Probabilistic description of coherent bremsstrahlung

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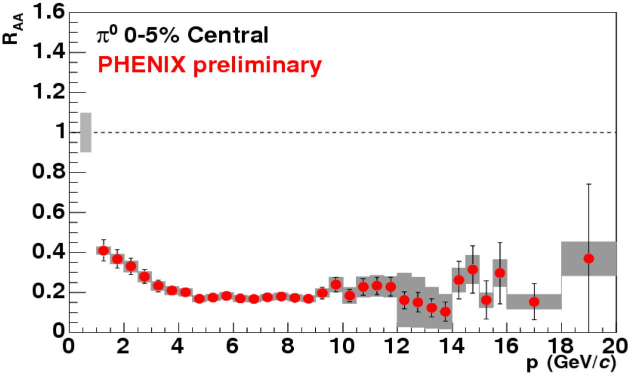
Introduction

π^0 spectrum in central Au+Au at RHIC divided by scaled p+p spectrum



strong suppression of high $p_\perp$ hadrons – commonly interpretated as QCD bremsstrahlung

Introduction

- so far only analytic calculations of QCD bremsstrahlung
- prominent interference effect: non-abelian LPM-effect
- analytic models largely limited to leading particles
- subleading jet fragments will be accessible at LHC
- subleading fragments likely to discriminate between different microscopic mechanisms cojectured to underly jet quenching $\rightarrow$ essential for characterisation of medium properties
- allows to characterise jet-induced modifications of medium and to disentangle jets from background

$\rightarrow$ need MC model for jet fragmentation in a dense QCD medium including coherent bremsstrahlung

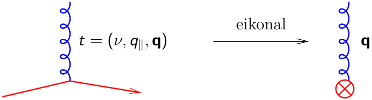
The Eikonal Approximation

Kinematical regime

$$E \gg \omega \gg |\boldsymbol{k}|, |\boldsymbol{q}_i| \geq \Lambda_{QCD}$$

Consequences

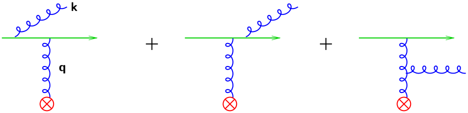
elastic scattering:



$A(\boldsymbol{q})$

potential scattering

inelastic scattering:



$$|A(\boldsymbol{q})|^2 \frac{\boldsymbol{q}^2}{\boldsymbol{k}^2(\boldsymbol{k}+\boldsymbol{q})^2}$$

$$= |A(\boldsymbol{q})|^2 R(\boldsymbol{k}, \boldsymbol{q})$$

Gunion-Bertsch cross section

Full Gluon Spectrum

BDMPS ASW gluon distribution in path integral formulation:

Wiedemann, Nucl. Phys. **B 588**(2000),303

$$\omega\frac{\mathrm{d}^3 I}{\mathrm{d}\omega\mathrm{d}\boldsymbol{k}} = \frac{\alpha_s C_R}{(2\pi)^2\omega^2}2\Re\int_{\xi_0}^{\infty}\mathrm{d}y_l\int_{y_l}^{\infty}\mathrm{d}\bar{y}_l\int\mathrm{d}\boldsymbol{u}\,e^{-\imath\boldsymbol{k}\cdot\boldsymbol{u}}e^{-\frac{1}{2}\int_{\bar{y}_l}^{\infty}\mathrm{d}\xi\,n(\xi)\sigma(\boldsymbol{u})}$$

$$\times\frac{\partial}{\partial\boldsymbol{y}}\frac{\partial}{\partial\boldsymbol{u}}\underbrace{\int_{\boldsymbol{y}=0}^{\boldsymbol{u}=\boldsymbol{r}(\bar{y}_l)}\mathcal{D}\boldsymbol{r}\,\exp\left[\imath\int_{\bar{y}_l}^{y_l}\mathrm{d}\xi\,\frac{\omega}{2}\left(\dot{\boldsymbol{r}}-\frac{n(\xi)\sigma(\boldsymbol{r})}{\imath\omega}\right)\right]}_{=\mathcal{K}(\boldsymbol{y}=0,y_l;\boldsymbol{u}=\boldsymbol{r}(\bar{y}_l),\bar{y}_l|\omega)}$$

dipole cross section:

$$\sigma(\boldsymbol{r}) = 2\int\frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^2}|A(\boldsymbol{q})|^2\left(1-e^{\imath\boldsymbol{q}\cdot\boldsymbol{r}}\right)$$

Opacity Expansion: Prologue

- expand gluon distribution in powers of $n(\xi)\sigma(\boldsymbol{r})$
- NB this is an expansion in numbers of interactions and not in numbers of scattering centres
- set $\xi_0 = -\infty$, where ξ_0 is the projectile production point
- set

$$n(\xi) = \begin{cases} n_0 & \text{for } 0 < \xi < L \\ 0 & \text{for } \xi < 0 \text{ or } \xi > L \end{cases}$$

- elastic cross section of a scattering potential

$$V_{\text{tot}} = \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^2} |A(\boldsymbol{q})|^2$$

- notation for transverse energies:

$$Q \equiv \frac{\boldsymbol{k}^2}{2\omega} \qquad Q_1 \equiv \frac{(\boldsymbol{k}+\boldsymbol{q})^2}{2\omega}$$

- convention: in each order N label the last interaction as $i = 1$ and the first as $i = N$

Opacity Expansion: Results

Zeroth Order

$$\omega \frac{\mathrm{d}^3 I^{(0)}}{\mathrm{d}\omega \mathrm{d}\boldsymbol{k}} = 0$$

no vacuum radiation for $\xi_0 = -\infty$

First Order

$$\omega \frac{\mathrm{d}^3 I^{(1)}}{\mathrm{d}\omega \mathrm{d}\boldsymbol{k}} = \frac{\alpha_s C_R}{\pi^2} n_0 L \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^2} \left(|A(\boldsymbol{q})|^2 - V_{\mathrm{tot}} \delta(\boldsymbol{q}) \right) R(\boldsymbol{k}, \boldsymbol{q})$$

Gunion-Bertsch + correction to zeroth order (which vanishes, as it should)

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Opacity Expansion: Results

Second Order

$$\omega \frac{\mathrm{d}^3 I^{(2)}}{\mathrm{d}\omega \mathrm{d}\boldsymbol{k}} = \frac{\alpha_s C_R}{\pi^2} \int \prod_{i=1}^{2} \frac{\mathrm{d}\boldsymbol{q}_i}{(2\pi)^2} \left(|A(\boldsymbol{q}_i)|^2 - V_{\mathrm{tot}} \delta(\boldsymbol{q}_i) \right)$$
$$\times \left[\frac{(n_0 L)^2}{2} R(\boldsymbol{k}+\boldsymbol{q}_1, \boldsymbol{q}_2) - n_0^2 \frac{1-\cos(LQ_1)}{Q_1^2} R(\boldsymbol{k}+\boldsymbol{q}_1, \boldsymbol{q}_2) + n_0^2 \frac{1-\cos(LQ_1)}{Q_1^2} R(\boldsymbol{k}, \boldsymbol{q}_1+\boldsymbol{q}_2) \right]$$

interference factor $n_0^2 \frac{1-\cos(LQ_1)}{Q_1^2}$ interpolates between totally coherent and totally incoherent production

$+$ again corrections to zeroth and first order

Opacity Expansion: Coherence

$$n_0^2 \frac{1-\cos(LQ_1)}{Q_1^2} = \begin{cases} 0 & \text{for } \tau_1 = 1/Q_1 \ll L \\ \frac{(n_0 L)^2}{2} & \text{for } \tau_1 = 1/Q_1 \gg L \end{cases}$$

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incoherent production ($\tau_1 \ll L$):

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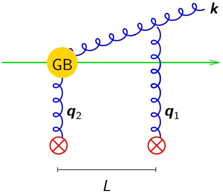

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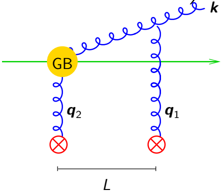

coherent production ($\tau_1 \gg L$):

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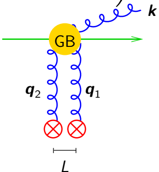


Number of radiated gluons

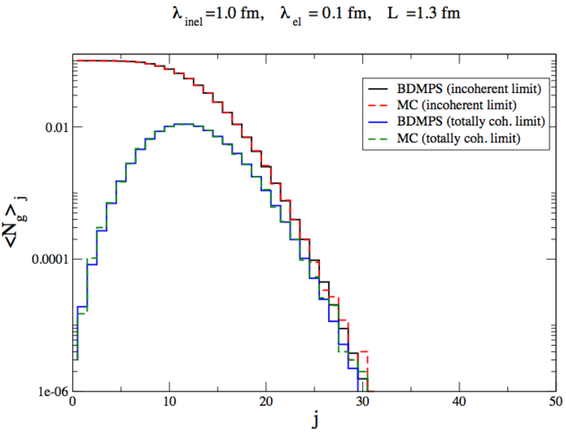
incoherent limit

- $\langle N_{\rm g}\rangle_j = \frac{1}{(j-1)!}\left(\frac{L}{\lambda_{\rm el}}\right)^{j-1} e^{-L/\lambda_{\rm el}} \frac{L}{\lambda_{\rm in}}$
- interpretation: produce gluons in inelastic processes and let them scatter elastically (formation time vanishes)

coherent limit

- $\langle N_{\rm g}\rangle_j = \frac{1}{j!}\left(\frac{L}{\lambda_{\rm el}}\right)^{j} e^{-L/\lambda_{\rm el}} \mathcal{P}_j^{\rm inel}$
- interpretation: probability for j momentum transfers times probability that coherent action of all momentum transfers is inelastic

Number of radiated gluons



very good agreement between analytic result and MC

Lessons learnt

- momentum transfers during gluon formation time act coherently
- during formation time of a gluon no radiation of additional gluons
- gluon radiation probabilities in coherent and incoherent limits

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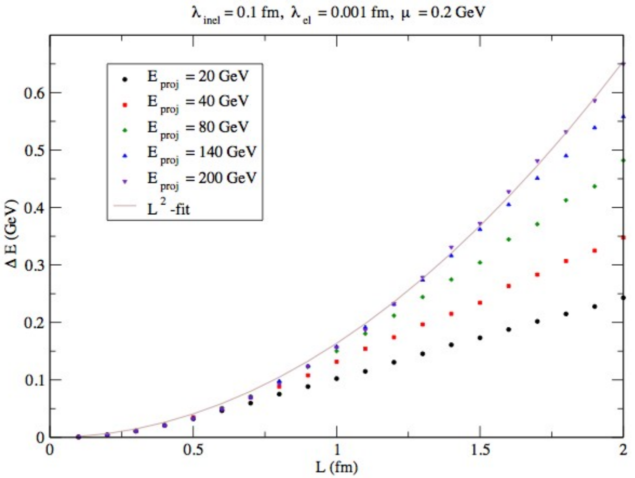
- momentum transfers during gluon formation time act coherently
- during formation time of a gluon no radiation of additional gluons
- gluon radiation probabilities in coherent and incoherent limits
- only gluons that are formed inside the medium can be radiated (from $\xi_0 = 0$ case)

MC algorithm for BDMPS-ASW case

1. generate trial emission $\{\omega, \mathbf{k}\}$ and determine its formation time τ
2. generate all momentum transfers during formation time keeping track of changes in $\mathbf{k}$ (done iteratively)
3. if no scattering was found reject emission
   otherwise weight emission with probability that coherent sum of momentum transfers is inelastic
4. reject radiation if formation outside medium
5. repeat with rest of path length in medium

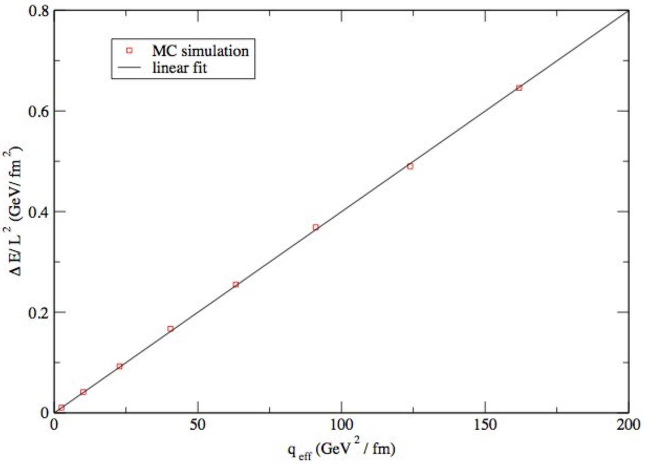
- in BDMPS-ASW vacuum radiation can be subtracted
- first test: $\Delta E \propto \hat{q} L^2$

Energy loss



$\lambda_{inel} = 0.1$ fm, $\lambda_{el} = 0.001$ fm, $\mu = 0.2$ GeV
$E_{proj} = 20$ GeV
$E_{proj} = 40$ GeV
$E_{proj} = 80$ GeV
$E_{proj} = 140$ GeV
$E_{proj} = 200$ GeV
L^2-fit
Δ E (GeV)
L (fm)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.5
1
1.5
2

Number of radiated gluons



Summay & outlook

- our MC algorithm quantitatively reproduces analytic results in eikonal approximation
  - number of radiated gluons
  - ΔE up to factor $\mathcal{O}(1)$
  - shape of gluon spectrum
  - transition from coherent to incoherent regimes
- can be generalised to realistic kinematics
  - done for case without vacuum radiation
  - interplay of bremsstrahlung with vacuum radiation requires extra care: work in progress
- treats radiative and collisional energy loss on equal footing
- part of JEWEL MC
- will be implemented in SHERPA

Opacity Expansion: Path Integral

$$\mathcal{K}(\boldsymbol{r}, y_l; \bar{\boldsymbol{r}}, \bar{y}_l|\omega) =$$
$$\mathcal{K}_0(\boldsymbol{r}, y_l; \bar{\boldsymbol{r}}, \bar{y}_l|\omega)$$
$$- \int_{y_l}^{\bar{y}_l} \mathrm{d}\xi \, n(\xi) \int \mathrm{d}\boldsymbol{\rho} \, \mathcal{K}_0(\boldsymbol{r}, y_l; \boldsymbol{\rho}, \xi|\omega) \frac{1}{2}\sigma(\boldsymbol{\rho}) \mathcal{K}_0(\boldsymbol{\rho}, \xi; \bar{\boldsymbol{r}}, \bar{y}_l|\omega)$$
$$+ \int_{y_l}^{\bar{y}_l} \mathrm{d}\xi_1 \, n(\xi_1) \int_{\xi_1}^{\bar{y}_l} \mathrm{d}\xi_2 \, n(\xi_2) \int \mathrm{d}\boldsymbol{\rho}_1 \int \mathrm{d}\boldsymbol{\rho}_2 \, \mathcal{K}_0(\boldsymbol{r}, y_l; \boldsymbol{\rho}_1, \xi_1|\omega)$$
$$\times \frac{1}{2}\sigma(\boldsymbol{\rho}_1) \mathcal{K}_0(\boldsymbol{\rho}_1, \xi_1; \boldsymbol{\rho}_2, \xi_2|\omega) \frac{1}{2}\sigma(\boldsymbol{\rho}_2) \mathcal{K}_0(\boldsymbol{\rho}_2, \xi_2; \bar{\boldsymbol{r}}, \bar{y}_l|\omega)$$
$$- \ldots$$

$$\mathcal{K}_0(\boldsymbol{r}, y_l; \bar{\boldsymbol{r}}, \bar{y}_l|\omega) = \frac{\omega}{2\pi\imath(\bar{y}_l - y_l)} e^{\frac{\imath\omega(\bar{\boldsymbol{r}}-\boldsymbol{r})^2}{2(\bar{y}_l - y_l)}}$$

generates the multiple scattering terms