



# *V-Gamma Production at LHC: NLO in Powheg Method*

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## Vector Boson-Photon Production at LHC

### Testing Standard Model and Search for New Physics:

- $W/Z\gamma$ : golden channel to test SM at LHC, we will have enough data to do significant analysis  $\sim 1\text{fb}^{-1}$  data for  $W\gamma$  (early next year?).
- Anomalous  $WW\gamma$  coupling: CP-conserving  $\kappa, \lambda, \tilde{\kappa}$  and  $\tilde{\lambda}$  ?
- Are there  $ZZ\gamma$  or  $Z\gamma\gamma$  couplings? Gauge symmetry breaking!
- Agree well with the standard model at Tevatron, and how about at LHC
- Learn to deal with photon at NLO.

### $W\gamma$ production at $D\bar{0}^*$ :

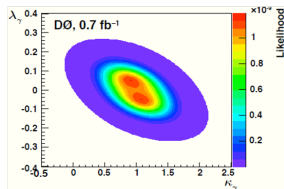
Best points:

$(-0.084, -0.05),$

$(+0.084, +0.05)$

$$|\Delta\kappa| \leq 0.51$$

$$-0.12 \leq \lambda \leq 0.13$$



\*Adam Lyon, ICHEP'08



## *NLO Matching ME with PS!*

- LHC: high energy scale and high luminosity: we need more precise NLO calculations
- NLO ME (BHO generator<sup>\*</sup>): additional parton radiation + parton shower MC  
⇒ double counting and invalid in IR phase space region
- Soft gluon radiation cut  $\delta_s$  and photon isolation cut  $\delta_c$  for quark radiation to cancel the IR divergence

Methods to match NLO matrix element  
with NLO parton shower:  
Implement Powheg in Herwig++

<sup>\*</sup> J.Ohnemus, Phys. Rev., D47, 940



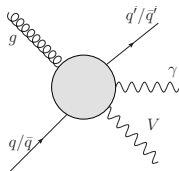
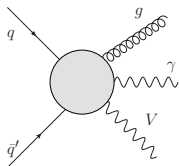
## NLO Matrix Element in Catani-Seymour framework

- The NLO cross-section for  $V\gamma$  matrix element:

$$d\sigma^{\bar{B}} = d\sigma_{V\gamma}^B + d\sigma_{V\gamma}^V + d\sigma_{V\gamma,g}^C + d\sigma_{q\bar{q}\rightarrow V\gamma g}^R + d\sigma_{V\gamma,q}^C + \underbrace{d\sigma_{V\gamma(qg)_i}^R + d\sigma_{V(\gamma q)_f}^R}_{\text{Catani-Seymour}} + d\sigma_{q\rightarrow\gamma}^{C,Brem}$$

- IR singularities: gluon soft and  $q/g$  collinear. We have *Catani-Seymour* subtraction framework to cancel divergences: Both  $\int d\Phi_{n+1}[\sigma_{V(\gamma g)}^R - \sigma^B \otimes V_{dipole}]$  and  $\int d\Phi^B[\sigma_{V\gamma}^V + \sigma_{V\gamma}^C + \sigma_{V\gamma}^A]$  are finite.
- Mapping from  $\Phi^B$  to  $\Phi_{n+1}$  associated to each dipole  $\mathcal{D}(z, u)$  that defined for an IR singular region  $\alpha_i$ : soft ( $z$ ) and collinear ( $u$ ).
- Multiply the weight factor with real radiation contribution in  $\alpha_r$  to cancel IR singularities with dipole and for Powhag:

$$R^{\alpha_r}(\bar{\Phi}^B, z^{\alpha_r}, u^{\alpha_r}) = \frac{\mathcal{D}^{\alpha_r}(\bar{\Phi}^B, z, u) \cdot R(\Phi_{n+1})}{\sum_j \mathcal{D}^j}$$





## PDF & Fragmentation Function Counterterm Remnants

- The dipole  $\mathcal{D}^{\alpha r}(\bar{\Phi}^B, z, u) \sim |\mathcal{M}^B(\bar{\Phi}^B)|^2 \cdot P_{ij}(z, u)/(2p_i \cdot p_j)$  has singularity in dimensional regularization ( $\sim 1/\epsilon$ )
- Consider dipoles  $\mathcal{D}^{ag,b}$ ,  $\mathcal{D}^{gq,\bar{q}}$ ,  $\mathcal{D}_{\gamma q}^a$ ,  $\mathcal{D}_{\gamma q,V}$  (and  $\mathcal{D}_q^{a\gamma}$ ,  $\mathcal{D}_V^{a\gamma}$ ?)
- After canceling the singularities the NLO PDF (and photon fragmentation function) remnants:
  - Gluon/quark PDF: standard, following C-S framework
  - Photon fragmentation function:

$$\int_{n+1} d\sigma_{\gamma q}^A + d\sigma_{q \rightarrow \gamma}^{C,FF}$$

$$= \int d[P_\gamma] \int_0^1 \frac{dz}{z^{2-2\epsilon}} \int d\Phi_{P_\gamma}^{m-1} |\mathcal{M}_{Vq}^B(p_V, P_\gamma/z)|^2 \frac{\alpha Q_C^2}{2\pi} [P_{q\gamma}(z) \ln\left(\frac{2P_\gamma \cdot \tilde{p}_V u_{lim}(z)z}{\mu_0^2(1-z)}\right) + z - 13.26]$$

where the non-perturbative photon fragmentation function fitted at ALEPH\*:

$$D_{q\gamma}(z, \mu_{FS}) = \frac{\alpha Q_C^2}{2\pi} [P_{q\gamma}(z) \ln(\mu_{FS}^2/\mu_0^2) - P_{q\gamma}(z) \ln(1-z)^2 - 13.26]$$

\* ALEPH Collaboration, Z. Phys., C69, 365



## Gluon and Quark QCD Radiation in Matrix Element

- 8 diagrams for real matrix element of  $d\sigma_{q\bar{q}\rightarrow V\gamma g}^R$  and  $d\sigma_{q(\bar{q})g\rightarrow V\gamma q'(\bar{q}')}^R$  respectively, calculated numerically.
- Subtraction terms: initial emitter-initial spectator ( $a, b = q, \bar{q}$ ) dipoles:

$$\mathcal{D}^{ag,b} = \frac{8\pi\alpha_S}{2k_a \cdot k_g} \frac{C_F}{x_{ga,b}} \cdot \frac{1 + x_{ga,b}^2}{1 - x_{ga,b}} \cdot |\mathcal{M}_{V\gamma}^B(\bar{\Phi}^B)|^2$$

$$\mathcal{D}^{gq,b} = \frac{8\pi\alpha_S}{2k_g \cdot k_q} \frac{T_R}{x_{qg,b}} \cdot [1 - 2x_{qg,b}(1 - x_{qg,b})] \cdot |\mathcal{M}_{V\gamma}^B(\bar{\Phi}^B)|^2$$

- Calculation of these subtracted real terms is straight-wards:

$$\left( \frac{R^{ag,b}}{\mathcal{D}^{qg,\bar{q}} + \mathcal{D}^{\bar{q}g,q}} - 1 \right) \mathcal{D}^{ag,b} \quad \left( \frac{R^{gq,b}}{\mathcal{D}^{gq,b} + \mathcal{D}_{\gamma q}^a + \mathcal{D}_{\gamma q,V} + \dots} - 1 \right) \mathcal{D}^{gq,b}$$

- For  $R^{gq,b}$  there is probability to produce soft photon that causes numerical instability, the region  $E_\gamma \leq \text{cut} \cdot \min(KT_\gamma \cosh(y_\gamma))$  should be cut out.



## Powhcg Method in Catani-Seymour framework

- NLO accuracy matching parton shower with matrix element
- Smooth IR region (low  $p_T$ ) to high  $p_T$  region, no phase-space slicing, no double counting
- Generate hardest radiation in specific singular region  $\alpha_r = (ag, b)$  or  $(gq, b)$  every time, by Powhcg Sudakov form factor:

$$\Delta^{f_b}(\Phi^B, p_T) = \prod_{\alpha_r \in \{\alpha_r | f_b\}} \exp\left\{ \int \frac{[d\Phi_{rad} R^{\alpha_r}(\Phi_{n+1}) \Theta(k_T(\Phi_{n+1}) - p_T)]_{\alpha_r}^{\bar{\Phi}_B^{\alpha_r} = \Phi^B}}{B^{f_b}(\Phi^B)} \right\}$$

- The cross-section of Powhcg:

$$d\sigma = \sum_{f_b} d\Phi^B \bar{B}^{f_b}(\Phi^B) \{ \Delta^{f_b}(\Phi^B, p_T^{\min}) + \sum_{\alpha_r} \frac{[d\Phi_{rad} \Theta(k_T - p_T^{\min}) \Delta^{f_b}(\Phi^B, k_T) R(\Phi_{n+1})]_{\alpha_r}^{\bar{\Phi}_B^{\alpha_r} = \Phi^B}}{B^{f_b}(\Phi^B)} \}$$



## Generate Radiation Events: Highest- $p_T$ -bid Method

- **Highest- $p_T$ -bid method** in Powheg:

$$\left[ \frac{R(\Phi_{n+1}^{\alpha_r})}{B_{f_b}(\Phi^B)} \Delta_{\alpha_r}^{f_b}(\Phi^B, k_T(\Phi_{n+1})) \right]^{\bar{\Phi}_B^{\alpha_r} = \Phi^B} d\Phi_{rad}^{\alpha_r}$$

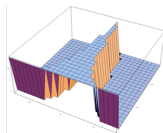
and choose the radiation in  $\alpha_r$  with highest  $p_T$ .

- **Veto technique**: estimate the **upper bounding function** of  $(R/B)_U$  according to dipoles  $\mathcal{D}^{ag,b}$  and  $\mathcal{D}^{gq,b}$  to generate  $k_T$  in MC numerical calculation.
- The prefactors for upper bounding function should be estimated suitable to have good efficiency.
- New constraint of radiation rapidity  $y$  rather than constant bounds  $y_{\max}, y_{\min}$  is applied

$$\kappa_T^2 \geq \frac{(1-\bar{x}_a)^2}{4\bar{x}_a} \equiv \kappa_{T0}^2$$

$$\sqrt{\bar{x}_b} e^y \in \left( \frac{\kappa_{T0}}{\kappa_T} - \sqrt{\frac{\kappa_{T0}^2}{\kappa_T^2} - 1}, \frac{\kappa_{T0}}{\kappa_T} + \sqrt{\frac{\kappa_{T0}^2}{\kappa_T^2} - 1} \right)$$

$$\text{where } \kappa_T^2 = \frac{p_T^2}{\hat{s}}$$



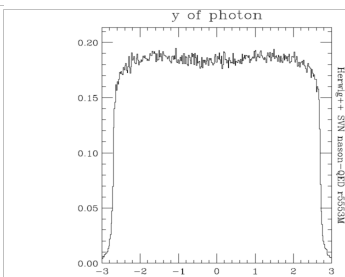
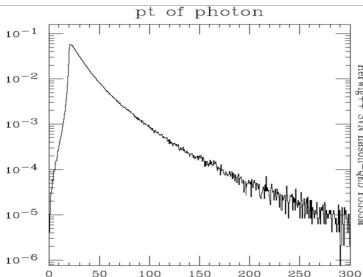




## QED Quark Real Radiation

Two schemes to deal with the bremsstrahlung component:

- **QED parton shower scheme:** model  $\gamma$  fragmentation with QED shower
- Non-perturbative effect of the fragmentation may come from hadronization
- We don't have soft photon problem.
- Since we model  $d\sigma_{Vjet}^{Brem}$  with QED Powheg shower, we cannot separate ME with PS to compare ME results with BHO generator.
- Try numerical result for  $Z\gamma$  at 14TeV with cuts:  $p_{T,\gamma} \geq 20\text{GeV}$ ,  $|\eta_\gamma| \leq 2.7$ :  
Events number: 800,000      Total Cross-section:  $46.78(\pm 0.06)\text{pb}$





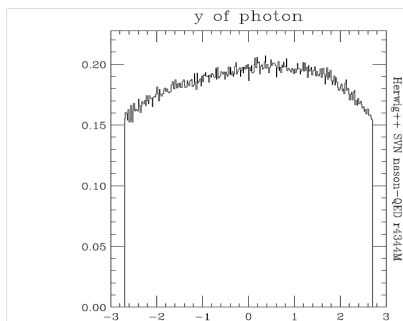
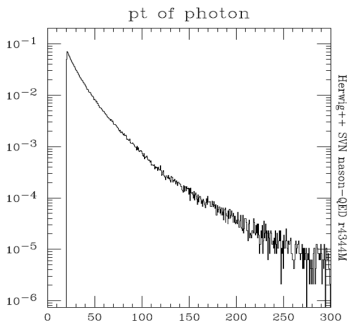
## QED Quark Real Radiation

Two schemes to deal with the bremsstrahlung component:

- **Photon fragmentation function scheme:** photon FF fitted from experiment
- Non-perturbative effect is fitted into photon FF (as we see before)
- But we have soft photon problem.
- Due to soft photon divergence we have to separate  $V\gamma$  events from  $Vjet$
- Try numerical result for  $Z\gamma$  at 14TeV with cuts:  $p_{T,\gamma} \geq 20\text{GeV}$ ,  $|\eta_\gamma| \leq 2.7$ ,  $z_{lim} = 0.5$  and  $z_{cut} = 0.1$ :

Events number: 800,000

Total Cross-section:  $43.93(\pm 0.02)\text{pb}$



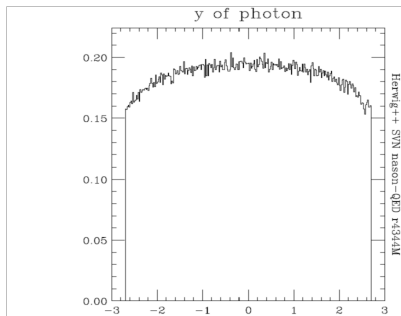
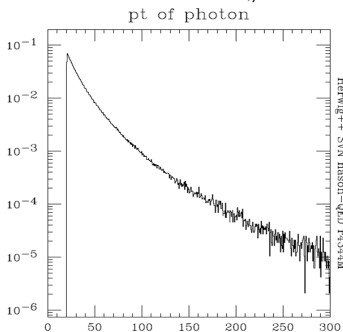


## Compare with Leading Order

The LO (LO matrix element + shower MC) numerical result for  $Z\gamma$  at  $14\text{TeV}$  with cuts:  $p_{T,\gamma} \geq 20\text{GeV}$ ,  $|\eta_\gamma| \leq 2.7$ :

Events number: 800,000      Total Cross-section:  $34.84(\pm 0.02)\text{pb}$

The distribution of photon  $p_{T,\gamma}$  and rapidity  $y_\gamma$  is





## Photon Shower to Model Photon Fragmentation

- Inspired by Photon-jet and diphoton merging work<sup>\*</sup>, in this scheme the non-perturbative effect of photon fragmentation comes from photon shower.
- Reminding the cross-section of our process:

$$d\sigma^{\bar{B}} = d\sigma_{V\gamma}^B + d\sigma_{V\gamma}^V + d\sigma_{V\gamma}^C + d\sigma_{q\bar{q} \rightarrow V\gamma g}^R + d\sigma_{V\gamma(qg)_i}^R + d\sigma_{V(\gamma q)_i}^R + d\sigma_{Vj}^{Brem}$$

$d\sigma_{V\gamma}^{\bar{B}}$                        $d\sigma_{qg \rightarrow V\gamma q}^R$                        $d\sigma_{Vjet}^{\bar{B}}$

- We treat QCD and QED parton shower democratically and model fragmentation component by QED parton shower,

$$d\sigma = d\Phi_B \bar{B}_{V\gamma} [\Delta_{V\gamma}(\Phi_B, p_T^{min}) + \Delta_{V\gamma}(\Phi_B, k_T(\Phi_B, \Phi_R)) \frac{R_{V\gamma g}(\Phi_B, \Phi_R) + R_{V\gamma(qg)_i}(\Phi_B, \Phi_R)}{B_{V\gamma}}] \\ + d\Phi'_B B_{Vjet} [\Delta_{Vjet}(\Phi'_B, p_T^{min}) + \Delta_{Vjet}(\Phi'_B, k_T(\Phi'_B, \Phi'_R)) \frac{R_{V(\gamma q)_i}(\Phi'_B, \Phi'_R)}{B_{Vjet}}]$$

<sup>\*</sup> Hoche, et al, Phys. Rev. D 81, 034026



## The Dipoles for QED Parton Shower Scheme

- Powheg QED shower to generate only one photon radiation and follow by QCD vetoed shower MC:

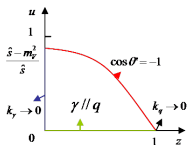
$$\Delta_{Vjet}^{\alpha r}(\Phi'_B, k_T(\Phi'_B, \Phi'_{rad,\gamma})) = \exp\left\{ \int \frac{[d\Phi_{rad,\gamma} R_{V(\gamma q)_f}^{\alpha r}(\Phi_{n+1}) \Theta(k_T(\Phi_{n+1}) - p_T)]_{\alpha r}^{\bar{\Phi}'_B = \Phi'^B}}{B_{Vjet}(\Phi'_B)} \right\}$$

- Photon can be collinear with initial and final state quark: QED dipoles  $\mathcal{D}_{q\gamma}^a$ ,  $\mathcal{D}_{q\gamma,V}$ ,  $\mathcal{D}_q^{a\gamma}$ ,  $\mathcal{D}_V^{a\gamma}$ . We don't need to consider:  $\mathcal{D}_{W\gamma}^a$ ,  $\mathcal{D}_{W\gamma,q}$  since the massive  $W$  doesn't lead to any singularity, and splitting function  $P_{W\gamma}(z)$  is ill-define. These QED dipoles are similar with QCD dipoles: only have **color charge products** replaced to **electric charge products**  $T_i \cdot T_j \rightarrow Q_i \cdot Q_j$ .
- Kinematics mapping: photon radiation momentum is calculated in Breit frame of  $a - q$  or  $a - V$  except  $\mathcal{D}_{W\gamma,q}$ . It's a little complicated when massive  $V$  involves.



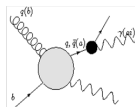
## Photon Fragmentation Function Scheme: Identified Photon

- Photon fragmentation function  $D_{q\gamma}(z, \mu_{FS})$  enters like PDF: treat photon as identified particle.
- We need dipoles  $\mathcal{D}_{\gamma q}^{a(n)}$ ,  $\mathcal{D}_{\gamma q, V}$  in this case. When we don't change  $\bar{k}_g$  in the mapping, the two dipoles is the same except electric charge:



$$\mathcal{D}_{\gamma q}^{a(n)}(z) + \mathcal{D}_{\gamma q, V}(z) = \frac{8\pi\mu^2\epsilon\alpha}{2k_q \cdot k_\gamma} \frac{Q_q^2}{\not{Q}_q} (Q_a \cdot Q_q - Q_V \cdot Q_q) \frac{1 + (1-z)^2}{z} |M_{Vq}^B|^2$$

$$\Delta_{Vq}^{\gamma q}(\Phi_B, k_T(\Phi_B, \Phi_R)) \left[ \frac{\mathcal{D}_{\gamma q}}{\mathcal{D}_{\gamma q} + \mathcal{D}^{gq, b}} \frac{R(\Phi_{n+1}^{\gamma q})}{B_{V\gamma}} \right]$$



- The collinear limit is correct in integration of  $u$  but not soft photon limit  $\Phi_{n+1}(z \rightarrow 0, u) \rightarrow \bar{\Phi}^B$ .
- In Powhcg the  $p_T$  and  $y$  and the constraints on them is quite complicated, and the upper bound of  $R/B$  is estimated more carefully.
- We can also use the unidentified photon dipole  $\mathcal{D}_{\gamma q}^a$  that has correct soft photon limit. The collinear singularity is extracted by changing  $\bar{\Phi}_{rad}$  variables to  $\bar{y}, \bar{v}$  and exchange the integration order:

$$\bar{v}^{-1-\epsilon} = -\frac{\bar{v}^{-\epsilon}}{\epsilon} \delta(\bar{v}) + \left(\frac{1}{\bar{v}}\right)_c - \epsilon \left(\frac{\ln \bar{v}}{\bar{v}}\right)_c + \mathcal{O}(\epsilon^2) \quad \left(\frac{1}{\bar{v}}\right)_c \rightarrow \int_0^1 \frac{f(\bar{v}) - f(0)\Theta(\bar{v}_{cut} - \bar{v})}{\bar{v}} d\bar{v}$$



## Soft Photon Problem

- When photon fraction  $z$  (or  $\bar{y}$ )  $\rightarrow 0$ , we reach soft photon singularity related to photon virtual loop.
- But photon FF  $D_{q\gamma}(z, \mu_{FS})$  is fitted down to the cutoff  $z_{lim}$ , and is insensitive to small fraction  $z$ .
- Inspired by *Wjet* QED correction\*, we need to distinguish  $V_\gamma$  and  $V_{jet}$  events by imposing this cut, but real contribution  $R_{\gamma q}$  in region  $z \in (0, z_{lim})$  should be treated carefully so that no soft singularity.
- We can have two ways to do that:
  - Simply cut off  $z \leq z_{lim}$ , both in photon FF and subtracted real piece  $R_{\gamma q}$ .
  - Cut off the photon FF remnant, but in real piece extract the remained collinear and soft divergences. Need to study how they are canceled with photon loop.
- To extract the remained collinear and soft divergences in subtracted real piece  $R_{\gamma q}$  we can use the similar technic of  $\bar{v}^{-1-\epsilon}$ :

$$\frac{1}{z^{1+2\epsilon}} = -\frac{z^{-2\epsilon}}{2\epsilon} \delta(z) + \left(\frac{1}{z}\right)_c - 2\epsilon \left(\frac{\ln z}{z}\right)_c + O(\epsilon^2)$$

- The trial numerical result we have is attained by throwing away these divergences that guarantee canceled with photon loop by KLN theorem.

\* Denner, et al, JHEP08, 075



## Outlook

- We have to study more carefully how the photon soft divergences should be handled.
- Compare the two scheme of photon fragmentation and see which is better.
- Compare with the previous WGamma MC tools and experimental data at LHC.
- Anomalous  $WW\gamma$  couplings and beyond SM
- Complete the process with  $W/Z$  leptonic decay.