2}{1-\cos\vartheta_{l,m+1}}$$

And let's have a special choice for soft partitioning function:

$$A'_{lk} = \theta(\vartheta_{l,m+1} < \vartheta_{l,k})\,\frac{1-\cos\vartheta_{m+1,k}}{1-\cos\vartheta_{l,k}} \quad\Rightarrow\quad A_{lk} + A_{kl} \approx 1$$

$$\Psi_l^{(\mathrm{a.o.})} = \frac{\alpha_s}{2\pi}\,\frac{2}{\hat{p}_l\cdot\hat{p}_{m+1}}\left[\frac{\hat{p}_l\cdot\hat{Q}}{\hat{p}_{m+1}\cdot\hat{Q}} + H_{ll}^{\mathrm{coll}}\big(\{\hat{f},\hat{p}\}_{m+1}\big)\right]$$

Independent of parton k!!!

One can perform the sum over the color connected parton analytically

$$-\sum_k \big(\{\hat{c}',\hat{c}\}_{m+1}\big|\mathcal{G}_\beta(l,k)\big|\{c',c\}_m\big) = \big(\{\hat{c}',\hat{c}\}_{m+1}\big|\mathcal{G}_\beta(l,l)\big|\{c',c\}_m\big)$$

$\Rightarrow$ *No complicated color structure.*

Leading Color Approx.

1. Don't have special choice for the evolution variable and the soft partitioning function

Anyway everybody uses transverse momentum and the simplest soft partitioning function :

$$t_\perp = T_l\big(\{\hat{p},\hat{f}\}_{m+1}\big) = \log\frac{\hat{Q}^2}{-k_\perp^2}$$

$$A_{lk} = \frac{\hat{p}_k\cdot\hat{p}_{m+1}}{\hat{p}_k\cdot\hat{p}_{m+1} + \hat{p}_l\cdot\hat{p}_{m+1}}$$

2. But do approximation in the color space by considering only the leading color contributions

$$\begin{aligned}
&\big(\{\hat{p},\hat{f},\hat{c}\}_{m+1}\big|\mathcal{H}(t)\big|\{p,f,c\}_m\big) \\
&\quad = \sum_{l=\mathrm{a},\mathrm{b},1,\dots,m} \delta\Big(t - T_l\big(\{\hat{p},\hat{f}\}_{m+1}\big)\Big)\,\big(\{\hat{p},\hat{f}\}_{m+1}\big|\mathcal{P}_l\big|\{p,f\}_m\big) \\
&\qquad\times \frac{n_\mathrm{c}(a)n_\mathrm{c}(b)\,\eta_\mathrm{a}\eta_\mathrm{b}}{n_\mathrm{c}(\hat{a})n_\mathrm{c}(\hat{b})\,\hat{\eta}_\mathrm{a}\hat{\eta}_\mathrm{b}}\,\frac{f_{\hat{a}/A}(\hat{\eta}_\mathrm{a},\mu_F^2)f_{\hat{b}/B}(\hat{\eta}_\mathrm{b},\mu_F^2)}{f_{a/A}(\eta_\mathrm{a},\mu_F^2)f_{b/B}(\eta_\mathrm{b},\mu_F^2)} \\
&\qquad\times (m+1)\sum_k \Psi_{lk}(\{\hat{f},\hat{p}\}_{m+1})\,\big\langle\{\hat{c}\}_{m+1}\big|a_{lk}^\dagger\big|\{c\}_m\big\rangle\ .
\end{aligned}$$

Antenna Dipole Shower

The antenna dipole shower is rather a *reorganization of the leading color* partitioned dipole *shower*.

$$\mathcal{H}_{lk}^{\mathrm{part}}(t) \propto [\mathcal{P}_l\, A_{lk} + \mathcal{P}_k\, A_{kl}]\, \frac{\hat{p}_l \cdot \hat{p}_k}{\hat{p}_{m+1} \cdot \hat{p}_l\;\hat{p}_{m+1} \cdot \hat{p}_k}$$

The antenna shower tries to remove the ambiguity of the soft partitioning function A_{lk} by using a new momentum mapping

$$\mathcal{H}_{lk}^{\mathrm{ant}}(t) \propto \mathcal{P}_{lk}\, \frac{\hat{p}_l \cdot \hat{p}_k}{\hat{p}_{m+1} \cdot \hat{p}_l\;\hat{p}_{m+1} \cdot \hat{p}_k}$$

Now the freedom to choose A_{lk} function resides in the freedom to choose P_{lk}. I think the best mapping for antenna shower would be

$$\mathcal{P}_{lk} = \theta(\vartheta_{l,m+1} < \vartheta_{k,m+1})\, \mathcal{P}_l + \theta(\vartheta_{k,m+1} < \vartheta_{l,m+1})\, \mathcal{P}_k$$

Parton Showers

There are basically three points where the essential differences lie, namely the *momentum mapping*, the *evolution parameter* and the choice of the *soft partitioning function*.

Angular ordered shower

- Full color evolution
- Easy to implement
- Loosing the full exclusiveness
- Angle doesn't control the goodness of the underlying approximation

Leading color shower

- More flexible
- Systematically improvable
- Easy to implement
- Leading color approximation

Solution of the Evolution Equation

The idea is split the splitting operator "*good*" and "*bad*" part and expand the evolution operator in the "bad" splitting operator.

$$\mathcal{H}_I(t) = \mathcal{H}_I^{(J)}(t) + \mathcal{H}_I^{(S)}(t)$$

Fully exponentiated *Subtracted*

The inclusive splitting operators are

$$\big(1\big|\mathcal{V}^{(J)}(t) = \big(1\big|\mathcal{H}_I^{(J)}(t) \qquad \text{and} \qquad \big(1\big|\mathcal{V}^{(S)}(t) = \big(1\big|\mathcal{H}_I^{(S)}(t)$$

Now the *good part* of the evolution operator is

$$\mathcal{U}^{(J)}(t,t') = \mathcal{N}^{(J)}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}^{(J)}(t,\tau)\mathcal{H}_I^{(J)}(\tau)\mathcal{N}^{(J)}(\tau,t')$$

The full evolution operator is given by

$$\mathcal{U}(t,t') = \mathcal{U}^{(J)}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}(t,\tau)\left[\mathcal{H}_I^{(S)}(\tau) - \mathcal{V}^{(S)}(\tau)\right]\mathcal{U}^{(J)}(\tau,t')$$

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The

$$\begin{aligned}\mathcal{U}(t,t') = {} & \mathcal{U}^{(J)}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}^{(J)}(t,\tau)\left[\mathcal{H}_I^{(S)}(\tau) - \mathcal{V}^{(S)}(\tau)\right]\mathcal{U}^{(J)}(\tau,t') \\ & + \int_{t'}^{t} d\tau_2 \int_{t'}^{\tau_2} d\tau_1\, \mathcal{U}^{(J)}(t,\tau_2)\left[\mathcal{H}_I^{(S)}(\tau_2) - \mathcal{V}^{(S)}(\tau_2)\right] \\ & \qquad \times \mathcal{U}^{(J)}(\tau_2,\tau_1)\left[\mathcal{H}_I^{(S)}(\tau_2) - \mathcal{V}^{(S)}(\tau_1)\right]\mathcal{U}^{(J)}(\tau_1,t') \\ & + \cdots\end{aligned}$$

Now

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$\mathcal{U}(t,t')$ ... $^{J)}(\tau, t')$

Now

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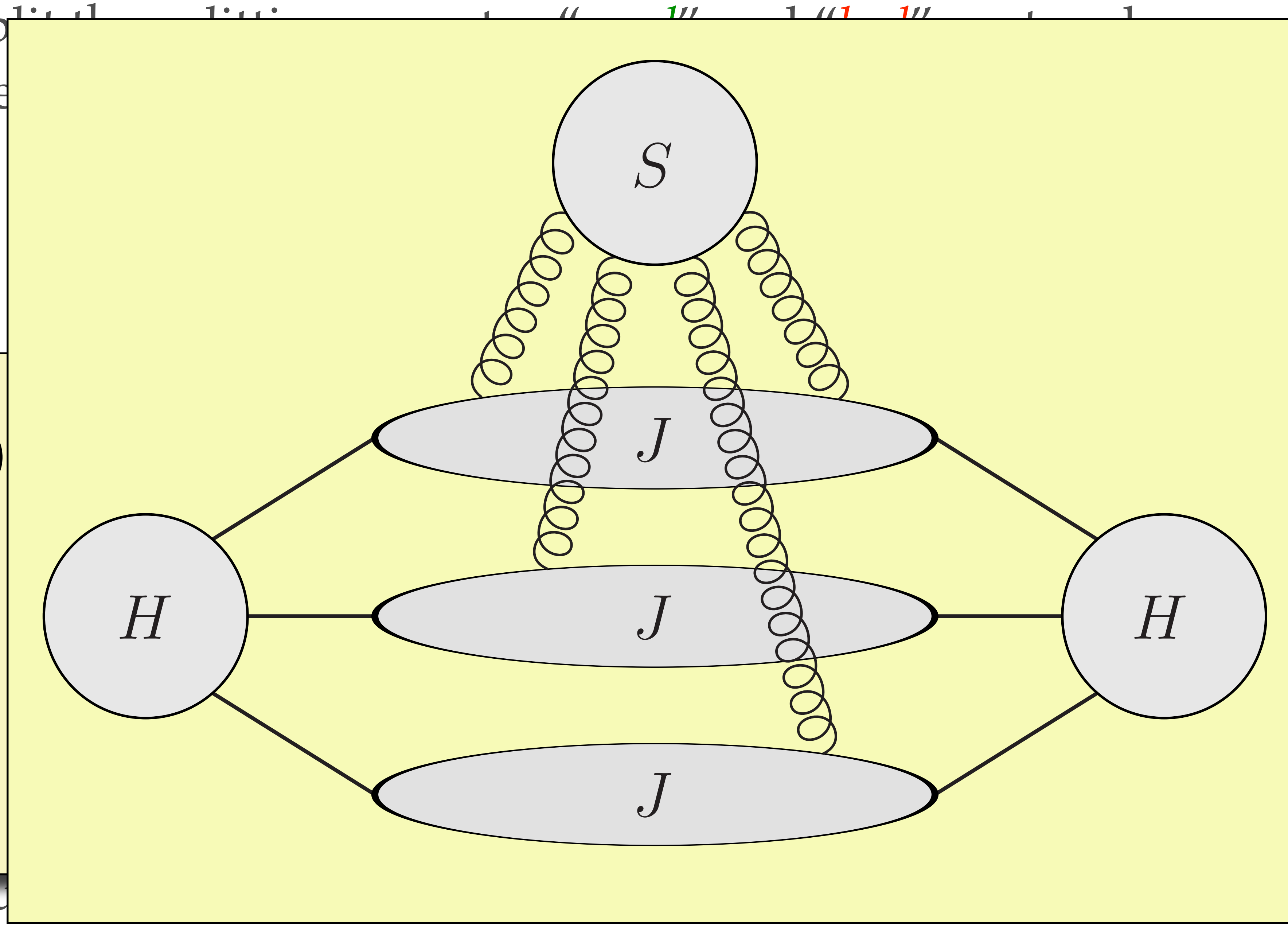

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\begin{aligned}
&\big(\{\hat p,\hat f,\hat c',\hat c\}_{m+1}\big|\mathcal{H}(t)\big|\{p,f,c',c\}_m\big)\\
&= \sum_{l=\mathrm{a,b},1,\dots,m} \delta\Big(t - T_l\big(\{\hat p,\hat f\}_{m+1}\big)\Big)\,\big(\{\hat p,\hat f\}_{m+1}\big|\mathcal{P}_l\big|\{p,f\}_m\big)\,\frac{m+1}{2}\\
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&\quad\times \sum_{\beta=L,R}(-1)^{1+\delta_{lk}}\big(\{\hat c',\hat c\}_{m+1}\big|\mathcal{G}_\beta(l,k)\big|\{c',c\}_m\big)
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$$

Important:

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A_{lk} + A_{kl} = 1
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Color operator for gluon emission is

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\begin{aligned}
&\big(\{\hat c',\hat c\}_{m+1}\big|\mathcal{G}_R(l,k)\big|\{c',c\}_m\big)\\
&\qquad= {}_D\big\langle\{\hat c\}_{m+1}\big|t_l^\dagger\big|\{c\}_m\big\rangle\,\big\langle\{c'\}_m\big|t_k\big|\{\hat c'\}_{m+1}\big\rangle_D\ .
\end{aligned}
$$

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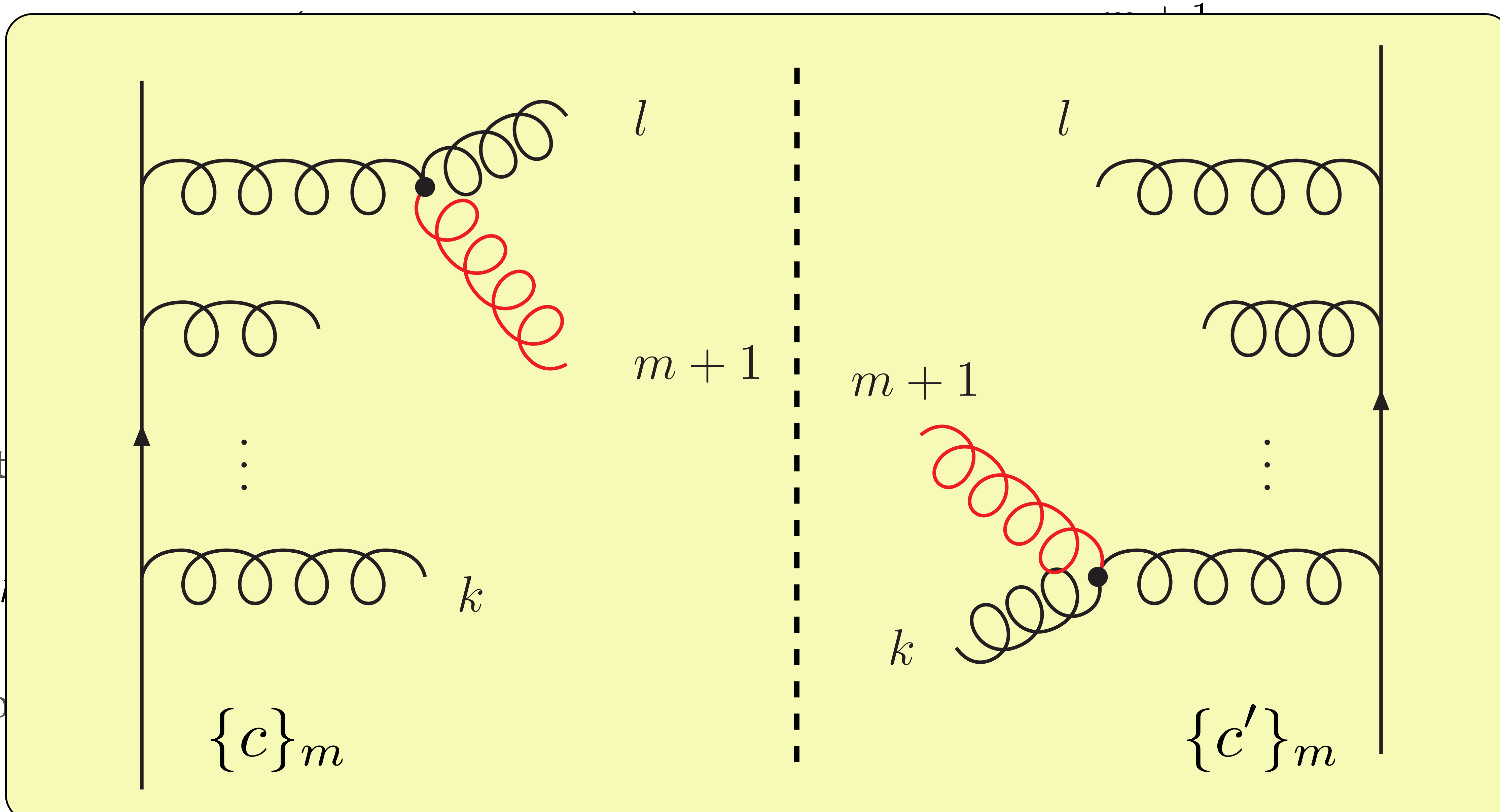
$$\left(\{\hat p, \hat f, \hat c', \hat c\}_{m+1}\middle|\mathcal{H}(t)\middle|\{p, f, c', c\}_m\right)$$



Split

Ψ_{l} $= 1$

d a name

Colo

$$= {}_D\big\langle\{\hat c\}_{m+1}\big|t_l^\dagger\big|\{c\}_m\big\rangle\,\big\langle\{c'\}_m\big|t_k\big|\{\hat c'\}_{m+1}\big\rangle_D\ .$$

Jet Splitting Operator

We approximate the color operator using a projection

$$\big(\{\hat{c}',\hat{c}\}_{m+1}\big|\mathcal{G}(k,l;\{\hat{f}\}_{m+1})\big|\{c',c\}_m\big) = \big(\{\hat{c}',\hat{c}\}_{m+1}\big|t_k^\dagger \otimes t_l\big|\{c',c\}_m\big)$$

$$\big(\{\hat{c}',\hat{c}\}_{m+1}\big|\mathcal{C}(l,m+1)\,\mathcal{G}(k,l;\{\hat{f}\}_{m+1})\big|\{c',c\}_m\big)$$

The projection keep the color connected part

$$\mathcal{C}(l,m+1)\big|\{c',c\}_{m+1}\big) = \begin{cases} \big|\{c',c\}_{m+1}\big) & l \text{ and } m+1 \text{ color connected} \\ & \text{in } \{c'\}_{m+1} \text{ and in } \{c\}_{m+1} \\ 0 & \text{otherwise} \end{cases}$$

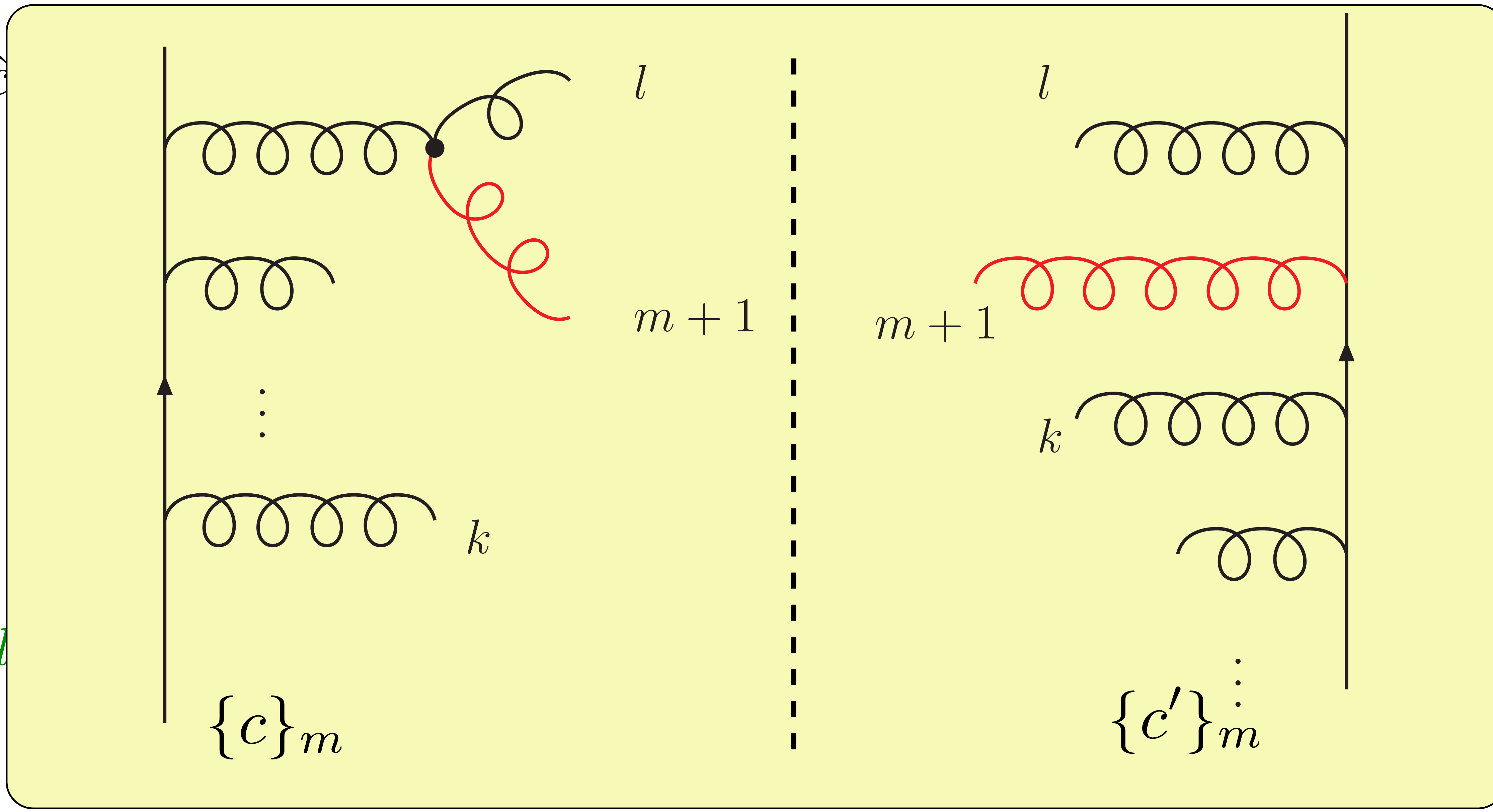
The corresponding quantum level operator is

$$\mathcal{C}(l,m+1) = C(l,m+1)^\dagger \otimes C(l,m+1)$$

Jet Splitting Operator

We approximate the color operator using a projection

$(\{\hat{c}$ … $_m)$



The … ted

$\mathcal{C}(l$ …

The corresponding quantum level operator is

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Jet Splitting Operator

- This operator can evolve interference contribution.
- Full collinear and soft+collinear contributions included.
- Wide angle pure soft contributions are not fully included. Omitted part is suppressed by $1/N_c^2$. It is treated perturbative.
- The corresponding inclusive splitting operator can be exponentiated easily.
- Leads to a quasi Markovian process.

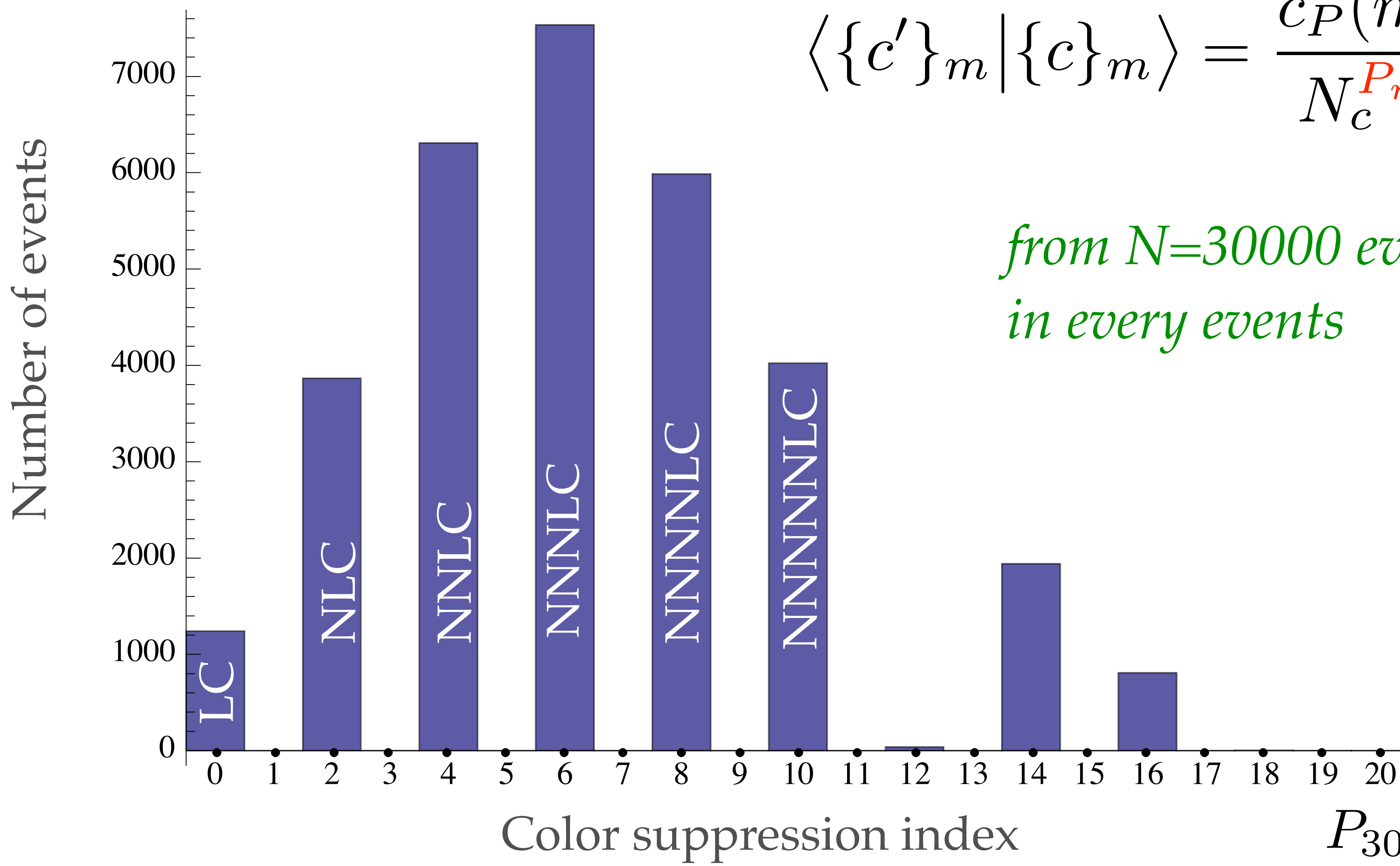
$$\mathcal{N}^{(\mathrm{J})}(t,t')\big|p,f,s',c',s,c\}_m\big) = \exp\left\{-\int_{t'}^{t} d\tau\,\left[\lambda_1(\{p,f,c\}_m) + \lambda_2(\{p,f,c'\}_m)\right]\right\} \times \big|p,f,s',c',s,c\}_m\big)$$

Numerical Efficiency

With a simple "color shower" we can estimate the importance of the subleading color contributions.

$$N_m(P) = \sum_{i=1}^{N} \left\langle \{c'_i\}_m \middle| \{c_i\}_m \right\rangle \delta(P - P_m(\{c'_i, c_i\}_m))$$

$$\left\langle \{c'\}_m \middle| \{c\}_m \right\rangle = \frac{c_P(m)}{N_c^{P_m}} \left\{ 1 + \mathcal{O}\left(\frac{1}{N_c^2}\right) \right\}$$



from N=30000 events; m=30 splitting in every events

Conclusions

- I think it is possible to go beyond the leading color approximation or include wide angle radiations.
- We have shown that it is possible to compute parton shower with full color *systematically* using mainly standard Monte Carlo techniques.
- We might have some numerical complication....
- It would be interesting to build in the coherence effect explicitly without imposing direct angular ordering.